# principles of Urban Science 12

## uncertainties and bias

dr.federica bianco | fbb.space | fedhere | fedhere

# 1 statistical uncertainties

1 formulate your prediction (NH)

2 identify all alternative outcomes (AH)

3 set confidence threshold ($p$-value)

4 find a measurable quantity which unde the Null has a known distribution (pivotal quantity)

5 calculate the pivotal quantity

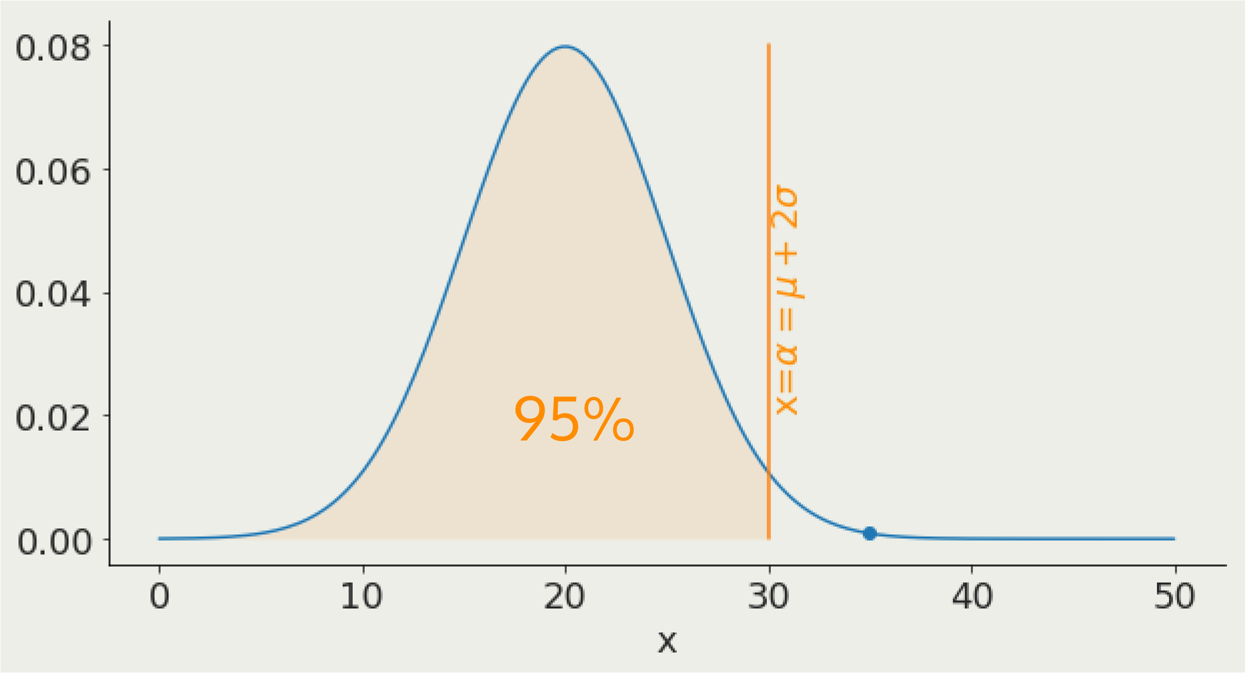
6 calculate probability of value obtained for the pivotal quantity under the NH

**if probability < $p$-value : reject Null**

**N**ull
**H**ypothesis
**R**ejection
**T**esting

# what is $\alpha$?

α is the x value corresponding to a chosen threshold




it represent the probability to get a result at least as extreme just by chance

6

test data against *alternative* outcomes

# stochastic or random errors

unpredictable uncertainty in a measurement due to lack of sensitivity in the measurement or to stochasticity in a process

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2.5 +/- 0.1 cm

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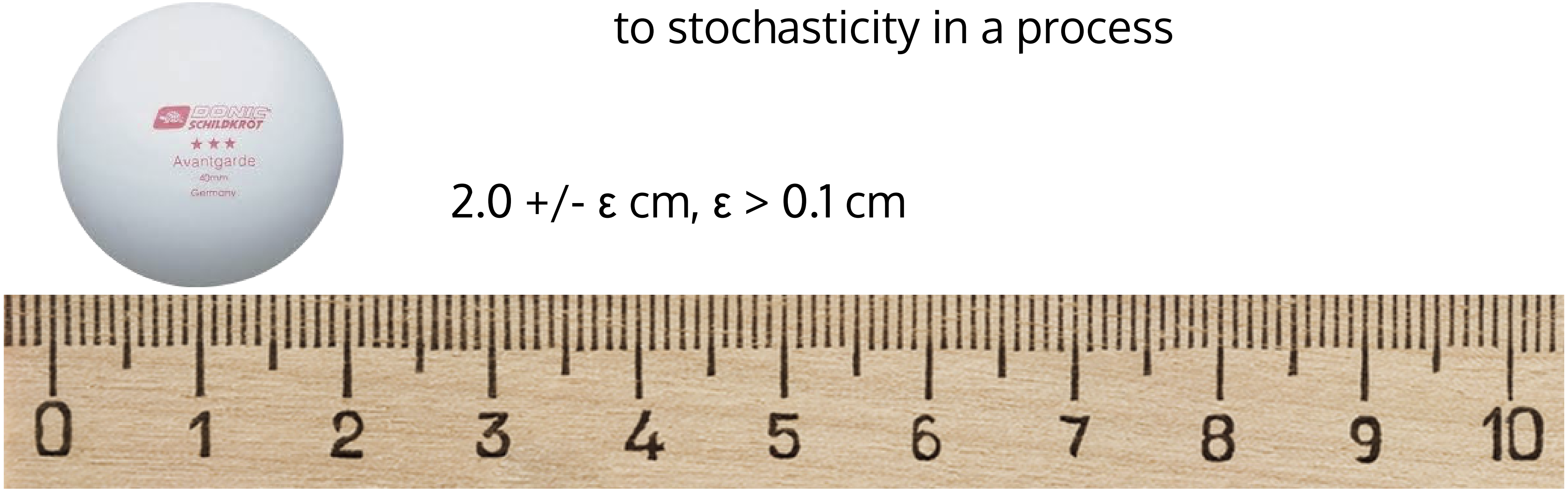
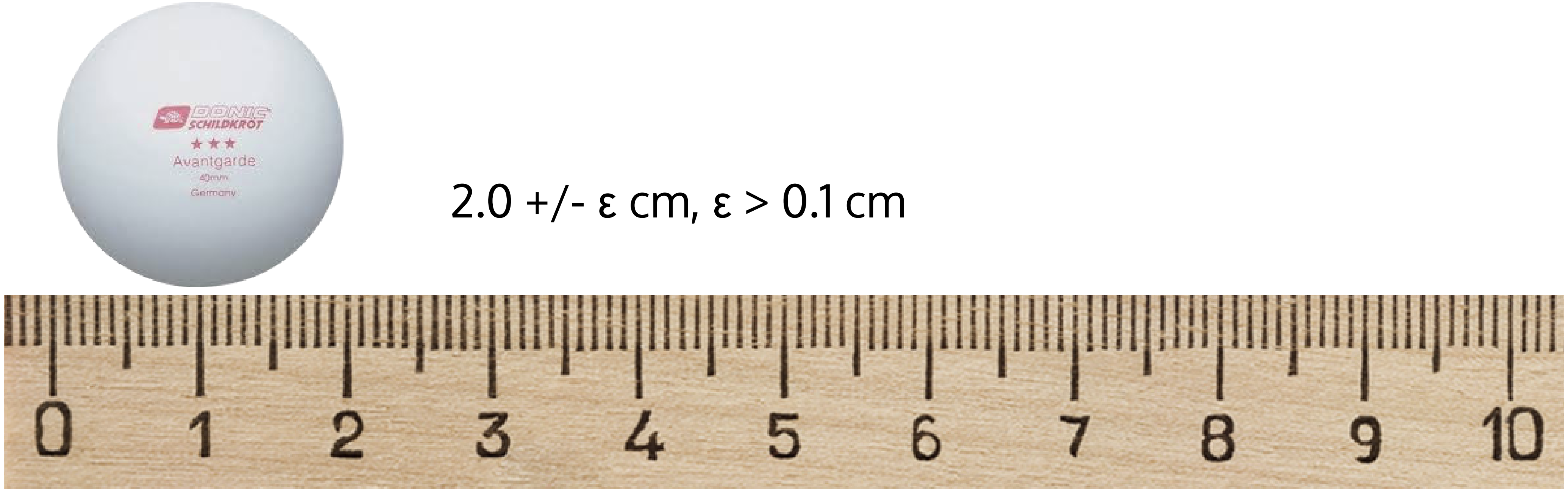
$2.0 \pm \varepsilon$ cm, $\varepsilon > 0.1$ cm



# ***stochastic*** or random errors

every measurement will be a bit different

2.0 +/- ε cm, ε > 0.1 cm



# *stochastic* or random errors

**Deterministic systems** have no randomness in their evolution. *Chaos* is deterministic....

**Stochastic processes** can be *completely random*: the probability of any event is disjoint from that of the previous one

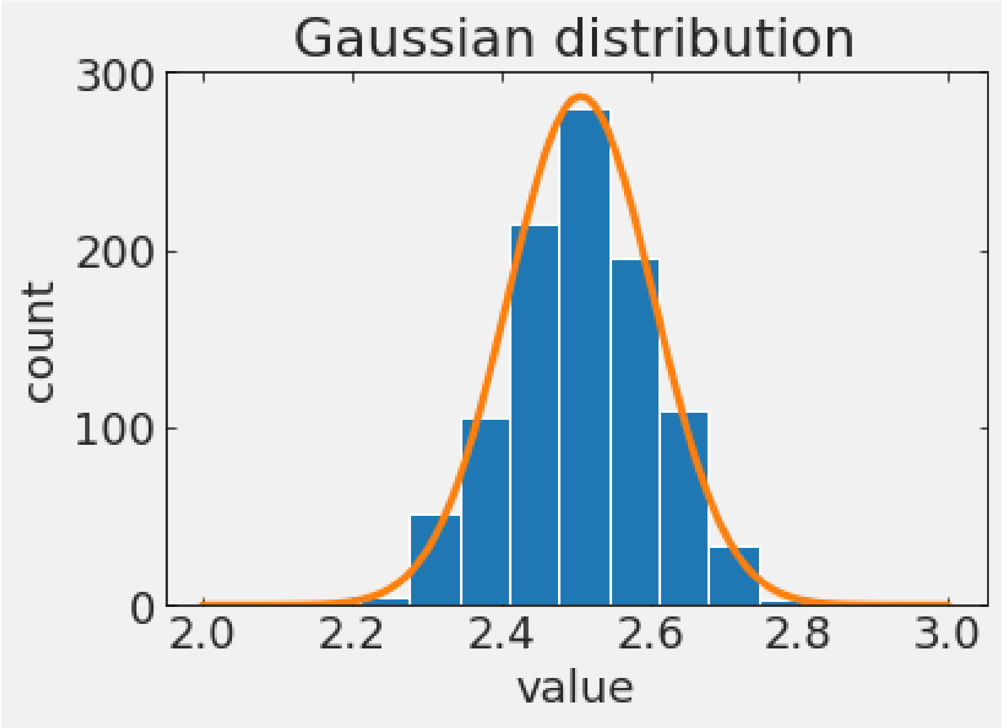
# *stochastic* or random errors

every measurement will be a bit different

```
2.4, 2.6, 2.5, 2.3, 2.4,
2.7, 2.3, 2.5, 2.6, 2.4
```

2.0 +/- ε cm, ε > 0.1 cm

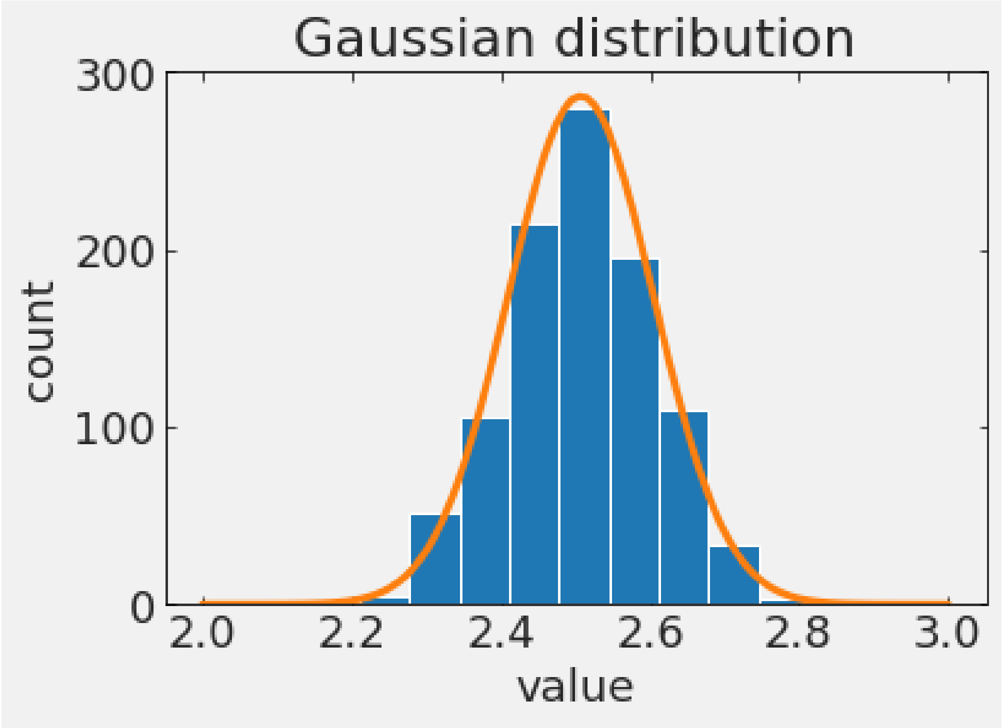
# *stochastic* or random errors




2.0 +/- ε cm, ε > 0.1 cm

# *stochastic* or random errors

$$p(y_i) = \frac{1}{\sigma_i\sqrt{2\pi}} \exp - \frac{(y_i - (mx_i + b))^2}{2\sigma_i^2}$$



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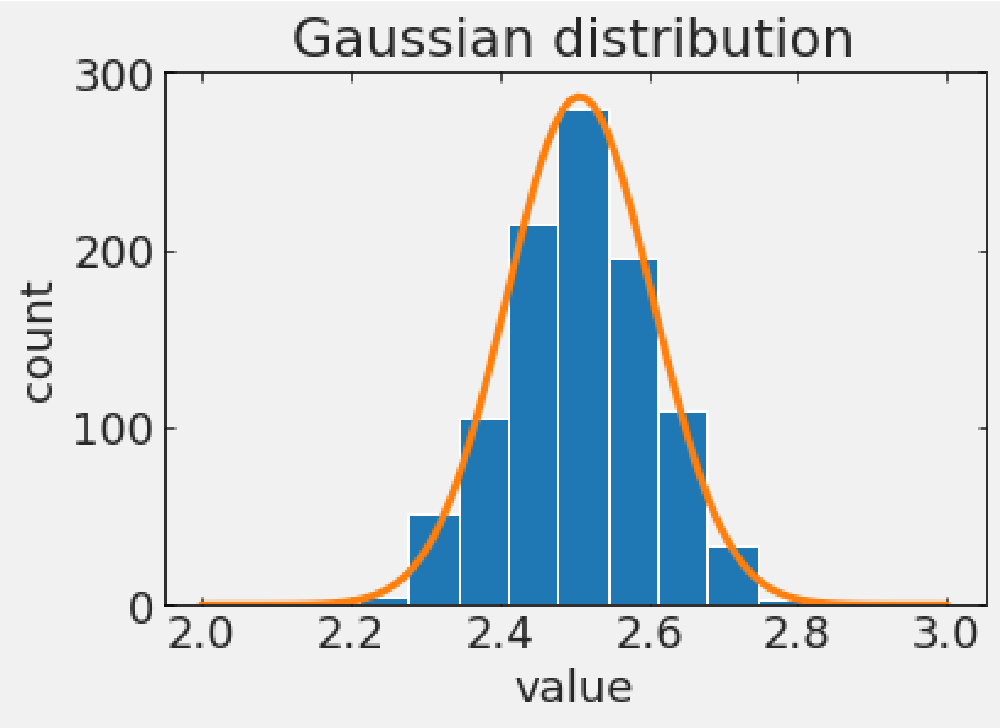
- symmetric



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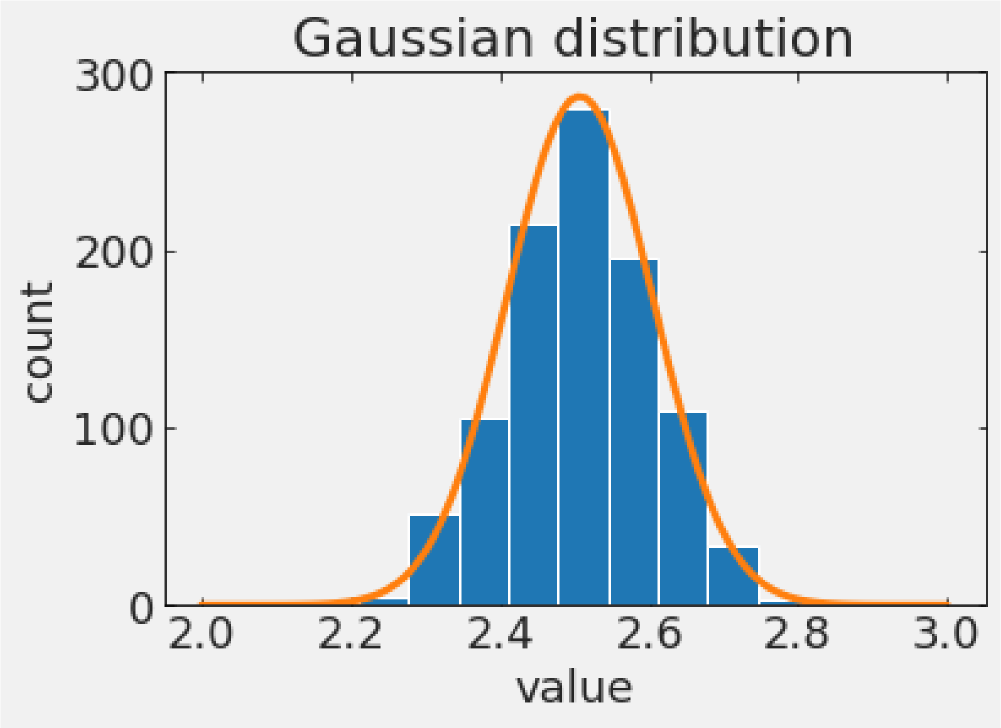
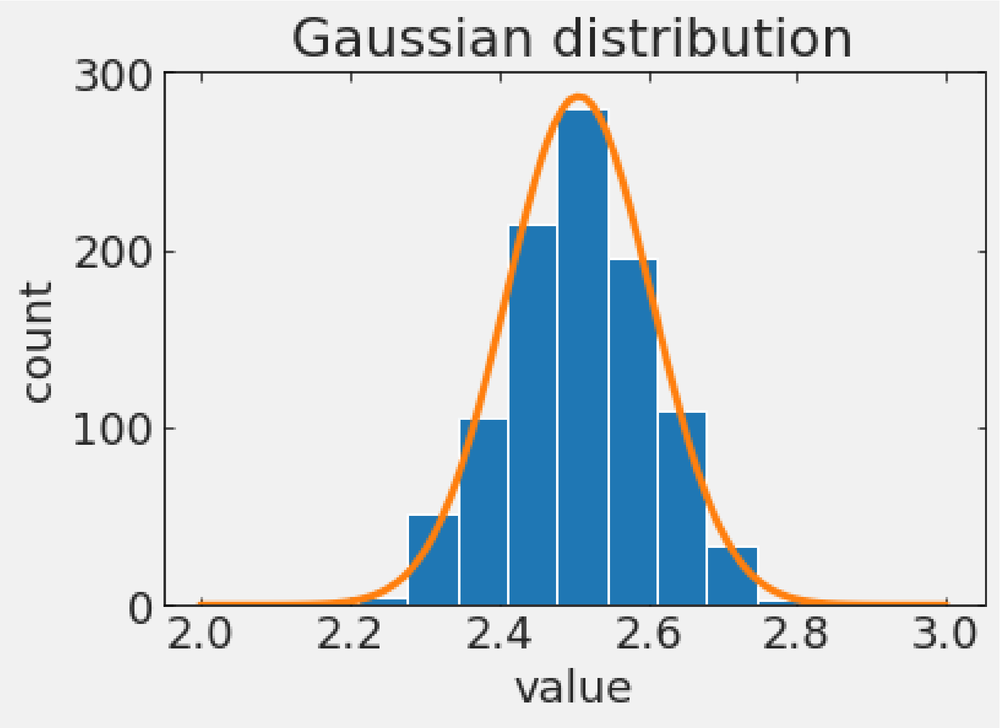



- symmetric
- max at $y_i = (mx_i + b)$

# *stochastic* or random errors

$$p(y_i) = \frac{1}{\sigma_i\sqrt{2\pi}} \exp -\frac{(y_i - (mx_i + b))^2}{2\sigma_i^2}$$

- symmetric
- max at $y_i = (mx_i + b)$
- bell shaped



# stochastic or random errors

of particular interest are ***Poisson processes***

*A discrete distribution that expresses the probability of a number of events*

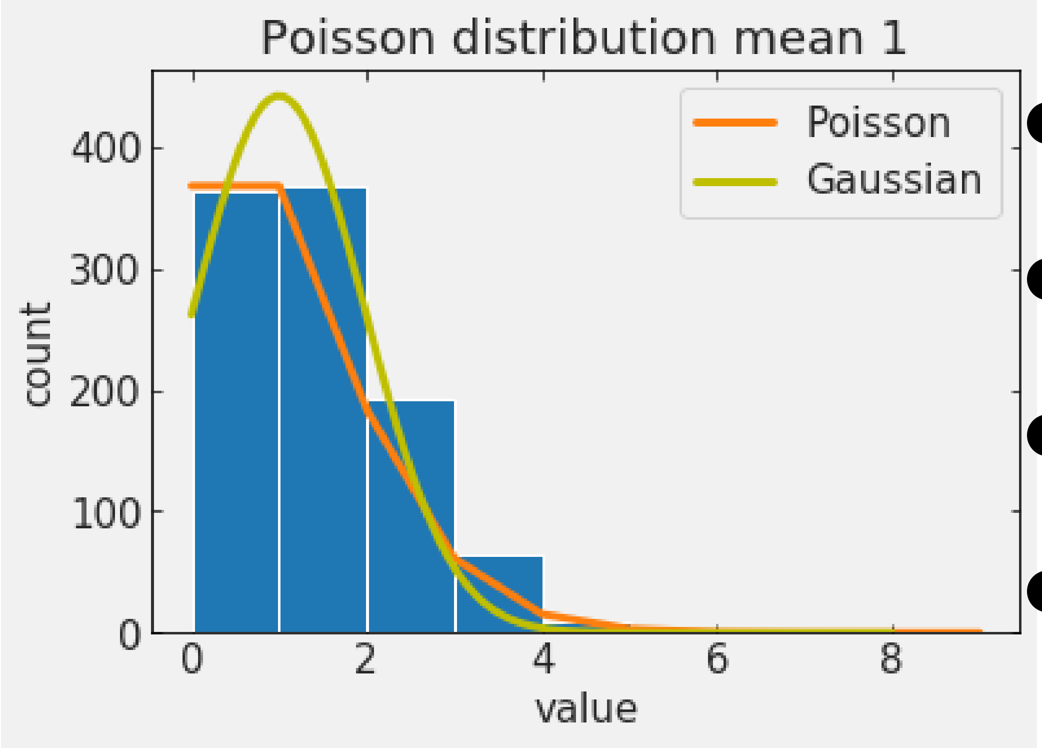
*occurring in a fixed period of time if these events occur with a known average rate*

*and independently of the time since the last event.*

# stochastic or random errors

of particular interest are ***Poisson processes***

$$P\left(x\right) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$




- asymmetric
- integer support
- support >0
- mean and stdev are related: $\mu : \lambda$ $\sigma : \sqrt{\lambda}$

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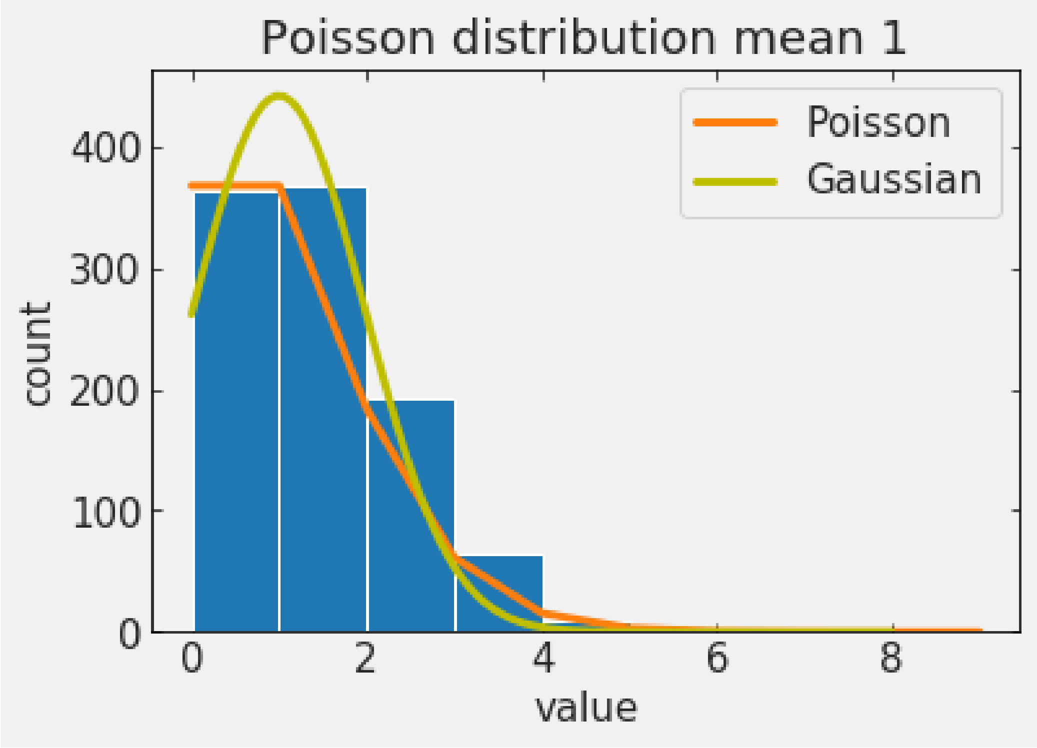


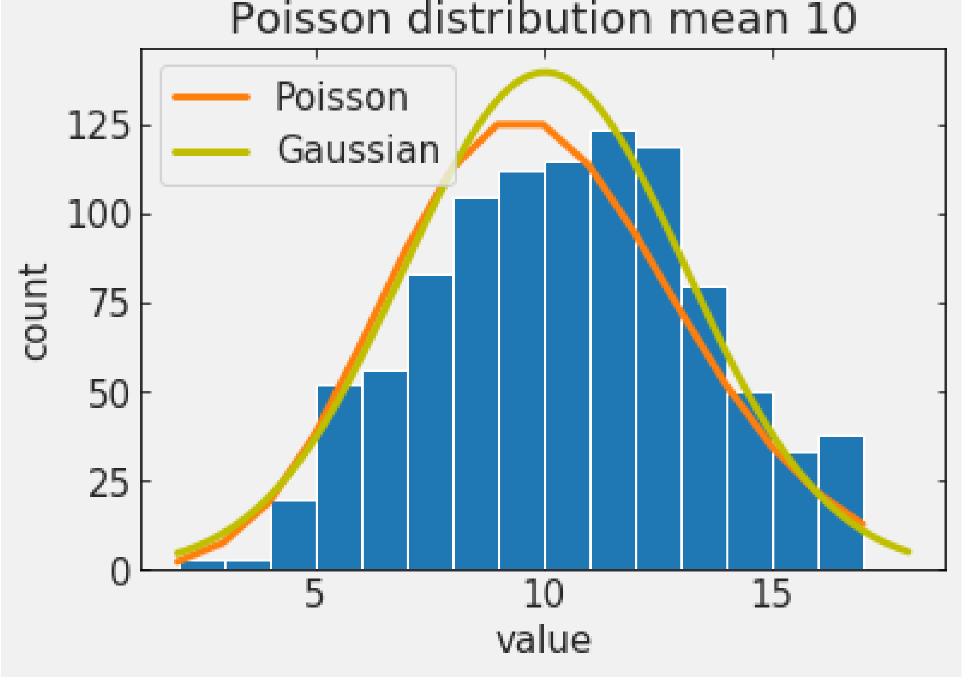

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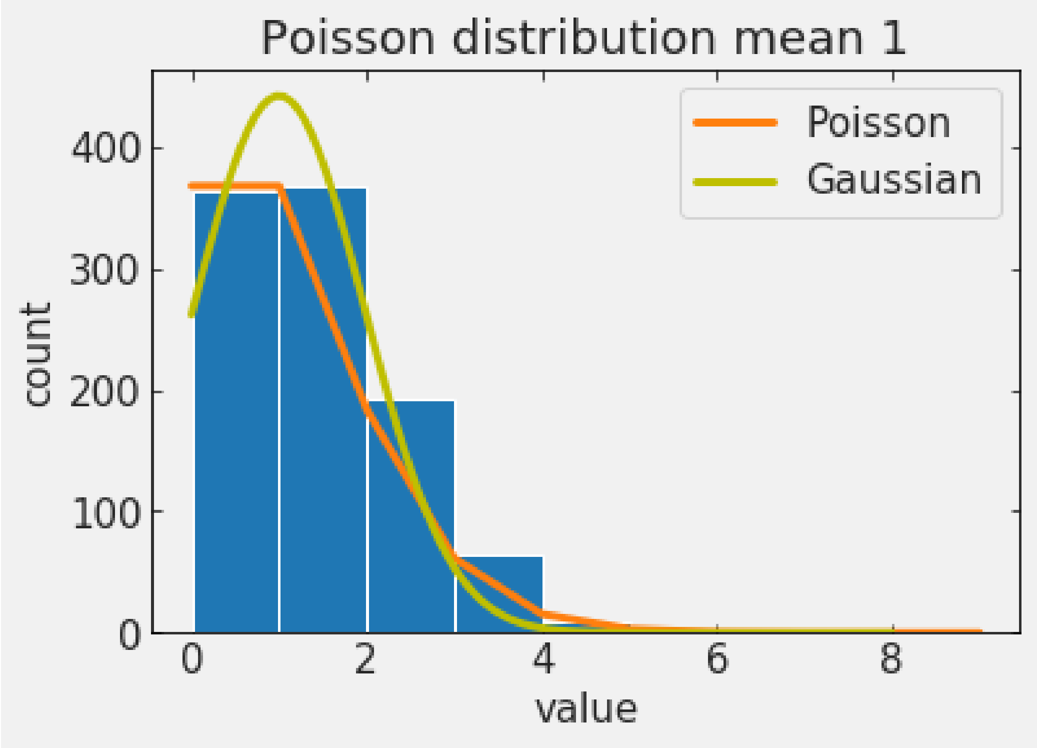
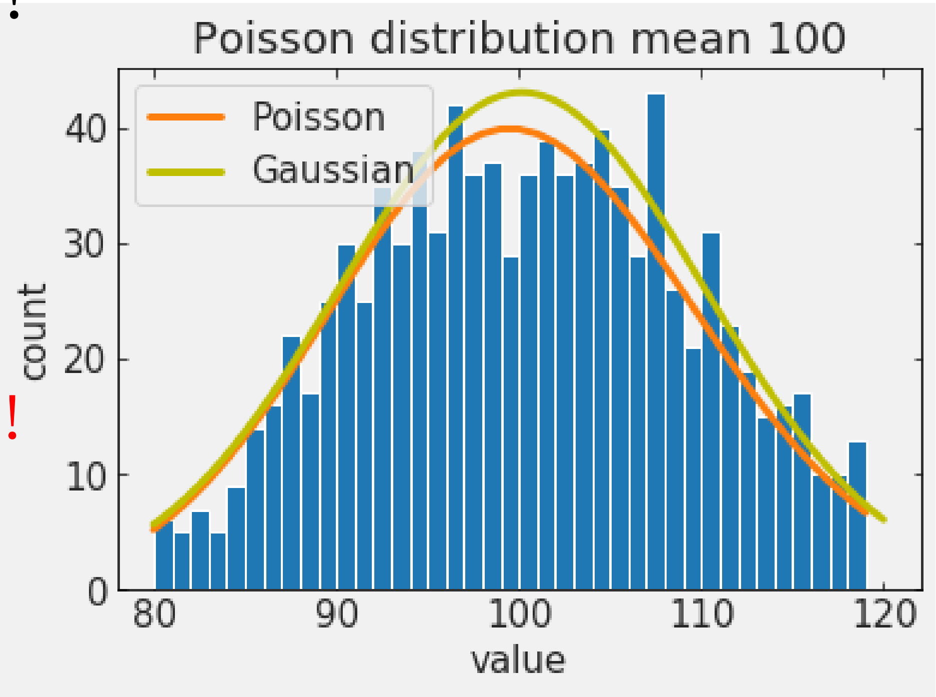



as the mean increases it starts looking like a Gaussian!



# 2 systematic uncertainties

# systematic errors

reproducible inaccuracy introduced by faulty equipment, calibration, or technique.

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~~2.5~~ 2.7 => 2.5 + 0.2 +/- 0.1

# systematic errors

REMOTE SENSING:

background, scanning efficiency, energy resolution, variation of counter efficiency with beam position, and energy, dead time, ...

reproducible inaccuracy introduced by faulty equipment, calibration, or technique.

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# systematic errors

reproducible inaccuracy introduced by faulty equipment, calibration, or technique.

- Measurements are taken at 22 C with a steel rule calibrated at 15 C. This is a systematic *bias* and not a systematic *uncertainty*

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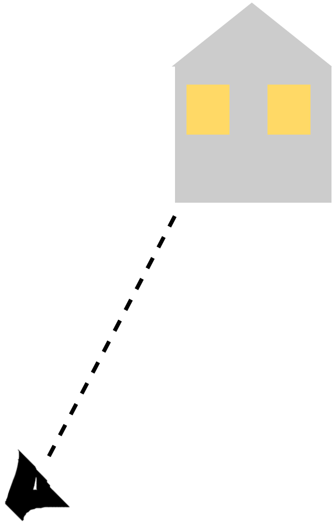

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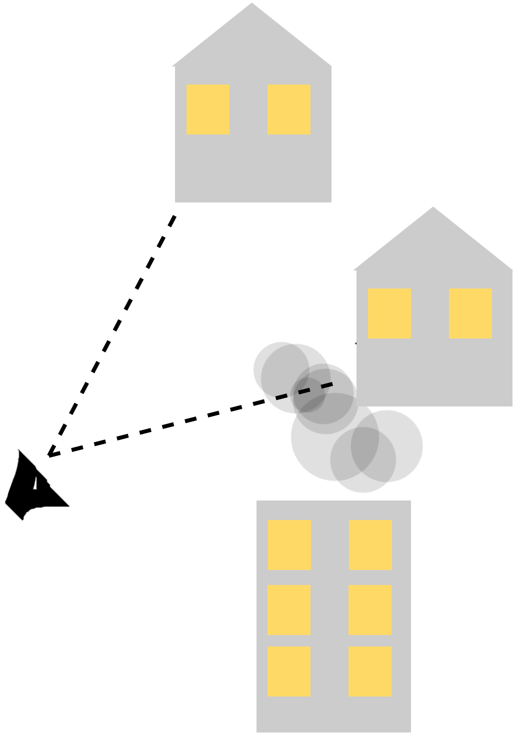

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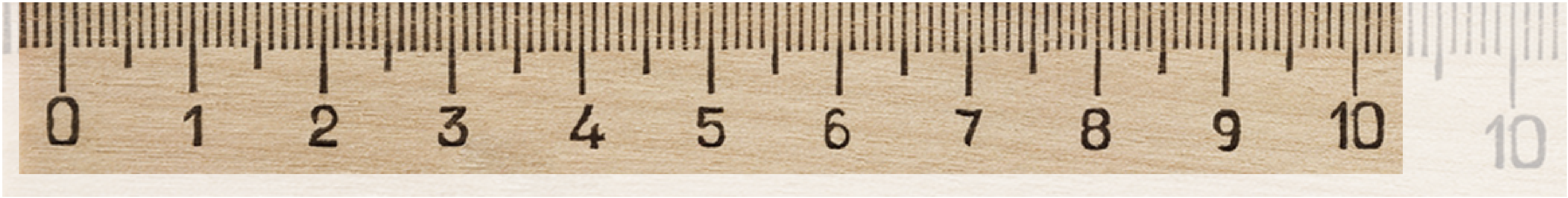
reproducible inaccuracy introduced by faulty equipment, calibration, or technique.

- Measurements are taken at 22 C with a steel rule calibrated at 15 C. This is a systematic *bias* and not a systematic *uncertainty*
- *Brightness* is measured but smoke between the sensor and the target can reduce its value artificially.

  systematic *uncertainty*

~~2.5~~ 2.7 => 2.5 + ?? +/- 0.1



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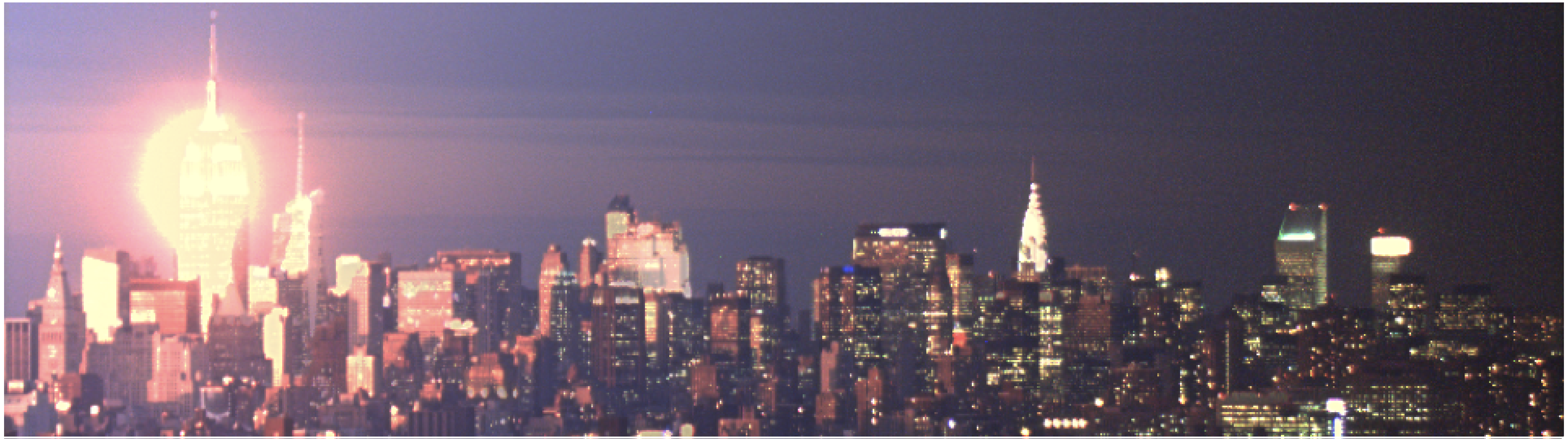

https://cuspuo.github.io/docs/dobler_urban_observatory.pdf

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# Bias in measurements: know your data

Some common biases:

**Undercoverage bias**

Self selection bias

Social desirability bias

Publication Bias

Data Dredging

# Bias in measurements: know your data

## Undercoverage bias

the surveyed segment of the population is lower in a sample than it is in the population. This can happen because the frame used to obtain the sample is incomplete or not representative of the population.

# Bias in measurements: know your data

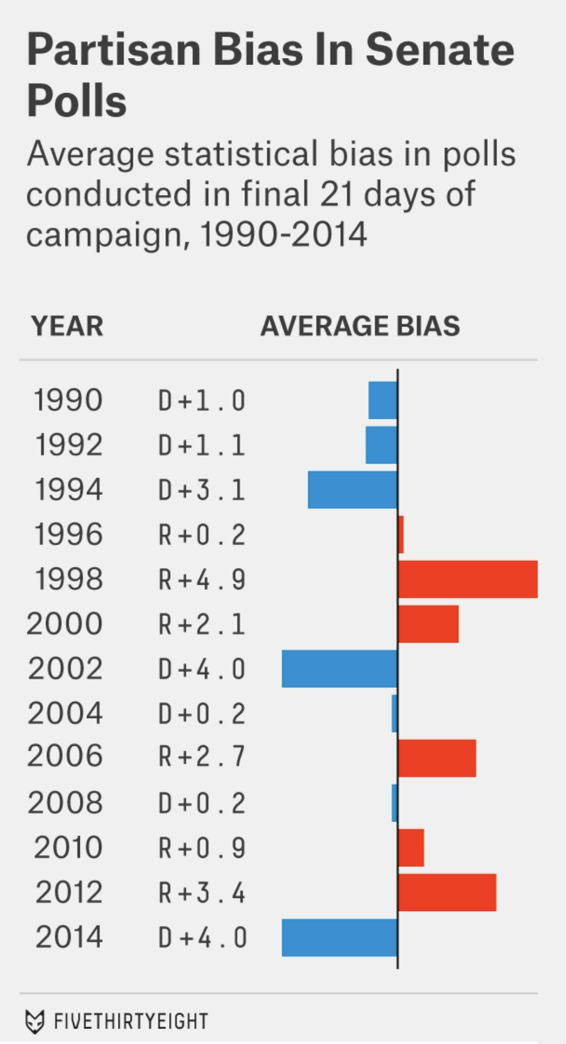
## Undercoverage bias

By Nate Silver

Filed under 2014 Midterms


Virginia Sen. Mark R. Warner gestures to reporters after entering his voting form, on Tuesday in Alexandria. BILL O'LEARY / GETTY IMAGES

For much of this election cycle, Democrats complained the polls were biased against them. They said the polls were failing to represent enough minority voters and applying overly restrictive likely-voter screens. They claimed early-voting data was proving the polls wrong. They cited the fact that polls were biased against Democrats in 2012.



# Bias in measurements: know your data

## Undercoverage bias

| POLLSTER | METHOD | LIVE CALLER WITH CELLPHONES | NCPP/ AAPOR/ ROPER | POLLS ANALYZED | SIMPLE AVERAGE ERROR | RACES CALLED CORRECTLY | ADVANCED +/- | PREDICTIVE +/- | 538 GRADE | BANNED BY 538 | MEAN-REVERTED BIAS |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **SurveyUSA** | IVR/ online/ live | ◐ | ● | 787 | 4.7 | 89% | -1.1 | **-0.8** | A | | D+0.1 |
| **Rasmussen Reports/ Pulse Opinion Research** | IVR/ online | | | 722 | 5.3 | 78% | +0.2 | **+0.8** | C+ | | R+1.5 |
| **Zogby Interactive/JZ Analytics** | Online | | | 473 | 5.4 | 77% | +0.4 | **+0.9** | C+ | | R+0.6 |
| **Mason-Dixon Polling & Strategy** | Live | ● | | 433 | 5.1 | 87% | -0.6 | **-0.3** | B+ | | R+0.6 |
| **Public Policy Polling** | IVR/ text | | | 423 | 5.0 | 80% | -0.4 | **+0.1** | B | | D+0.3 |
| **YouGov** | Online | | | 416 | 4.9 | 88% | -0.2 | **+0.3** | B | | D+0.4 |
| **Research 2000** | Live* | | | 280 | 5.5 | 88% | -0.1 | **+0.3** | F | ✕ | D+1.3 |

| POLLSTER | METHODOLOGY | NO. OF POLLS | AVERAGE ERROR |
|---|---|---|---|
| Monmouth University | Live | 7 | 7.5 |
| YouGov | Online | 9 | 7.6 |
| Suffolk University | Live | 6 | 8.0 |
| Emerson College | IVR/Online/Text | 13 | 8.5 |
| AtlasIntel | Online | 8 | 8.8 |
| Data for Progress | Online/Text | 29 | 9.3 |
| Point Blank Political | Online/Text | 6 | 10.7 |
| Swayable | Online | 22 | 12.0 |
| Marist College | Live | 6 | 13.3 |
| University of Massachusetts Lowell | Online | 6 | 14.0 |
| Change Research | Online | 7 | 16.1 |

# Bias in measurements: know your data

Some common biases:

Undercoverage bias

**Self-selection bias**

Social desirability bias

Publication Bias

Data Dredging

# Bias in measurements: know your data

## Self-Selection bias

people willing to participate in a survey have a specific "interest" in it

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*people willing to answer a survey about climate are more likely concerned citizens that care about the climate*

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*higher test scores observed among students who participate in a test preparation courses, but due to self-selection, people who choose to take the course may be more motivated, have more support...*

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## Social desirability bias

tendency of survey respondents to answer in a manner that may be viewed favorably: over-reporting "good behavior”, under-reporting undesirable behavior (e.g. drug+alcohol use).

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## Social desirability bias

tendency of survey respondents to answer in a manner that may be viewed favorably: over-reporting "good behavior”, under-reporting undesirable behavior (e.g. drug+alcohol use).

it is known that the shy Trump supporter effect played a role in the 2016 poll results

**Did ‘Shy Trump Voters’ throw off the polls? Maybe not.**

# Bias in measurements: know your data

## Social desirability

Organizational Research Methods
00(0) 1-34

DOI: 10.1177/1094428113493120
orm.sagepub.com
SAGE

**Random and Systematic Error Effects of Insomnia on Survey Behavior**

**Larissa K. Barber[1], Christopher M. Barnes[2], and Kevin D. Carlson[2]**

**Abstract**

Insomnia is a prevalent experience among employees and survey respondents. Drawing from research on sleep and self-regulation, we examine both random (survey errors) and systematic (social desirability) effects of research participant insomnia on survey responses. With respect to random effects, we find that insomnia leads to increased survey errors, and that this effect is mediated by a lack of self-control and a lack of effort. However, insomnia also has a positive systematic effect, leading to lower levels of social desirability. This effect is also mediated by self-control depletion and a lack of

http://www.researchgate.net/publication/244478619_Random_and_Systematic_Error_Effects_of_Insomnia_on_Survey_Behavior

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**Publication Bias**

Data Dredging

# Bias in measurements: know your data

## Publication Bias

significant (i.e. rejection of NH) results are more likely to be published

nature International weekly journal of science

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News & Comment > News > 2015 > August > Article

NATURE | NEWS

### Social sciences suffer from severe publication bias

**Survey finds that 'null results' rarely see the light of the day.**

Mark Peplow

28 August 2014

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null results (i.e. cannot reject NH) are less likely to be published

*NATURE* | NEWS

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28 August 2014

28 August 2014

Mark Peplow

Survey finds that 'null results' rarely see the light of the day.

His team investigated the fate of 221 sociological studies conducted between 2002 and 2012, which were recorded by Time-sharing Experiments for the Social Sciences (TESS), a US project that helps social scientists to carry out large-scale surveys of people's views.

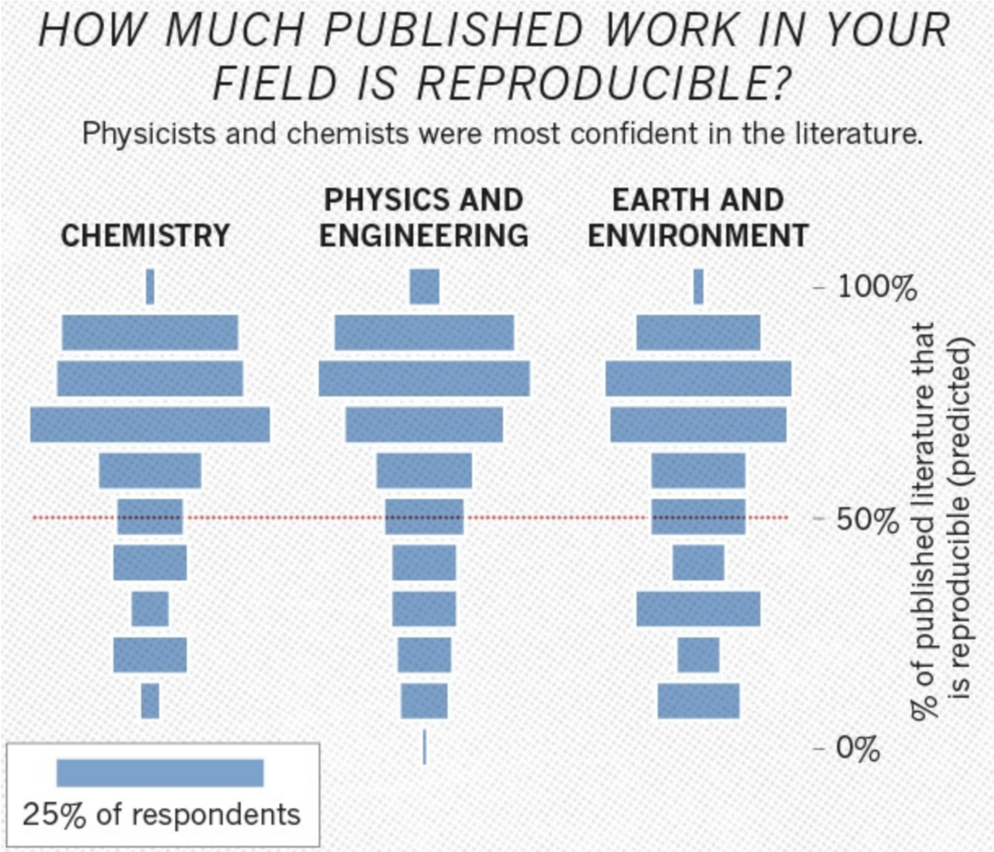
Only 48% of the completed studies had been published. So the team contacted the remaining authors to find out whether they had written up their results, or submitted them to a journal or conference. They also asked whether the results supported the researchers' original hypothesis.

Of all the null studies, just 20% had appeared in a journal, and 65% had not even been written up. By contrast, roughly 60% of studies with strong results had been published. Many of the researchers contacted by Malhotra's team said that they had not written up their null results because they thought that journals would not publish them, or that the findings were neither interesting nor important enough to warrant any further effort.

# Bias in measurements: know your data

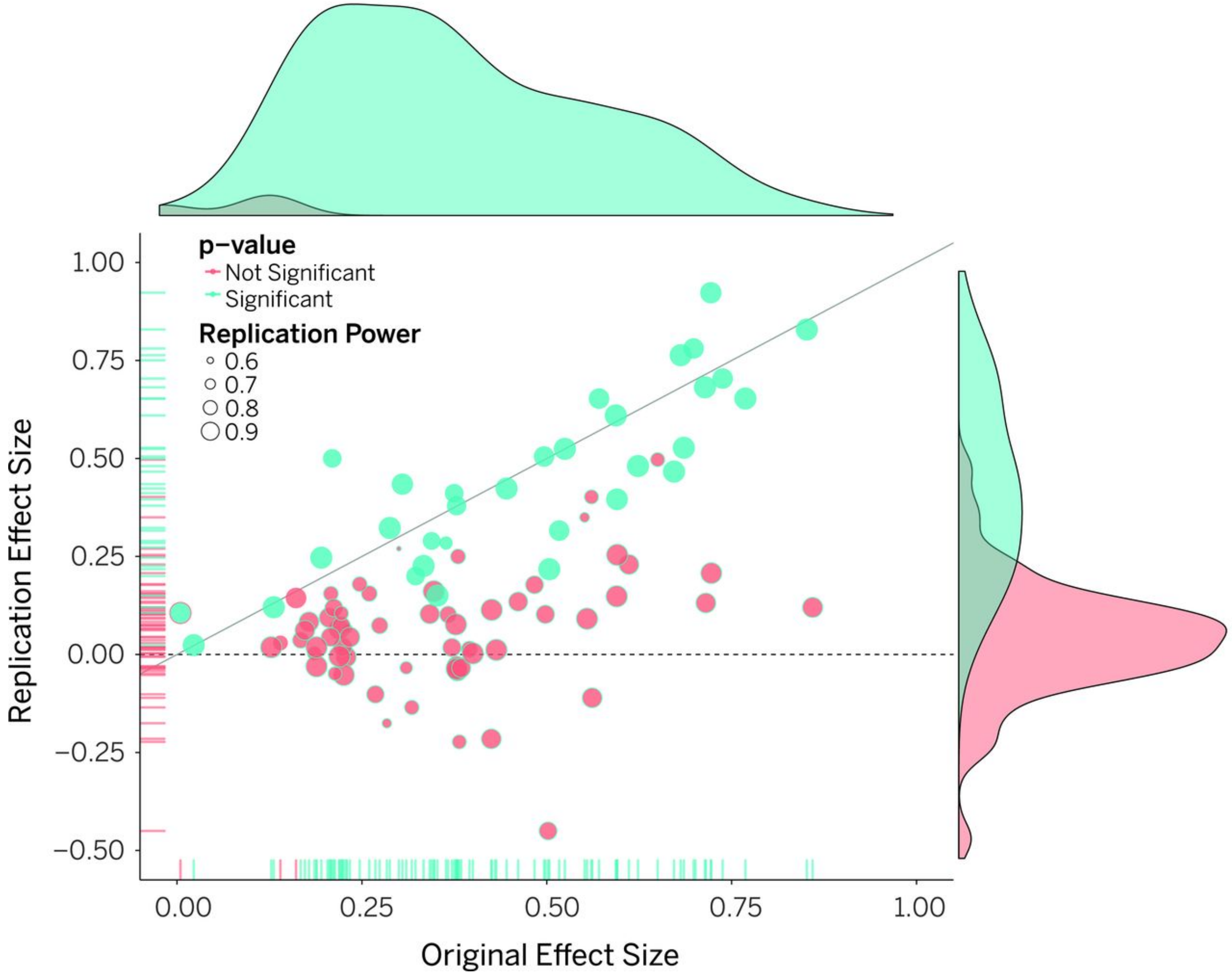
## Publication Bias

*reproducibility*




https://www.nature.com/news/1-500-scientists-lift-the-lid-on-reproducibility-1.19970

*effect size: difference between two groups*




http://science.sciencemag.org/content/349/251/aac4716

# Bias in measurements: know your data

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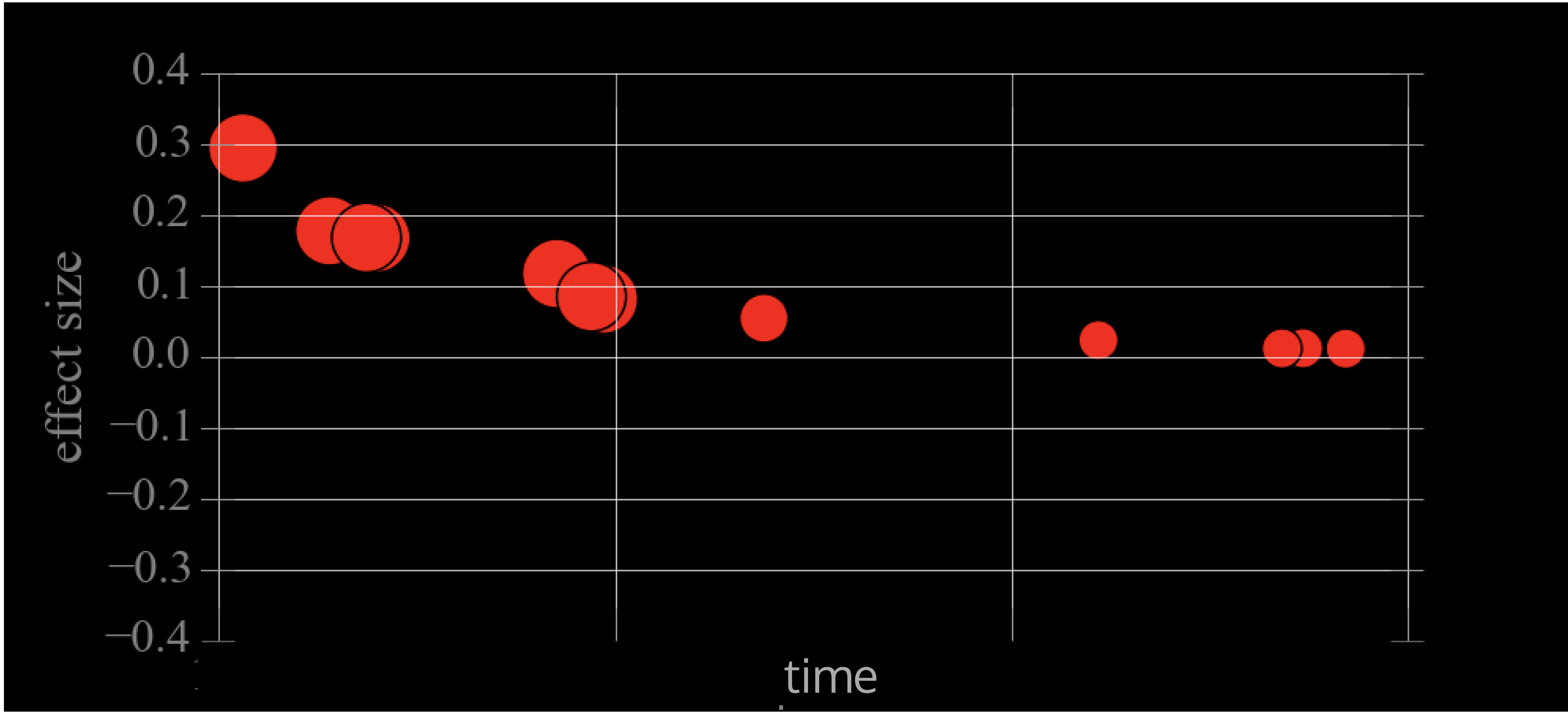

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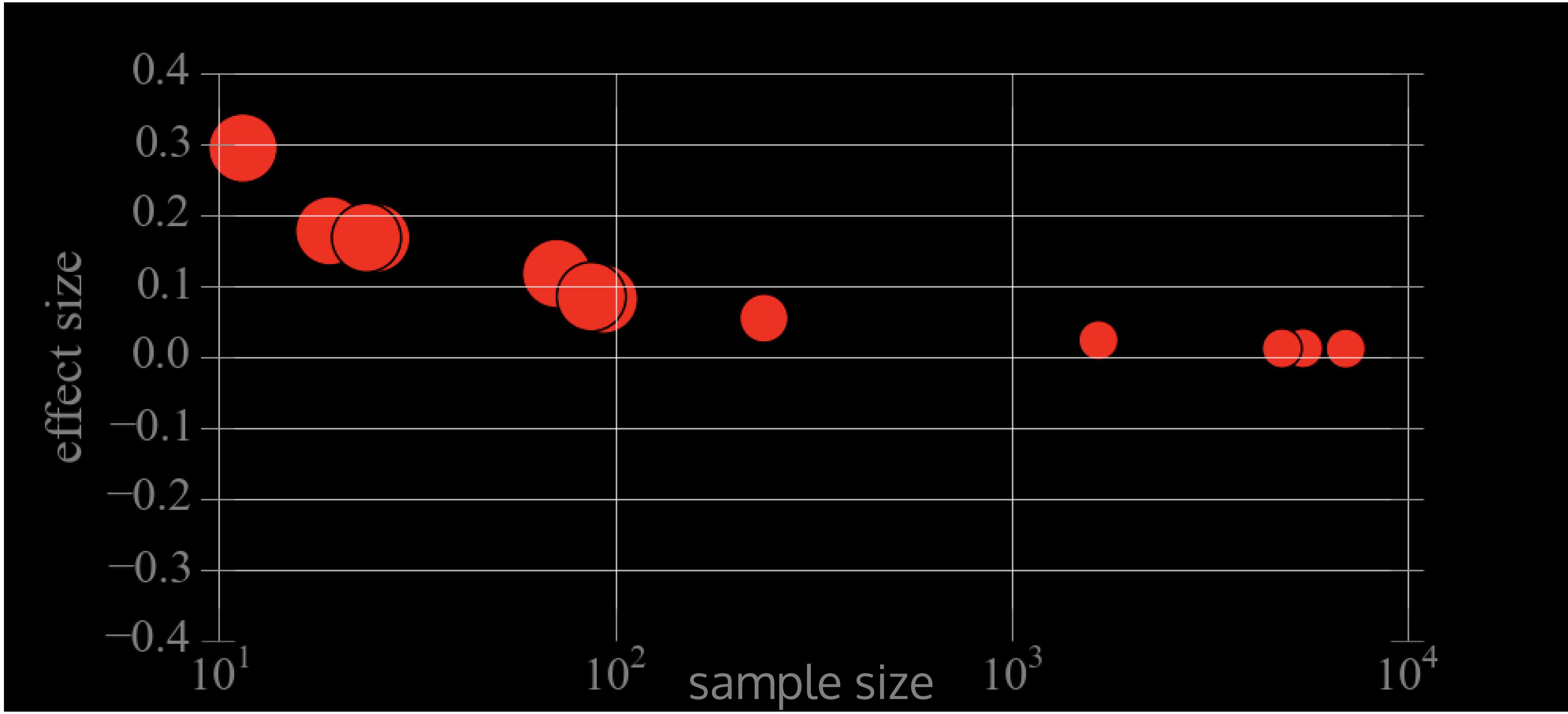
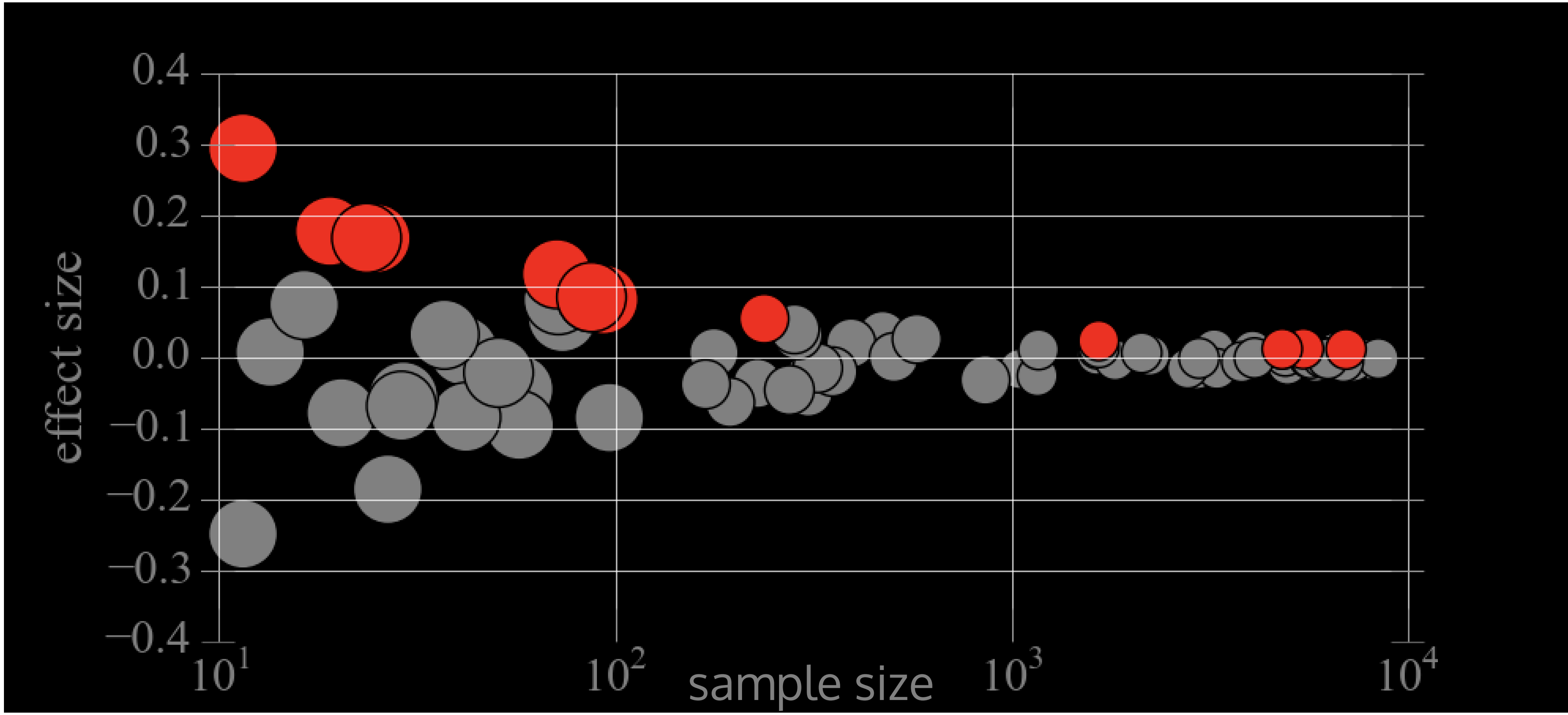



sample size can grw if funding grows!

# Bias in measurements: know your data

## Publication Bias



# Bias in measurements: know your data

## Publication Bias

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### Publication Bias in Measuring Climate Sensitivity

*Dominika Rečková* and *Zuzana Iršová* (zuzana.irsova@ies-prague.org)

No 2015/14, Working Papers IES from Charles University Prague, Faculty of Social Sciences, Institute of Economic Studies

**Abstract:** We present a meta-regression analysis of the relation between the concentration of carbon dioxide in the atmosphere and changes in global temperature. The relation is captured by "climate sensitivity", which measures the response to a doubling of carbon dioxide concentrations compared to pre-industrial levels. Estimates of climate sensitivity play a crucial role in evaluating the impacts of climate change and constitute one of the most important inputs into the computation of the social cost of carbon, which reflects the socially optimal value of a carbon tax. Climate sensitivity has been estimated by many researchers, but their results vary significantly. We collect 48 estimates from 16 studies and analyze the literature quantitatively. We find evidence for publication selection bias: researchers tend to report preferentially large estimates of climate sensitivity. Corrected for publication bias, the bulk of the literature is consistent with climate sensitivity lying between 1.4 and 2.3C.

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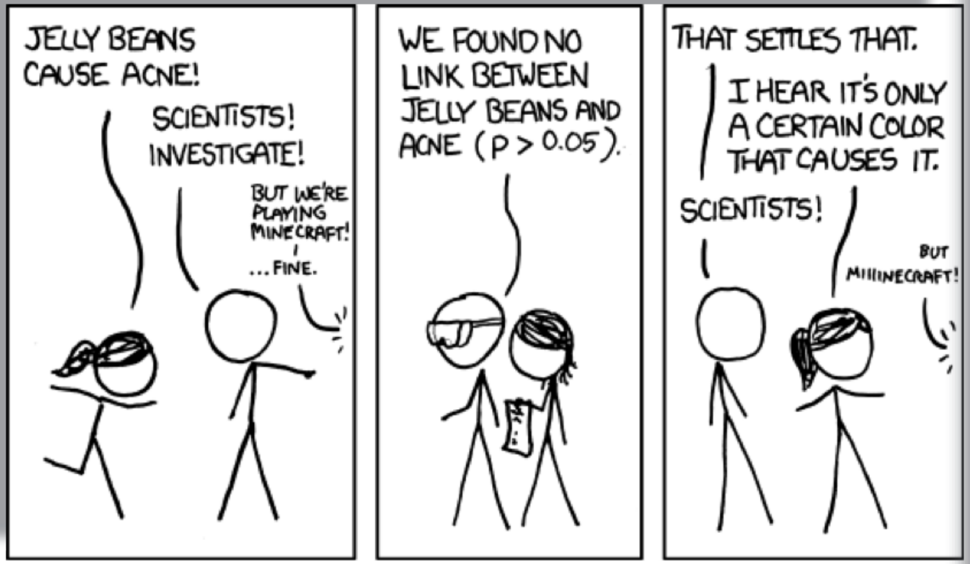
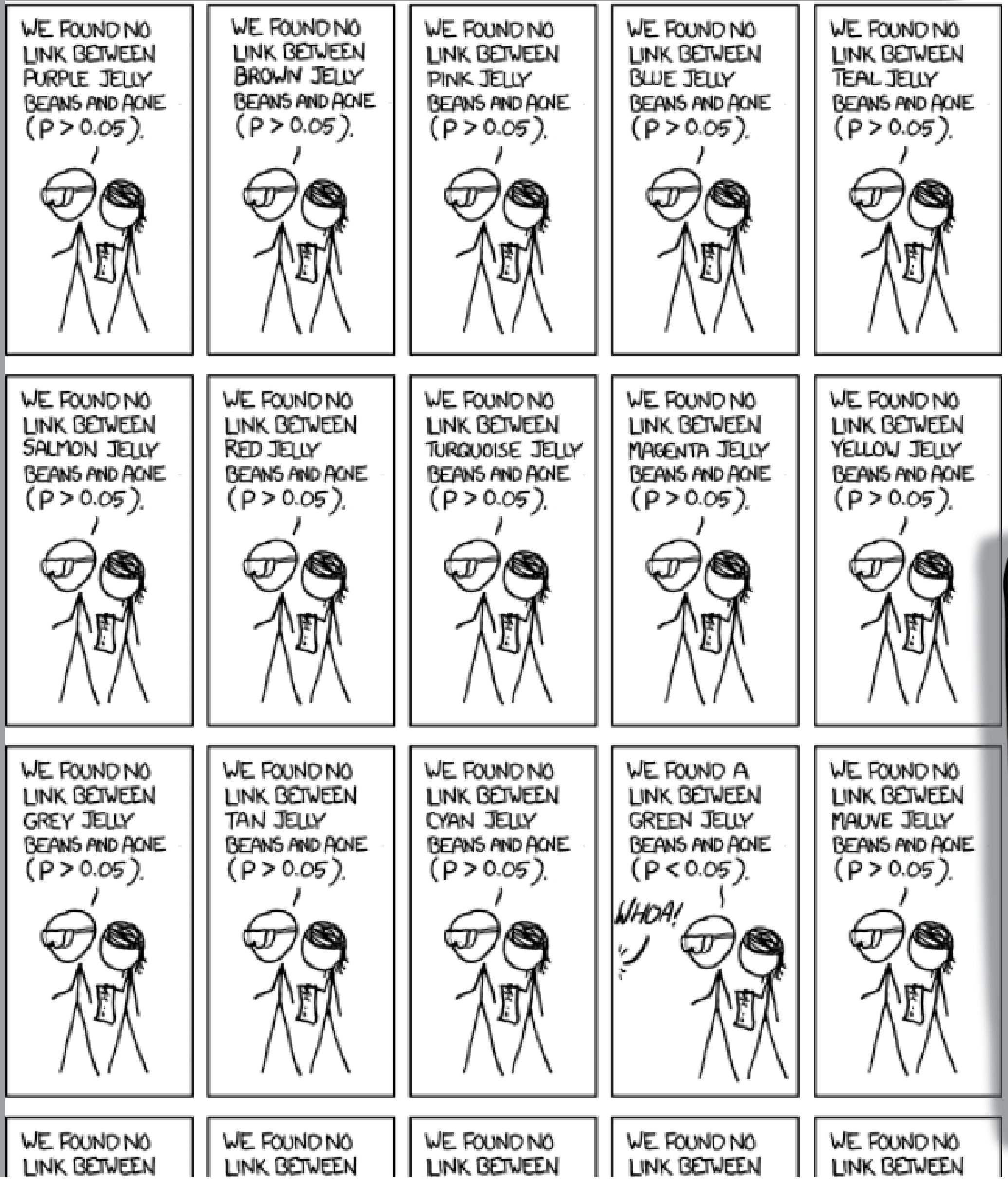
**Data Dredging**

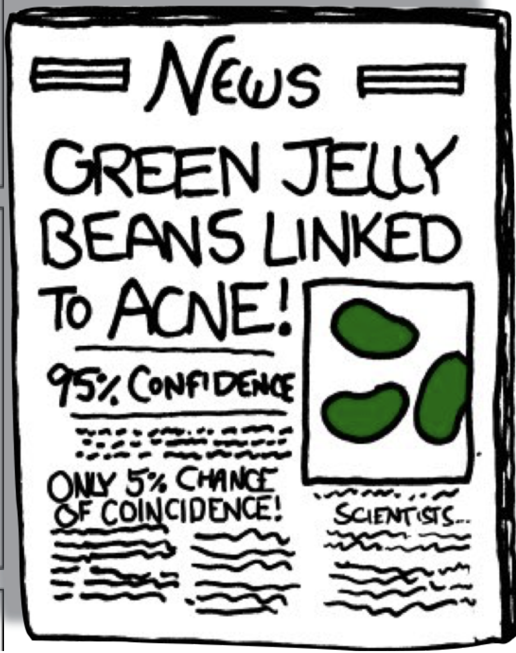
# Bias in measurements: know your data

## Data Dredging




https://xkcd.com/882/



# Bias in measurements: know your data

## Data Dredging

each test has a

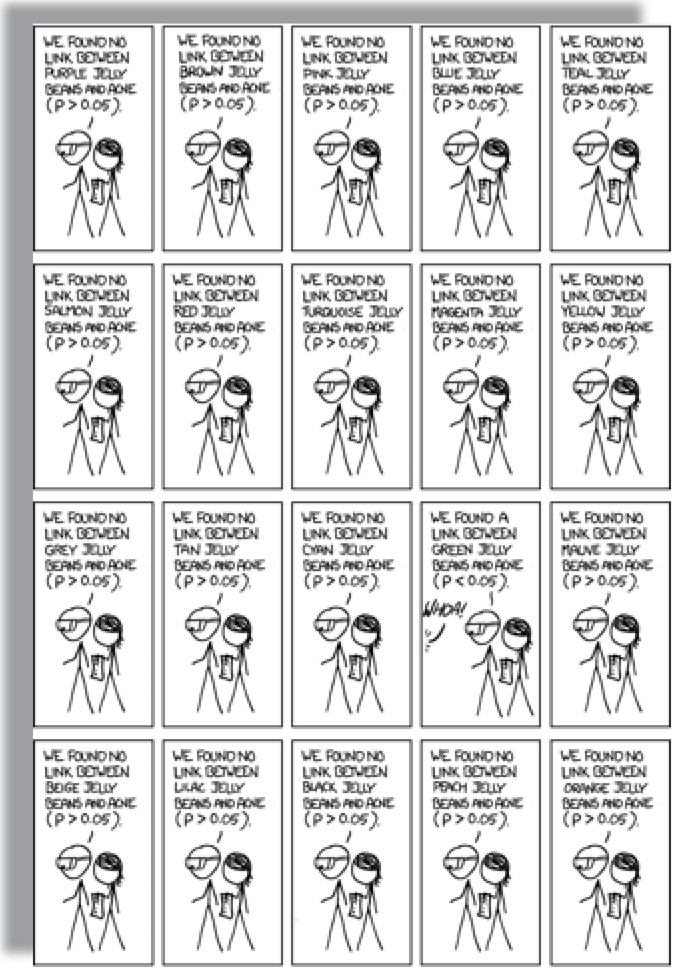
probability $p$<=0.05

of Type I error

significance 95%

5 tests out of 100 will succeed just out of chance

... if 20 tests are preformed...



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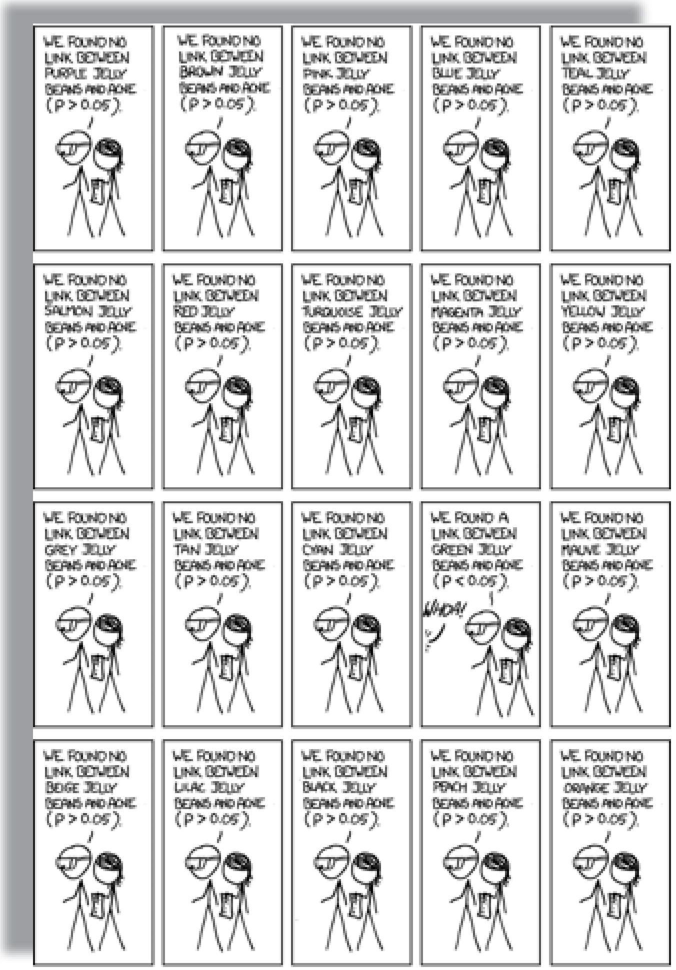



new significance threshold (assuming independent tests)

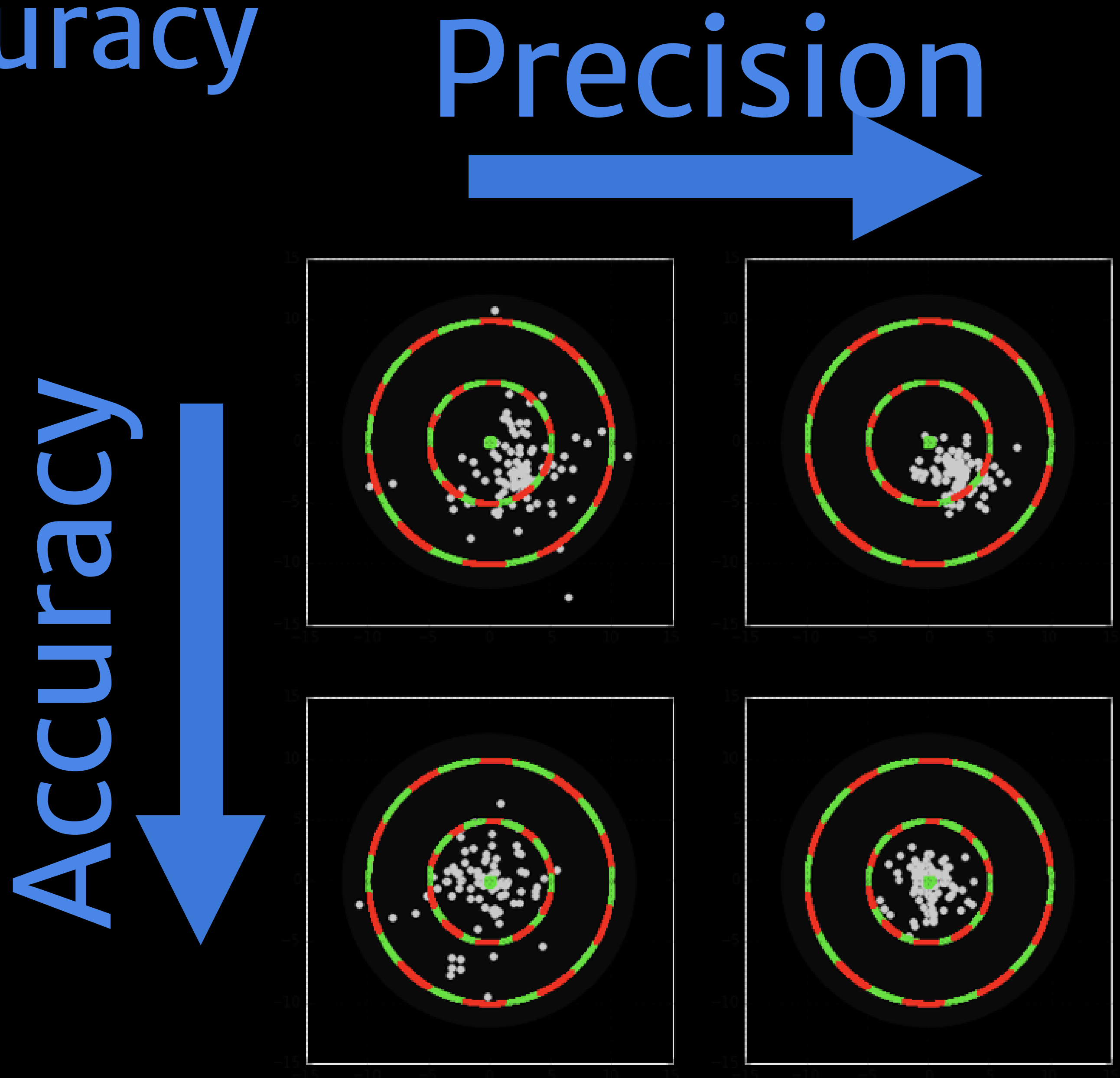
$$p_{v_{tot}} = \prod_i p_i$$

$$p_{v_{tot}} = p_i^{20}$$

# Stochastic vs Systematics

| **Systematic** | **Statistical** |
| --- | --- |
| Biases the measurement *in one direction* | No preferred direction |
| Affects the sample *regardless of the size* | Shrinks with the sample size (typically as $\sqrt{N}$ ) |
| Any distribution (usually we use Gaussian though) | Typically Gaussian or Poisson |

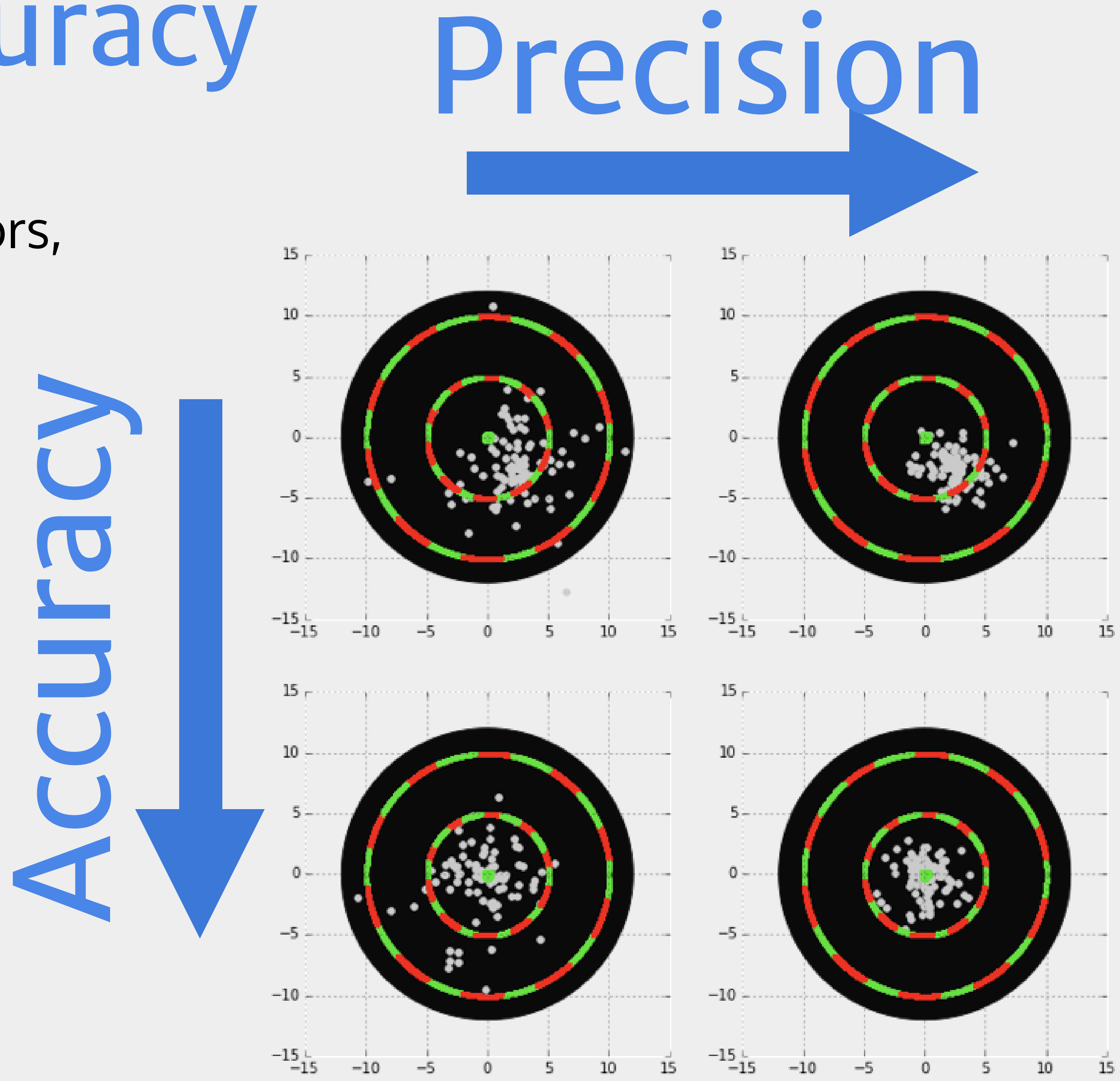
# Precision vs Accuracy



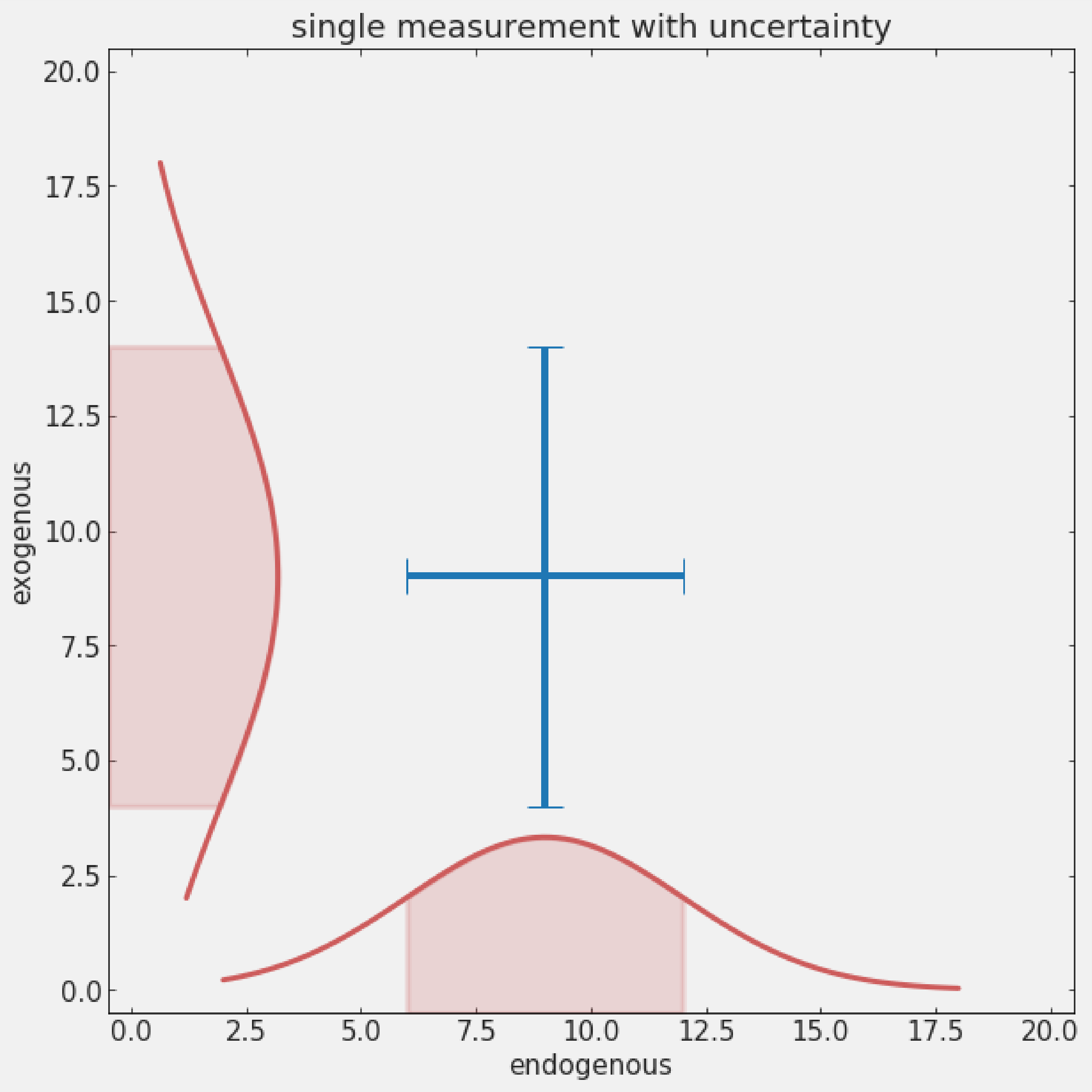
# Precision vs Accuracy

which relates to stochastic errors,

which to stsystematic?



# the meaning of "uncertainty"



# 3 combining and reporting uncertainties

# combining uncertainties

If ***x,…,w*** are measured with ***independent*** and ***random*** uncertainties

***Δx,…,Δw*** the uncertainty in a linear combination of ***x,…,w*** is the quadratic sum:

# combining uncertainties

If ***x*,…,*w*** are measured with ***independent*** and ***random*** uncertainties ***Δx*,…,*Δw*** the uncertainty in a linear combination of ***x*,…,*w*** is the quadratic sum:

| | | |
|---|---|---|
| **Addition/Subtraction** | $z = x \pm y$ | $\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ |
| **Multiplication** | $z = xy$ | $\Delta z = \lvert xy \rvert \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$ |
| **Division** | $z = \frac{x}{y}$ | $\Delta z = \left\lvert \frac{x}{y} \right\rvert \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$ |
| **Power** | $z = x^n$ | $\Delta z = \lvert n \rvert x^{n-1} \Delta x$ |
| **Multiplication by a Constant** | $z = cx$ | $\Delta z = \lvert c \rvert \Delta x$ |
| **Function** | $z = f(x,y)$ | $\Delta z = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 (\Delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\Delta y)^2}$ |

# reporting

## uncertainties in a result

It is essential that the systematic error be reported separately from the imprecision part of the reported valus

(Statistical Concepts and Procedures

by United States. National Bureau of Standards 1969)

Keep statistical, systematic errors separate. Report results as

something like:

x = [965 ± 30(stat) ± 12(sys)]

Add in quadrature (note that this assumes Gaussian distribution) to compare with known values $\sqrt{30^2 + 12^2} = 32$

x = [965 ± 32(total)] number of car accidents

# combining uncertainties

If ***x,y,..,w*** are measured with ***independent*** and ***random*** uncertainties

***Δx,Δy,…,Δw*** the uncertainty in a linear combination of ***x,y,…,w*** is the quadratic sum:

$$f(x, y, \ldots w):$$

$$\Delta_f = \sqrt{\left(\frac{\partial f}{\partial x}^2\right)\Delta_x^2 + \left(\frac{\partial f}{\partial y}^2\right)\Delta_y^2 + \ldots + \left(\frac{\partial f}{\partial w}^2\right)\Delta_w^2}$$

http://ipl.physics.harvard.edu/wp-uploads/2013/03/PS3_Error_Propagation_sp13.pdf

# combining uncertainties

derivation

$$f_k = \sum_{i=1}^{n} A_{ki} x_i \text{ or } \mathrm{f} = \mathrm{Ax}$$

$$\Sigma^x = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \cdots \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} & \cdots \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} \Sigma_{11}^x & \Sigma_{12}^x & \Sigma_{13}^x & \cdots \\ \Sigma_{12}^x & \Sigma_{22}^x & \Sigma_{23}^x & \cdots \\ \Sigma_{13}^x & \Sigma_{23}^x & \Sigma_{33}^x & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

# combining uncertainties

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$$\Sigma_{ij}^f = \sum_{k}^{n} \sum_{\ell}^{n} A_{ik} \Sigma_{k\ell}^x A_{j\ell}$$

# combining uncertainties

derivation

$$f_k = \sum_{i=1}^{n} A_{ki} x_i \text{ or } \mathrm{f} = \mathrm{Ax}$$

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$$\Sigma = \mathrm{A} \Sigma^x \mathrm{A}^\top$$

# combining uncertainties

derivation

$$f_k = \sum_{i=1}^{n} A_{ki} x_i \text{ or } \mathrm{f} = \mathrm{Ax}$$

$$\Sigma^x = \begin{pmatrix} \sigma_1^2 & 0 & 0 & \cdots \\ 0 & \sigma_2^2 & 0 & \cdots \\ 0 & 0 & \sigma_3^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} \Sigma_{11}^x & 0 & 0 & \cdots \\ 0 & \Sigma_{22}^x & 0 & \cdots \\ 0 & 0 & \Sigma_{33}^x & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\boxed{\Sigma_{ij}^f = \sum_{k}^{n} A_{ik} \Sigma_k^x A_{jk}}$$

sum in quadrature:

$$\Delta_f = \sqrt{\left(\frac{\partial f}{\partial x}^2\right) \Delta_x^2 + \left(\frac{\partial f}{\partial y}^2\right) \Delta_y^2 + \ldots$$

# combining uncertainties

derivation

$$f_k = \sum_{i=1}^{n} A_{ki} x_i \text{ or } \mathbf{f} = \mathbf{A}\mathbf{x}$$

$$f = \sum_{i}^{n} a_i x_i : f = \mathbf{a}x$$

$$\sigma_f^2 = \sum_{i}^{n} \sum_{j}^{n} a_i \Sigma_{ij}^{x} a_j = \mathbf{a} \Sigma^{x} \mathbf{a}^\top$$

https://www.math.upenn.edu/~deturck/m170/wk4/lecture/case1.html

# The Literary Digest

NEW YORK AUGUST 22, 1936

*Topics of the day*

**"THE DIGEST" PRESIDENTIAL POLL IS ON!**

Famous Forecasting Machino Is Thrown Into Gear for 1936

# reading

https://fivethirtyeight.com/features/philadelphia-is-trying-to-connect-with-its-least-heard-residents/

# listening