

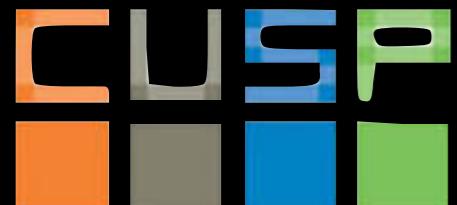
Urban Informatics

Fall 2018

dr. federica bianco fianco@nyu.edu

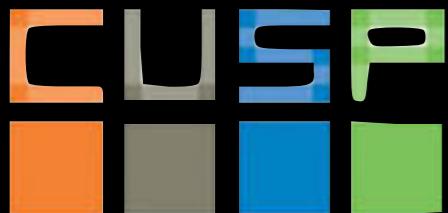


@fedhere



Recap:

- Good practices with data: falsifiability, reproducibility
- Basic data retrieving and munging: APIs, Data formats
- Basic statistics: distributions and their moments
- Hypothesis testing: p -value, statistical significance

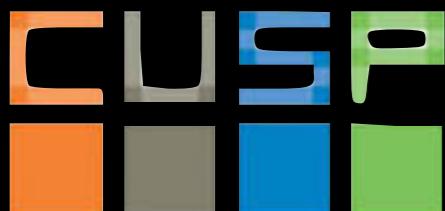


Recap:

- Good practices with data: falsifiability, reproducibility
- Basic data retrieving and munging: APIs, Data formats
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This class:

- How to choose the right statistical test
- Z, t, F test and tests for correlation
- Correlation vs Causation

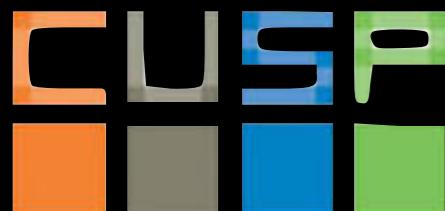


hypothesis testing

did we detect a phenomenon
(e.g. as a result of an implemented policy)?

null hypothesis: no relationship between (two) measured phenomena
or no difference between (two) groups
if you have a test control sample: test sample and
control sample are the same - no effect

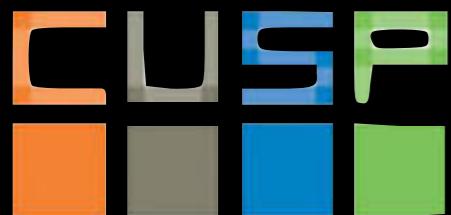
falsify the null hypothesis: do you see an effect?
do you see a difference b/w samples?



A simple (too simple?) answer

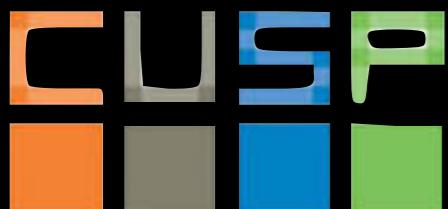
did we detect a phenomenon
(e.g. as a result of an implemented policy)?

p-value a measure of the probability that the result you observed could have been observed by chance under the *Null hypothesis*



Steps in Null-Rejection Hypothesis Testing

1. Formulate Null (and alternative) Hypothesis
2. Choose a significance level α
3. Measure a *statistic* for a *sample* to be compared to the *parameter of a population*
OR
Measure a *statistic* for *two or more samples* to be compared to *each other*
4. Assess if your statistics is significant or not. In practice: compare the statistics (Z, t, F, chisq) with a distribution table



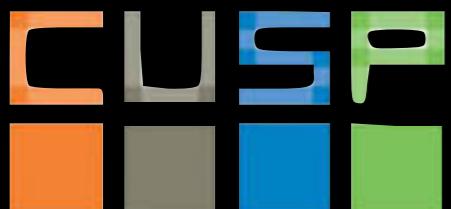
Steps in Null-Rejection Hypothesis Testing

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3. Measure a *statistic* for a *sample* to be compared to the *parameter of a population*

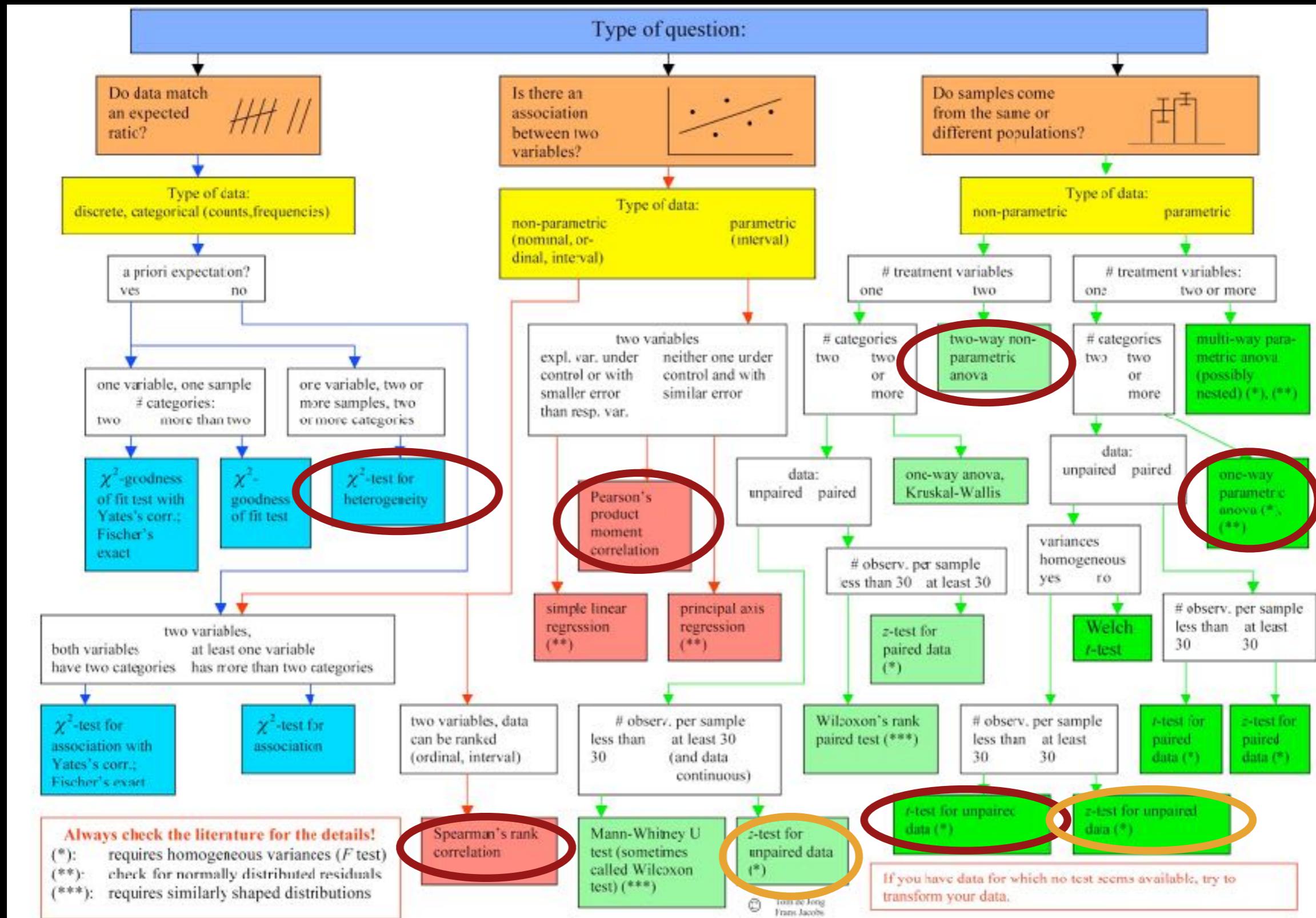
OR

Measure a *statistic* for *two or more samples* to be compared to *each other*

- 4. Assess if your statistics is significant or not. In practice: compare the statistics (Z, t, F, chisq) with a distribution table



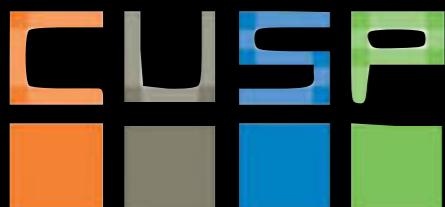
Steps in Null-Rejection Hypothesis Testing



how to choose the right statistical test

[http://www.csun.edu/~amarenco/Fcs%20682/
When%20to%20use%20what%20test.pdf](http://www.csun.edu/~amarenco/Fcs%20682/When%20to%20use%20what%20test.pdf)

Statistical Analyses	Independent Variables		Dependent Variables		Control Variables	Question Answered by the Statistic
	# of IVs	Data Type	# of DVs	Type of Data		
Chi square	1	categorical	1	categorical	0	Do differences exist between groups?
t-Test	1	dichotomous	1	continuous	0	Do differences exist between 2 groups on one DV?
ANOVA	1 +	categorical	1	continuous	0	Do differences exist between 2 or more groups on one DV?
ANCOVA	1 +	categorical	1	continuous	1 +	Do differences exist between 2 or more groups after controlling for CVs on one DV?
MANOVA	1 +	categorical	2 +	continuous	0	Do differences exist between 2 or more groups on multiple DVs?
MANCOVA	1 +	categorical	2 +	continuous	1 +	Do differences exist between 2 or more groups after controlling for CVs on multiple DVs?
Correlation	1	dichotomous or continuous	1	continuous	0	How strongly and in what direction (i.e., +, -) are the IV and DV related?
Multiple regression	2 +	dichotomous or continuous	1	continuous	0	How much variance in the DV is accounted for by linear combination of the IVs? Also, how strongly related to the DV is the beta coefficient for each IV?
Path analysis	2 +	continuous	1 +	continuous	0	What are the direct and indirect effects of predictor variables on the DV?
Logistic Regression	1 +	categorical or continuous	1	dichotomous	0	What is the odds probability of the DV occurring as the values of the IVs change?



Regression Logistic	1 +	continuous to dichotomous	1	dichotomous	0	What is the odds probability of the DV occurring as the values of the IVs change?
Path analysis	2 +	continuous	1 +	continuous	0	What are the direct and indirect effects of predictor variables on the DV?

IV: Statistical analysis

how to choose the right statistical test

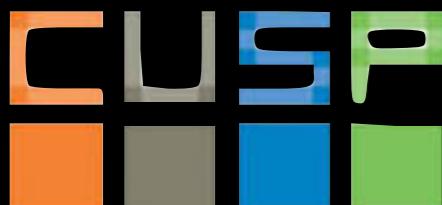
Assignment 1:

Browse PlosOne's abstracts to find 1 paper for each of the 3 tests:

[http://www.csun.edu/~amarenco/Fcs%20682/
When%20to%20use%20what%20test.pdf](http://www.csun.edu/~amarenco/Fcs%20682/When%20to%20use%20what%20test.pdf)

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choose 1
choose 1
choose 1 ←

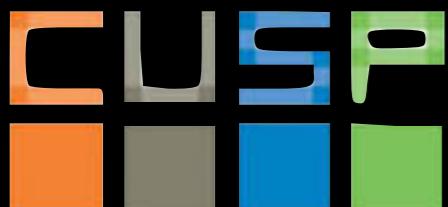


how to choose the right statistical test

Assignment 1:

Example: ANCOVA

Statistical Analyses	Dependent Variables	Independent Variables	Control Variables	Question Answered
ANCOVA	Ratings about their values <i>type: ordinal</i>	did Self Affirmation or not <i>type: categorical</i>	age <i>type: continuous</i>	self-affirmation group rates the value significantly more than control group



<https://journals.plos.org/plosone/article/figure?id=10.1371/journal.pone.0062593.t001>

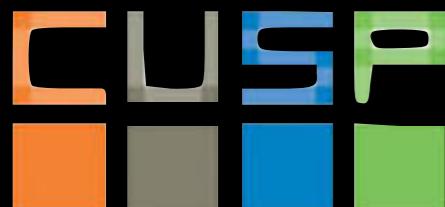
IV: Statistical analysis

how to choose the right statistical test

Assignment 1: Prepare a markdown table describing the use of the test in each of the papers you chose (3 rows in your table)

Example: ANCOVA

Statistical Analyses	Dependent Variables	Independent Variables	Control Variables	Question Answered
ANCOVA	Ratings about their values <i>type: ordinal</i>	did Self Affirmation or not <i>type: categorical</i>	age <i>type: continuous</i>	self-affirmation group rates the value significantly more than control group



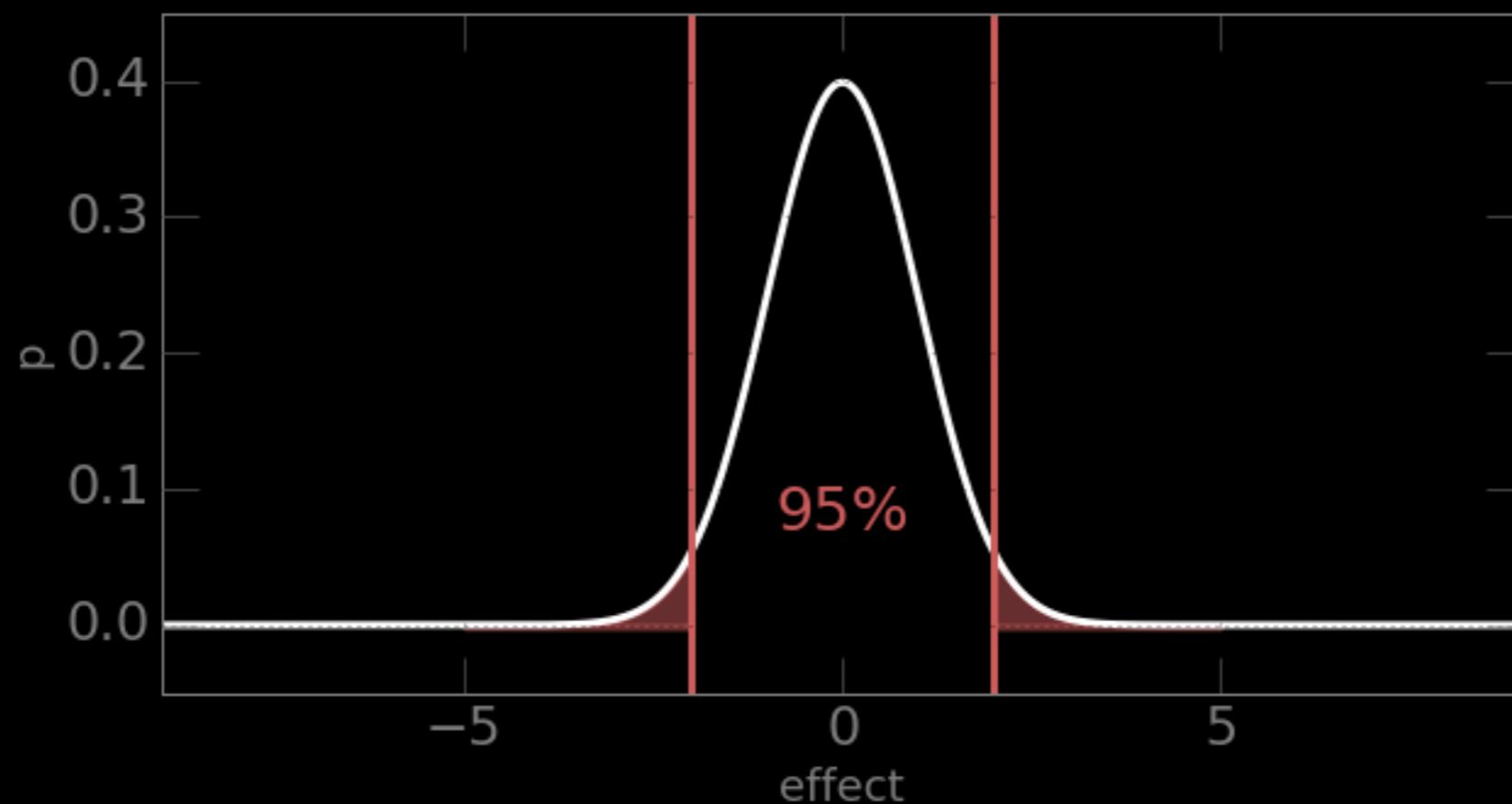
<https://journals.plos.org/plosone/article/figure?id=10.1371/journal.pone.0062593.t001>

IV: Statistical analysis

if you knew how your statistics should be distributed...

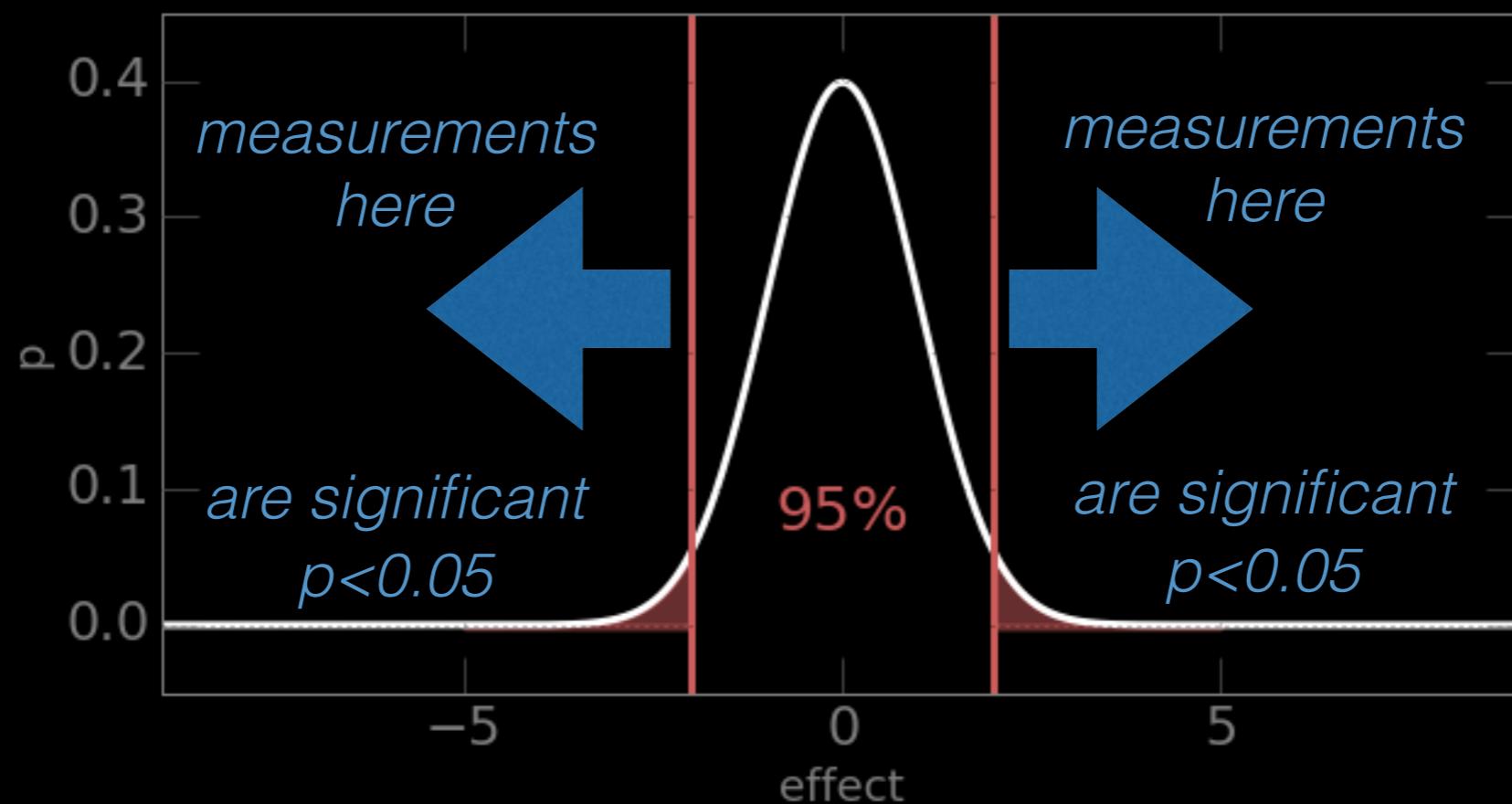
$$\alpha = 0.05$$

$$1 - 0.05 = 0.95 \Rightarrow 95\%$$



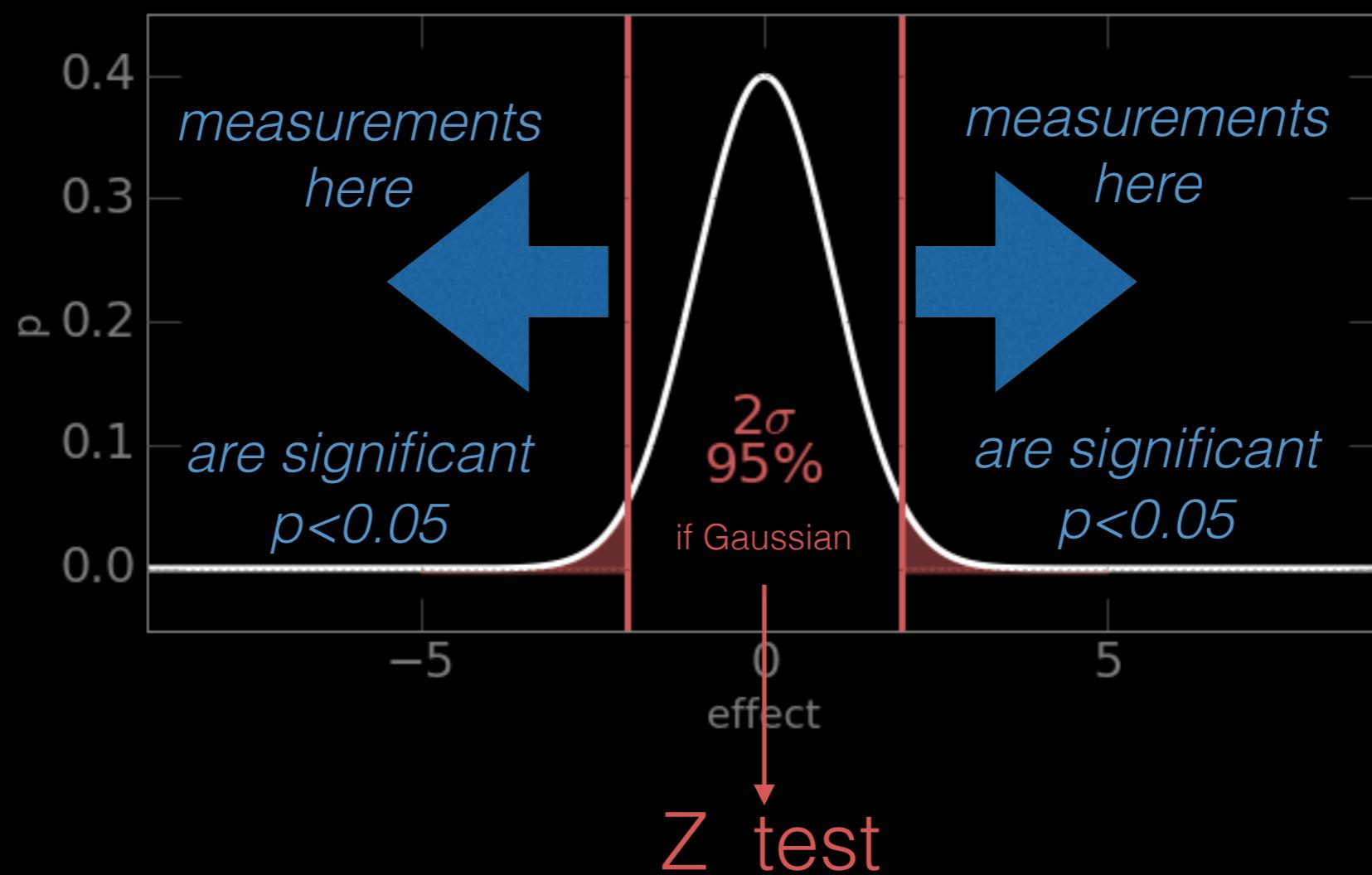
$$\alpha = 0.05$$

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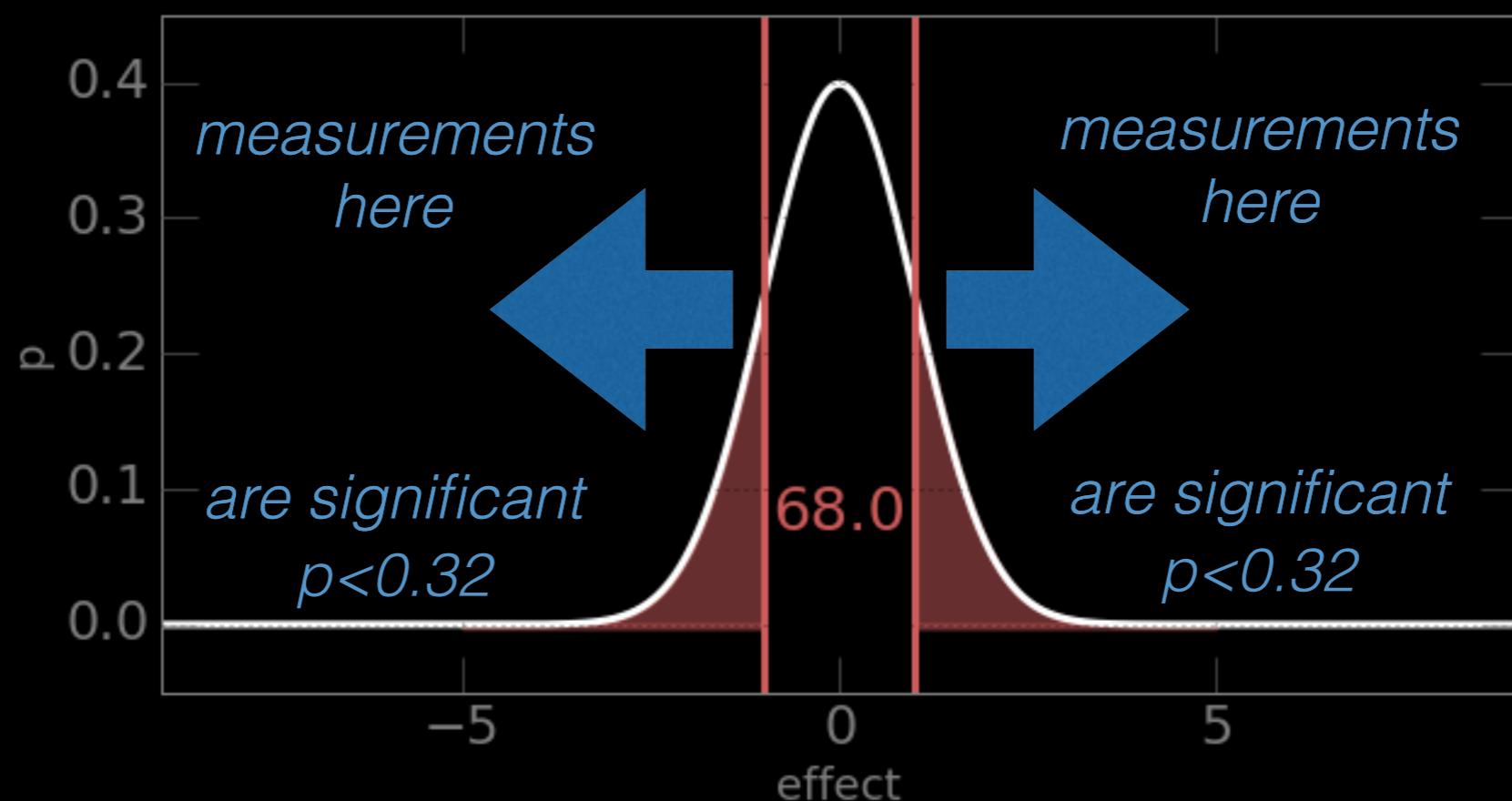


$$\alpha = 0.05$$

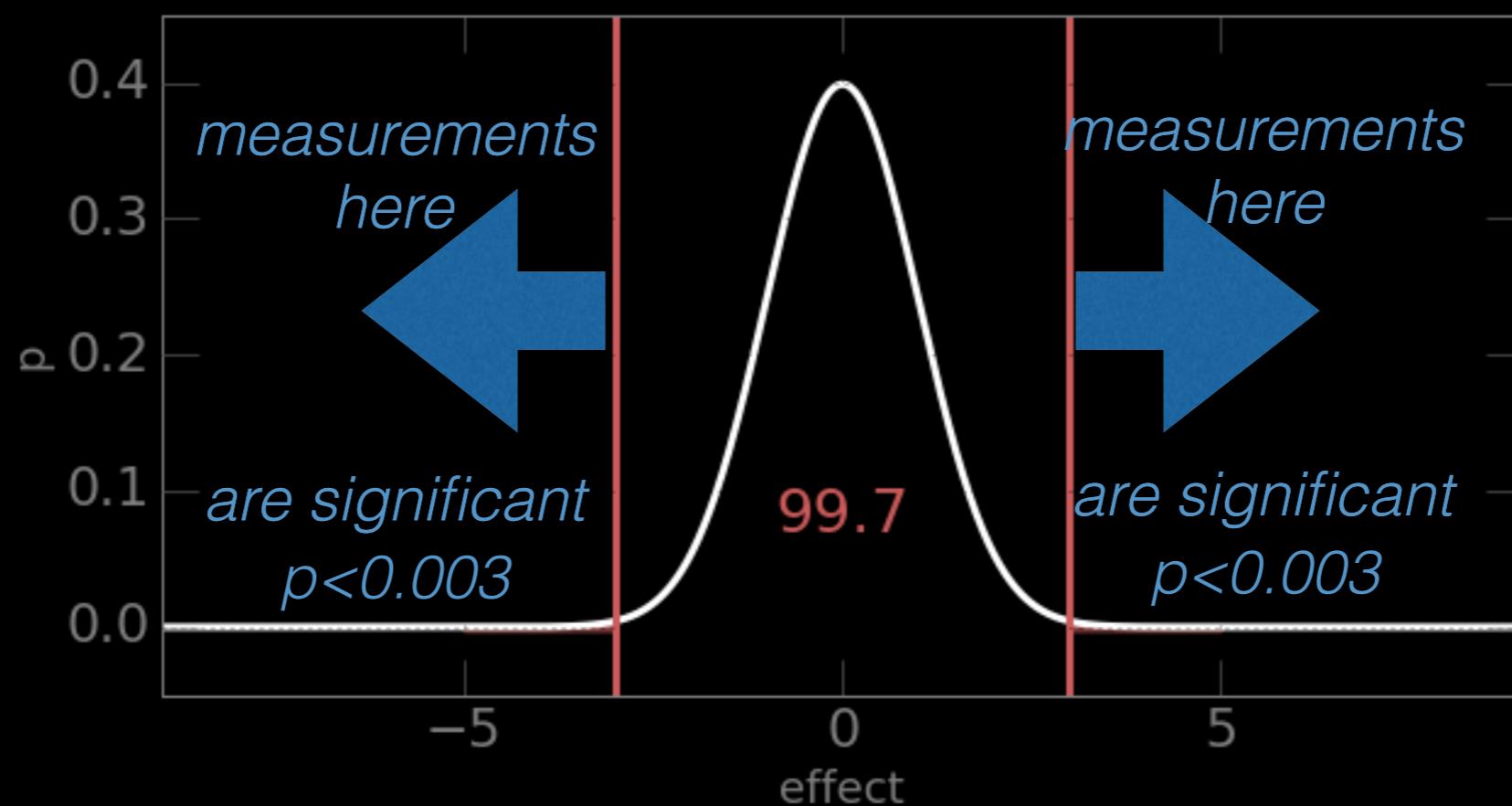
$$1 - \alpha = 0.95 \Rightarrow 95\%$$



$$a = 0.32$$



$$\alpha = 0.003$$



What is the *probability distribution* of a *statistics*?

To measure the probability of the value of a statistics that was measured after an experiment we need to know *how the statistics is distributed* under the alternative hypothesis.

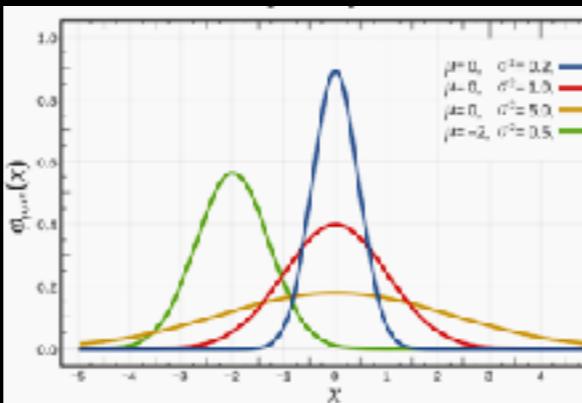
Quantities that follow a specific distribution under the alternative hypothesis are called *pivotal*.

Each *statistics* follows some distribution.

Which one though?

Z statistics Gaussian

$$Z = \frac{\mu - \bar{x}}{\sigma / \sqrt{n}}$$

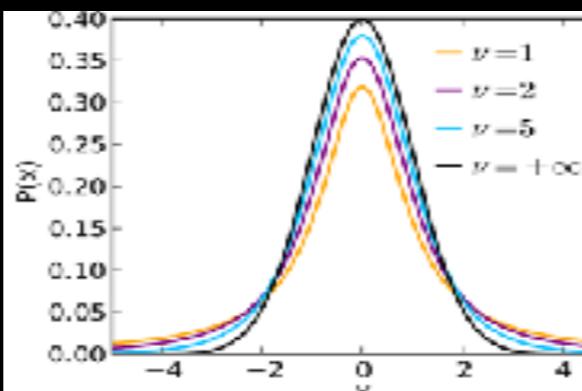


Notation	$N(\mu, \sigma^2)$
Parameters	$\mu \in \mathbb{R}$ — mean (location) $\sigma^2 > 0$ — variance (squared scale)
Support	$x \in \mathbb{R}$
PDF	$\frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
CDF	$\frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}} \right) \right]$
Quantile	$\mu + \sigma\sqrt{2} \operatorname{erf}^{-1}(2F - 1)$
Mean	μ
Median	μ
Mode	μ
Variance	σ^2

Quantile	$\mu + \sigma\sqrt{2} \operatorname{erf}^{-1}(2F - 1)$
Mean	μ
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Student's t

$$t = \frac{\mu - \bar{x}}{s / \sqrt{n}}$$

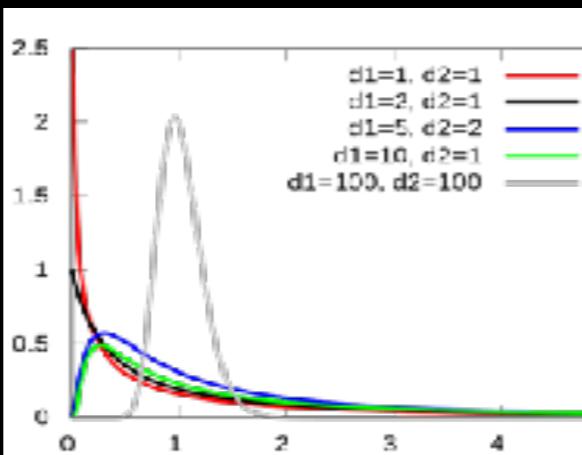


Parameters	$\nu > 0$ degrees of freedom (real)
Support	$x \in (-\infty; +\infty)$
PDF	$\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$
CDF	$\frac{1}{2} + x\Gamma\left(\frac{\nu+1}{2}\right) \times \\ {}_2F_1\left(\frac{1}{2}, \frac{\nu+1}{2}; \frac{3}{2}; -\frac{x^2}{\nu}\right) \\ \sqrt{\frac{\nu}{\nu+2}} \Gamma\left(\frac{\nu}{2}\right)$
where ${}_2F_1$ is the hypergeometric function	

Mean	0 for $\nu > 1$, otherwise undefined
Median	0
Mode	0
Variance	$\frac{\nu}{\nu-2}$ for $\nu > 2$, = for $1 < \nu \leq 2$, otherwise undefined

F statistics

$$F = \frac{\sum_i n_i (\bar{x}_i - \bar{\bar{x}})^2 / (K-1)}{\sum_{ij} (x_{ij} - \bar{x}_i)^2 / (N-K)}$$

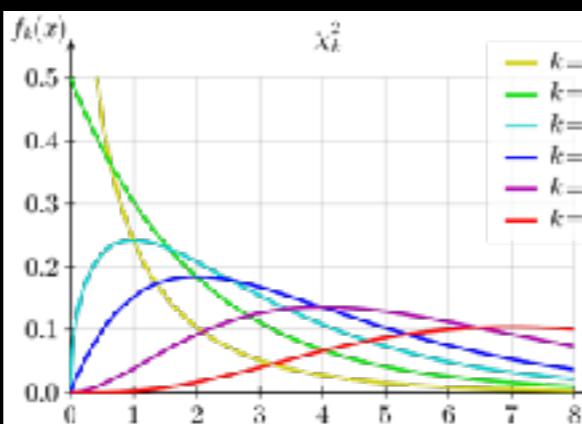


Parameters	$d_1, d_2 > 0$ deg. of freedom
Support	$x \in [0, +\infty)$
PDF	$\frac{\sqrt{(d_1 x)^{d_1} d_2^{d_2}}}{\sqrt{(d_1 + d_2)^{d_1+d_2}}} \\ x \operatorname{B}\left(\frac{d_1}{2}, \frac{d_2}{2}\right)$
CDF	$I \frac{d_1 x}{d_1 x + d_2} \left(\frac{d_1}{2}, \frac{d_2}{2}\right)$

Mean	$\frac{d_2}{d_2 - 2}$ for $d_2 > 2$
Mode	$\frac{d_1 - 2}{d_1 - 4} \frac{d_2}{d_2 + 2}$ for $d_1 > 2$
Variance	$\frac{2 d_2^2 (d_1 + d_2 - 2)}{d_1 (d_2 - 2)^2 (d_2 - 4)}$ for $d_2 > 4$
Skewness	$\frac{(2d_1 + d_2 - 2)\sqrt{8(d_2 - 4)}}{(d_2 - 6)\sqrt{d_1(d_1 + d_2 - 2)}}$ for $d_2 > 6$

Pearson's χ^2

$$\chi_P^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$



Notation	$\chi^2(k)$ or χ_k^2
Parameters	$k \in \mathbb{N}_{>0}$ (known as "degrees of freedom")
Support	$x \in [0, +\infty)$
PDF	$\frac{1}{2^{\frac{k}{2}} \Gamma\left(\frac{k}{2}\right)} x^{\frac{k}{2}-1} e^{-\frac{x}{2}}$
CDF	$\frac{1}{\Gamma\left(\frac{k}{2}\right)} \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$

Mean	k
Median	$\approx k \left(1 - \frac{2}{9k}\right)^3$
Mode	$\max\{k-2, 0\}$
Variance	$2k$
Skewness	$\sqrt{\frac{8}{k}}$

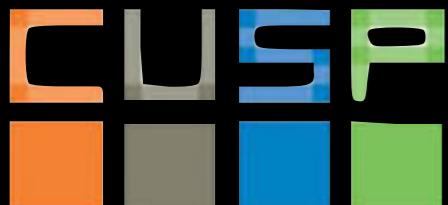
see

goodness of fit χ^2 $\chi_F^2 = \sum_i \frac{(m_i - x_i)^2}{e_i}$ - Statistics in a Nutshell
IV: Statistical analysis

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<https://documents.software.dell.com/statistics/textbook/distribution-tables>



Is there a difference between means or population and sample,
difference between proportion in 2 samples?

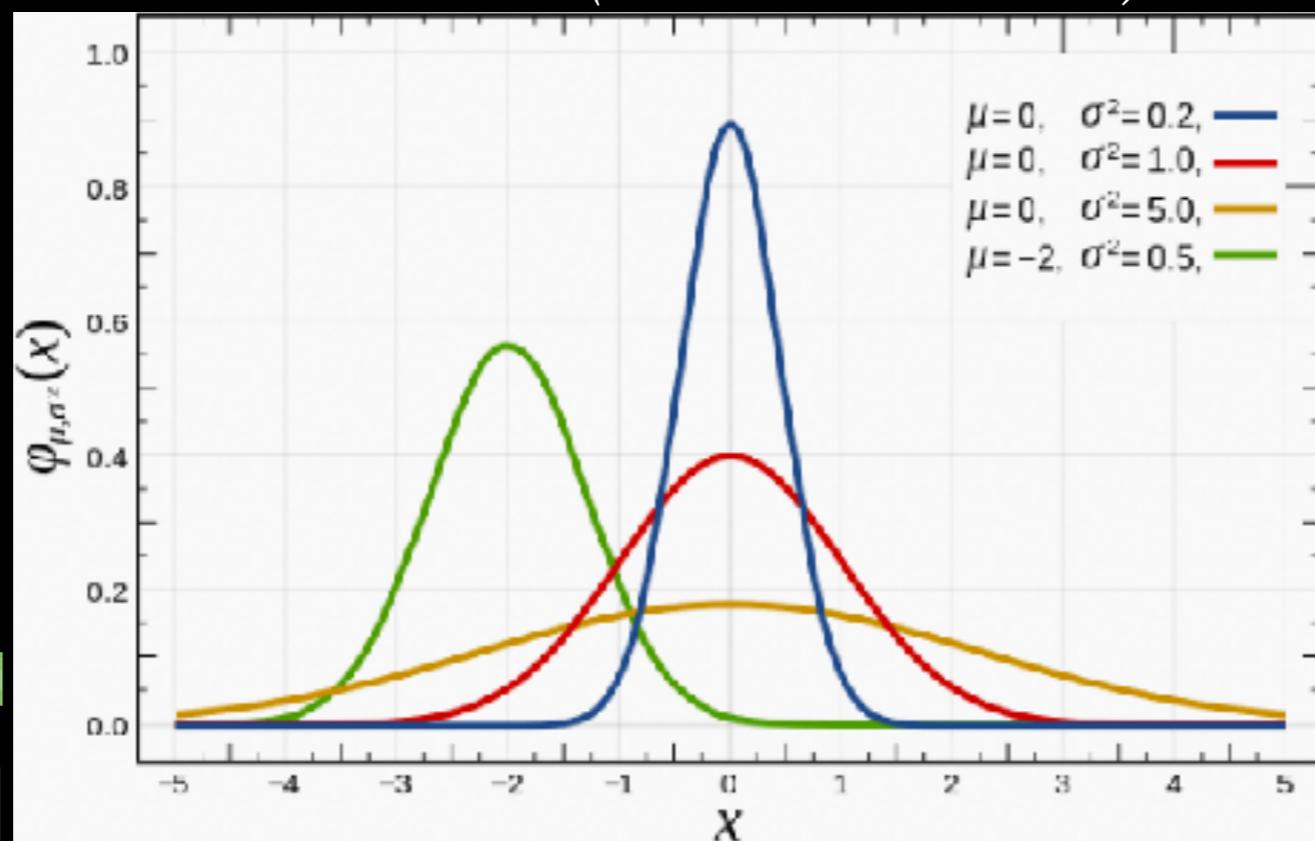
Z statistics

$$Z = \frac{\mu - \bar{x}}{\sigma / \sqrt{n}}$$

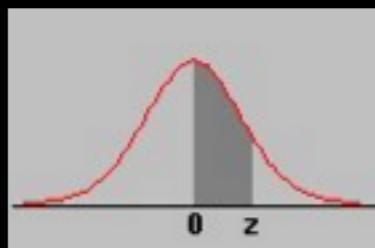
In absence of effect (i.e. under the Null)

Z is distributed according to a **Standard Normal** $N(\mu=0, \sigma=1)$

(i.e. Gaussian)



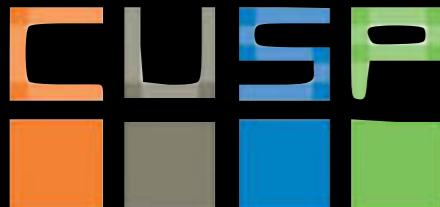
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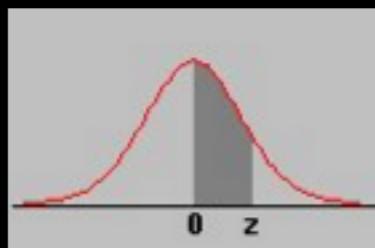


$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 2.56$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
											3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990

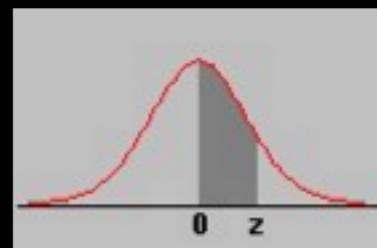
<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>





$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 2.56$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
											3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990

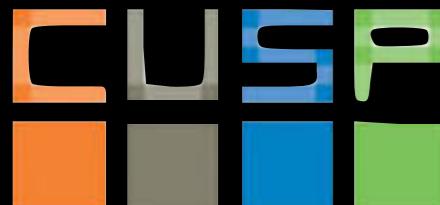


$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 2.56$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4951	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
											3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990

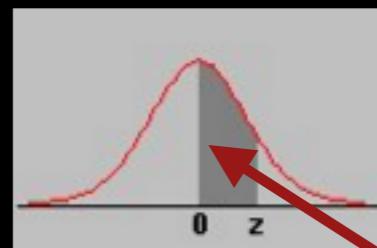
<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

1 sided test $1 - 0.4948 - 0.5 = 0.0052$
 $p < 0.05$



H_0 IS REJECTED ($p < 0.05$)

IV: Statistical analysis

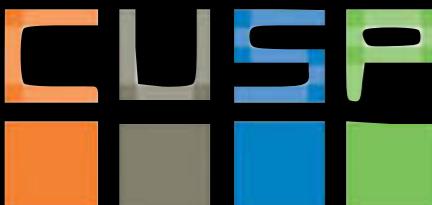


$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 2.56$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359		1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753		1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141		1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517		1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879		1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224		2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549		2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852		2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133		2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389		2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621		2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830		2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4951	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015		2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177		2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319		2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
												3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990

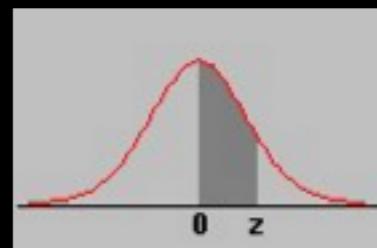
<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

1 sided test $1 - 0.4948 - 0.5 = 0.0052$
 $p < 0.05$



H_0 IS REJECTED ($p < 0.05$)

IV: Statistical analysis

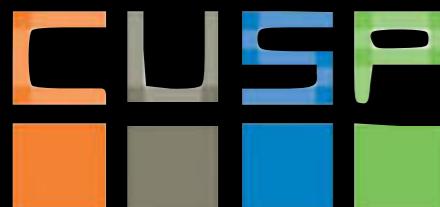


$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 2.55$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
											3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4990	0.4990

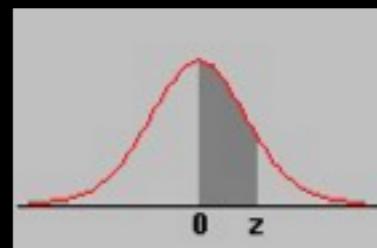
<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

1 sided test $1 - 0.4946 - 0.5 = 0.0054$
 $p < 0.05$



H_0 IS REJECTED ($p < 0.05$)

IV: Statistical analysis

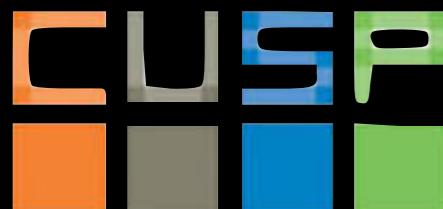


$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 1.57$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
											3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990

<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

2 sided test $1 - 0.4418^2 = 0.1164$
 $p > 0.05$



H_0 CANNOT BE REJECTED

IV: Statistical analysis

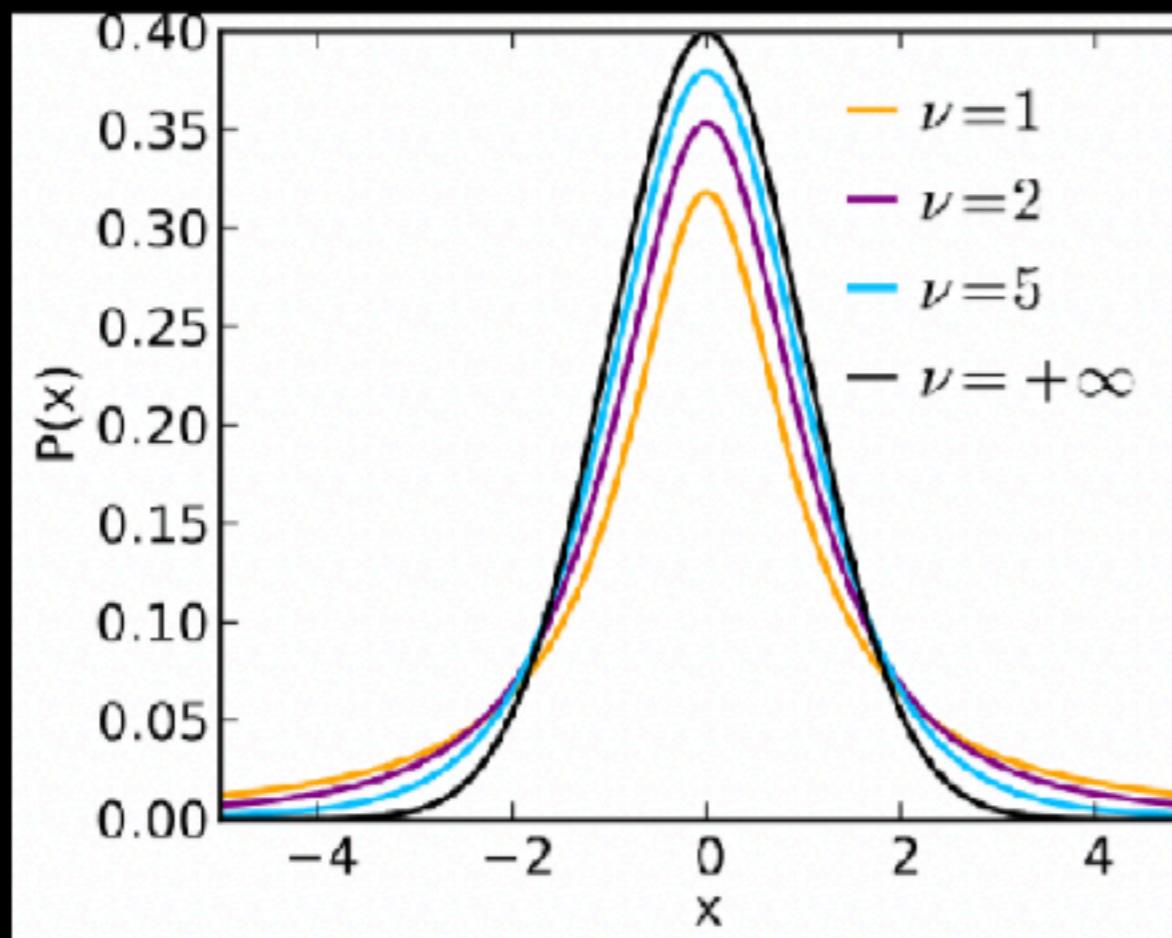
Is there a difference between means of 2 sample?

In absence of effect (i.e. under the Null)

Student's t

$$t = \frac{\mu - \bar{x}}{s/\sqrt{n}}$$

t is distributed according to a **t -distribution** with $v =$ degrees of freedom of the problem

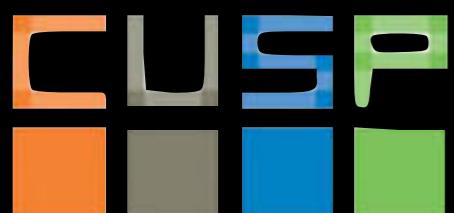


Parameters	$\nu > 0$ degrees of freedom (real)
Support	$x \in (-\infty; +\infty)$
PDF	$\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$
CDF	$\frac{1}{2} + x\Gamma\left(\frac{\nu+1}{2}\right) \times \frac{{}_2F_1\left(\frac{1}{2}, \frac{\nu+1}{2}; \frac{3}{2}; -\frac{x^2}{\nu}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)}$ <p>where ${}_2F_1$ is the hypergeometric function</p>
Mean	0 for $\nu > 1$, otherwise undefined
Median	0
Mode	0
Variance	$\frac{\nu}{\nu-2}$ for $\nu > 2$, ∞ for $1 < \nu \leq 2$, otherwise undefined

Comparing to population

Standard DEVIATION of Sample Estimates

Sample mean, \bar{x}	$Z = \frac{\mu_{\text{pop}} - \bar{x}}{\sigma / \sqrt{n}}$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
Sample proportion, p	$z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$\sigma_p = \sqrt{\frac{p_0(1-p_0)}{n}}$
Difference between proportions, $p_1 - p_2$	$z = \frac{(p_2 - p_1)}{\sqrt{p(1-p)(\frac{1}{n_2} + \frac{1}{n_1})}}, p = \frac{p_2 n_2 + p_1 n_1}{n_2 + n_1}$	$\sigma_{p_1 - p_2} = \sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}$

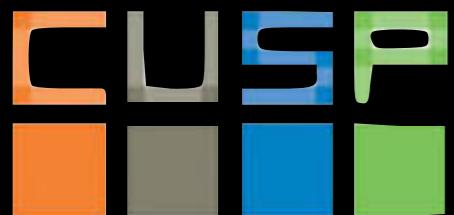


Use if you know the population *parameters*
e.g. Z -test

Comparing samples

Standard ERROR of Sample Estimates

Sample mean, \bar{x}	$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$
Sample proportion, p	$SE_p = \frac{p(1-p)}{n}$
Difference between means, $x_1 - x_2$	$SE_{x_1 - x_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
Difference between proportions, $p_1 - p_2$	$SE_{p_1 - p_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$



Use if you DO NOT know the population *parameters*
e.g. t - test

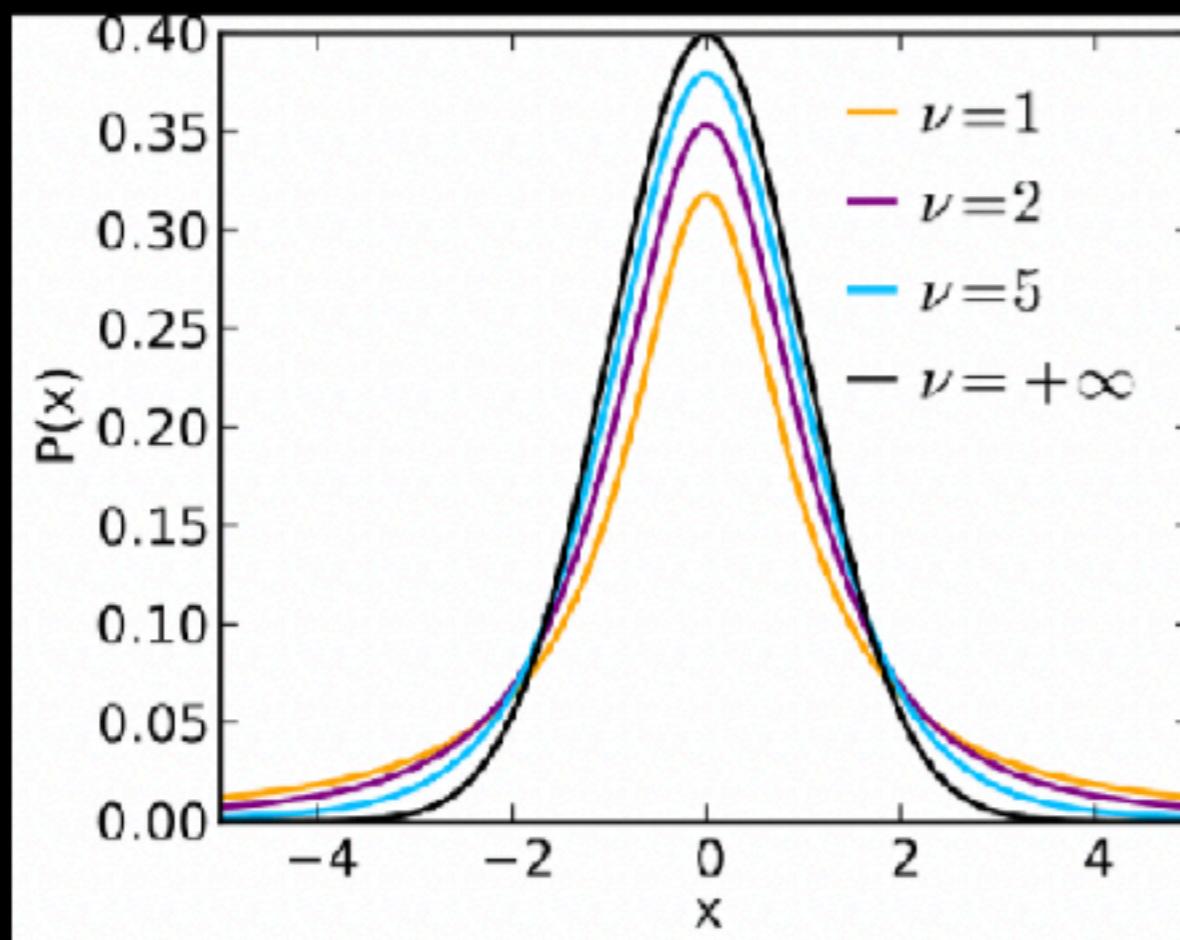
Is there a difference between means of 2 sample?

In absence of effect (i.e. under the Null)

Student's t

$$t = \frac{\mu - \bar{x}}{s/\sqrt{n}}$$

t is distributed according to a **t -distribution** with $v =$ degrees of freedom of the problem

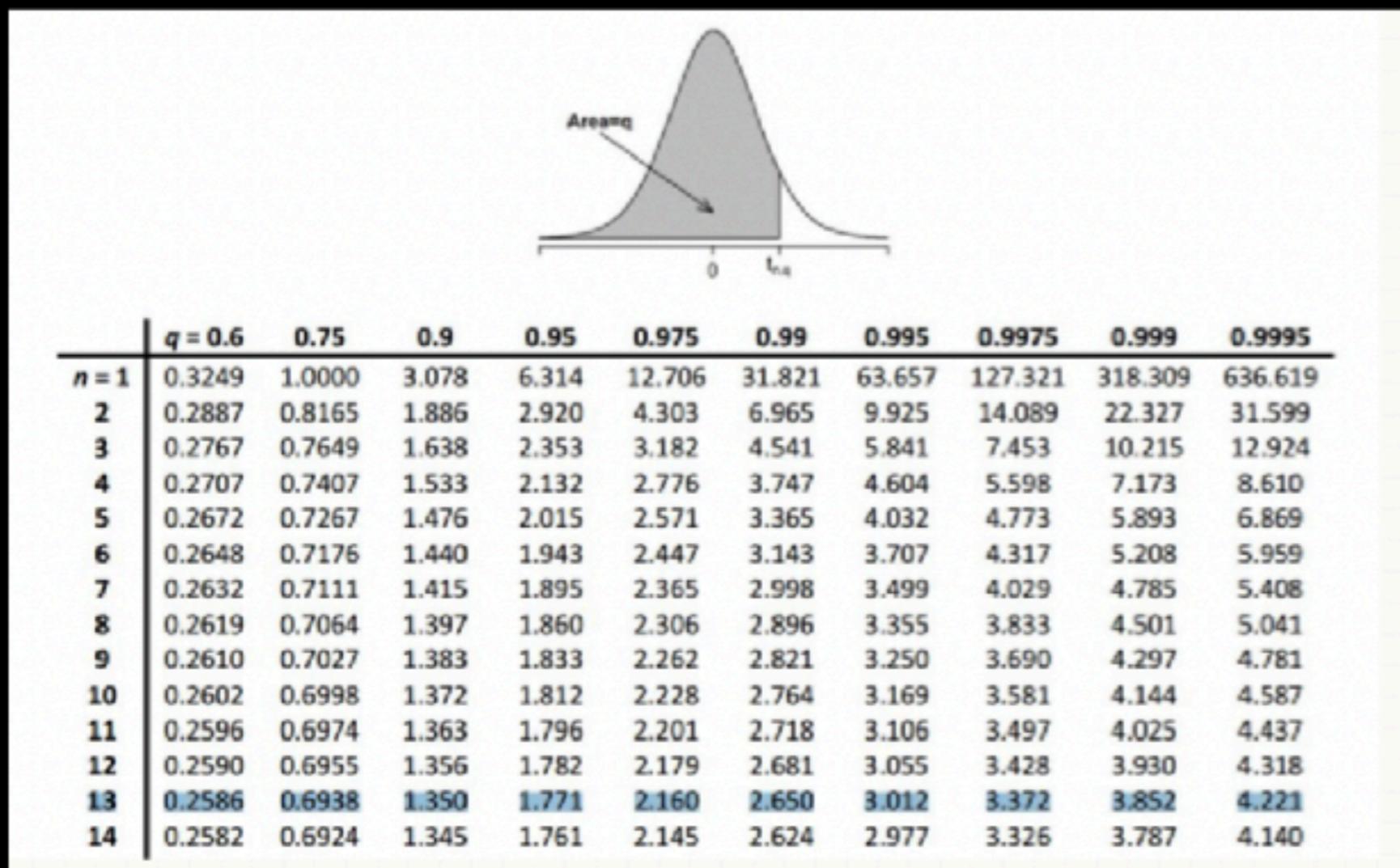


Parameters	$\nu > 0$ degrees of freedom (real)
Support	$x \in (-\infty; +\infty)$
PDF	$\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$
CDF	$\frac{1}{2} + x\Gamma\left(\frac{\nu+1}{2}\right) \times \frac{{}_2F_1\left(\frac{1}{2}, \frac{\nu+1}{2}; \frac{3}{2}; -\frac{x^2}{\nu}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)}$ <p>where ${}_2F_1$ is the hypergeometric function</p>
Mean	0 for $\nu > 1$, otherwise undefined
Median	0
Mode	0
Variance	$\frac{\nu}{\nu-2}$ for $\nu > 2$, ∞ for $1 < \nu \leq 2$, otherwise undefined

Is there a difference between means of 2 sample?

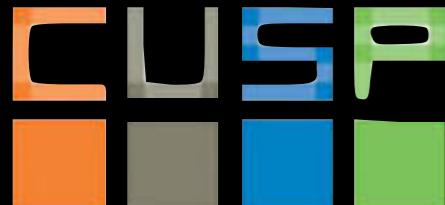
$$t = \frac{\mu - x}{s/\sqrt{n}} = 1.75$$

$n=13$,
 $q=.95$



$\leftarrow 1-A$

d.o.f
↑



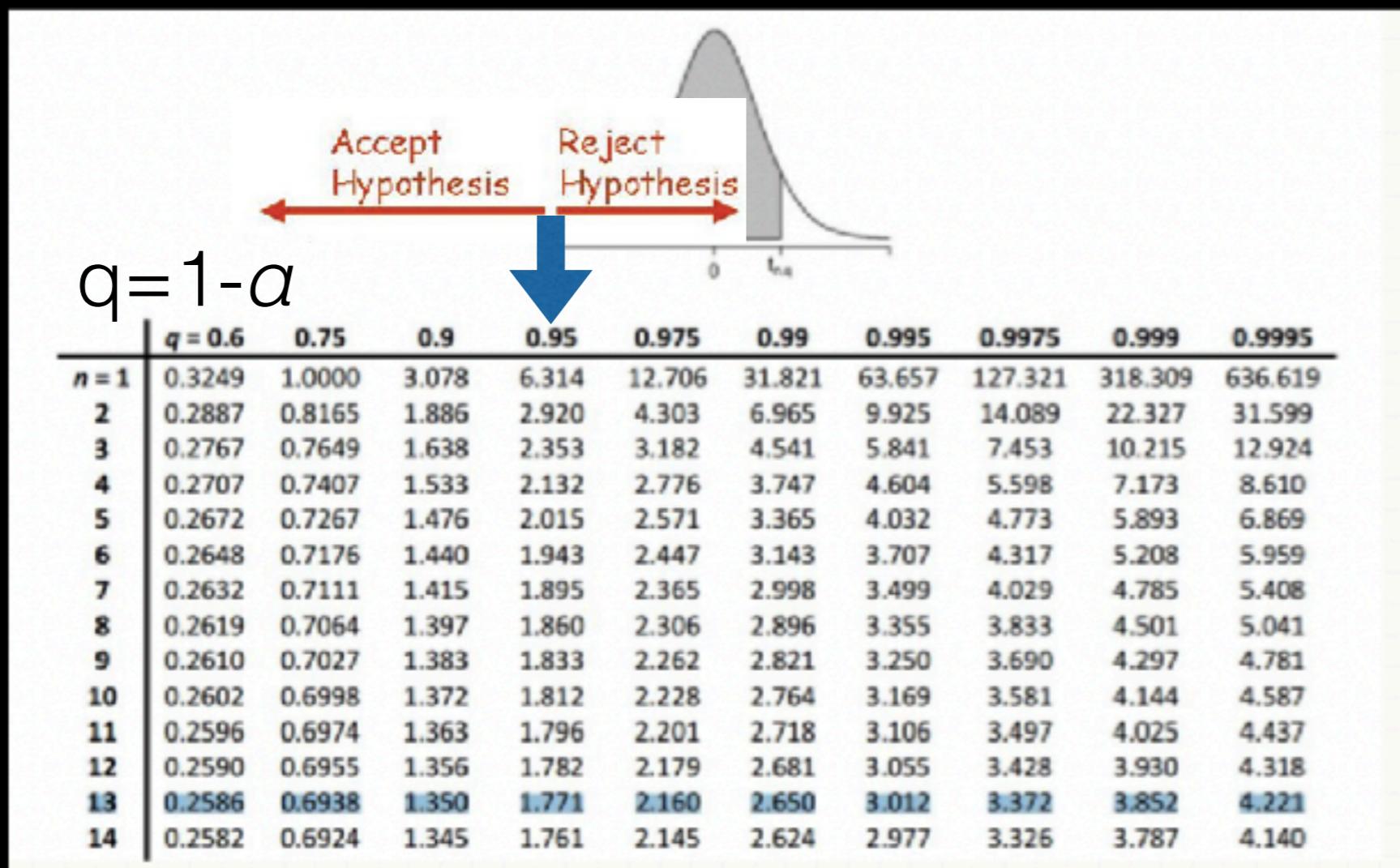
Generally d.o.f = $n_{observations} - 1$

IV: Statistical analysis

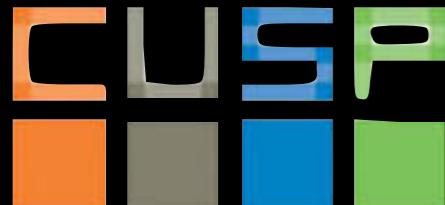
Is there a difference between means of 2 sample?

$$t = \frac{\mu - x}{s/\sqrt{n}} = 1.75$$

$n=13, q=.95$



d.o.f



Generally d.o.f = $n_{observations} - 1$

IV: Statistical analysis

Is there a difference between means or population and sample,
difference between proportion in 2 samples?

χ^2 statistics

$$\chi_P^2 = \sum_{i=1}^N \frac{(O_i - E_i)^2}{E_i}$$

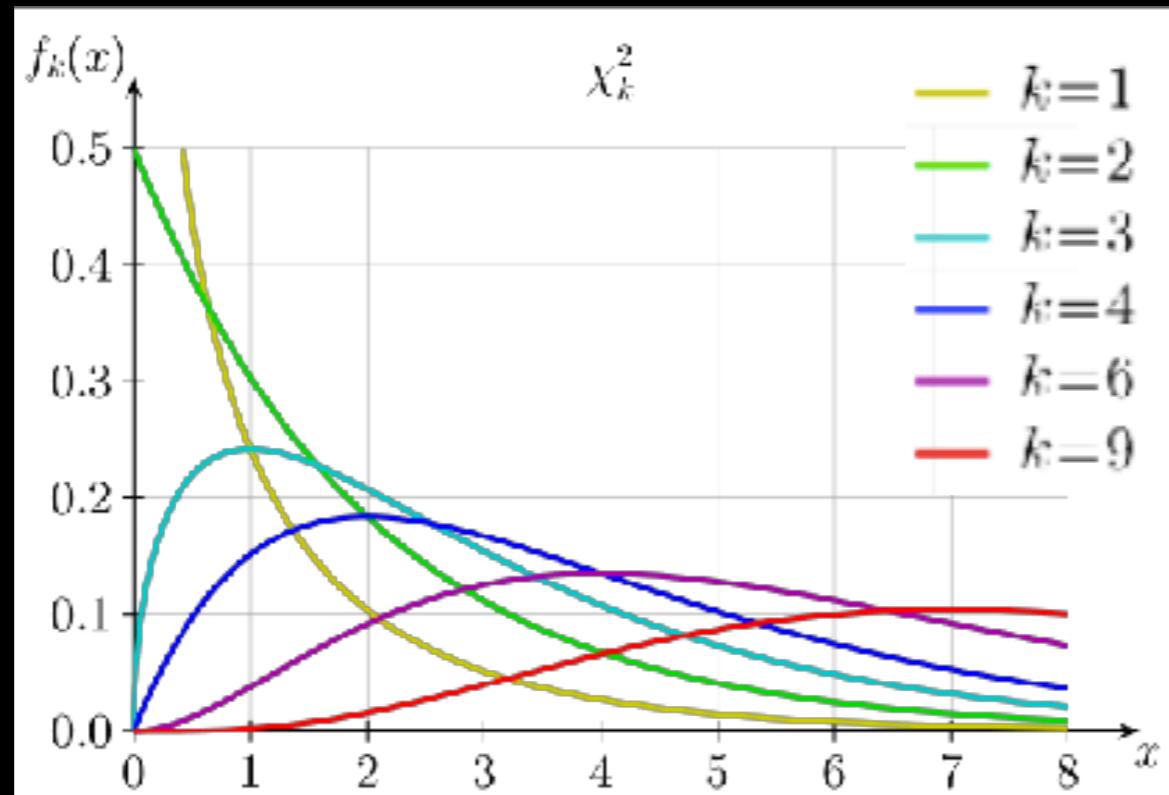
In absence of effect (i.e. under the Null)

χ^2 is distributed according to a χ^2 distribution with k =degrees of freedom

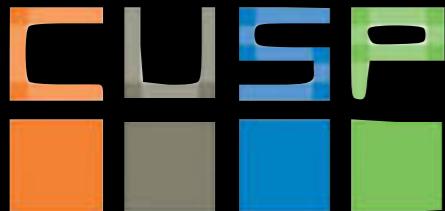
O : observed

E : expected (also model prediction)

N : observations



Notation	$\chi^2(k)$ or χ_k^2
Parameters	$k \in \mathbb{N}_{>0}$ (known as "degrees of freedom")
Support	$x \in [0, +\infty)$
PDF	$\frac{1}{2^{\frac{k}{2}} \Gamma(\frac{k}{2})} x^{\frac{k}{2}-1} e^{-\frac{x}{2}}$
CDF	$\frac{1}{\Gamma(\frac{k}{2})} \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$
Mean	k
Median	$\approx k \left(1 - \frac{2}{9k}\right)^3$
Mode	$\max(k-2, 0)$
Variance	$2k$
Skewness	$\sqrt{8/k}$



observed → *expected*

$$\chi_P^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

For test of proportion

expected

4 observations - 1 independent variable =
3 degreeed of freedom

Accept Hypothesis Reject Hypothesis

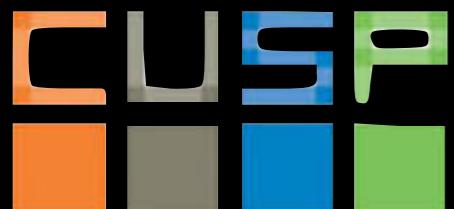
← →

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09

$$\alpha = 0.05$$

$$p < 0.05$$



H_0 CAN BE REJECTED

IV: Statistical analysis

For goodness of fit

$$model \quad \chi^2_F = \sum_{i=1}^4 \frac{(m_i - x_i)^2}{e_i} \quad data \quad error$$

4 observations - 1 independent variable =
3 degrees of freedom

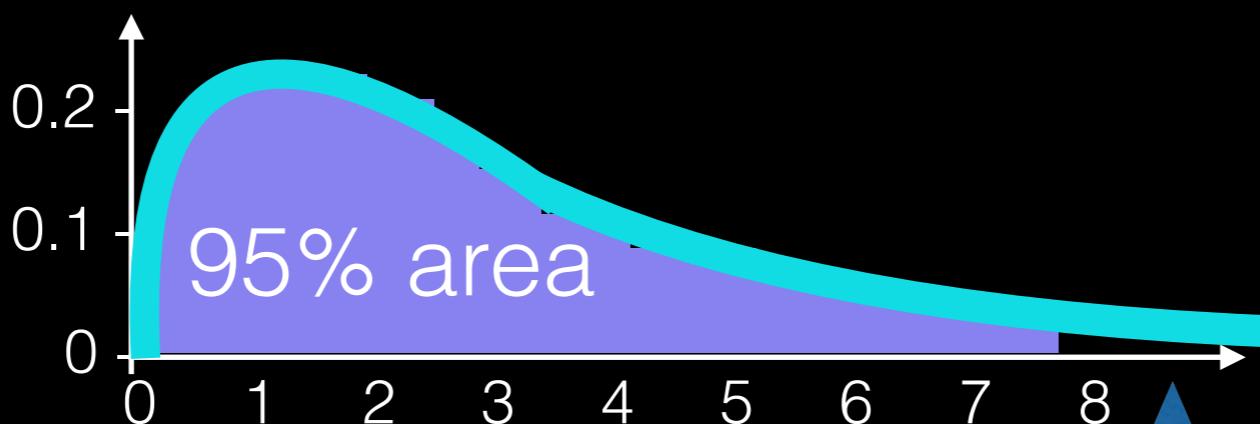
The table shows the Chi-Square distribution with columns for degrees of freedom (1, 2, 3, 4, 5) and rows for probability levels (0.99, 0.95, 0.90, 0.75, 0.50, 0.25, 0.10, 0.05, 0.01). The 0.05 column is circled in red. A horizontal arrow at the top indicates the range of values where the null hypothesis is accepted (left) or rejected (right).

Degrees of Freedom	Probability of a larger value of χ^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
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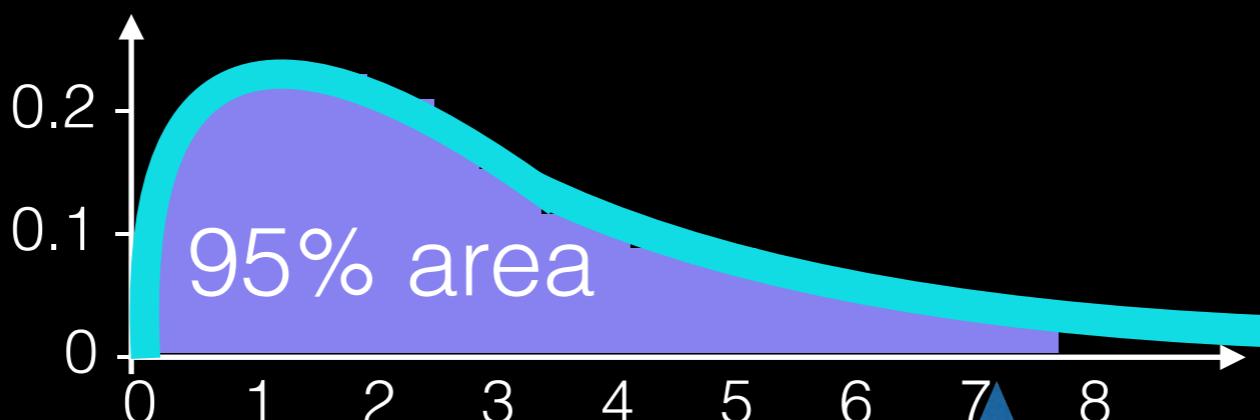
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$$\alpha = 0.05$$

$$\chi_P^2 = \sum_i \frac{(O_i - E_i)^2}{E_i} = 8.57 \quad 8.57 > 7.81 \\ p < 0.05$$



4 observations - 1 independent variable =
3 degrees of freedom

Percentage Points of the Chi-Square Distribution

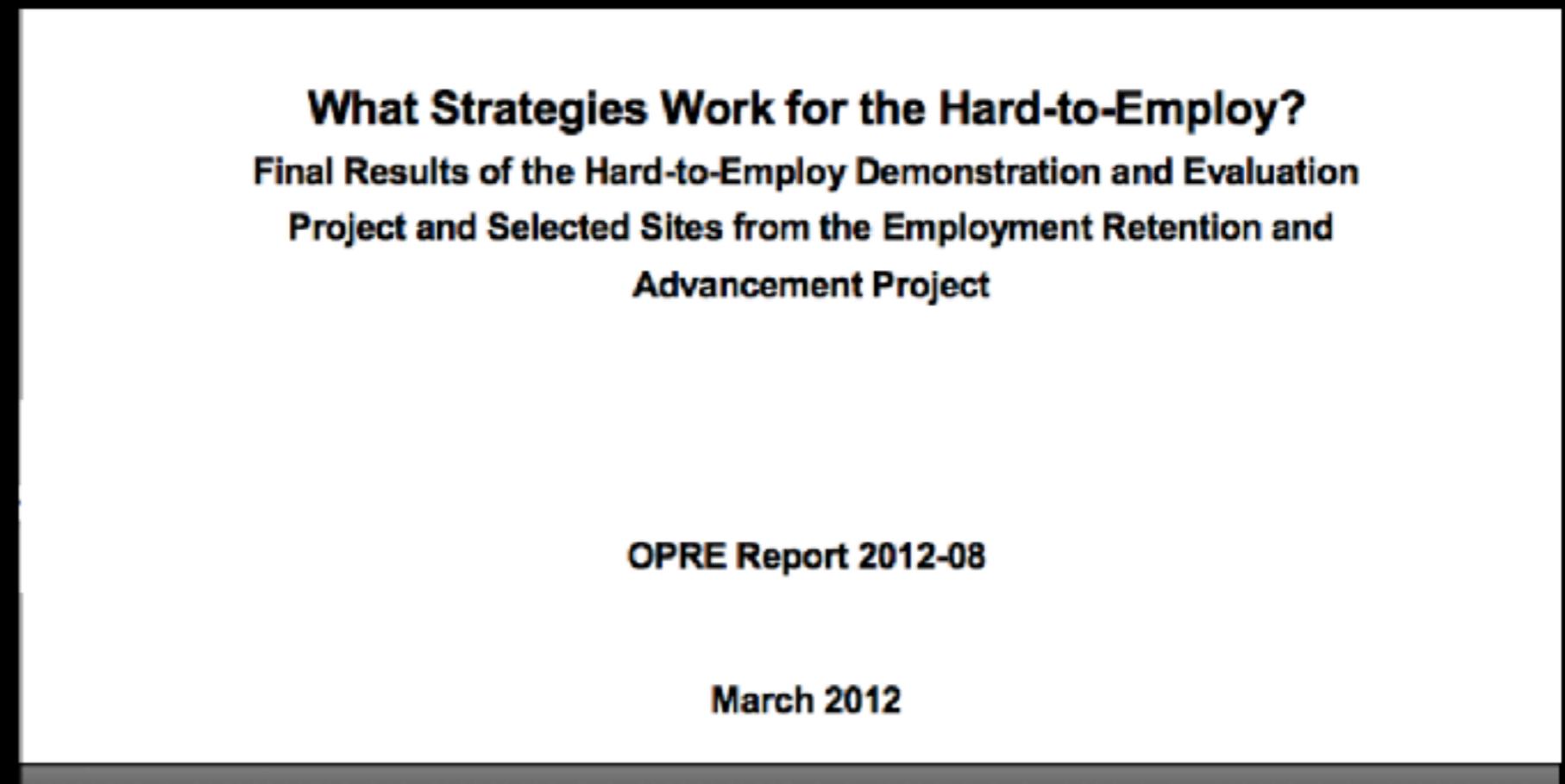
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$$\alpha = 0.05$$

$$\chi_P^2 = \sum_i \frac{(O_i - E_i)^2}{E_i} = 7.11 \quad 7.11 < 7.81 \\ p >= 0.05$$

H_0 CANNOT BE REJECTED

Example: **NULL HYPOTHESIS:** the % of former prisoners employed 3 years after release is *the same or lower* for candidates who participated in the program as for the control group,
significance level p=0.05



<https://www.mdrc.org/sites/default/files/What%20Strategies%20Work%20for%20the%20Hard%20FR.pdf>

NULL HYPOTHESIS: the % of former prisoners employed 3 years after release is *the same or lower* for candidates who participated in the program as for the control group,
significance level p=0.05

The Enhanced Services for the Hard-to-Employ Demonstration and Evaluation Project

Table 2.1

Summary of Impacts, New York City Center for Employment Opportunities

Outcome	Program Group	Control Group	Difference (Impact)	P-Value
<u>Employment (Years 1-3) (%)</u>	P₁	P₀		
Ever employed	83.8	70.4	13.4 ***	0.000
Ever employed in a CEO transitional job ^a	70.1	3.5	66.6 ***	0.000
Ever employed in an unsubsidized job	63.7	69.0	-5.3 *	0.078
<u>Postprogram unsubsidized employment (Years 2-3)</u>				
Ever employed in an unsubsidized job (%)	53.3	52.1	1.2	0.713
Employed in an unsubsidized job, average per quarter (%)	28.2	27.2	1.1	0.618
Employed for six or more consecutive quarters (%)	14.7	11.9	2.8	0.195
Total UI-covered earnings ^b (\$)	10,435	9,846	589	0.658
Sample size (total = 973) ^c	564	409		

$$H_0: P_0 - P_1 \geq 0$$

$$H_a: P_0 - P_1 < 0$$

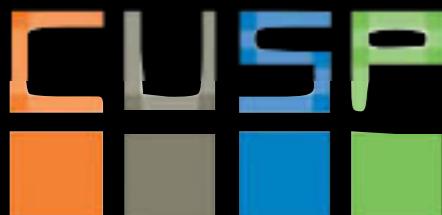
$$\alpha = 0.05$$

<http://www.mdrc.org/sites/default/files/What%20Strategies%20Work%20for%20the%20Hard%20FR.pdf>

SOURCES: MDRC earnings calculations from the National Directory of New Hires (NDNH) database and employment calculations from the unemployment insurance (UI) wage records from New York State, MDRC calculations using data from the New York State Division of Criminal Justice Services (DCJS) and the New York City Department of Correction (DOC).

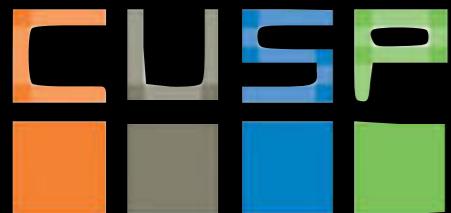
NOTES: Statistical significance levels are indicated as: *** = 1 percent; ** = 5 percent; * = 10 percent.

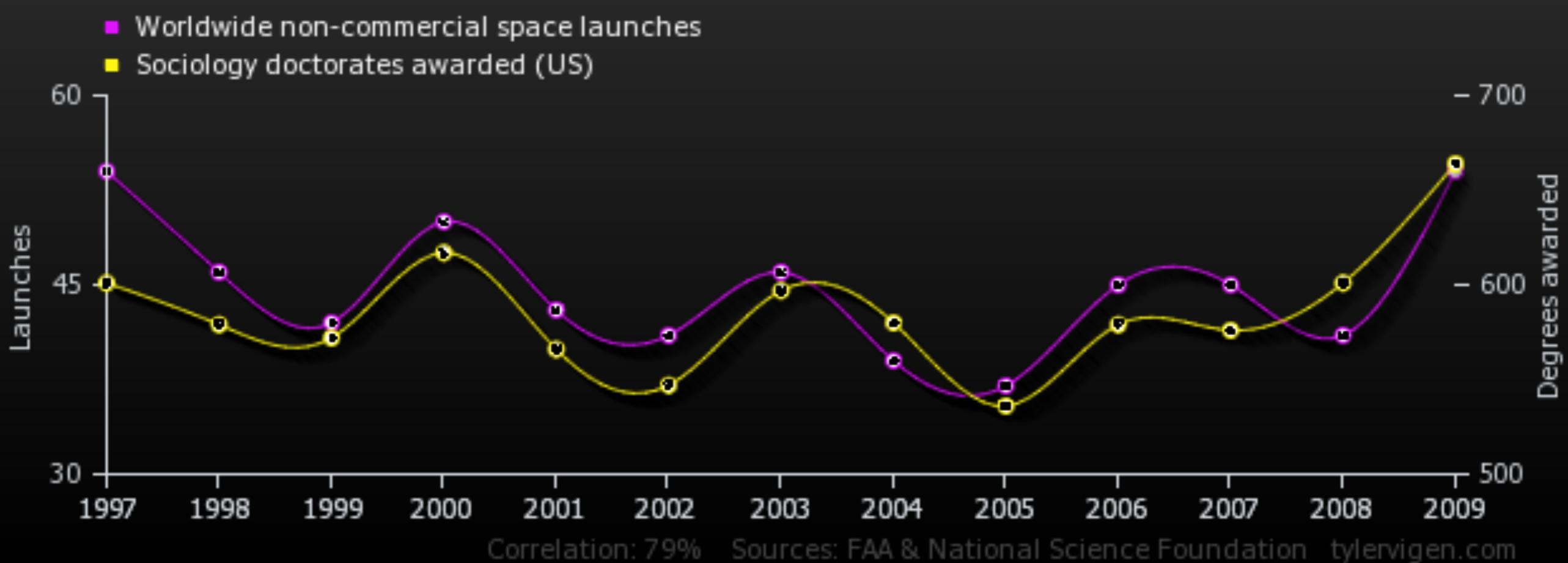
The p-value indicates the likelihood that the difference between the program and control groups arose by chance.





[https://github.com/fedhere/PUI2018_fb55/blob/master/
Lab5_fb55/effectiveness%20of%20NYC%20Post-
Prison%20Employment%20Programs.ipynb](https://github.com/fedhere/PUI2018_fb55/blob/master/Lab5_fb55/effectiveness%20of%20NYC%20Post-Prison%20Employment%20Programs.ipynb)

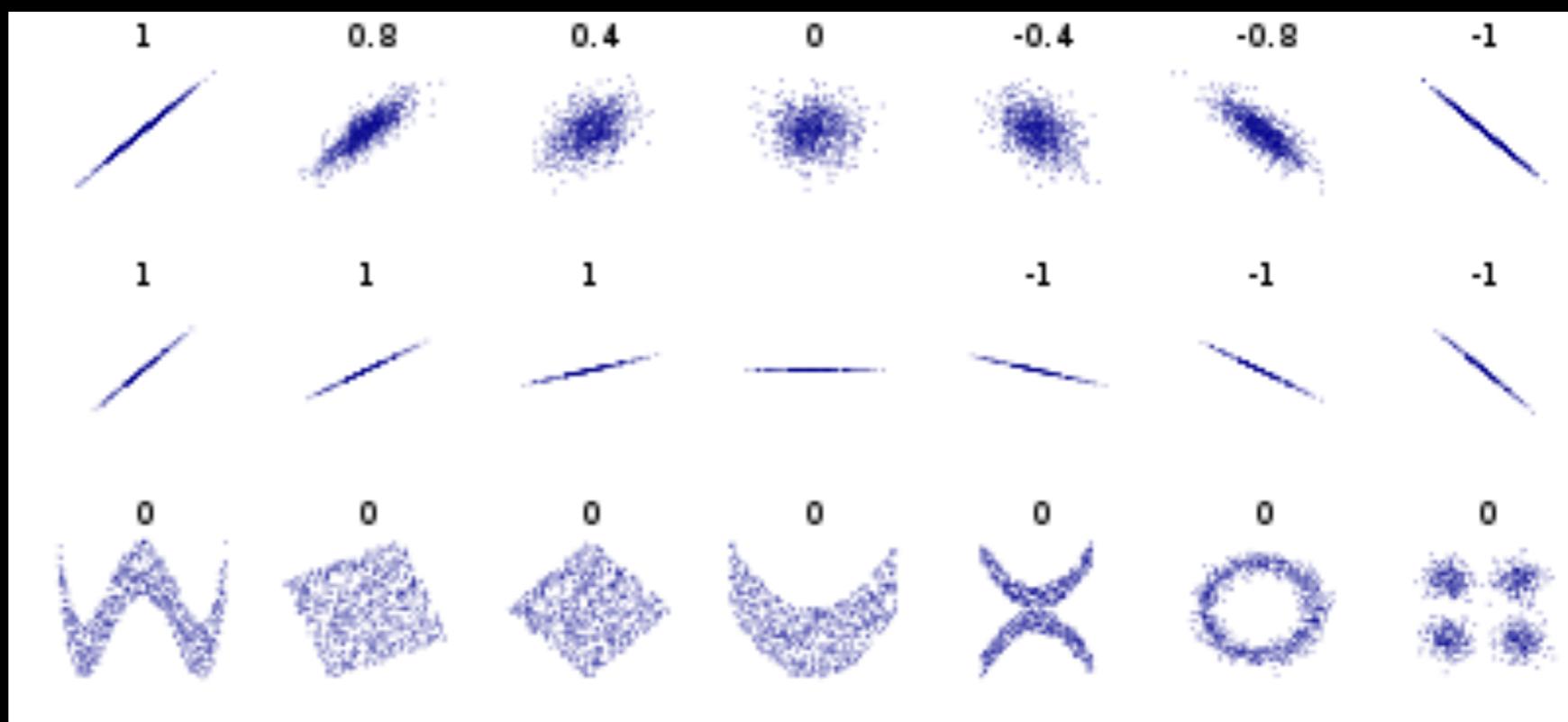




Correlation

Pearson's test:

$$r_{xy} = \frac{1}{n-1} \sum_{i=1}^n \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$
$$s_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$



Pearson's test:

$$r_{xy} = \frac{1}{n-1} \sum_{i=1}^n \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

Spearman's test:
(Pearson's for ranks)

$$\rho = 1 - \frac{6 \sum (x_i - y_i)^2}{n(n^2 - 1)}$$

Choosing the test

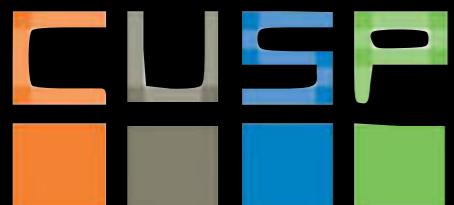
Use the table below to choose the test. See below for further details.

How many dichotomous* (binary) variables?				
Both variables interval or ratio?				
0	Y	Measures are linear? (No = monotonic*)		
		Y Pearson correlation		
0	N	N Spearman correlation		
		Both variables are ordinal?		
1	Y	Kendall correlation		
		Both variables can be ranked?		
1	N	Y Kendall correlation		
		N Convert to frequency data and use Chi-square test for independence		
1 serial Correlation Coefficient				
2 x 2 table?				
2	Y	Phi		
		N Cramer's V		
Data has frequency values for each category?				
Y	Chi-square test for independence			

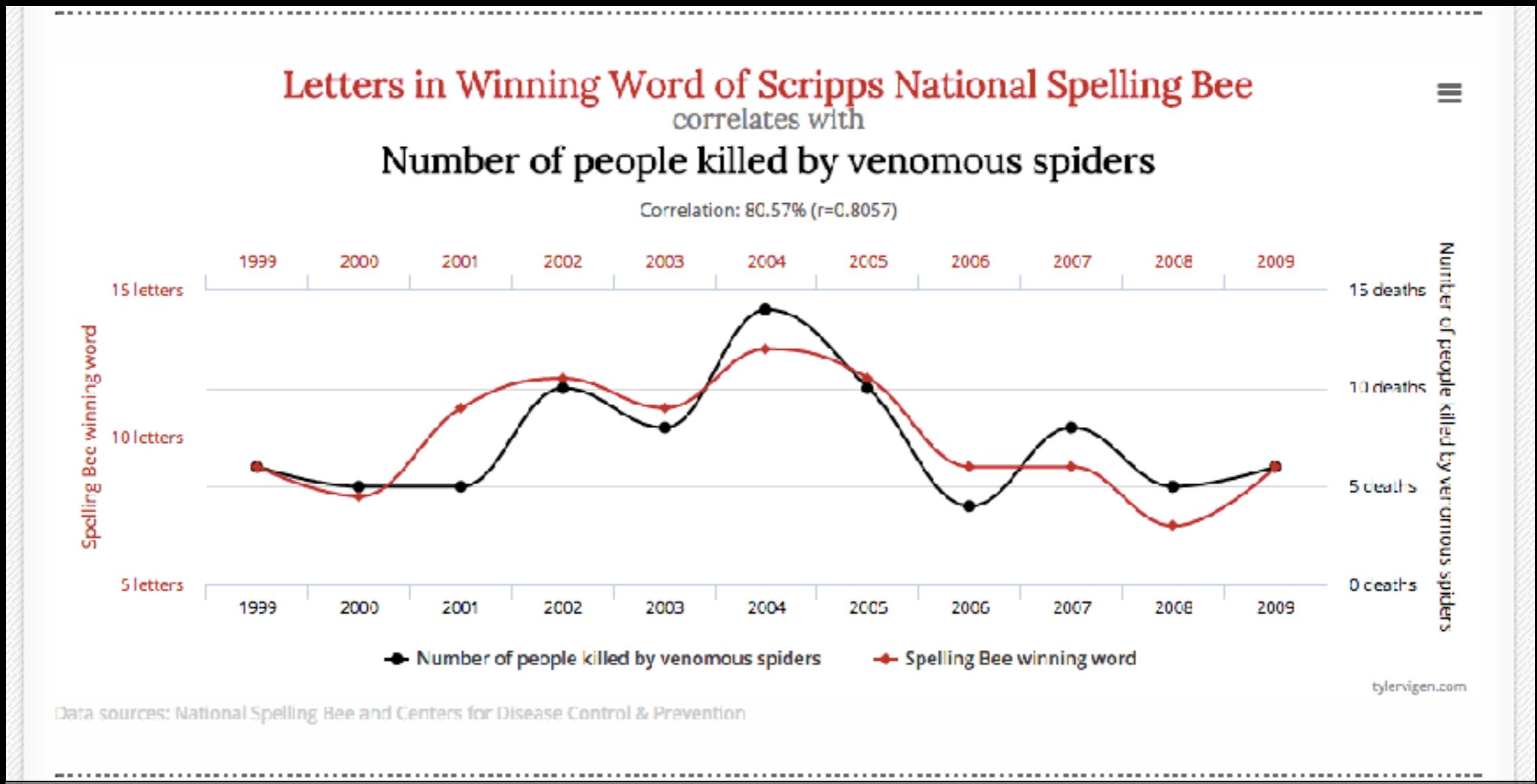
*dichotomous = 'can have only two values' (eg. yes/no or 0/1).

†monotonic = constantly increasing or decreasing.

http://changingminds.org/explanations/research/analysis/choose_correlation.htm



WARNING: Correlation is not causation!



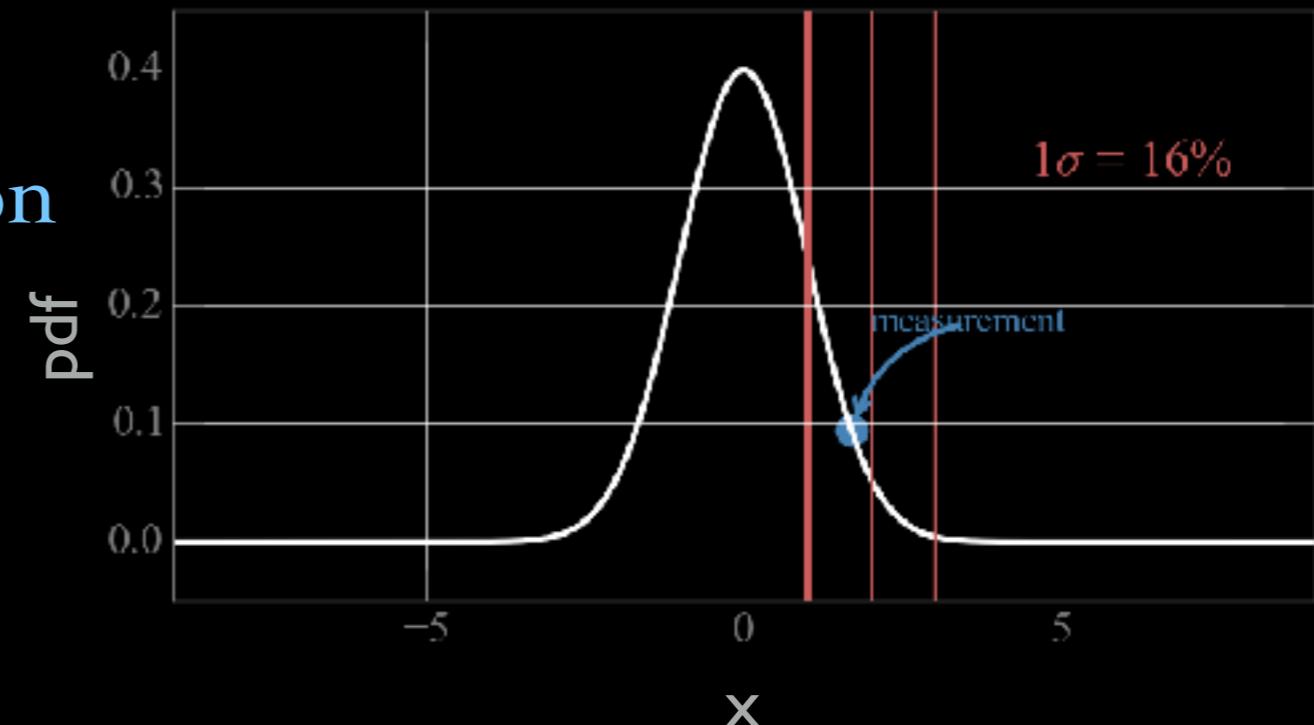
<http://www.tylervigen.com/spurious-correlations>

Tests for correlation and independence (continuous variables)

Probability Distribution Function

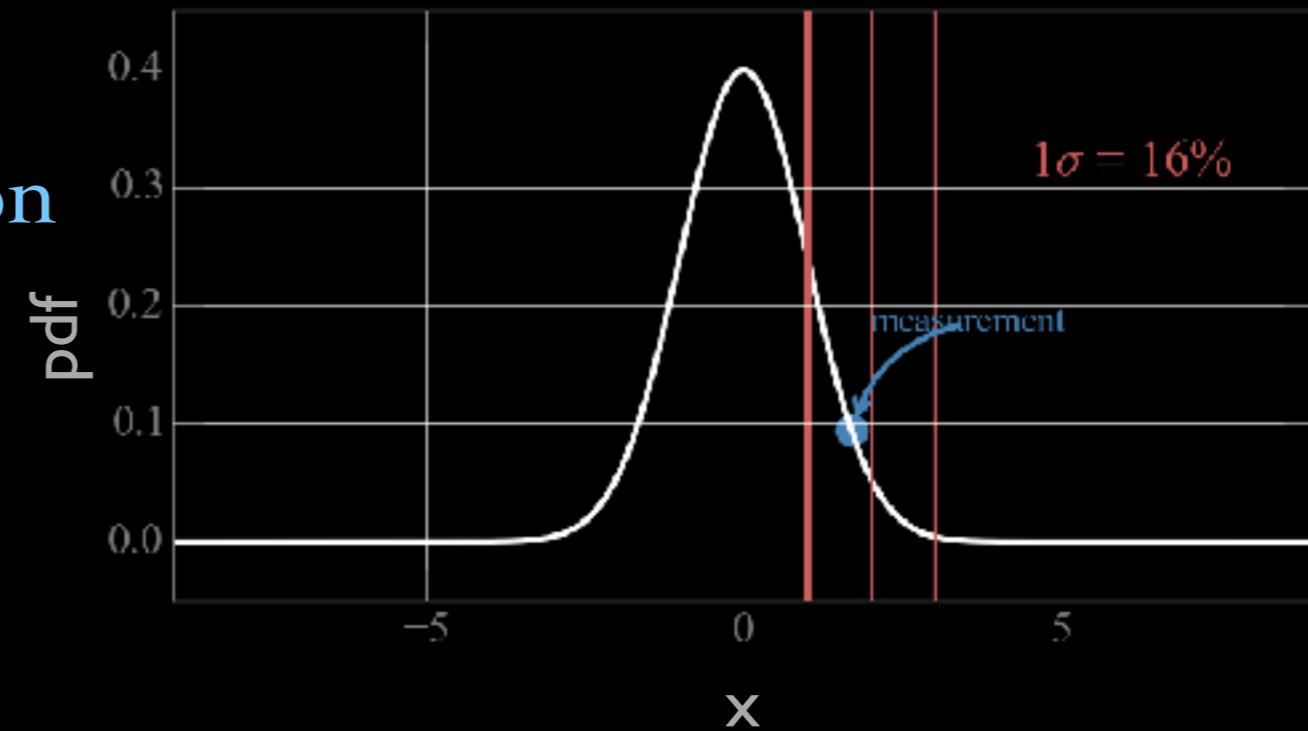
$$f_{x_0}(x) \sim p(x=x_0)$$

$$f_{x_0}(x) \sim p(x > x_0 - dx) \cap p(x < x_0 + dx)$$



Probability Distribution Function

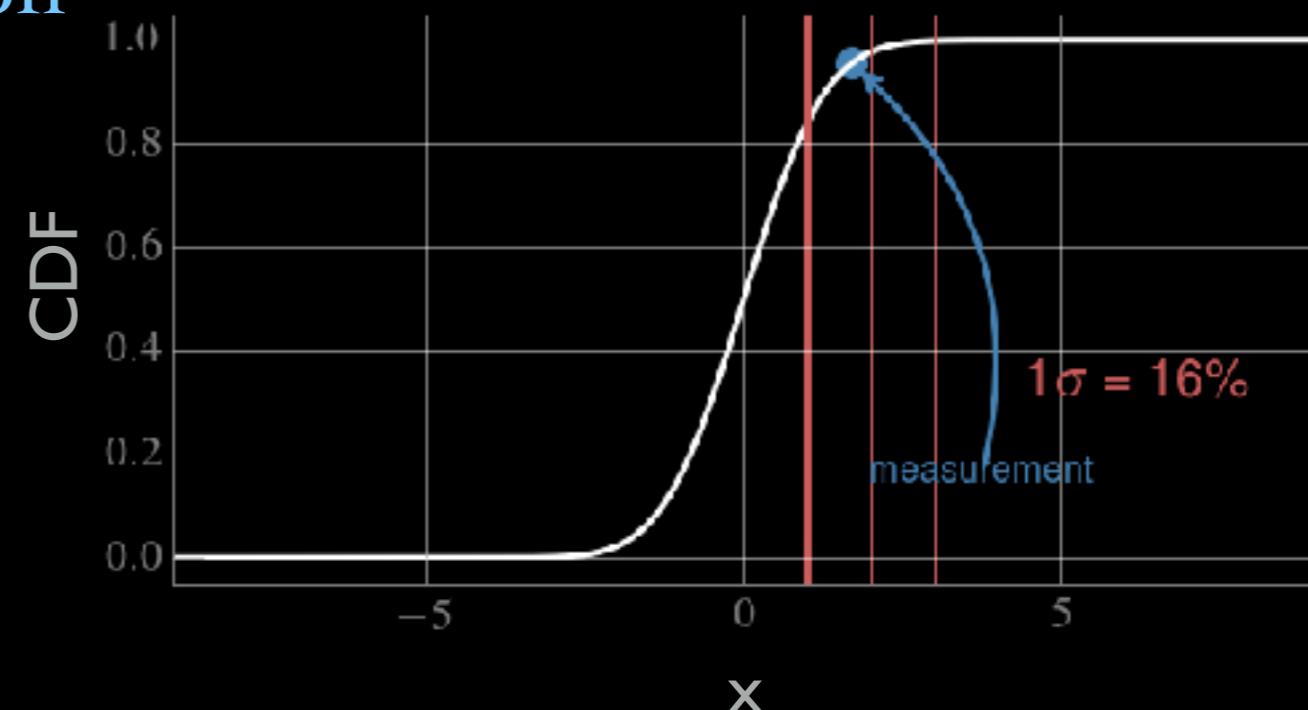
$$f_{x_0}(x) \sim p(x=x_0)$$

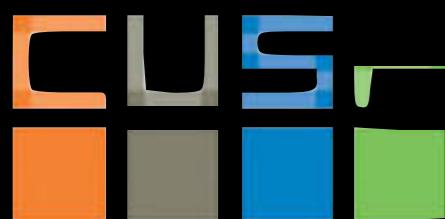
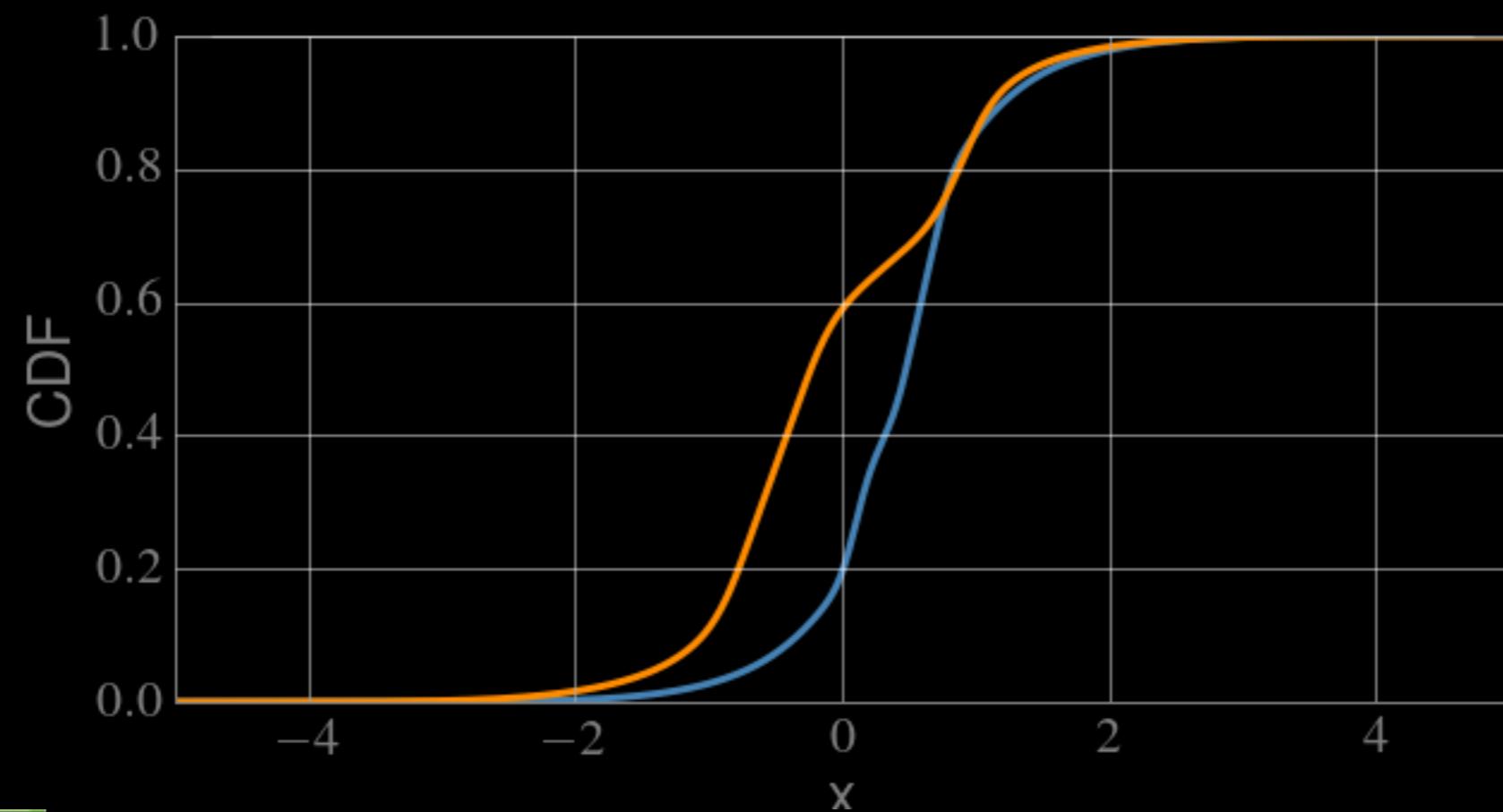
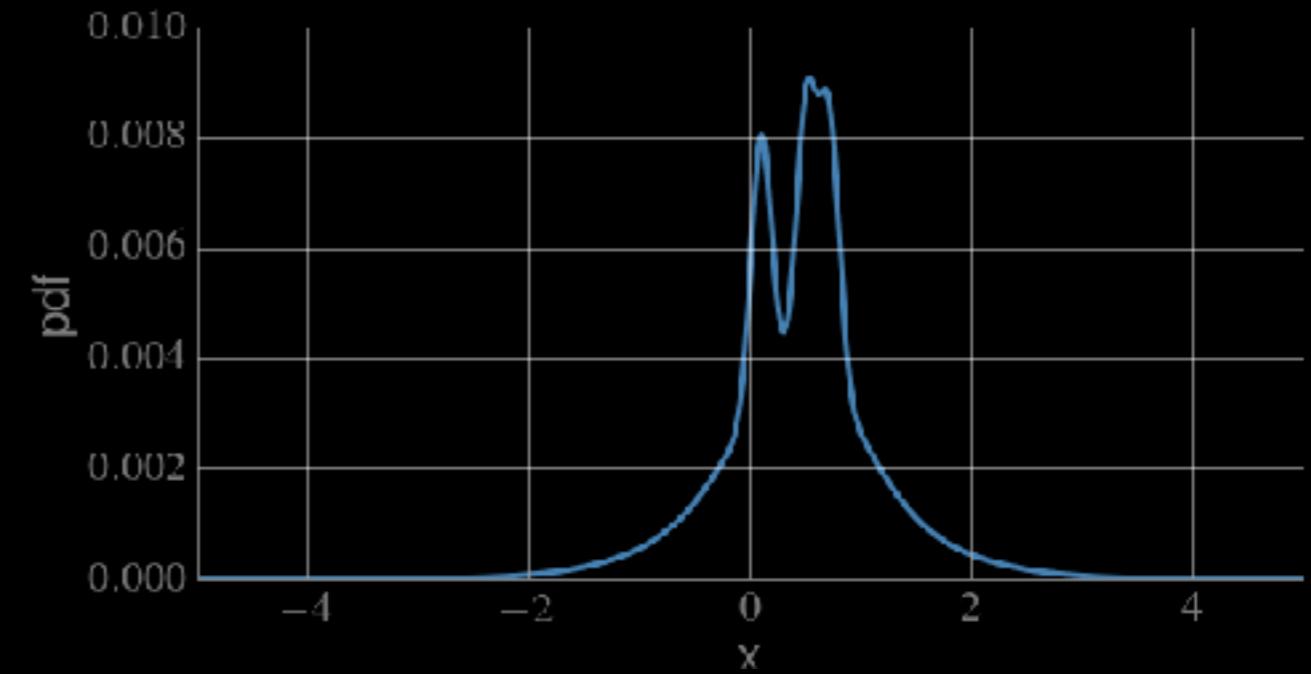
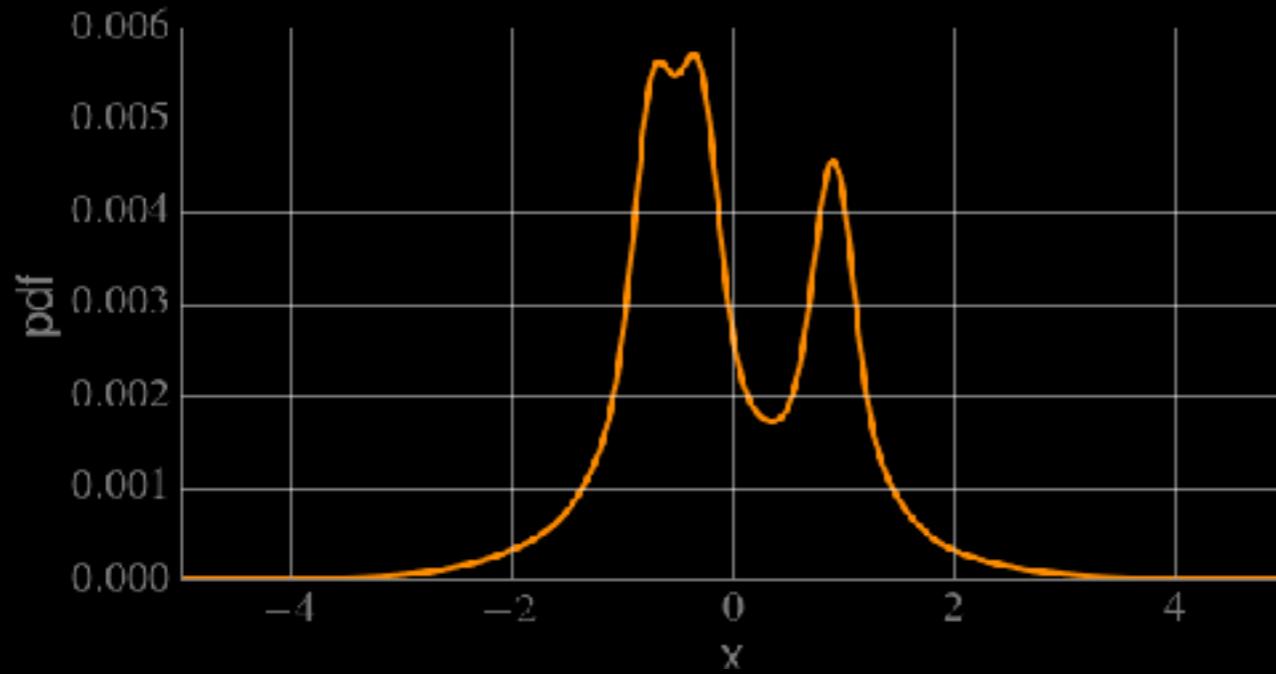


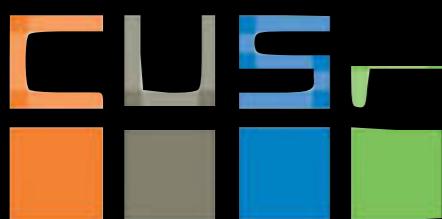
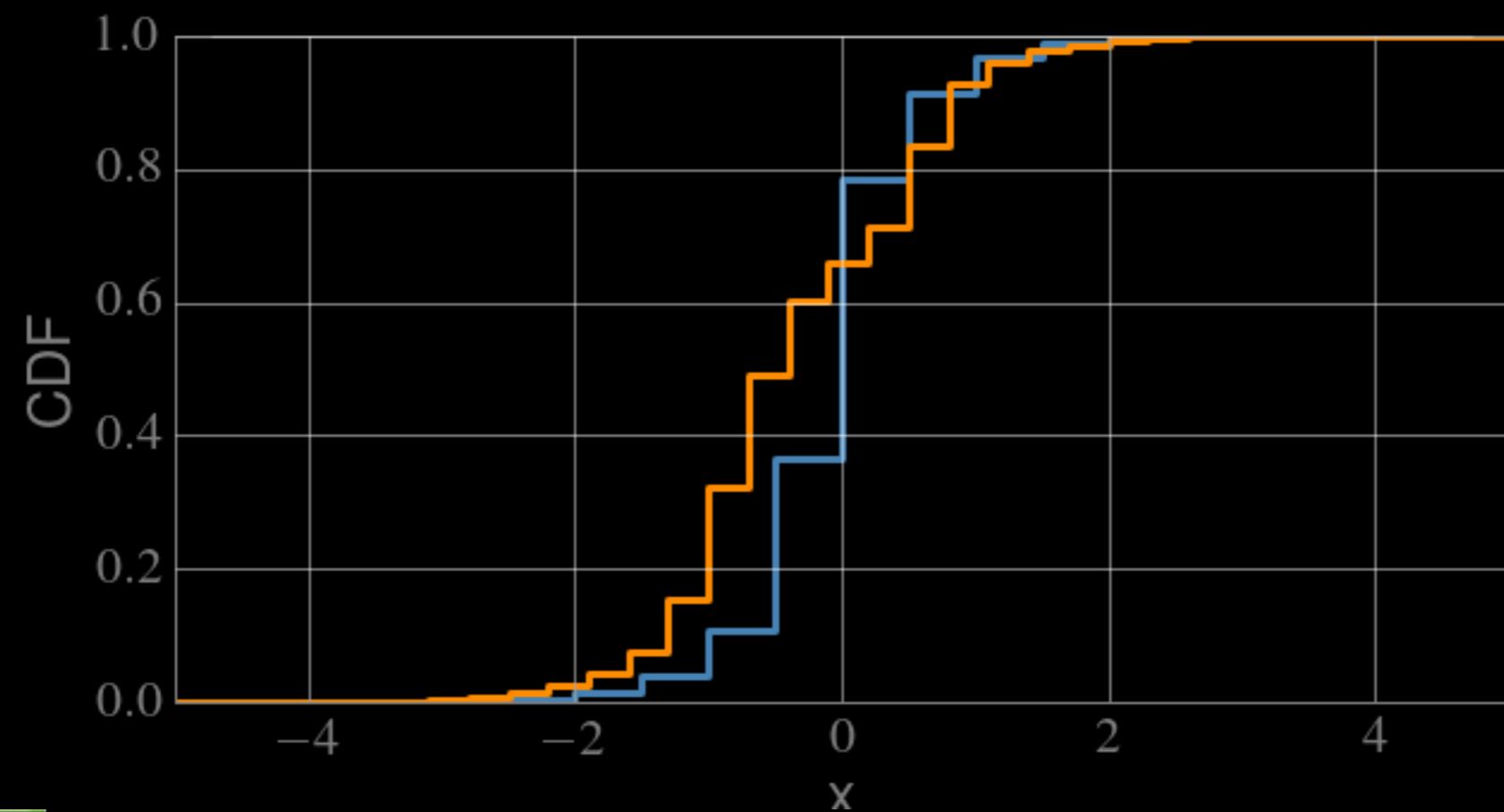
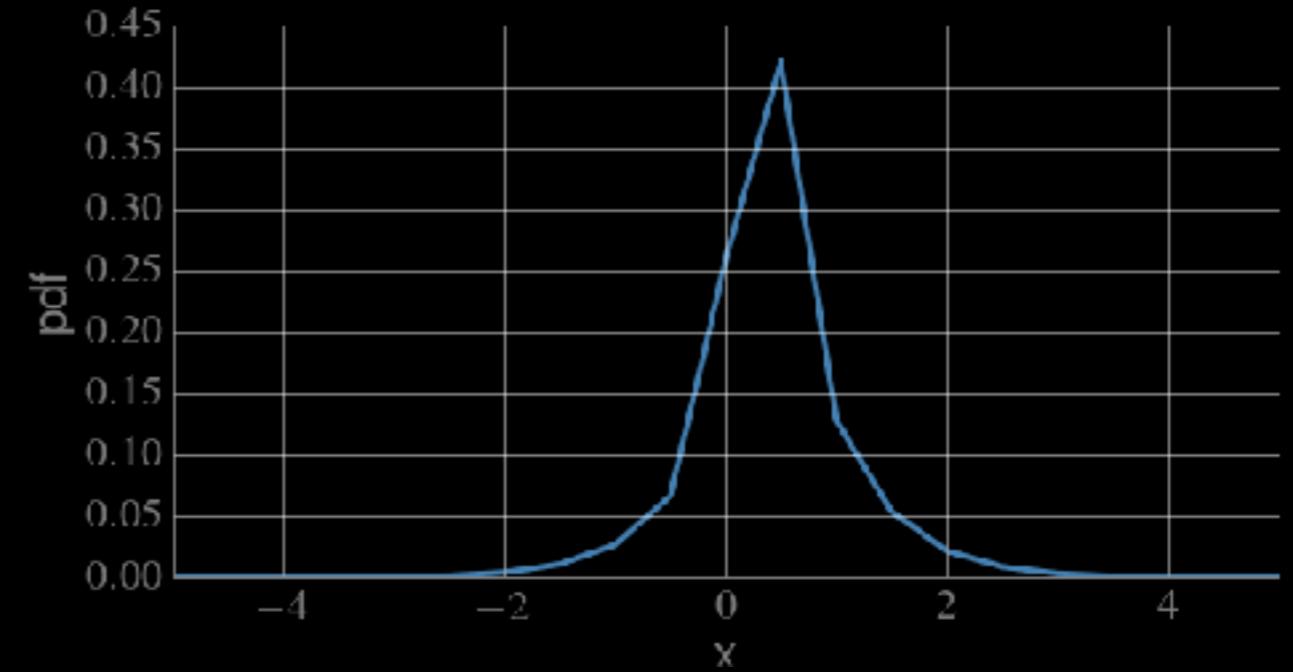
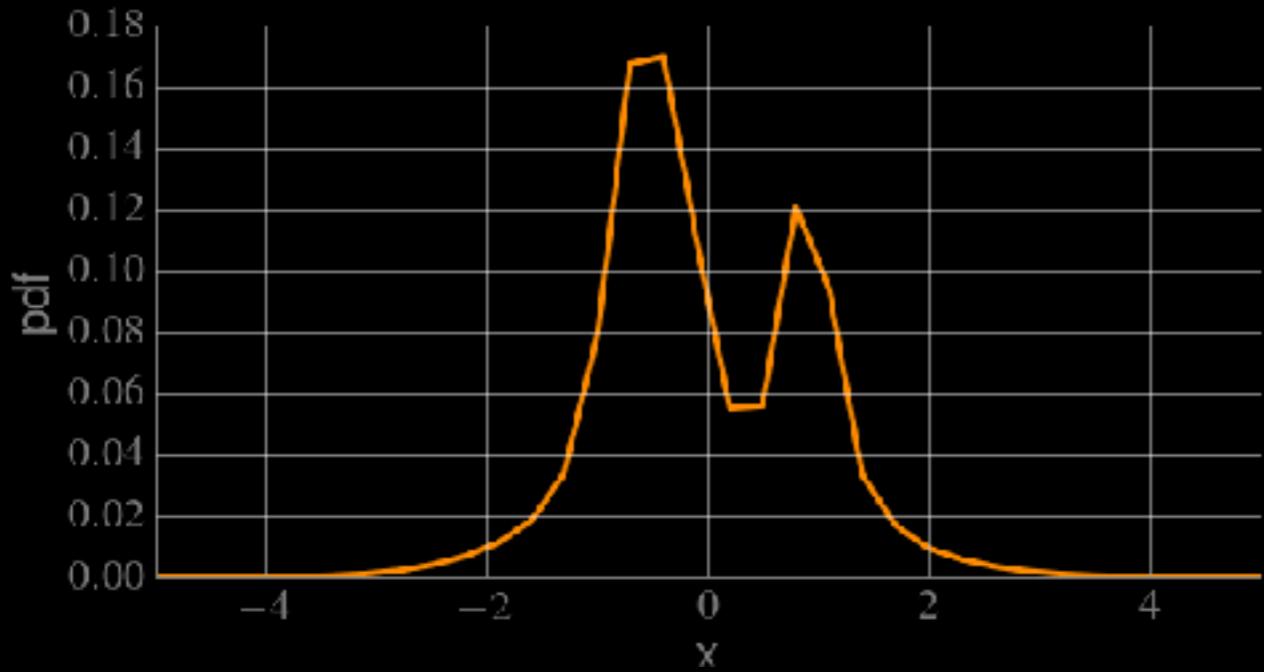
$$f_{x_0}(x) \sim p(x > x_0 - dx) \cap p(x < x_0 + dx)$$

Cumulative Distribution Function

$$F_{x_0}(x) = P(x < x_0)$$







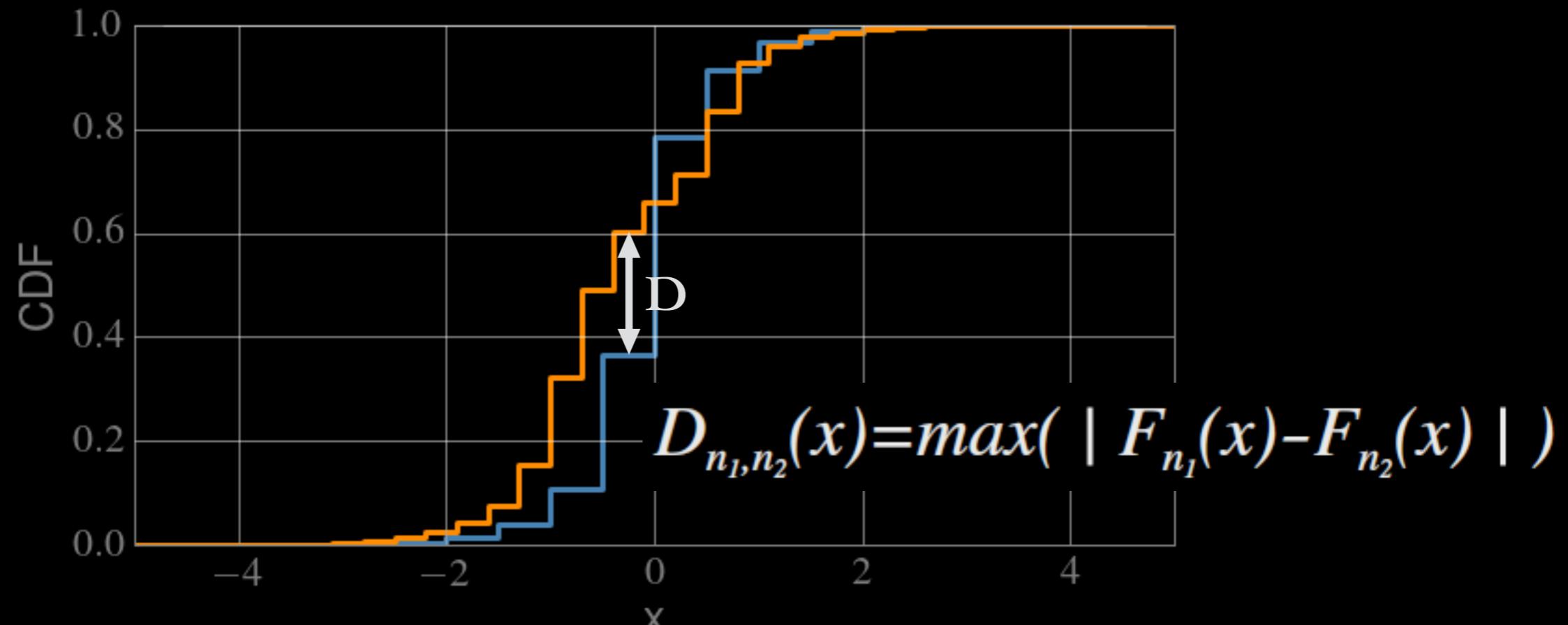
Two sample Kolmogorov Smirnoff test:

null hypothesis H_0 : the samples come from the same parent distribution

H_0 is rejected at level α if $D(n_1, n_2) > c(\alpha) \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$

with $c(\alpha)$ given by a table

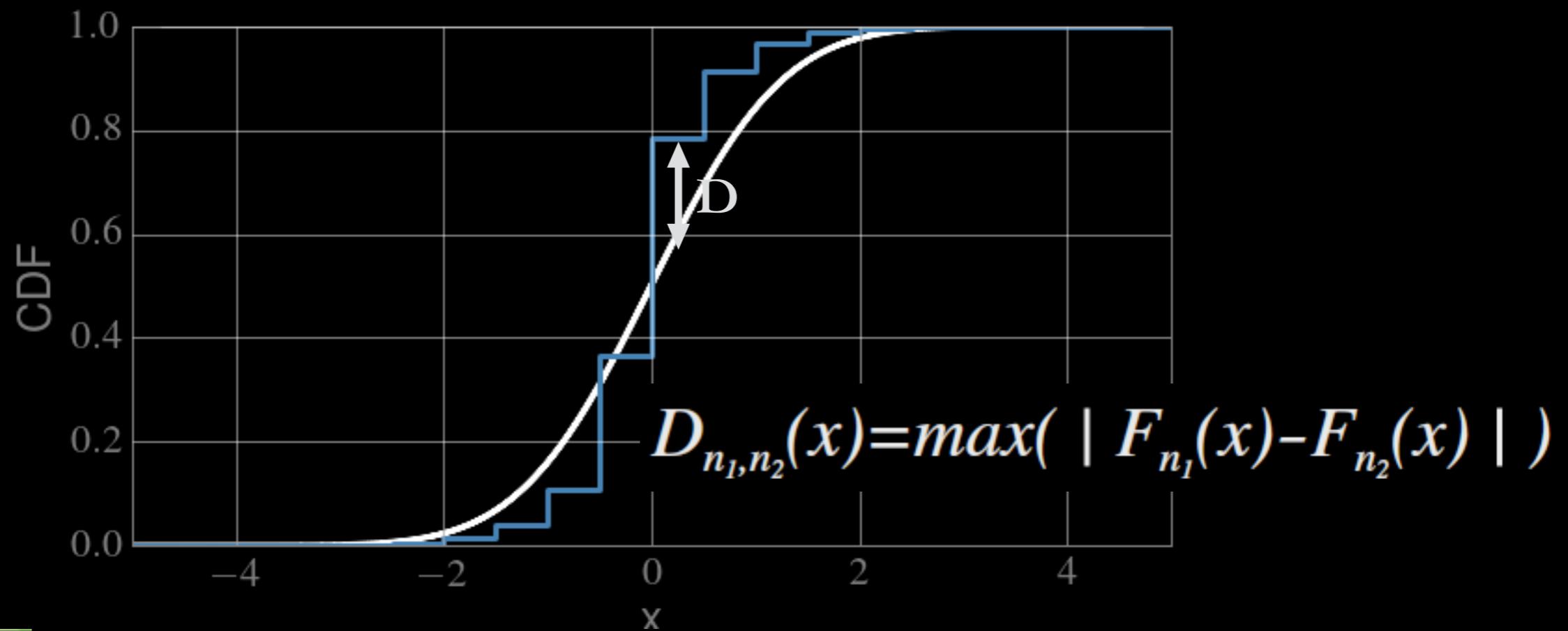
NOTE: it ONLY works in 2D where the Euclidian distance is uniquely defined!



Goodness-of-fit Kolmogorov Smirnoff test:

null hypothesis H_0 : the sample does comes from the model distribution

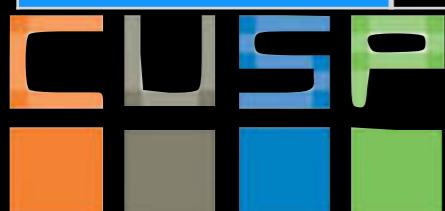
H_0 is rejected at level α if $\sqrt{n} D_n > K_\alpha$ where $P(K \leq K_\alpha) = 1 - \alpha$



Tests Cheat Sheet:

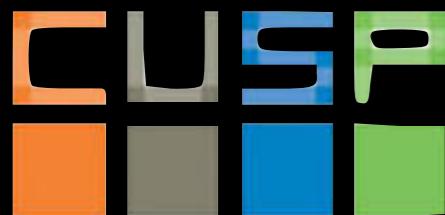
2 (+) samples comparison

	metric (statistic)	compare to	
KS	$D_{n_1, n_2}(x) = \max(F_{n_1}(x) - F_{n_2}(x))$	$c(\alpha) \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$	Non parametric 2 samples only
K-sample Anderson-Darling	$ADK = \frac{n-1}{n^2(k-1)} \sum_{i=1}^k \frac{1}{n(i)} \left(\sum_{j=1}^L h_j \frac{(nF_{ij} - n_i H_j)^2}{H_j(n-H_j) - nh_j/4} \right)$	• AK table	Non parametric, N samples
Pearson's	$r_{xy} = \frac{1}{n-1} \sum_{i=1}^n \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$	The interpretation of a correlation coefficient depends on the context and purpose	-1 anticorrelated 0 uncorrelated 1 correlated .
Spearman's	$\rho = 1 - \frac{6 \sum (x_i - y_i)^2}{n(n^2 - 1)}$	t test $t = r \sqrt{\frac{n-2}{1-r^2}}$	ranked data only p-value from t-test, Fisher's transformation +z score, permutation test



assignment 3: Z-test and chi sq test

- Reproduce the analysis of the Hard to Employ program. Reproduce the results in cell 2 and 10. Follow the notebook in the HW directory (turn in the python notebook in the HW4_<netID> directory)



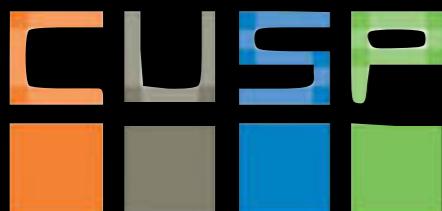
assignment 4. Compare Tests for Correlation

The following are 3 tests that assess correlation between 2 samples of citibike data:

- **Pearson's test** (answer: are the 2 samples correlated?)
- **Spearman's test** (answer: are the 2 samples correlated?)
- **K-S test** (answer: are the samples likely to come from the same parent distribution?)

Use: age of bikers for 2 genders. State your result in words in terms of the Null Hypothesis

Extra Credit: Use the age of bikers in day vs night and assess the correlation/independence of the 2 samples in each case..



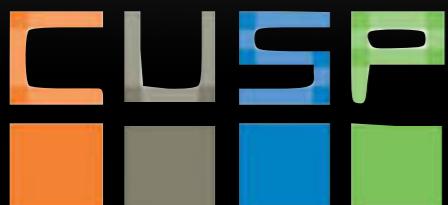
Beyond Significance Based Hypothesis Rejection

BEYOND SIGNIFICANCE TESTING

**REFORMING DATA ANALYSIS METHODS
IN BEHAVIORAL RESEARCH**

REX B. KLINE

РЕХ В. КЛІН



Recommendations

Specific suggestions are listed and then discussed afterward:

1. Only in very exploratory research where it is unknown whether effects exist may a primary role for NHST be appropriate.
2. If statistical tests are used, (a) information about power must be reported, and (b) the null hypothesis must be plausible.
3. In any kind of behavioral research, it is not acceptable anymore to describe results solely in terms of NHST outcomes.
4. Drop the word “significant” from our data analysis vocabulary. Use it only in its everyday sense to describe something actually noteworthy or important.
5. It is the researcher’s responsibility to report and interpret, whenever possible, effect size estimates and confidence intervals for primary results. This does not mean to report effect sizes only for H_0 rejections.
6. It is also the researcher’s responsibility to demonstrate the substantive (theoretical, clinical, or practical) significance of the results. Statistical tests are inadequate for this purpose.
7. Replication is the best way to deal with sampling error.
8. Education in statistical methods needs to be reformed, too. The role of NHST should be greatly deemphasized so that more time can be spent showing students how to determine whether a result has substantive significance and how to replicate it.
9. Researchers need more help from their statistical software to compute effect sizes and confidence intervals.

compute effect sizes and confidence intervals.
d. Researchers need more help from their statistical software to
use it.
whether a result has substantive significance and how to replicate it.
more time can be spent showing students how to determine whether a result has substantive significance and how to replicate it.

IV: Statistical analysis

Arguments against NHST

$$1, P(H_0|D) = \frac{P(H_0)P(D|H_0)}{P(H_0)} \Rightarrow P(H_0|D) \neq P(D|H_0)$$

2. Significance depends on sample size, e.g.:

$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{N}}$$

if you measure the same effect in two samples of different size the significance you measure will be different

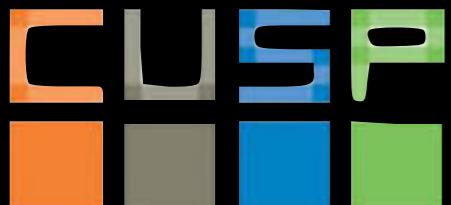
Effect size to measure the strength of a phenomenon

Estimate the **strength** of, for example, an apparent relationship, rather than assigning a significance level.

The effect size does not directly determine the significance level, or vice versa.

Jacob Cohen (1960) "***A coefficient of agreement for nominal scales***" Educational and Psychological Measurement, 20 (1): 37–46,

[https://github.com/fedhere/PUI2016_fb55/blob/master/
HW3_fb55/citibikes_gender.ipynb](https://github.com/fedhere/PUI2016_fb55/blob/master/ HW3_fb55/citibikes_gender.ipynb)

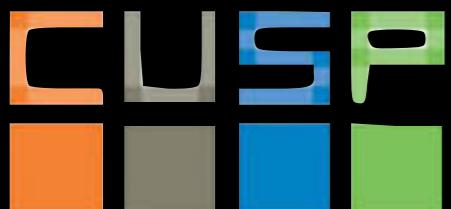


small,medium,large

Effect size to measure the strength of a phenomenon



Cohen's <i>d</i>	$d = \frac{\bar{x}_1 - \bar{x}_2}{s} \quad s = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$	for Z and t	.20 .50 .80
Cohen's <i>w</i>	$w = \sqrt{\sum_{i=1}^N \frac{(p_{oi} - p_{1i})^2}{p_{oi}}}$	for X²	.10 .30 .50
Cohen's <i>h</i>	$h = 2(\arcsin \sqrt{p_1} - \arcsin \sqrt{p_2})$	for proportions	.20 .50 .80
<i>f</i> -squared	$\hat{f}_{effect} = \sqrt{(df_{effect}/N)(F_{effect}-1)}$	for F-test/ ANOVA	.10 .25 .40
R ² coefficient of determination		for regression	.02 .15 .35



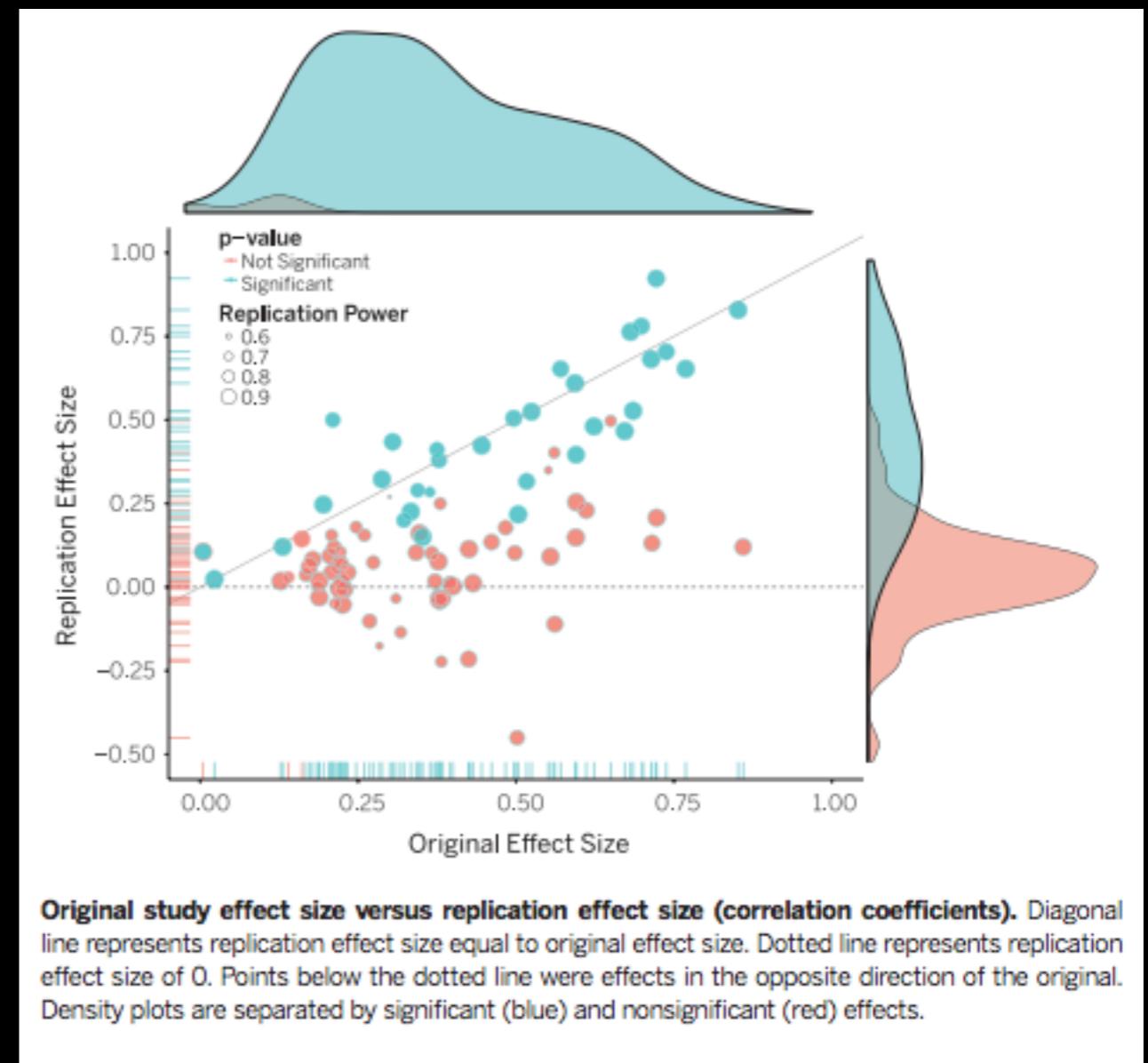
Homework: READING

RESEARCH ARTICLE SUMMARY

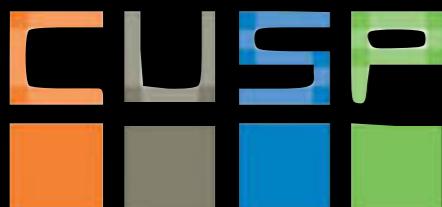
PSYCHOLOGY

Estimating the reproducibility of psychological science

Open Science Collaboration*



<http://www.sciencemag.org/content/349/6251/aac4716.full.pdf>



IV: Statistical analysis

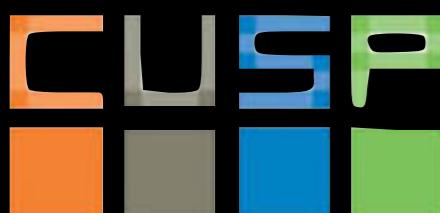
Homework: 2. Compare Tests for Goodness of fit (synthetic data)

The following are 5 tests that can be used to assess the goodness of fit of a model

- **K-S**
- **Pearson's Chi squared**
- **Anderson-Darling**
- **K-L Divergence**
- **(Likelihood ratio, you do not need to do this yet!)**

Use KS, K-L divergence, and one more test (AD or Chisq) to quantify the difference between a binomial & Gaussian distribution and a Poisson & Gaussian distribution as a function of the parameters of the first distribution (np for binomial, λ for poisson)

For each test plot the relevant parameter (the K-L parameter, Anderson-Darling statistics, p-value for KS, Chi-sq parameter), against the distribution parameter (np, λ)

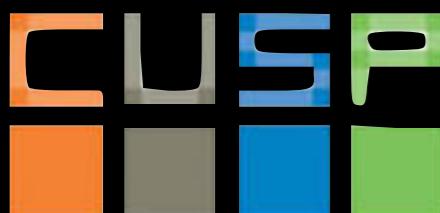


Homework: 3. Compare Tests for Goodness of fit (real data)

Test whether a gaussian model for the age distribution of citibike drivers is a sensible model, or if you can find a better fit with another distribution. Use 2 tests (from the previous exercise) to do this. Test at least 2 distributions.

Divide your riders sample by seasons: SpringSummer vs FallWinter and see if you notice any changes.

Optional (extra credit): Divide your sample geographically: by Borrow + split Manhattan in an Uptown and a Downtown sample (use your discretion to do so) and see if you notice any differences in how the age distribution can be modeled.



Compare Tests for Correlation and Goodness of fit:

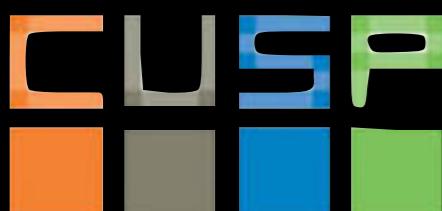
The following are 3 tests that assess correlation between 2 samples:

- Pearson's test e.g. Estimate number of MTA Bus passengers at different hours
- Spearman's test (morning, afternoon, or in time chunks as 7:30-10:30, 10:30-1:30, 1:30-3, 3:6, 6:9, you can do it per bus line, per origin or destination neighborhood...)
- K-S test

The following are 5 tests that can be used to assess the goodness of fit of a model

- K-S
- Pearson's Chi squared
- Anderson-Darling e.g. Estimate number of MTA Bus passengers per bus line within an interval of time: are the passengers randomly distributing on busses.
- K-L Divergence
- Likelihood ratio

In the lab/homework you will 2 out of these tests to assess if 2 samples are related (measure their correlation, or decide if they come from the same parent distributions) and 2 out of the goodness of fit tests to see if a dataset comes from a normal distribution, or from another distribution (where possible) of your choice.



Compare Tests for Correlation and Goodness of fit:

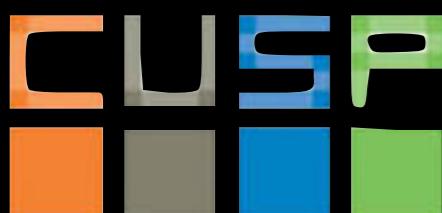
The following are 3 tests that assess correlation between 2 samples:

- Pearson's test e.g. Age distribution of male vs female Citibikes riders. Age
- Spearman's test distribution in different seasons. Age distribution for long/short
- K-S test trips

The following are 5 tests that can be used to assess the goodness of fit of a model

- K-S
 - Pearson's Chi squared
 - Anderson-Darling
 - K-L Divergence
 - Likelihood ratio
- e.g. Estimate Age of riders: could be Gaussian, could be lognormal, power law, bimodal

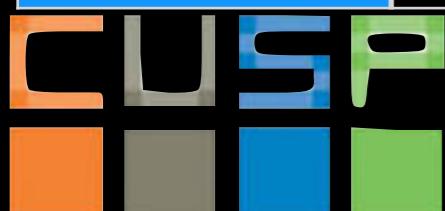
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Tests Cheat Sheet:

2 (+) samples comparison

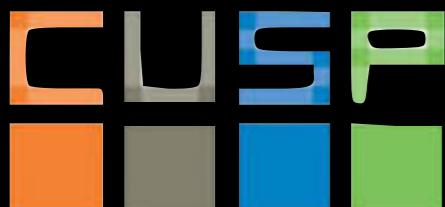
	metric (statistic)	compare to	
KS	$D_{n_1, n_2}(x) = \max(F_{n_1}(x) - F_{n_2}(x))$	$c(\alpha) \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$	Non parametric 2 samples only
K-sample Anderson-Darling	$ADK = \frac{n-1}{n^2(k-1)} \sum_{i=1}^k \frac{1}{n(i)} \left(\sum_{j=1}^L h_j \frac{(nF_{ij} - n_i H_j)^2}{H_j(n-H_j) - nh_j/4} \right)$	• AK table	Non parametric, N samples
Pearson's	$r_{xy} = \frac{1}{n-1} \sum_{i=1}^n \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$	The interpretation of a correlation coefficient depends on the context and purpose	-1 anticorrelated 0 uncorrelated 1 correlated .
Spearman's	$\rho = 1 - \frac{6 \sum (x_i - y_i)^2}{n(n^2 - 1)}$	t test $t = r \sqrt{\frac{n-2}{1-r^2}}$	ranked data only p-value from t-test, Fisher's transformation +z score, permutation test



Tests Cheat Sheet:

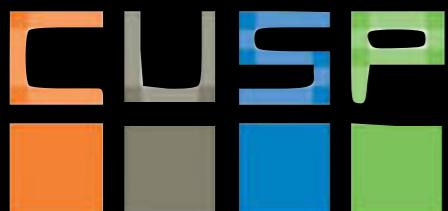
goodness of fit

	metric (statistic)	compare to	
KS	$D_{n_1, n_2}(x) = \max(F_n(x) - F(x))$	$\frac{K_\alpha}{\sqrt{n}}$	power in the core only
Pearson's chi square	$\chi^2_{red} = \frac{\chi^2}{df} = \frac{1}{df} \sum \frac{(O-E)^2}{\sigma^2}$	scipy.stats.chisquare(f_obs, f_exp=None, ddof=0, axis=0)[0]	
Anderson-Darling	$A = n \int_{-\infty}^{\infty} \frac{(F_n(x) - F(x))^2}{F(x) (1-F(x))} dF(x)$	scipy.stats.anderson(x, dist='norm')	power in the tails
K-L divergence	$D_{KL} = - \int_x p(x) \log(q(x)) + p(x) \log(p(x))$	scipy.stats.entropy(pk, qk=<not None>)	relates to information entropy
Likelihood ratio	$\frac{L(\text{model 1} \text{data})}{L(\text{model 2} \text{data})}$		suitable to bayesian analysis



MUST KNOWS:

- How to choose (and perform) a statistical test
- Statistical errors
- how to perform Z and chisel test
- PDF vs CDF
- correlation vs causation
- KS test for 2 samples, Pearson's, Spareman



Resources:

Sarah Boslaugh, Dr. Paul Andrew Watters, 2008

Statistics in a Nutshell (Chapters 3,4,5)

https://books.google.com/books/about/Statistics_in_a_Nutshell.html?id=ZnhgO65Pyl4C

David M. Lane et al.

Introduction to Statistics (XVIII)

http://onlinestatbook.com/Online_Statistics_Education.epub

<http://onlinestatbook.com/2/index.html>

Reckova & Irsova

Publication Bias in Measuring Climate Sensitivity

IES Working Paper: 14/2015

http://salserver.org.aalto.fi/vanhat_sivut/Opinnot/Mat-2.4108/pdf-files/emet03.pdf

