DD2434/FDD3434 Machine Learning, Advanced Course
Assignment 2 - Tree Formulations

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1 Tree Formulations

There are three tasks in the assignment which are related to trees; 2.3, 2.5 and 2.8. The notations used in these questions are very similar to each other (with minor differences). Still, the questions might be hard to understand at the first glance. This short document is created to help you understand the basics.

Below, you see an example representation of a tree T and its features for guidance.

- A rooted tree T
- The root is $r = V_0$
- The vertex set is $V(T) = \{V_0, V_1, V_2, V_3, V_4, V_5, V_6\}$
- The leaf set is $L(T) = \{V_1, V_3, V_4, V_6\}$
- Each vertex $v \in V(T)$ has an associated random variable X_v, which takes value in $c = \{0, \ldots, C-1\}$
- The Categorical Distribution probabilities corresponding to the random variables depend on the parent node's value $\theta_v = p(X_v | X_{pa(v)} = x_{pa(v)})$ where $pa(v)$ is the parent of node v
- All the CPDs on the tree edges are represented by $\Theta = \{\theta_0, \ldots, \theta_6\}$

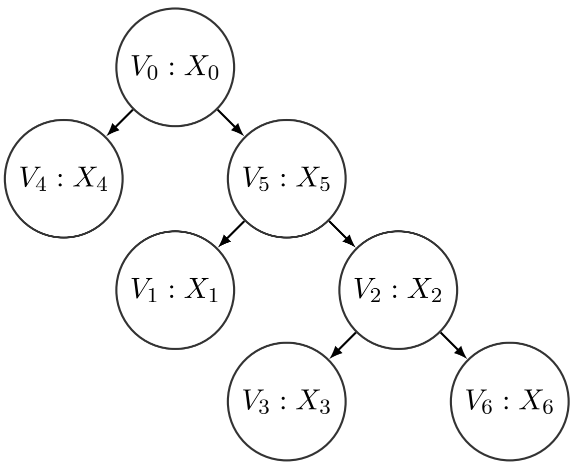

Figure 1: Example binary tree T

1.1 Tree Formulation for Task 2.3

In Task 2.3 the tree is binary, therefore each node can have either zero or two children. The goal for this task is, given the tree topology (T) and CPDs (Θ) one needs to calculate the likelihood of a specific leaf assignment $p(\beta|T, \Theta)$ where $\beta = \{x_l : l \in L(T)\}$. For example, one needs to compute $p(x_1 = 0, x_3 = 2, x_4 = 1, x_6 = 1|T, \Theta)$ or $p(x_1 = 1, x_3 = 1, x_4 = 1, x_6 = 1|T, \Theta)$ etc.

1.2 Tree Formulation for Task 2.5

In this task, we have a mixture of K trees, represented by $\mathcal{M}$.

- There are K mixture components
- Each mixture weight is positive $\pi_k > 0$ and $\sum_{k=1}^{K} \pi_k = 1$
- Each component is a (tree,CPD) tuple $\tau_k = (T_k, \Theta_k)$
- The trees do not have to binary (i.e see Figure 2)
- The random variables of the nodes can only have *binary* values ($X_v = c$ where $c \in \{0, 1\}$)

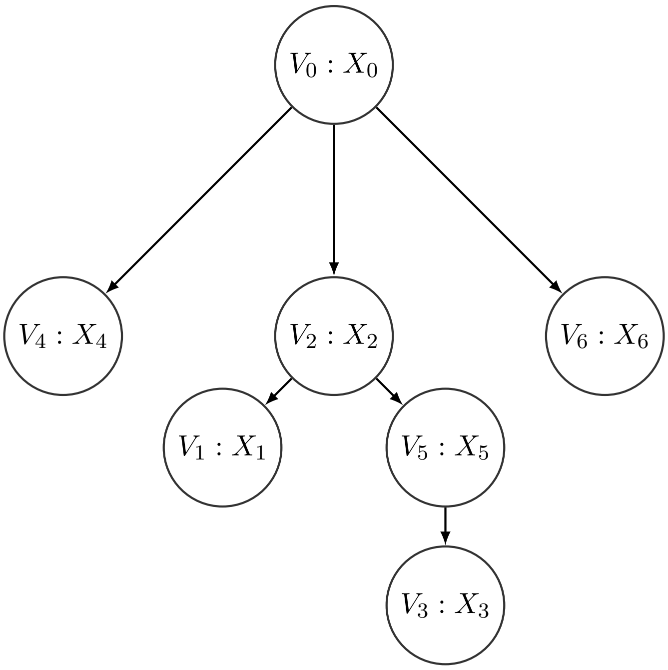

Figure 2: Example tree T

The objective is to infer the tree mixture ($\mathcal{M}$) given observed data D. The data has N observations (x^n) where each observation consists of the states of each node. An example dataset D (with $N = 5$ samples) is shown below:

Table 1: Example simulated data D

V_0	V_1	V_2	V_3	V_4	V_5	V_6
0	0	0	0	0	1	0
0	0	0	1	0	1	0
0	0	1	0	1	1	0
0	0	1	1	0	1	1
1	0	1	1	0	0	1

1.3 Tree Formulation for Task 2.8

In the Task 2.8, the tree structure is binary. Each node has either zero or two children nodes. Here, we will not explicitly write the desired probability expression (you need to formulate it). In this task, you will work with the assignment of values to the leaves, whose sum is odd ($x = \{x_l : l \in L(T)\}$ such that $\sum_{l \in L(T)}$ is odd). For example, x might correspond to $x = \{x_1 = 0, x_3 = 2, x_4 = 1, x_6 = 0\}$ or $x = \{x_1 = 1, x_3 = 1, x_4 = 1, x_6 = 2\}$ etc.

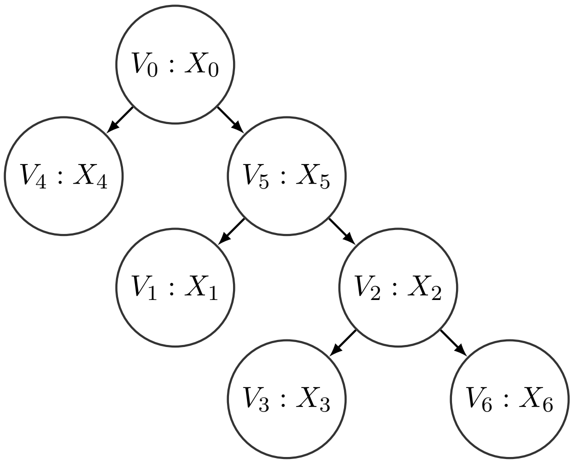

Figure 3: Example binary tree T