TUTORATO 30/11/2022

- es. 8.8.1 FOGLIO 8:

$$f(x) = \frac{x^2-1}{x(x+2)}$$

$$\boxed{\frac{d}{dx}\left[\frac{g(x)}{h(x)}\right] = \frac{g'(x)\,h(x) - g(x)\,h'(x)}{(h(x))^2}}$$

$g(x) = x^2-1 \Rightarrow g'(x) = 2x$

$h(x) = x(x+2) \Rightarrow h'(x) = x+2+x = 2x+2$

$$\Rightarrow f'(x) = \frac{2x^2(x+2) - (x^2-1)(2x+2)}{x^2(x+2)^2} = \frac{\cancel{2x^3} + 4x^2 - \cancel{2x^3} - 2x^2 + 2x + 2}{x^2(x+2)^2}$$

$$= \frac{2x^2+2x+2}{x^2(x+2)^2}$$

- es. 8.8.3 FOGLIO 8:

$$f(x) = \text{sen}\left(x^{2e-x}\right)$$

$$h(x) = g(f(x)) \Rightarrow \boxed{\frac{d}{dx}h(x) = g'(f(x))\cdot f'(x)}$$

$$\boxed{\frac{d}{dx}f(x)^{g(x)} = f(x)^{g(x)}\left[g'(x)\cdot \log f(x) + \frac{g(x)\cdot f'(x)}{f(x)}\right]}$$

$$\Rightarrow f'(x) = \cos\left(x^{2e-x}\right)\, x^{2e-x}\left[-\log x + \frac{2e-x}{x}\right]$$

- es. 8.8.14 FOGLIO 8:

$$f(x) = \arctan\sqrt{\frac{1-x}{1+x}}$$

$$g(x) = \sqrt{\frac{1-x}{1+x}} = \left(\frac{1-x}{1+x}\right)^{1/2} \Rightarrow g'(x) = \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2}\left[-\frac{1}{1+x} - (1-x)\frac{1}{(1+x)^2}\right]$$

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2}\left[-\frac{1+\cancel{x}+1-\cancel{x}}{(1+x)^2}\right]$$

$$= -\frac{1}{\cancel{2}}\left(\frac{1-x}{1+x}\right)^{-1/2}\frac{\cancel{2}}{(1+x)^2}$$

$$\Rightarrow f'(x) = \frac{1}{1+\frac{1-x}{1+x}}\left[-\sqrt{\frac{1+x}{1-x}}\;\frac{1}{(1+x)^2}\right]$$

$$= \frac{\cancel{1+x}}{1+\cancel{x}+1-\cancel{x}}\left[-\sqrt{\frac{\cancel{1+x}}{1-x}}\;\frac{1}{(1+x)^{\cancel{2}}}\right]\times\frac{1}{2}$$

$$= -\frac{1}{2\sqrt{1-x}\cdot\sqrt{1+x}} = -\frac{1}{2\sqrt{1-x^2}}$$

- es. 124 FOGLIO 8

$f(x) = 2\log x - 5\arctan x$

Dominio: $(0, +\infty)$

$$f'(x) = \frac{2}{x} - \frac{5}{1+x^2} = \frac{2(1+x^2)-5x}{x(1+x^2)} = \frac{2x^2-5x+2}{x(1+x^2)}$$

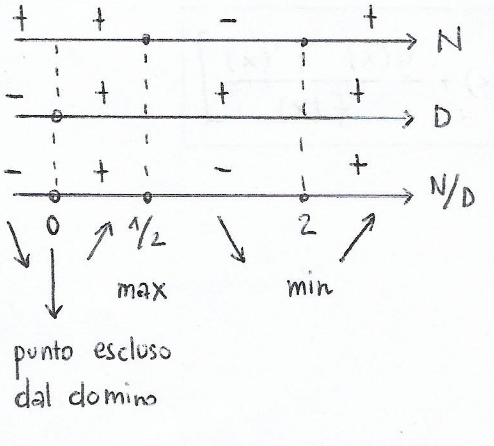
Studio il segno di $f'(x)$:

$N \geq 0$: $2x^2 - 5x + 2 \geq 0 \Rightarrow x \leq 1/2$ e $x \geq 2$

$\Delta = 25 - 16 = 9$

$$x = \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4} = \begin{cases} 2 \\ 1/2 \end{cases}$$

$D > 0$: $x(1+x^2) > 0 \Rightarrow x > 0$



$x = \frac{1}{2} \Rightarrow$ max

$x = 2 \Rightarrow$ min

- es. 115 FOGLIO 8:

$f(x) = x^k (x-1)^m \qquad k, m \in \mathbb{Z}\ ;\ k, m > 1$

$$f'(x) = k x^{k-1}(x-1)^m + x^k m (x-1)^{m-1}$$

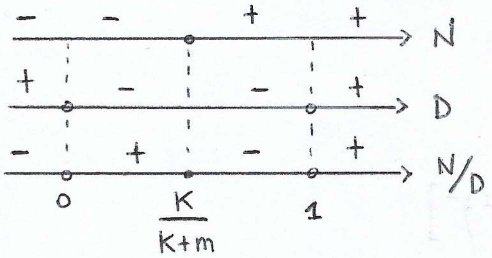
$$= x^k (x-1)^m \left(\frac{k}{x} + \frac{m}{x-1}\right) = x^k (x-1)^m \left[\frac{k(x-1)+mx}{x(x-1)}\right]$$

$$= x^k (x-1)^m \frac{kx - k + mx}{x(x-1)} = x^k (x-1)^m \frac{x(k+m)-k}{x(x-1)}$$

Studio il segno di $\dfrac{x(K+m)-K}{x(x-1)}$:

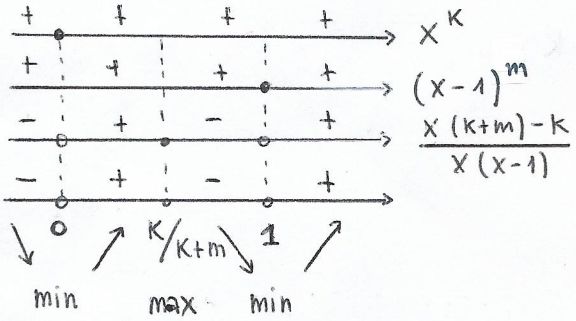
$N \geq 0$: $x(K+m)-K \geq 0 \Rightarrow x \geq \dfrac{K}{K+m} \quad (\leq 1)$

$D > 0$: $x(x-1) > 0 \Rightarrow x < 0$ e $x > 1$



Studio il segno di $f'(x)$ al variare di K e m :

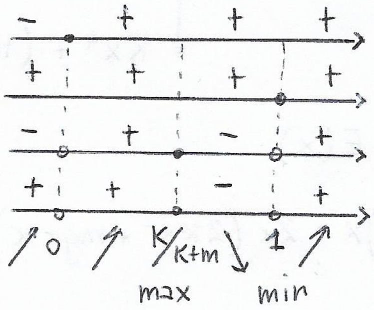
1) K pari, m pari :



min $\Rightarrow$ $x = 0, 1$

max $\Rightarrow$ $x = K/K+m$

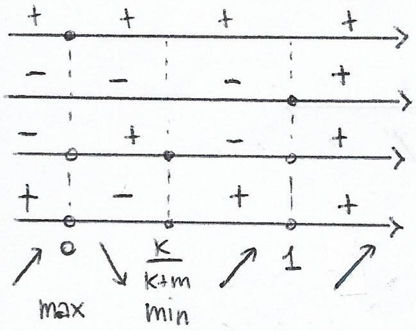
3) K dispari, m pari :



min $\Rightarrow$ 1

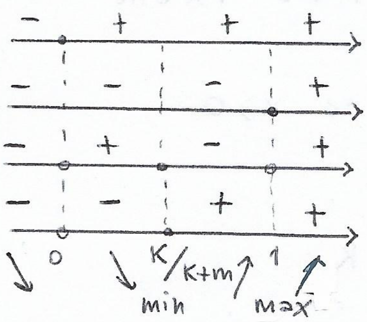
max $\Rightarrow$ $K/K+m$

2) K pari, m dispari :



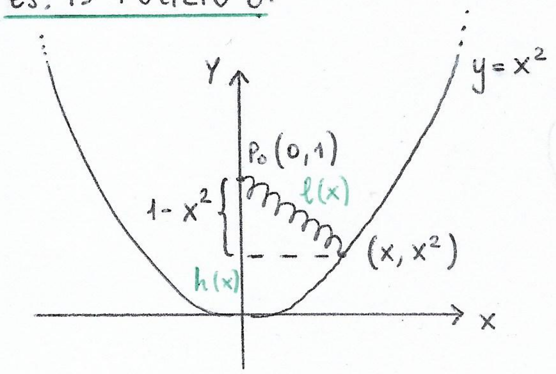
min $\Rightarrow \dfrac{K}{K+m}$ max $\Rightarrow$ 0

4) K dispari, m dispari



min $\Rightarrow \dfrac{K}{K+m}$

- es. 93 FOGLIO 8:



$$\left.\begin{aligned} h(x) &= x^2 \\ \ell(x) &= \sqrt{x^2 + (1-x^2)^2} \end{aligned}\right\} \quad E(x) = \overbrace{mg\,h(x)}^{E_{grav}} + \overbrace{K\,\ell(x)^2}^{E_{el}}$$

$$= mgx^2 + K\left[x^2 + (1-x^2)^2\right]$$

$$= mgx^2 + Kx^2 + K\left(1 + x^4 - 2x^2\right)$$

$$= mgx^2 + Kx^2 + K + Kx^4 - 2Kx^2$$

$$= Kx^4 + (mg - K)x^2 + K$$

Calcolo la derivata di $E(x)$:

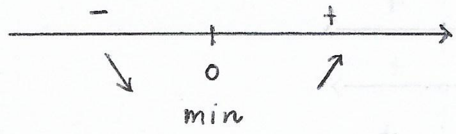
$$E'(x) = 4Kx^3 + 2(mg-K)x = 2x\left(2Kx^2 + mg - K\right)$$

Candidati minimi:

$$E'(x) = 0 \Rightarrow 2x\left(2Kx^2 + mg - K\right) = 0 \left\langle \begin{array}{l} x = 0 \\ 2Kx^2 + mg - K = 0 \Rightarrow x = \pm\sqrt{\dfrac{K-mg}{2K}} \end{array}\right.$$

1) $mg \geq K \Rightarrow 2Kx^2 + mg - K \geq 0 \quad \forall x \in \mathbb{R}$

$2x\left(2Kx^2 + mg - K\right) \geq 0$ se $x \geq 0$

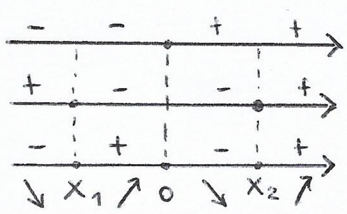


min: $x = 0 \Rightarrow E_{min} = K$

2) $mg < K \Rightarrow 2x\left(2Kx^2 + mg - K\right) \geq 0$

$2x \geq 0 \Rightarrow x \geq 0$

$2Kx^2 + mg - K \geq 0 \Rightarrow x < -\sqrt{\dfrac{K-mg}{2K}}$ (x_1) e $x > \sqrt{\dfrac{K-mg}{2K}}$ (x_2)



min: $x = \pm\sqrt{\dfrac{K-mg}{2K}} \Rightarrow E_{min} = K\left(\dfrac{K-mg}{2K}\right)^2 + (mg-K)\dfrac{K-mg}{2K} + K = *$

$$* = \frac{(K-mg)^2}{4K} - \frac{(K-mg)^2}{2K} + K = \frac{(K-mg)^2 - 2(K-mg)^2 + 4K^2}{4K} = \frac{-(K-mg)^2 + 4K^2}{4K} =$$

$$= \frac{-K^2 - m^2g^2 + 2Kmg + 4K^2}{4K} = \frac{3K^2 + 2Kmg - m^2g^2}{4K} = \frac{3K^2 + 3Kmg - Kmg - m^2g^2}{4K}$$

$$= \frac{3K(K+mg) - mg(K+mg)}{4K} = \frac{(3K-mg)(K+mg)}{4K}$$

- es. 2 APPELLO 15/09/2022

$$f(x) = |x|\sqrt{1-x^3}$$

- Dominio: $1 - x^3 \geq 0 \Rightarrow x^3 \leq 1 \Rightarrow x < 1$

$D = (-\infty, 1]$

- Asintoti: $\lim_{x\to-\infty} |x|\sqrt{1-x^3} = \lim_{x\to-\infty} \sqrt{x^2(1-x^3)} = \lim_{x\to-\infty} \sqrt{x^2 - x^5} =$

$= \lim_{x\to-\infty} \sqrt{\underbrace{x^5}_{\to-\infty}\underbrace{\left(-1 + 1/x^3\right)}_{\to-1}} = +\infty \Rightarrow$ no asintoti orizzontali

Asintoti obliqui?

$m = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\overset{-1}{|x|}\sqrt{1-x^3}}{x} = -\infty \Rightarrow$ no asintoti obliqui

- Derivata: $f(x)$ e' derivabile in $(-\infty, 1] \setminus \{0\}$ → $|x|$ non e' derivabile in 0.

$|x| = x\,\mathrm{sgn}(x) \Rightarrow f'(x) = \mathrm{sgn}(x)\sqrt{1-x^3} - x\,\mathrm{sgn}(x)\frac{3x^2}{2}(1-x^3)^{-1/2}$

$= \mathrm{sgn}(x)\sqrt{1-x^3} - \frac{3x^3\,\mathrm{sgn}(x)}{2\sqrt{1-x^3}}$

$= \frac{\mathrm{sgn}(x)}{2\sqrt{1-x^3}}\left(2 - 2x^3 - 3x^3\right) = \frac{\mathrm{sgn}(x)}{2\sqrt{1-x^3}}\left(2 - 5x^3\right)$

Studio il segno di $f'(x)$:

		0		$\left(\frac{2}{5}\right)^{1/3}$		1
$\mathrm{sgn}(x)$	$-$		$+$		$+$	
$2-5x^3$	$+$		$+$		$-$	
$f'(x)$	$-$		$+$		$-$	
	↘	0	↗	$\left(\frac{2}{5}\right)^{1/3}$	↘	1

min $\Rightarrow 0$

max $\Rightarrow \left(\frac{2}{5}\right)^{1/3}$

$$\lim_{x\to 0^+} f'(x) = \lim_{x\to 0^+} \underbrace{\frac{1}{2\sqrt{1-x^3}}}_{\to 1/2} \underbrace{(2-5x^3)}_{\to 2} = 1$$

$$\lim_{x\to 0^-} f'(x) = -1 \qquad \lim_{x\to 1^-} f'(x) = \lim_{x\to 1^-} \underbrace{\frac{1}{2\sqrt{1-x^3}}}_{\to +\infty} \underbrace{(2-5x^3)}_{\to -3} = -\infty$$