TUTORATO 07/12/2022

$$\lim_{x\to 0} \frac{x - \operatorname{sen} x}{x^3}$$

Per quanto visto con i limiti notevoli: $\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1 \Rightarrow \operatorname{sen} x = x + o(x)$

$$\Rightarrow \lim_{x\to 0} \frac{x - \operatorname{sen} x}{x^3} = \lim_{x\to 0} \frac{\cancel{x} - \cancel{x} + o(x)}{x^3} = \lim_{x\to 0} \frac{o(x)}{x^3} = ?$$

Qui il primo grado non basta $\longrightarrow$ vado al termine successivo

$$\operatorname{sen} x = x - \frac{x^3}{6} + o(x^3)$$

$$\Rightarrow \lim_{x\to 0} \frac{x - \operatorname{sen} x}{x^3} = \lim_{x\to 0} \frac{\cancel{x} - \cancel{x} + \frac{\cancel{x^3}}{6} + o(x^3)}{\cancel{x^3}} = \frac{1}{6}$$

$$\lim_{x\to 0} \frac{\log(1+x)^3}{\operatorname{sen} 5x + x^{4/3} \operatorname{sen} x}$$

- $\log(1+x)^3 = 3\log(1+x) = 3(x + o(x))$
- $\operatorname{sen} 5x = 5x + o(x)$
- $\operatorname{sen} x = x + o(x)$

$$\Rightarrow \lim_{x\to 0} \frac{\log(1+x)^3}{\operatorname{sen} 5x + x^{4/3}\operatorname{sen} x} = \lim_{x\to 0} \frac{3(x+o(x))}{5x + \underbrace{o(x) + x^{4/3}(x + o(x))}_{= o(x)}} = \lim_{x\to 0} \frac{3x + o(x)}{5x + o(x)} = \frac{3}{5}$$

$$\Downarrow$$

1) $x^\alpha = o(x)$ se $\alpha > 1$, $x > 0$

$$\lim_{x\to 0} \frac{x^\alpha}{x} = \lim_{x\to 0} x^{\alpha-1} \underset{\alpha>1}{=} 0 \Rightarrow x^\alpha = o(x)$$

2) $o(o(f(x))) = o(x)$

$$\Rightarrow o(x) + \underbrace{x^{4/3}\cdot x}_{= x^{7/3} = o(x)} + \underbrace{x^{4/3}\cdot o(x)}_{= o(x^{7/3}) = o(x)} = o(x)$$

Cambiamo leggermente l'esercizio:

$$\lim_{x\to 0} \frac{\log(1+x)^3 - 3\,\text{sen}\,x + \frac{3}{2}x^2}{\text{sen}\,5x - 5x\cos x}$$

Con gli sviluppi dei "limiti notevoli":

$$\frac{\cancel{3x} + o(x) - \cancel{3x} + o(x) + \frac{3}{2}x^2}{\cancel{5x} + o(x) - 5x\left(\cancel{1} - \frac{x^2}{2} + o(x^2)\right)} = \frac{o(x) + \frac{3}{2}x^2}{\underbrace{o(x) + \frac{5}{2}x^3 + o(x^3)}_{=\,o(x)}} = \frac{o(x)}{o(x)} \xrightarrow{x\to 0} ?$$

Sviluppo di ordine 3 di $f(x)$:

- $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3) \Rightarrow 3\log(1+x) = 3x - \frac{3}{2}x^2 + x^3 + o(x^3)$
- $\text{sen}\,x = x - \frac{x^3}{6} + o(x^3)$
- $\text{sen}\,5x = 5x - \frac{125}{6}x^3 + o(x^3)$

$$\Rightarrow f(x) = \frac{\cancel{3x} - \cancel{\frac{3}{2}x^2} + x^3 + o(x^3) - \cancel{3x} + \frac{x^3}{2} + o(x^3) + \cancel{\frac{3}{2}x^2}}{\cancel{5x} - \frac{125}{6}x^3 + o(x^3) - 5x\left(\cancel{1} - \frac{x^2}{2} + o(x^2)\right)} = \frac{o(x^3) + \frac{3x^3}{2} + o(x^3)}{\left(\frac{5}{2} - \frac{125}{6}\right)x^3 + o(x^3)}$$

$$= \frac{\frac{3}{2}x^3 + o(x^3)}{-\frac{110}{6}x^3 + o(x^3)} \xrightarrow{x\to 0} -\frac{9}{110}$$

$$\Rightarrow \lim_{x\to 0} \frac{\log(1+x)^3 - 3\,\text{sen}\,x + \frac{3}{2}x^2}{\text{sen}\,5x - 5x\cos x} = -\frac{9}{110}$$

$$f(x) = e^{-2x} - (1+\sin(2x))^{-1} + 2\log(1+x^2)$$

$$e^{y} = 1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + o(y^3)$$

$$\Rightarrow e^{-2x} = 1 - 2x + \frac{4x^2}{2} - \frac{8x^3}{6} + o(x^3) = 1 - 2x + 2x^2 - \frac{4}{3}x^3 + o(x^3)$$

$$\log(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} + o(y^3)$$

$$\Rightarrow \log(1+x^2) = x^2 + o(x^3)$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + o(x^3)$$

$$\sin x = x - \frac{x^3}{3!} + o(x^3) \Rightarrow \sin 2x = 2x - \frac{8x^3}{6} + o(x^3)$$

$$\Rightarrow \left(1+\sin(2x)\right)^{-1} = 1-\sin(2x)+\sin^2(2x)-\sin^3(2x)+o(x^3)$$

$$= 1-2x+\frac{4x^3}{3}+4x^2-8x^3+o(x^3)$$

$$f(x) = \cancel{1}-\cancel{2x}+\cancel{2x}^2-\frac{4}{3}x^3+o(x^3)-\cancel{1}+\cancel{2x}-\frac{4x^3}{3}-\cancel{4x}^2+8x^3+o(x^3)+\cancel{2x}^2+o(x^3)$$

$$= x^3\left(-\frac{4}{3}-\frac{4}{3}+8\right)+o(x^3) = \frac{16}{3}x^3+o(x^3)$$

$$\Rightarrow \lim_{x\to 0}\frac{e^{-2x}-\left(1+\sin(2x)\right)^{-1}+2\log(1+x^2)}{\tan^3 x} = \lim_{x\to 0}\frac{\frac{16}{3}x^3+o(x^3)}{x^3+o(x^3)} = \frac{16}{3}$$

$$f(x) = e^{\sin(4x)}-e^{4x}$$

$$e^y = 1+y+\frac{y^2}{2}+\frac{y^3}{6}+o(y^3)$$

$$\sin y = y-\frac{y^3}{6}+o(y^3) \Rightarrow \sin 4x = 4x-\frac{32}{3}x^3+o(x^3)$$

$$\Rightarrow e^{\sin(4x)} = 1+\sin(4x)+\frac{1}{2}\sin^2(4x)+\frac{1}{6}\sin^3(4x)+o(x^3)$$

$$= 1+4x-\frac{32}{3}x^3+\frac{1}{2}16x^2+\frac{1}{6}64x^3+o(x^3)$$

$$= 1+4x-\cancel{\frac{32}{3}x^3}+8x^2+\cancel{\frac{32}{3}x^3}+o(x^3)$$

$$\Rightarrow e^{4x} = 1+4x+\frac{16x^2}{2}+\frac{64x^3}{6}+o(x^3) = 1+4x+8x^2+\frac{32}{3}x^3+o(x^3)$$

$$\Rightarrow f(x) = \cancel{1}+\cancel{4x}+\cancel{8x}^2-\cancel{1}-\cancel{4x}-\cancel{8x}^2-\frac{32}{3}x^3+o(x^3) = -\frac{32}{3}x^3+o(x^3)$$

$$\Rightarrow \lim_{x\to 0}\frac{1-\cos(x^2)}{e^{\sin 4x}-e^{4x}}\cdot\frac{x^4}{x^4} = \lim_{x\to 0}\underbrace{\frac{1-\cos x^2}{x^4}}_{\to 1/2}\cdot\underbrace{\frac{x^4}{-\frac{32}{3}x^3+o(x^3)}}_{\to 0} = 0$$

$g(x) = \sin(\sin^3 x)$

$f(x) = \sin x - \sinh x + \frac{1}{3}\sin(\sin^3 x)$

$\sin y = y - \frac{y^3}{6} + o(y^3)$

$$\Rightarrow g(x) = \sin^3 x - \frac{(\sin^3 x)^3}{6} + o(x^9) = \sin^3 x + o(x^9) = \left(x - \frac{x^3}{6} + o(x^3)\right)^3 = x^3 - 3x^2\frac{x^3}{6} + o(x^5) = x^3 - \frac{x^5}{2} + o(x^5)$$

$\sinh x = x + \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)$

$$\Rightarrow f(x) = \cancel{x} - \cancel{\frac{x^3}{6}} + \cancel{\frac{x^5}{120}} - \cancel{x} - \cancel{\frac{x^3}{6}} - \cancel{\frac{x^5}{120}} + \cancel{\frac{1}{3}}\cancel{x^3} - \frac{1}{6}x^5 + o(x^5) = -\frac{x^5}{6} + o(x^5)$$

$$\Rightarrow \lim_{x\to 0} \frac{\sin x - \sinh x + \frac{1}{3}\sin(\sin^3 x)}{\tan(x^5)} = \lim_{x\to 0} \underbrace{\frac{x^5}{\tan(x^5)}}_{\to 1} \underbrace{\frac{1}{x^5}\left(-\frac{x^5}{6} + o(x^5)\right)}_{\to -\frac{1}{6}} = -\frac{1}{6}$$

$g(x) = e^{\sin^3 x} - 1$

$f(x) = \log(1 - 2x^3)$

$e^y = 1 + y + o(y)$

$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)$

$$\Rightarrow \sin^3 x = \left(x - \frac{x^3}{6} + o(x^3)\right)^3 = x^3 - \cancel{3}x^2\frac{x^3}{\cancel{6}_{2}} + o(x^5) = x^3 - \frac{x^5}{2} + o(x^5)$$

$$\Rightarrow g(x) = e^{\sin^3 x} - 1 = \cancel{1} + x^3 - \frac{x^5}{2} + o(x^5) - \cancel{1} = x^3 - \frac{x^5}{2} + o(x^5)$$

$\cancel{\log(1+x) = x - \frac{x^2}{2} + o(x^2)}$ $\log(1+y) = y - \frac{y^2}{2} + o(y^2)$

$$\Rightarrow f(x) = -2x^3 - \frac{(-2x^3)^2}{2} + o(x^6) = -2x^3 + o(x^5)$$

$$\Rightarrow \lim_{x\to 0} \frac{e^{\sin^3 x} - 1}{\log(1 - 2x^3)} = \lim_{x\to 0} \frac{x^3 - \frac{x^5}{2} + o(x^5)}{-2x^3 + o(x^5)} = \frac{\cancel{x^3}}{-2\cancel{x^3}} = -\frac{1}{2}$$

$g(x) = \log(1-\sin^2 x)$

$f(x) = \cos x^2 - \cosh x^2$

$\log(1+x) = x - \frac{x^2}{2} + o(x^2)$

$\sin x = x - \frac{x^3}{6} + o(x^3)$

$$\Rightarrow \log(1-\sin^2 x) = -\sin^2 x - \frac{\sin^4 x}{2} + o(x^4)$$

$$= -\left(x - \frac{x^3}{6} + o(x^3)\right)^2 - \frac{1}{2}\left(x + o(x^2)\right)^4 + o(x^5)$$

$$= -x^2 + \frac{x^4}{3} - \frac{x^4}{2} + o(x^5) = -x^2 - \frac{x^4}{6} + o(x^5)$$

$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5) \Rightarrow \cos x^2 = 1 - \frac{x^4}{2} + o(x^5)$

$\cosh x = 1 + \frac{x^2}{2} + \frac{x^4}{24} + o(x^5) \Rightarrow \cosh x^2 = 1 + \frac{x^4}{2} + o(x^5)$

$$\Rightarrow f(x) = \cancel{1} - \frac{x^4}{2} - \cancel{1} - \frac{x^4}{2} + o(x^5) = -x^4 + o(x^5)$$

$$\Rightarrow \lim_{x\to 0} \frac{x^2 + \log(1-\sin^2 x)}{\cos x^2 - \cosh x^2} = \lim_{x\to 0} \frac{\cancel{x^2} - \cancel{x^2} + x^4/6 + o(x^5)}{-x^4 + o(x^5)} = \frac{1}{6}$$