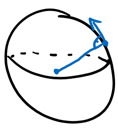
At any time there is a location on earth where the wind is not blowing.



Mathematical interpretation:

For any continuous vector field on S^2

$\tau : S^2 \to \mathbb{R}^3$

$(\langle \tau(x), x\rangle = 0$ for $x \in S^2 \subset \mathbb{R}^3)$

there is a zero.

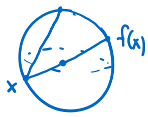
Lemma: Let $n \geq 2$ even. For each map $S^n \xrightarrow{f} S^n$ there is a $x \in S^n$ such that

$$f(x) \in \{x, -x\}.$$

Proof: Assume that $f(x) \notin \{x, -x\}$ for all $x \in S^n$.

Define

$$F(x,t) = \frac{(1-t)x + t \cdot f(x)}{\|(1-t)x + t\cdot f(x)\|}$$ is homotopy $\mathrm{id}_{S^n} \simeq f$



$$G(x,t) = \frac{(1-t)f(x) + t(-x)}{\|(1-t)f(x) + t(-x)\|}$$ is a homotopy $-\mathrm{id}_{S^n} \simeq f$.

$-\mathrm{id}_{S^n}$ is the composition of $n+1$ reflections.

Hence $H_n(-\mathrm{id}_{S^n}) : H_n(S^n) \to H_n(S^n)$

is multiplication with $(-1)^{n+1} = -1$.

Hence $-\mathrm{id}_{S^n} \not\simeq \mathrm{id}_{S^n}$ ↯

$\square$

Theorem Let $n \geq 2$ be even. Every continuous vector field on S^n vanishes at some point.

[every map $\tau : S^n \to \mathbb{R}^{n+1}$ with $\langle \tau(x), x\rangle = 0$ $\forall x \in S^n$ vanishes at some point]

Proof. Assume $\tau(x) \neq 0$ for all $x \in S^n$.

Define $f(x) = \dfrac{\tau(x)}{\|\tau(x)\|} \in S^n$.

By the lemma there is $x_0 \in S^n$ such that

$$f(x_0) \in \{x_0, -x_0\}.$$

This contradicts

$$0 = \langle \tau(x_0), x_0\rangle = \|\tau(x_0)\| \cdot \langle f(x_0), x_0\rangle$$

$\square$