# TUTORATO 16/11/2022

$$\lim_{x\to 0} \frac{3\arctan x + (1-\cos(2x))\sin^2 x}{27x^4 + 5\sin x}$$

$\lim_{x\to 0} \frac{\arctan x}{x} = 1$

Dim: Pongo $t = \arctan x \Rightarrow x = \tan t$ per $t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\lim_{t\to 0} \frac{t}{\tan t} = \lim_{t\to 0} \left(\underbrace{\frac{t}{\operatorname{sen} t}}_{\to 1} \cdot \underbrace{\cos t}_{\to 1}\right) = 1$$

$$\lim_{x\to 0} \left(\frac{\arctan x - x}{x}\right) = 0 \Rightarrow \arctan x = x + o(x)$$

- $\arctan x = x + o(x)$
- $\cos(2x) = 1 - \frac{(2x)^2}{2} + o\left((2x)^2\right) = 1 - 2x^2 + o(4x^2) = 1 - 2x^2 + o(x^2)$

$o(\alpha \cdot f(x)) = o(f(x))$

$\Rightarrow 1 - \cos(2x) = \cancel{1} - \cancel{1} + 2x^2 + o(x^2) = 2x^2 + o(x^2)$

- $\sin x = x + o(x) \Rightarrow \sin^2 x = (x + o(x))^2 = x^2 + o(x)^2 + 2x\,o(x) = x^2 + o(x^2)$

$o(x^m)^n = o(x^{m\cdot n})$

$f(x) \cdot o(g(x)) = o(f(x)\cdot g(x))$

Sostituisco nel limite:

$$\lim_{x\to 0} \frac{3(x+o(x)) + (2x^2+o(x^2))\cdot(x^2+o(x^2))}{27x^4 + 5(x+o(x))} = \lim_{x\to 0} \frac{3x + \overbrace{3o(x)}^{o(x)} + 2x^4 + \overbrace{2x^2 o(x^2) + o(x^2)x^2 + o(x^2)o(x^2)}^{o(x^4)}}{27x^4 + 5(x+o(x))}$$

$$= \lim_{x\to 0} \frac{3x + o(x) + 2x^4 + o(x^4)}{27x^4 + 5x + o(x)} = \lim_{x\to 0} \frac{3x + \overbrace{o(x) + 2o(x) + o(o(x))}^{=o(x)}}{27o(x) + 5x + o(x)} = \lim_{x\to 0} \frac{3x + o(x)}{5x + o(x)} = \frac{3}{5}$$

$x^4 = o(x)$ per $x\to 0$ poiche' $\lim_{x\to 0} \frac{x^4}{x} = x^3 = 0$

---

$$\lim_{x\to 0} \frac{(1-\cos(5x))\tan(3x)}{(\operatorname{sen} x - x^3)^3}$$

- $\cos(5x) = 1 - \frac{(5x)^2}{2} + o\left((5x)^2\right) = 1 - \frac{25x^2}{2} + o(x^2)$
- $\tan(3x) = 3x + o(3x) = 3x + o(x)$
- $\operatorname{sen} x = x + o(x)$

$$\lim_{x\to 0} \frac{\left(\frac{25}{2}x^2 + o(x^2)\right)(3x + o(x))}{(x + o(x) - x^3)^3} = *$$

$$N:\ \left(\frac{25}{2}x^2+o(x^2)\right)\left(3x+o(x)\right)=\frac{75}{2}x^3+\underbrace{\frac{25}{2}x^2o(x)}_{(1)}+\underbrace{o(x^2)3x}_{(2)}+\underbrace{o(x^2)o(x)}_{(3)}=\frac{75}{2}x^3+o(x^3)$$

$$(1)\ \frac{25}{2}x^2o(x)=o\left(\frac{25}{2}x^3\right)=o(x^3)$$

$g\cdot o(f)=o(f\cdot g)$ $\qquad$ $o(\alpha f)=o(f)$

$$(2)\ o(x^2)3x=o(3x^3)=o(x^3)$$

$$(3)\ o(x^2)o(x)=o(x^3)$$

$o(f)\cdot o(g)=o(f+g)$

$$(1)+(2)+(3):\ o(x^3)+o(x^3)+o(x^3)=o(x^3)$$

$o(f)+o(f)=o(f)$

$$D:\ (x+o(x)-x^3)^3=(x+o(x)-o(x))^3=(x+o(x))^3$$

$x^3=o(x)$

$$=x^3+3x^2o(x)+3xo(x^2)+o(x^3)$$

$$=x^3+o(x^3)$$

$$*=\lim_{x\to 0}\frac{\frac{75}{2}x^3+o(x^3)}{x^3+o(x^3)}=\frac{75}{2}$$

---

$$\lim_{x\to 0}\frac{e^{\tan^3 x}-1}{x(\cos x-e^{x^2})}=*$$

$$N:\ e^{\tan^3 x}-1=e^{x^3+o(x)}-1=x^3+o(x^3)+o(x^3+o(x^3))=x^3+o(x^3)$$

$$\tan^3 x=(x+o(x))^3=x^3+3x^2o(x)+3x\,o(x^2)+o(x^3)=x^3+o(x^3)$$

$$D:\ \cos x=1-\frac{x^2}{2}+o(x^2)\qquad e^{x^2}=1+x^2+o(x^2)$$

$$\Rightarrow \cos x-e^{x^2}=\cancel{1}-\frac{x^2}{2}+o(x^2)-\cancel{1}-x^2+o(x^2)=-\frac{3}{2}x^2+o(x^2)$$

$$*=\lim_{x\to 0}\frac{x^3+o(x^3)}{-\frac{3}{2}x^3+o(x^3)}=-\frac{2}{3}$$

$$\lim_{x\to 0} \frac{\sin(\pi \cos x)}{x \sin x}$$

Riscrivo il numeratore:

$$\underset{\substack{\downarrow \\ \cos x = 1 - \frac{x^2}{2} + o(x^2)}}{\sin(\pi \cos x) =} \sin\left(\pi - \frac{\pi x^2}{2} + o(x^2)\right) \underset{\substack{\downarrow \\ \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}}{=} \overset{\overset{0}{\parallel}}{\operatorname{sen}\pi} \cos\left(-\frac{\pi x^2}{2} + o(x^2)\right) + \overset{\overset{-1}{\parallel}}{\cos\pi} \sin\left(-\frac{\pi x^2}{2} + o(x^2)\right) =$$

$$= -\sin\left(-\frac{\pi x^2}{2} + o(x^2)\right) \underset{\substack{\downarrow \\ \sin(-\alpha) = -\sin\alpha}}{=} \sin\left(\frac{\pi x^2}{2} + o(x^2)\right)$$

$$\Rightarrow \lim_{x\to 0} \frac{\sin\left(\frac{\pi x^2}{2} + o(x^2)\right)}{x \sin x} = *$$

- $\sin\left(\frac{\pi x^2}{2} + o(x^2)\right) = \frac{\pi x^2}{2} + o(x^2) + \overbrace{o\left(\frac{\pi x^2}{2} + o(x^2)\right)}^{= o(x^2)} = \frac{\pi x^2}{2} + o(x^2)$
- $x \sin x = x(x + o(x)) = x^2 + o(x^2)$

$$* = \lim_{x\to 0} \frac{\frac{\pi x^2}{2} + o(x^2)}{x^2 + o(x^2)} = \frac{\pi}{2}$$

---

$$\lim_{x\to 0} \frac{\tan(Ax^5 + 4Bx^4)}{\sin^2(2x)\log(1+x^2)}$$

- $\tan(Ax^5 + 4Bx^4) = Ax^5 + 4Bx^4 + o(Ax^5 + 4Bx^4) \underset{\substack{\downarrow \\ x^5 = o(x^4)}}{=} A\,o(x^4) + 4Bx^4 + o(A\,o(x^4) + 4Bx^4) =$

$$= o(x^4) + 4Bx^4 + o(x^4) = 4Bx^4 + o(x^4)$$

- $(\sin(2x))^2 = (2x + o(2x))^2 = (2x + o(x))^2 = 4x^2 + o(x^2) + 4x\,o(x^2) = 4x^2 + o(x^2)$
- $\log(1+x^2) = x^2 + o(x^2)$

Sostituisco nel limite:

$$\lim_{x\to 0} \frac{4Bx^4 + o(x^4)}{(4x^2 + o(x^2))(x^2 + o(x^2))} = \lim_{x\to 0} \frac{4Bx^4 + o(x^4)}{4x^4 + \underbrace{4x^2 o(x^2) + o(x^2)x^2 + o(x^2)o(x^2)}_{= o(x^4)}} =$$

$$= \lim_{x\to 0} \frac{4Bx^4 + o(x^4)}{4x^4 + o(x^4)} = B$$

$$\lim_{x\to\infty} (2+x^3)^{1/3} - (1+2x^2+x^3)^{1/3}$$

- $\sqrt[3]{2+x^3} = \sqrt[3]{x^3\left(1+\frac{2}{x^3}\right)} = x\sqrt[3]{1+\frac{2}{x^3}}$ $\quad \frac{2}{x^3} \xrightarrow{x\to\infty} 0$

$$\sqrt[3]{1+\frac{2}{x^3}} = \left(1+\frac{2}{x^3}\right)^{1/3} = 1 + \frac{1}{3}\frac{2}{x^3} + o\left(\frac{2}{x^3}\right) = 1 + \frac{2}{3x^3} + o\left(\frac{1}{x^3}\right)$$

$$\Rightarrow \sqrt[3]{2+x^3} = x\left(1+\frac{2}{3x^3}+o\left(\frac{1}{x^3}\right)\right) = x + \frac{2}{3x^2} + o\left(\frac{1}{x^2}\right)$$

- $\sqrt[3]{1+2x^2+x^3} = \sqrt[3]{x^3\left(1+\frac{2}{x}+\frac{1}{x^3}\right)} = x\sqrt[3]{1+\frac{2}{x}+\frac{1}{x^3}}$

$$\sqrt[3]{1+\frac{2}{x}+\frac{1}{x^3}} = \left[1+\left(\frac{2}{x}+\frac{1}{x^3}\right)\right]^{1/3} = 1 + \frac{1}{3}\left(\frac{2}{x}+\frac{1}{x^3}\right) + o\left(\frac{2}{x}+\frac{1}{x^3}\right)$$

$$= 1 + \frac{2}{3x} + \frac{1}{3x^3} + o\left(\frac{1}{x}+\frac{1}{x^3}\right)$$

$$\Rightarrow \sqrt[3]{1+2x^2+x^3} = x\left(1+\frac{2}{3x}+\frac{1}{3x^3}+o\left(\frac{1}{x}+\frac{1}{x^3}\right)\right) = x + \frac{2}{3} + \frac{1}{3x^2} + o\left(1+\frac{1}{x^2}\right)$$

Sostituisco nel limite:

$$\lim_{x\to\infty} \cancel{x} + \frac{2}{3x^2} + o\left(\frac{1}{x^2}\right) - \cancel{x} - \frac{2}{3} - \frac{1}{3x^2} - o\left(1+\frac{1}{x^2}\right) =$$

$$\lim_{x\to\infty} -\frac{2}{3} + \underbrace{\frac{1}{3x^2} + o\left(\frac{1}{x^2}\right) - o\left(1+\frac{1}{x^2}\right)}_{o(1)} = -\frac{2}{3}$$

$$o(1) \quad \lim_{x\to\infty} \frac{1/x^2}{1} = \lim_{x\to\infty} \frac{1}{x^2} = 0$$