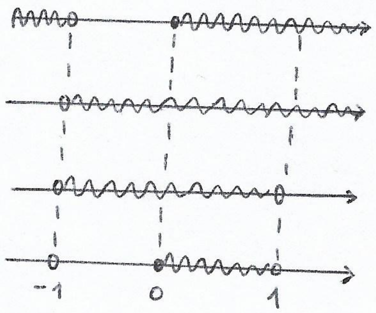
TUTORATO 02/11/2022

$$f(x) = \arcsin\left(\frac{x-1}{x+1}\right) + \ln\left(1-x^2\right)$$

Trovare l'insieme di definizione di $f(x)$.

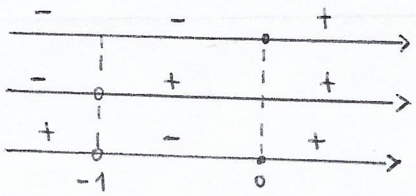
$$\begin{cases} \dfrac{x-1}{x+1} \geq -1 \Rightarrow x<-1 \text{ e } x \geq 0 \\ \dfrac{x-1}{x+1} \leq 1 \Rightarrow x > -1 \\ 1-x^2 > 0 \Rightarrow -1 < x < 1 \\ x+1 \neq 0 \Rightarrow x \neq -1 \end{cases}$$



- $\dfrac{x-1}{x+1} \geq -1 \Rightarrow \dfrac{x-\cancel{1}+x+\cancel{1}}{x+1} \geq 0 \Rightarrow \dfrac{2x}{x+1} \geq 0 \Rightarrow x < -1$ e $x \geq 0$

  $N \geq 0: \quad 2x \geq 0 \Rightarrow x \geq 0$

  $D > 0: \quad x+1 > 0 \Rightarrow x > -1$



- $\dfrac{x-1}{x+1} \leq 1 \Rightarrow \dfrac{\cancel{x}-1-\cancel{x}-1}{x+1} \leq 0 \Rightarrow \dfrac{-2}{x+1} \leq 0 \Rightarrow x > -1$

  $N \geq 0: \quad -2 \geq 0 \rightarrow$ mai verificata in $\mathbb{R}$

  $D > 0: \quad x+1 > 0 \rightarrow x > -1$

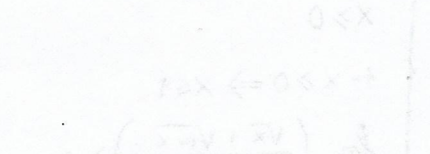
- $1-x^2 > 0 \Rightarrow x^2-1 > 0 \Rightarrow -1 < x < 1$

Il dominio di $f(x)$ è : $[0, 1)$

$$f(x) = \sqrt{|x-3| - |x+4|}$$

Trovare l'insieme di definizione di $f(x)$.

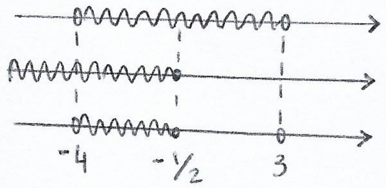
$$|x-3| = \begin{cases} x-3 & \text{se } x \geq 3 \\ -x+3 & \text{se } x < 3 \end{cases} \qquad |x+4| = \begin{cases} x+4 & \text{se } x \geq -4 \\ -x-4 & \text{se } x < -4 \end{cases}$$



1) $\begin{cases} x \leq -4 \\ -\cancel{x}+3+\cancel{x}+4 \geq 0 \rightarrow 7 \geq 0 \rightarrow \text{sempre verificata} \end{cases}$

$D_1: \ x \leq -4$

2) $\begin{cases} -4 < x < 3 \\ -x+3-x-4 \geq 0 \Rightarrow -2x-1 \geq 0 \Rightarrow 2x+1 \leq 0 \Rightarrow x \leq -\frac{1}{2} \end{cases}$



$D_2 : -4 < x \leq -\frac{1}{2}$

3) $\begin{cases} x \geq 3 \\ x-3-x-4 \geq 0 \Rightarrow -7 \geq 0 \Rightarrow \text{mai verificata in } \mathbb{R} \end{cases}$

$D_3 : \emptyset$

Il dominio di $f(x)$ e': $D = D_1 \cup D_2 \cup D_3 = \left(-\infty, -\frac{1}{2}\right]$

$f(x) = \sqrt{\ln \frac{\sqrt{x}+\sqrt{1-x}}{\alpha}} \qquad (\alpha > 0)$

Trovare l'insieme di definizione di $f(x)$.

$$\begin{cases} x \geq 0 \\ 1-x \geq 0 \Rightarrow x \leq 1 \\ \ln\left(\frac{\sqrt{x}+\sqrt{1-x}}{\alpha}\right) \geq 0 \end{cases}$$

$$\ln\left(\frac{\sqrt{x}+\sqrt{1-x}}{\alpha}\right) \geq 0 = \ln(1) \implies \frac{\sqrt{x}+\sqrt{1-x}}{\alpha} \geq 1 \Rightarrow \sqrt{x}+\sqrt{1-x} \geq \alpha$$

Elevo entrambi i membri al quadrato:

$$\cancel{x}+1-\cancel{x}+2\sqrt{x(1-x)} \geq \alpha^2 \Rightarrow 2\sqrt{x(1-x)} \geq \alpha^2-1$$

Di nuovo: $4x(1-x) \geq (\alpha^2-1)^2$

$$4x-4x^2-(\alpha^2-1)^2 \geq 0 \Rightarrow 4x^2-4x+(\alpha^2-1)^2 \leq 0$$

$$\Delta = 16-16(\alpha^2-1)^2 = 16\left[1-(\alpha^2-1)^2\right] = 16\left[\cancel{1}-\alpha^4-\cancel{1}+2\alpha^2\right] = 16\alpha^2(2-\alpha^2)$$

$$x = \frac{4 \pm 4\alpha\sqrt{2-\alpha^2}}{8} = \frac{1 \pm \alpha\sqrt{2-\alpha^2}}{2}$$

Soluzione: $\frac{1-\alpha\sqrt{2-\alpha^2}}{2} \leq x \leq \frac{1+\alpha\sqrt{2-\alpha^2}}{2}$

$\hookrightarrow$ valida solo se: $2-\alpha^2 \geq 0 \Rightarrow \alpha^2-2 \leq 0 \Rightarrow -\sqrt{2} \leq \alpha \leq \sqrt{2}$

Poiche' $\alpha > 0$ la condizione si restringe a: $0 < \alpha \leq \sqrt{2}$

Dunque, il dominio di $f(x)$ è:

- $\emptyset$ se $\alpha > \sqrt{2}$
- $\left\{\frac{1}{2}\right\}$ se $\alpha = \sqrt{2}$
- Nell'intervallo $(0, \sqrt{2})$:

$\alpha = 1$: $x = \frac{1}{2} \pm \frac{1}{2}\sqrt{2-1} = \frac{1}{2} \pm \frac{1}{2} = \left\langle \begin{matrix} 1 \\ 0 \end{matrix} \right\} \; 0 \leq x \leq 1$

Per $0 < \alpha \leq 1$: $[0, 1]$

Per $1 < \alpha \leq \sqrt{2}$: $\frac{1-\alpha\sqrt{2-\alpha^2}}{2} \leq x \leq \frac{1+\alpha\sqrt{2-\alpha^2}}{2}$

$f(x) = |x-1|$

Calcolare $f((0,5))$.

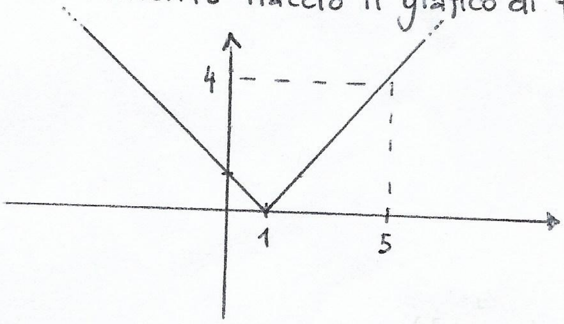
$$y = |x-1| = \begin{cases} x-1 & \text{se } x \geq 1 \\ -x+1 & \text{se } x < 1 \end{cases}$$

$$\begin{cases} x \geq 1 \\ y = x-1 \Rightarrow x = y+1 \Rightarrow 1 \leq y+1 < 5 \Rightarrow 0 \leq y < 4 \end{cases}$$

$$\begin{cases} x < 1 \\ y = -x+1 \Rightarrow x = 1-y \Rightarrow 0 < 1-y < 1 \Rightarrow 0 < y < 1 \end{cases}$$

$\Rightarrow f((0,5)) = [0,4)$

Alternativamente traccio il grafico di $f(x)$:

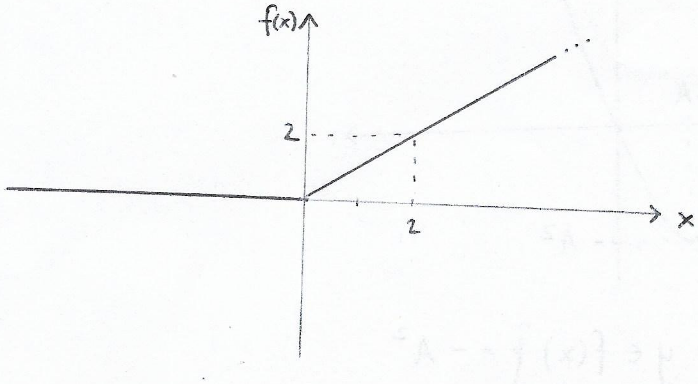


$f((0,5)) = [0,4)$

$f(x) = \frac{x+|x|}{2}$

Calcolare $f((-2,2))$.

$$f(x) = \begin{cases} \frac{2x}{2} = x & \text{se } x \geq 0 \\ 0 & \text{se } x < 0 \end{cases}$$

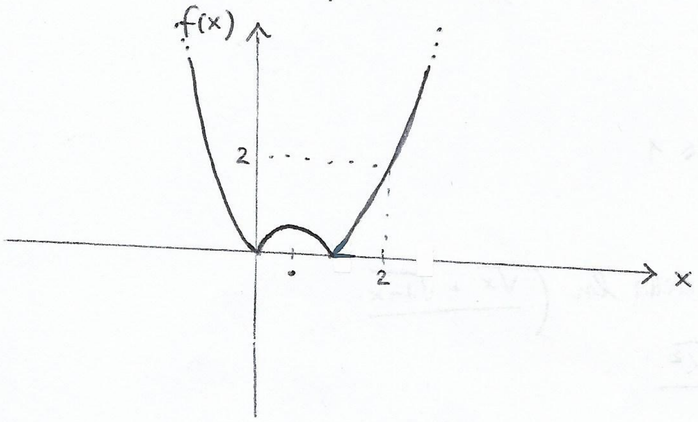


$\Rightarrow f((-2,2)) = [0,2)$

$f(x) = |x^2 - x|$

Calcolare $f((0,2))$.

$f(x) = x^2 - x$ e' una parabola di vertice $V = \left(-\frac{b}{2a}; -\frac{\Delta}{4a}\right) = \left(\frac{1}{2}; -\frac{1}{4}\right)$



$\Rightarrow f((0,2)) = [0,2)$

es. 2 QUIZ 10/11/2021

Data la funzione $f: \mathbb{R} \to \mathbb{R}$

$$f(x) = x^2 + 2Ax$$

determinare il minimo dell'immagine di $f(x)$

$$\min\{y \in \mathbb{R} : y \in f(x)\}$$

$$f(x) = x^2 + 2Ax = \underbrace{x^2 + 2Ax + A^2}_{(x+A)^2} - A^2 = \underbrace{(x+A)^2}_{\geq 0} - A^2$$

Quindi $\min\{f(x) \mid x \in \mathbb{R}\}$ verrà realizzato quando:

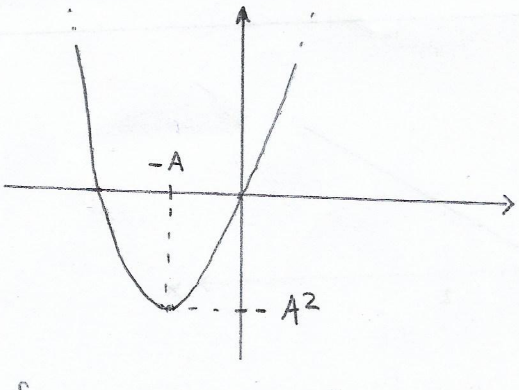
$(x+A)^2 = 0 \Rightarrow x = -A$

$f(-A) = A^2 - 2A^2 = -A^2$

$\Rightarrow \min\{y \in \mathbb{R} : y \in f(x)\} = -A^2$

Alternativamente, traccio il grafico:

$y = x^2 + 2Ax \Rightarrow$ parabola di vertice $V = \left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right) = (-A, -A^2)$



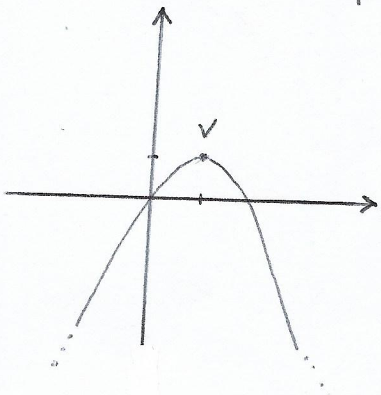
$\Rightarrow \min\{y \in \mathbb{R} : y \in f(x)\} = -A^2$

$f(x) = 2x - x^2$

non e' iniettiva in $\mathbb{R}$.

Trovare un intervallo $[a,b]$ tale che la restrizione di f a tale intervallo sia invertibile, e scrivere la funzione inversa.

$f(x) = 2x - x^2$ e' una parabola di vertice $V = \left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right) = (1, 1)$



La funzione e' invertibile in ogni intervallo:

- $[a,b] \subset (-\infty, 1]$
- $[a,b] \subset [1, +\infty)$

Calcolo la funzione inversa:

$y = 2x - x^2 \rightarrow x^2 - 2x = -y \rightarrow x^2 - 2x + 1 - 1 = -y \rightarrow (x-1)^2 = 1 - y \rightarrow x = 1 \pm \sqrt{1-y}$

Dunque avremo:
- $f^{-1}(x) = 1 - \sqrt{1-x}$ per $[a,b] \subset (-\infty, 1]$
- $f^{-1}(x) = 1 + \sqrt{1-x}$ per $[a,b] \subset [1, +\infty)$