
Esercizi aggiuntivi (continuità)

① Studiare la continuità delle seguenti funzioni:

(a) $\sin(x)$;

(b) $|x|$;

(c) e^x ;

(d) $\arctan(x)$;

(e) $\sqrt[3]{7 + e^x}$;

(f) $\log(9 + e^{1-x})$;

(g) $\frac{1}{x-3}$;

(h) $\frac{x-3}{x^2-3x+2}$;

(i) $\Theta(x) = \begin{cases} 0 & \text{se } x < 0 \\ \frac{1}{2} & \text{se } x = 0 \\ 1 & \text{se } x > 0 \end{cases}$

(j) $\begin{cases} x^2 & \text{se } x \neq 0; \\ 1 & \text{se } x = 0 \end{cases}$

(k) $\begin{cases} x - 3 & \text{se } x \leq 2 \\ -x^2 + 3 & \text{se } 2 < x < 5 \\ \frac{1}{x} & \text{se } x \geq 5 \end{cases}$

(l) $\text{sinc}(x) = \begin{cases} \frac{\sin(x)}{x} & \text{se } x \neq 0 \\ 1 & \text{se } x = 0 \end{cases}$

(m) $\chi(x) = \begin{cases} 1 & \text{se } x \in \mathbb{Q} \\ 0 & \text{se } x \in (\mathbb{R} - \mathbb{Q}) \end{cases}$

(n) $\delta(x) = \begin{cases} +\infty & \text{se } x = 0 \\ 0 & \text{se } x \neq 0 \end{cases}$

(o) $\begin{cases} x + 2 & \text{se } -2 < x < 3 \\ -x - 2 & \text{se } x \leq -2 \\ \sqrt[3]{x-4} & \text{se } 3 < x \leq 5 \\ 2x - 9 & \text{se } x > 5 \end{cases}$

(p) $\begin{cases} \sin(\frac{1}{x}) & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases}$

(q) $\frac{e^x - 1}{x}$;

$$(r) \begin{cases} \frac{e^x-1}{x} \text{ se } x \neq 0 \\ 1 \text{ se } x = 0 \end{cases}$$

② Determinare, al variare di $\alpha \in \mathbb{R}$ e $\beta \in \mathbb{R}$, la continuità delle seguenti funzioni:

$$(a) \begin{cases} 3x^2 + x + 2 - \alpha \text{ se } x \leq 0 \\ \sqrt{x^4 + 1} \text{ se } x > 0 \end{cases}$$

$$(b) \begin{cases} x^5 - 3\alpha \text{ se } x < 0 \\ 2\alpha \cdot e^{x-1} \text{ se } x \geq 0 \end{cases}$$

$$(c) \begin{cases} 2 - \sqrt{x} \text{ se } x \geq 0 \\ 4(x + \alpha) \text{ se } x < 0 \end{cases}$$

$$(d) \begin{cases} |x - 1|^\alpha \left[\sin\left(\frac{1}{x-1}\right) + 2 \right] \text{ se } x \neq 1 \\ 0 \text{ se } x = 1 \end{cases}$$

$$(e) \begin{cases} \frac{\alpha}{2-x} \text{ se } x < 1 \\ \beta \text{ se } x = 1 \\ \alpha x^2 + x + \beta \text{ se } x > 1 \end{cases}$$

$$(f) \begin{cases} \alpha x^2 + \beta \text{ se } x \geq 0 \\ \log(\alpha x + \beta) \text{ se } x < 0 \end{cases}$$