TUTORATO 09/11/2022

$$\lim_{x\to 0} \frac{\ln(1+x)^3}{\sin 5x + \sqrt[3]{x^4}\sin x} = \lim_{x\to 0} \frac{3\ln(1+x)}{\sin 5x + \sqrt[3]{x^4}\sin x} = \lim_{x\to 0} \frac{3\ln(1+x)}{\frac{5x}{5x}\sin 5x + \sqrt[3]{x^4}\sin x} =$$

$\ln(x^\alpha) = \alpha \ln x$

$$= \lim_{x\to 0} \frac{3\ln(1+x)}{x\left(5\frac{\sin 5x}{5x} + \sqrt[3]{x^4}\frac{\sin x}{x}\right)} = \lim_{x\to 0} 3 \underbrace{\frac{\ln(1+x)}{x}}_{\to 1} \underbrace{\frac{1}{\left(5\underbrace{\frac{\sin 5x}{5x}}_{\to 1} + \underbrace{\sqrt[3]{x^4}\underbrace{\frac{\sin x}{x}}_{\to 1}}_{\to 0}\right)}}_{\to 5} = \frac{3}{5}$$

$$\lim_{x\to 0}\left[\frac{1}{x\tan x} - \frac{1}{x\sin x}\right] = \lim_{x\to 0}\left[\frac{\cos x}{x\sin x} - \frac{1}{x\sin x}\right] = \lim_{x\to 0}\left[\frac{\cos x - 1}{x\sin x}\cdot\frac{x}{x}\right] =$$

$\tan x = \frac{\operatorname{sen} x}{\cos x}$

$$= \lim_{x\to 0}\left[-\frac{1-\cos x}{x^2}\cdot\frac{x}{\operatorname{sen} x}\right] = \lim_{x\to 0}\left[-\underbrace{\frac{1-\cos x}{x^2}}_{\to 1/2}\cdot\frac{1}{\underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1}}\right] = -\frac{1}{2}$$

$$\lim_{x\to 0} \frac{e^{\tan^3 x} - 1}{x(\cos x - e^{x^2})} = \lim_{x\to 0} \frac{e^{\tan^3 x} - 1}{x(\cos x - e^{x^2})}\cdot\frac{\tan^3 x}{\tan^3 x} = \lim_{x\to 0} \frac{e^{\tan^3 x} - 1}{\tan^3 x}\cdot\frac{\tan^3 x}{x\frac{x^2}{x^2}(\cos x - e^{x^2})}$$

$$= \lim_{x\to 0} \frac{e^{\tan^3 x} - 1}{\tan^3 x}\cdot\frac{\tan^3 x}{x^3\left(\frac{\cos x - e^{x^2} - 1 + 1}{x^2}\right)} = \lim_{x\to 0} \underbrace{\frac{e^{\tan^3 x} - 1}{\tan^3 x}}_{\to \ln e = 1}\cdot\underbrace{\left(\frac{\tan x}{x}\right)^3}_{\to 1}\frac{1}{\underbrace{\left(-\underbrace{\frac{1-\cos x}{x^2}}_{\to 1/2} - \underbrace{\frac{e^{x^2}-1}{x^2}}_{\to 1}\right)}_{\to -3/2}}$$

$$= -3/2$$

$$\lim_{x\to 0} \frac{e^{\alpha x} - \sqrt{1-x}}{\sin x} = \lim_{x\to 0} \frac{e^{\alpha x} - \sqrt{1-x}}{\sin x}\cdot\frac{e^{\alpha x} + \sqrt{1-x}}{e^{\alpha x} + \sqrt{1-x}} = \lim_{x\to 0} \frac{e^{2\alpha x} - 1 + x}{\sin x}\cdot\frac{1}{e^{\alpha x} + \sqrt{1-x}} =$$

$$= \lim_{x\to 0}\left(\frac{e^{2\alpha x}-1}{\sin x} + \frac{x}{\sin x}\right)\cdot\frac{1}{e^{\alpha x}+\sqrt{1-x}} =$$

$$= \lim_{x\to 0}\left(\underbrace{\frac{e^{2\alpha x}-1}{2\alpha x}}_{\to 1} 2\alpha \frac{x}{\sin x} + \frac{x}{\sin x}\right)\cdot\underbrace{\frac{1}{e^{\alpha x}+\sqrt{1-x}}}_{\to 1/2} = (2\alpha + 1)\,1/2 = \alpha + \frac{1}{2}$$

$$\frac{x}{\sin x} = \frac{1}{\underbrace{\frac{\sin x}{x}}_{\to 1}}$$

$$\lim_{x\to 0} (\sin x^2)^{\frac{1}{\log_5 x^2}} = \lim_{x\to 0} e^{(\ln \sin x^2)^{1/\log_5 x^2}} = \lim_{x\to 0} e^{\frac{1}{\log_5 x^2}\cdot \ln(\sin x^2)}$$

$x = e^{\ln x}$ $\qquad$ $(a^n)^m = a^{n\cdot m}$

$$= \lim_{x\to 0} e^{\frac{\ln 5}{\ln x^2} \ln\left(\sin x^2 \frac{x^2}{x^2}\right)} = \lim_{x\to 0} e^{\frac{\ln 5}{\ln x^2}\ln\left(\frac{\sin x^2}{x^2}\cdot x^2\right)}$$

$\log_a(b) = \dfrac{\log_c(b)}{\log_c(a)}$

$$= \lim_{x\to 0} e^{\frac{\ln 5}{\ln x^2}\left(\underbrace{\ln \underbrace{\frac{\operatorname{sen} x^2}{x^2}}_{\to 1}}_{\to 0} + \ln x^2\right)} = e^{\ln 5} = 5$$

$\ln(a\cdot b) = \ln(a) + \ln(b)$

ES. 4 QUIZ 10/11/2021

$$\lim_{x\to 0} \frac{\tan(Ax^5+4Bx^4)}{\sin^2(2x)\log(1+x^2)} = \lim_{x\to 0} \frac{\tan(Ax^5+4Bx^4)}{Ax^5+4Bx^4} \overbrace{(Ax^5+4Bx^4)}^{x^4(Ax+4B)} \frac{1}{\sin^2(2x)\log(1+x^2)}$$

$$= \lim_{x\to 0} \frac{\tan(Ax^5+4Bx^4)}{Ax^5+4Bx^4}\left(\frac{2x}{2\sin(2x)}\right)^2 \frac{x^2}{\log(1+x^2)} (Ax+4B)$$

$$= \lim_{x\to 0} \underbrace{\frac{\tan(Ax^5+4Bx^4)}{Ax^5+4Bx^4}}_{\to 1} \frac{1}{4} \underbrace{\left(\frac{1}{\frac{\sin(2x)}{2x}}\right)^2}_{\to 1} \underbrace{\frac{1}{\frac{\log(1+x^2)}{x^2}}}_{\to 1} \underbrace{(Ax+4B)}_{\to 4B} = \frac{1}{\cancel{4}}\cdot\cancel{4}B = B$$

ES. 2 ESAME 28/08/19

$$\lim_{x\to 0} \frac{x^{(3x^3)} - 1}{(\tan 4x^3)(\log x)} = \lim_{x\to 0} \frac{e^{3x^3\log x} - 1}{(\tan 4x^3)(\cancel{\log x})}\cdot\frac{3x^3 \cancel{\log x}}{3x^3\log x} =$$

$x^{3x^3} = e^{\log x^{3x^3}} = e^{3x^3\log x}$

$$= \lim_{x\to 0} \frac{e^{3x^3\log x} - 1}{3x^3\log x}\cdot\frac{3x^3}{\tan 4x^3}\cdot\frac{4}{4} = \lim_{x\to 0} \underbrace{\frac{e^{3x^3\log x} - 1}{3x^3\log x}}_{\to 1}\cdot\underbrace{\frac{4x^3}{\tan 4x^3}}_{\to 1}\cdot\frac{3}{4} = \frac{3}{4}$$

ES. 2 ESAME 17/06/19

$$\lim_{x\to 0} \frac{e^{\cos(3x^3)} - e}{x^6} = \lim_{x\to 0} e\,\frac{\left(e^{\cos(3x^3)-1} - 1\right)}{x^6}\cdot\frac{\cos(3x^3)-1}{\cos(3x^3)-1} =$$

$$= \lim_{x\to 0} e\,\frac{e^{\cos(3x^3)-1} - 1}{\cos(3x^3)-1}\cdot\frac{\cos(3x^3)-1}{(x^3)^2}\cdot\frac{9}{9} =$$

$$\lim_{x\to 0} 9e \underbrace{\frac{e^{\cos(3x^3)-1}-1}{\cos(3x^3)-1}}_{\to 1} \underbrace{\frac{\cos(3x^3)-1}{(3x^3)^2}}_{\to -1/2} = -\frac{9}{2}e$$

ES. 2 ESAME 11/02/2019

$$\lim_{x\to +\infty} \left(\frac{4+7x^\alpha}{3+7x}\right)^x \quad \text{ove} \quad \alpha>0$$

Dato che: $\lim_{x\to +\infty} \frac{4+7x^\alpha}{3+7x} = \lim_{x\to +\infty} \frac{x^\alpha}{x} \underbrace{\frac{(7+4/x^\alpha)}{(7+3/x)}}_{\to 1} = \begin{cases} +\infty & \text{se } \alpha>1 \\ 1 & \text{se } \alpha=1 \\ 0 & \text{se } \alpha<1 \end{cases}$

troviamo che: $\lim_{x\to +\infty} \left(\frac{4+7x^\alpha}{3+7x}\right)^x = \begin{cases} +\infty & \text{se } \alpha>1 \\ 0 & \text{se } \alpha<1 \end{cases}$

Nel caso $\alpha=1$ abbiamo:

$$\lim_{x\to +\infty} \left(\frac{4+7x}{3+7x}\right)^x = \lim_{x\to +\infty} \left(\frac{1+3+7x}{3+7x}\right)^x = \lim_{x\to +\infty} \left(\frac{1}{3+7x} + \overset{1}{\cancel{\frac{3+7x}{3+7x}}}\right)^x$$

$$= \lim_{x\to +\infty} \left[\left(1+\frac{1}{3+7x}\right)^{3+7x}\right]^{\frac{x}{3+7x}} =$$

$$= \lim_{x\to +\infty} \left[\underbrace{\left(1+\frac{1}{3+7x}\right)^{3+7x}}_{\to e}\right]^{\underbrace{\frac{\cancel{x}}{\cancel{x}(7+3/x)}}_{\to 1/7}} = e^{1/7} = \sqrt[7]{e}$$

QUIZ 4 ESAME 16/06/2021

$$\lim_{x\to +\infty} \left(\frac{x+6}{x}\right)^{\frac{x^2+5}{x+3}} = \lim_{x\to +\infty} \left[1+\frac{1}{(x/6)}\right]^{\frac{x^{\cancel{2}}(1+5/x^2)}{\cancel{x}(1+3/x)}}$$

$$= \lim_{x\to +\infty} \left[\underbrace{\left(1+\frac{1}{(x/6)}\right)^{x/6}}_{\to e}\right]^{\underbrace{\frac{6}{\cancel{x}}\frac{x(1+5/x^2)}{1+3/x}}_{\to 6}} = e^6$$

QUIZ 3 ESAME 01/09/2020

$$\lim_{x\to+\infty} x \log\left(e^{2/x}+\frac{5}{x}\right) = \lim_{x\to+\infty} x \log\left[e^{2/x}\left(1+\frac{1}{\frac{xe^{2/x}}{5}}\right)\right] =$$

$$e^{2/x}+\frac{5}{x} = e^{2/x}\left(1+\frac{5}{xe^{2/x}}\right) = e^{2/x}\left(1+\frac{1}{\frac{xe^{2/x}}{5}}\right)$$

$$= \lim_{x\to+\infty} x \left\{\log e^{2/x} + \log\left[\left(1+\frac{1}{\frac{xe^{2/x}}{5}}\right)^{\frac{xe^{2/x}}{5}}\right]^{\frac{5}{xe^{2/x}}}\right\}$$

$$= \lim_{x\to+\infty} x \left\{\frac{2}{x} + \frac{5}{xe^{2/x}} \log\left[\left(1+\frac{1}{\frac{xe^{2/x}}{5}}\right)^{\frac{xe^{2/x}}{5}}\right]\right.$$

$$= \lim_{x\to+\infty} 2 + \underbrace{\frac{5}{e^{2/x}}}_{\to 5} \underbrace{\log\underbrace{\left[\left(1+\frac{1}{\frac{xe^{2/x}}{5}}\right)^{\frac{xe^{2/x}}{5}}\right]}_{\to e}}_{\to 1} = 7$$