Urban Informatics

Fall 2017

dr. federica bianco fbianco@nyu.edu

@fedhere

CUSP

Recap:

- Good practices with data: falsifiability, reproducibility
- Basic data retrieving and munging: APIs, Data formats
- Basic statistics: distributions and their moments
- Hypothesis testing: *p*-value, statistical significance

Today:

- How to choose the right statistical test
- Z, t, F test and tests for correlation
- Correlation vs Causation

hypothesis testing

null hypothesis 2 tailed: no relationship between two measured phenomena, no difference among groups
if you have a test control sample: test sample and control sample are the same - no effect

falsify the null hypothesis: do you see an effect?
do you see a difference b/w samples?

hypothesis testing

null hypothesis 1 tailed: measured phenomena larger/smaller for one group than the other groups

falsify the null hypothesis: do you see an effect in a specific direction?
do you see a difference b/w samples
with the correct sign?

A simple (too simple?) answer

p-value a measure of the probability that the result you observed could have been observed by chance under the *Alternative hypothesis*

Steps in Hypothesis Testing

1. Formulate Null (and alternative) Hypothesis
2. Choose a significance level α
3. Measure a statistic for *a sample* to be compared to the *parameter of a population*

OR

Measure a statistic for *two or more samples* to be compared to *each other*

4. Assess if your statistics is significant or not. In practice: compare the statistics (Z, t, F, chisq) with a distribution table

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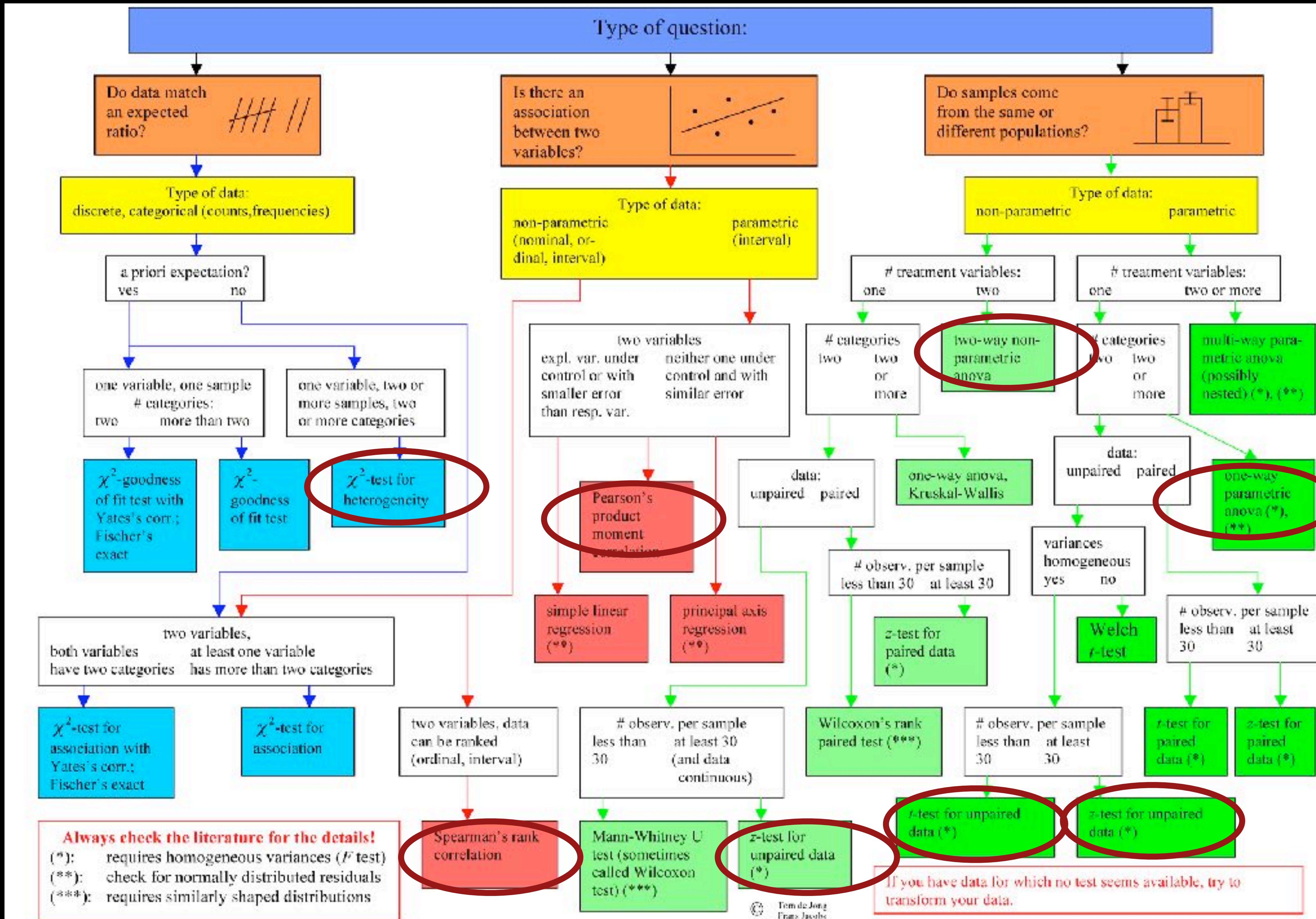
Which statistical test?

What type of question?

What type of data?



When to Use a Particular Statistical Test

http://www.csun.edu/~amarenco/Fcs%20682/When%20to%20use%20what%20test.pdf

Statistical Analyses	Independent Variables		Dependent Variables		Control Variables	Question Answered by the Statistic
	# of IVs	Data Type	# of DVs	Type of Data		
Chi square	1	categorical	1	categorical	0	Do differences exist between groups?
***t*-Test**	1	dichotomous	1	continuous	0	Do differences exist between 2 groups on one DV?
ANOVA	1 +	categorical	1	continuous	0	Do differences exist between 2 or more groups on one DV?
ANCOVA	1 +	categorical	1	continuous	1 +	Do differences exist between 2 or more groups after controlling for CVs on one DV?
MANOVA	1 +	categorical	2 +	continuous	0	Do differences exist between 2 or more groups on multiple DVs?
MANCOVA	1 +	categorical	2 +	continuous	1 +	Do differences exist between 2 or more groups after controlling for CVs on multiple Dvs?
Correlation	1	dichotomous or continuous	1	continuous	0	How strongly and in what direction (i.e., +, -) are the IV and DV related?
Multiple regression	2 +	dichotomous or continuous	1	continuous	0	How much variance in the DV is accounted for by linear combination of the IVs? Also, how strongly related to the DV is the beta coefficient for each IV?
Path analysis	2 +	continuous	1 +	continuous	0	What are the direct and indirect effects of predictor variables on the DV?
Logistic Regression	1 +	categorical or continuous	1	dichotomous	0	What is the odds probability of the DV occurring as the values of the IVs change?

assignment 1:

Browse **PlosOne** for papers that uses 3 of the tests

http://www.csun.edu/~amarenco/Fcs%20682/When%20to%20use%20what%20test.pdf

choose 1 choose 1

choose 1 ->

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assignment 2:

Example: ANCOVA

Prepare an MD file NHST.md with the following table for each of the three papers/tests:

we tested whether […] self-affirmation improves problem-solving performance in chronically stressed participants.

http://journals.plos.org/plosone/article?id=10.1371/journal.pone.0062593

Statistical Analyses	Dependent Variables	Independent Variables	Control Variables	Question Answered
ANCOVA	Rating of their values (ordinal)	Did Self Affirmation or not (categorical)	age (continuous)	Do self-affirmation group rates their value more than control group

Comparing to population

Standard DEVIATION of Sample Estimates

Sample mean, $\overline{x}$	$Z = \dfrac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{n}}$	$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$
Sample proportion, p	$z = \dfrac{p - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$	$\sigma_p = \sqrt{\dfrac{P(1-P)}{n}}$
Difference between proportions, p1 - p2	$z = \dfrac{(p_2 - p_1)}{\sqrt{p(1-p)(\dfrac{1}{n_2} + \dfrac{1}{n_1})}}, p = \dfrac{p_2 n_2 + p_1 n_1}{n_2 + n_1}$	$\sigma_{p_1 - p_2} = \sqrt{\dfrac{P_1(1-P_1)}{n_1} + \dfrac{P_2(1-P_2)}{n_2}}$

use if you know the *parameters* of the *population*:

e.g. in the Z test

Comparing sample to population or between samples

Standard ERROR of Sample Estimates

Sample mean, $\bar{x}$	$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$
Sample proportion, p	$SE_p = \frac{p(1-p)}{n}$
Difference between means, x1 - x2	$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
Difference between proportions, p1 - p2	$SE_{p_1 - p_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$

use if you DO NOT know the parameters of the population:

e.g. in the t-test

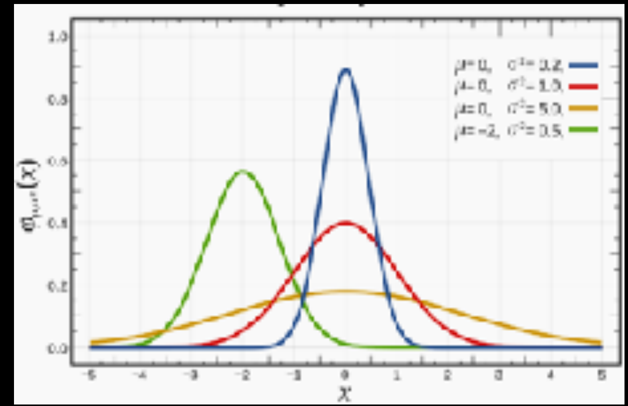
What is the distribution of a statistics?

To measure the probability of the measured value of a satistic I need to know *how the statistics is distributed* (under the alternative hypothesis)

Each statistics follows some distribution, which though?

Z statistics Gaussian

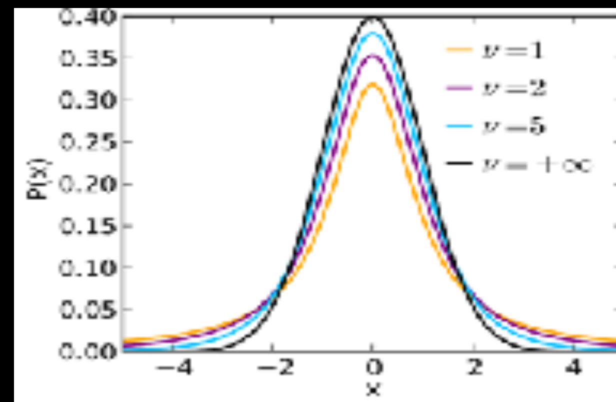
$$Z = \frac{\mu - \bar{x}}{\sigma/\sqrt{n}}$$



Notation	$\mathcal{N}(\mu,\sigma^2)$
Parameters	$\mu \in \mathbb{R}$ — mean (location) $\sigma^2 > 0$ — variance (squared scale)
Support	$x \in \mathbb{R}$
PDF	$\frac{1}{\sqrt{2\sigma^2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
CDF	$\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$
Quantile	$\mu+\sigma\sqrt{2}\,\mathrm{erf}^{-1}(2F-1)$
Mean	μ
Median	μ
Mode	μ
Variance	σ^2

Student's *t*

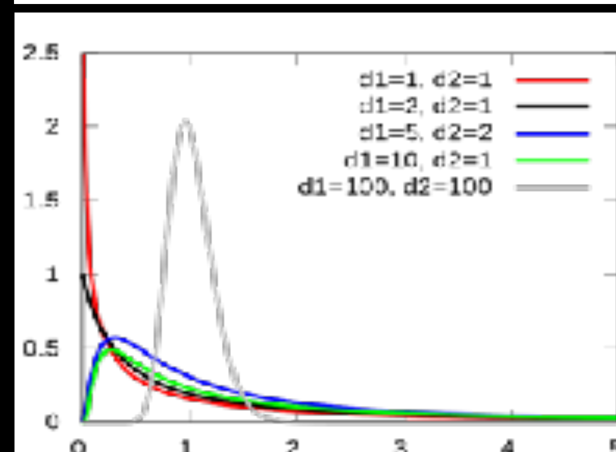
$$t = \frac{\mu - \bar{x}}{s/\sqrt{n}}$$



Parameters	$\nu > 0$ degrees of freedom (real)
Support	$x \in (-\infty; +\infty)$
PDF	$\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$
CDF	$\frac{1}{2}+x\Gamma\left(\frac{\nu+1}{2}\right)\times\frac{{}_2F_1\left(\frac{1}{2},\frac{\nu+1}{2};\frac{3}{2};-\frac{x^2}{\nu}\right)}{\sqrt{\pi\nu}\,\Gamma\left(\frac{\nu}{2}\right)}$ where ${}_2F_1$ is the hypergeometric function
Mean	0 for $\nu > 1$, otherwise undefined
Median	0
Mode	0
Variance	$\frac{\nu}{\nu-2}$ for $\nu > 2$, ∞ for $1 < \nu \le 2$, otherwise undefined

F statistics

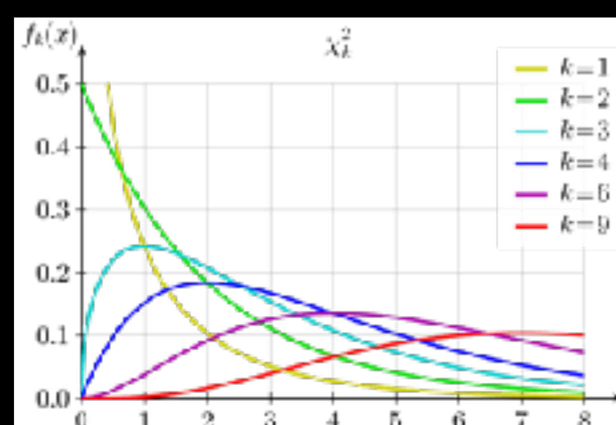
$$F = \frac{\sum_i n_i(\bar{x}_i - \bar{x})^2/(K-1)}{\sum_{ij}(x_{ij} - \bar{x}_i)^2/(N-K)}$$



Parameters	$d_1, d_2 > 0$ deg. of freedom
Support	$x \in [0, +\infty)$
PDF	$\frac{\sqrt{\frac{(d_1 x)^{d_1}\, d_2^{d_2}}{(d_1 x + d_2)^{d_1+d_2}}}}{x\,\mathrm{B}\left(\frac{d_1}{2},\frac{d_2}{2}\right)}$
CDF	$I_{\frac{d_1 x}{d_1 x + d_2}}\left(\frac{d_1}{2},\frac{d_2}{2}\right)$
Mean	$\frac{d_2}{d_2-2}$ for $d_2 > 2$
Mode	$\frac{d_1-2}{d_1}\,\frac{d_2}{d_2+2}$ for $d_1 > 2$
Variance	$\frac{2\,d_2^2\,(d_1+d_2-2)}{d_1(d_2-2)^2(d_2-4)}$ for $d_2 > 4$
Skewness	$\frac{(2d_1+d_2-2)\sqrt{8(d_2-4)}}{(d_2-6)\sqrt{d_1(d_1+d_2-2)}}$ for $d_2 > 6$

Pearson's χ^2

$$\chi^2_P = \sum_i \frac{(O_i - E_i)^2}{E_i}$$



Notation	$\chi^2(k)$ or χ^2_k
Parameters	$k \in \mathbb{N}_{>0}$ (known as "degrees of freedom")
Support	$x \in [0, +\infty)$
PDF	$\frac{1}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)}\, x^{\frac{k}{2}-1} e^{-\frac{x}{2}}$
CDF	$\frac{1}{\Gamma\left(\frac{k}{2}\right)}\,\gamma\left(\frac{k}{2},\frac{x}{2}\right)$
Mean	k
Median	$\approx k\left(1-\frac{2}{9k}\right)^3$
Mode	$\max(k-2, 0)$
Variance	$2k$
Skewness	$\sqrt{8/k}$

goodness of fit χ^2

$$\chi^2_F = \sum_i \frac{(m_i - x_i)^2}{e_i}$$

see Statistics in a Nutshell - V: Inferential Statistics

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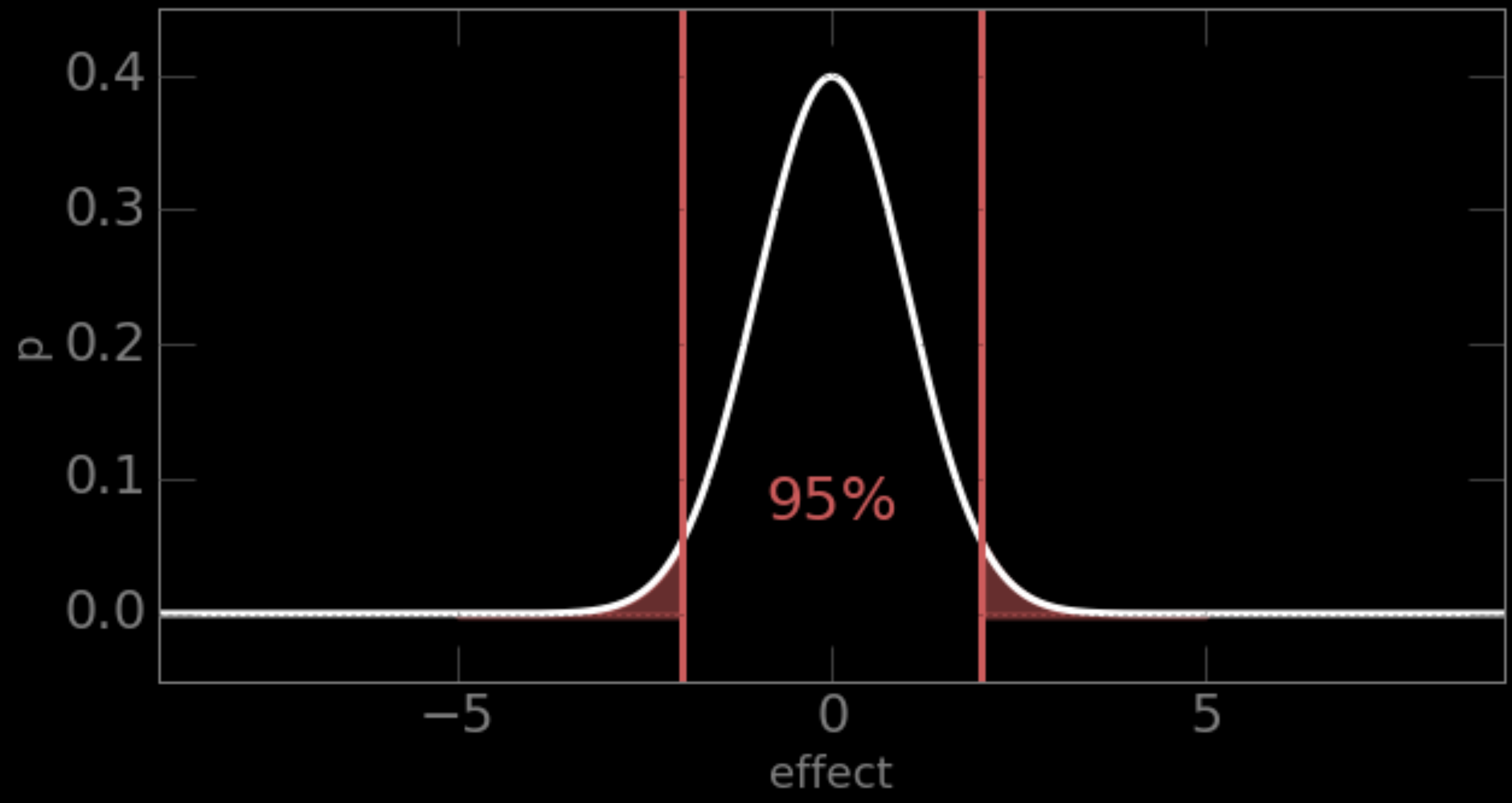
4. Assess if your statistics is significant or not.

if you knew how your statistics should be distributed…

$\alpha = 0.05$

$1-0.05 = 0.95 \quad \Rightarrow 95\%$



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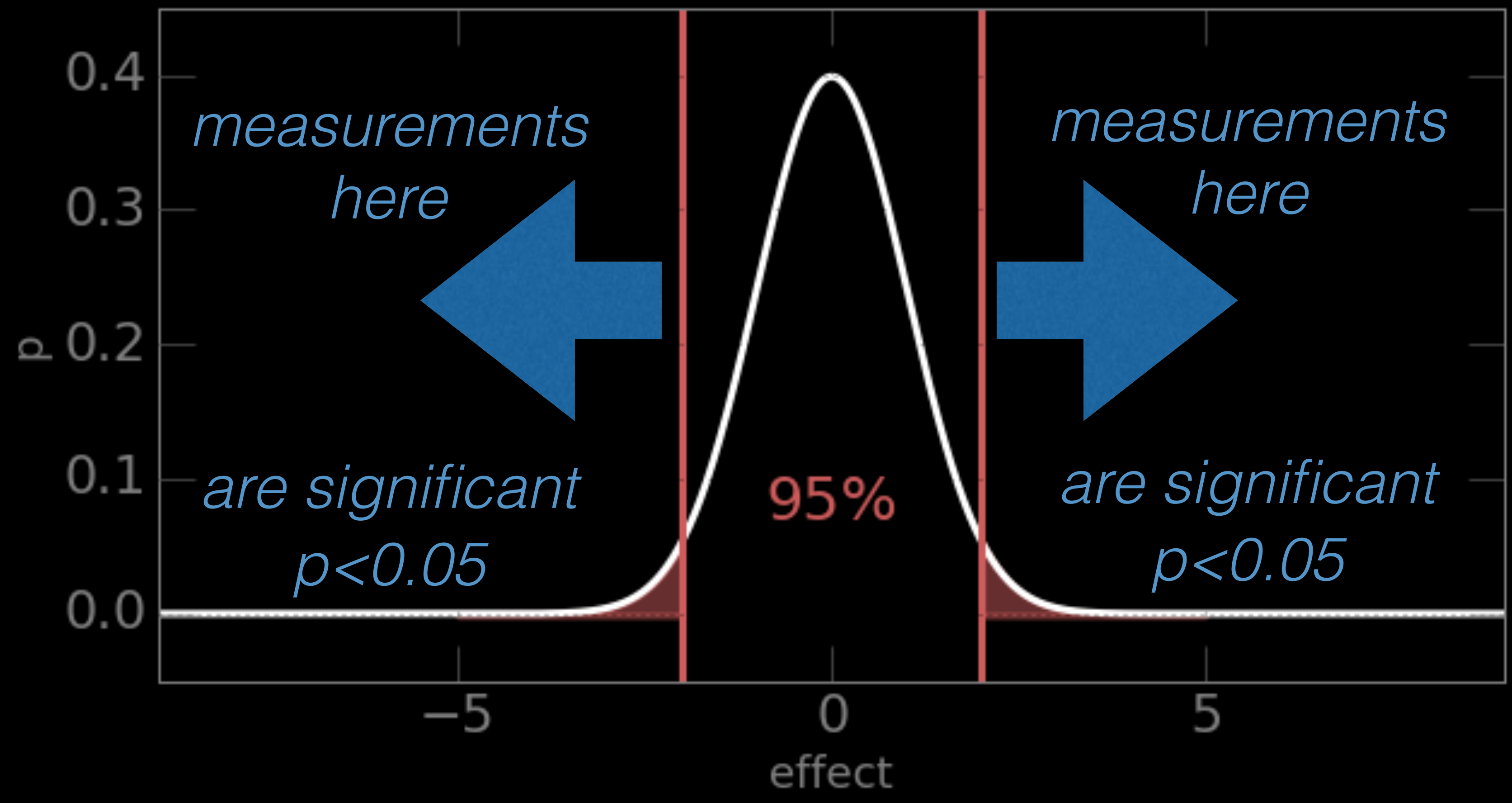
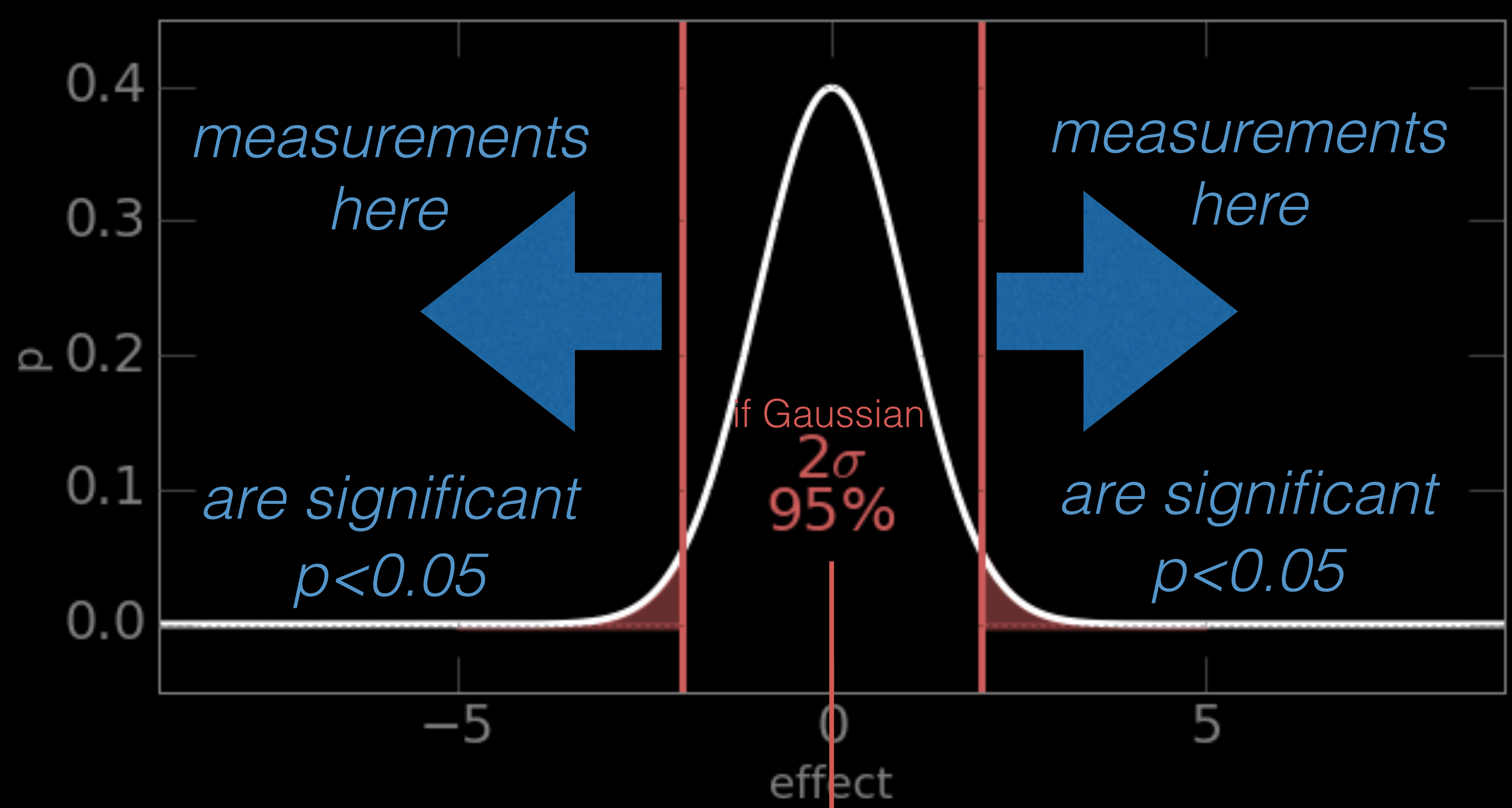


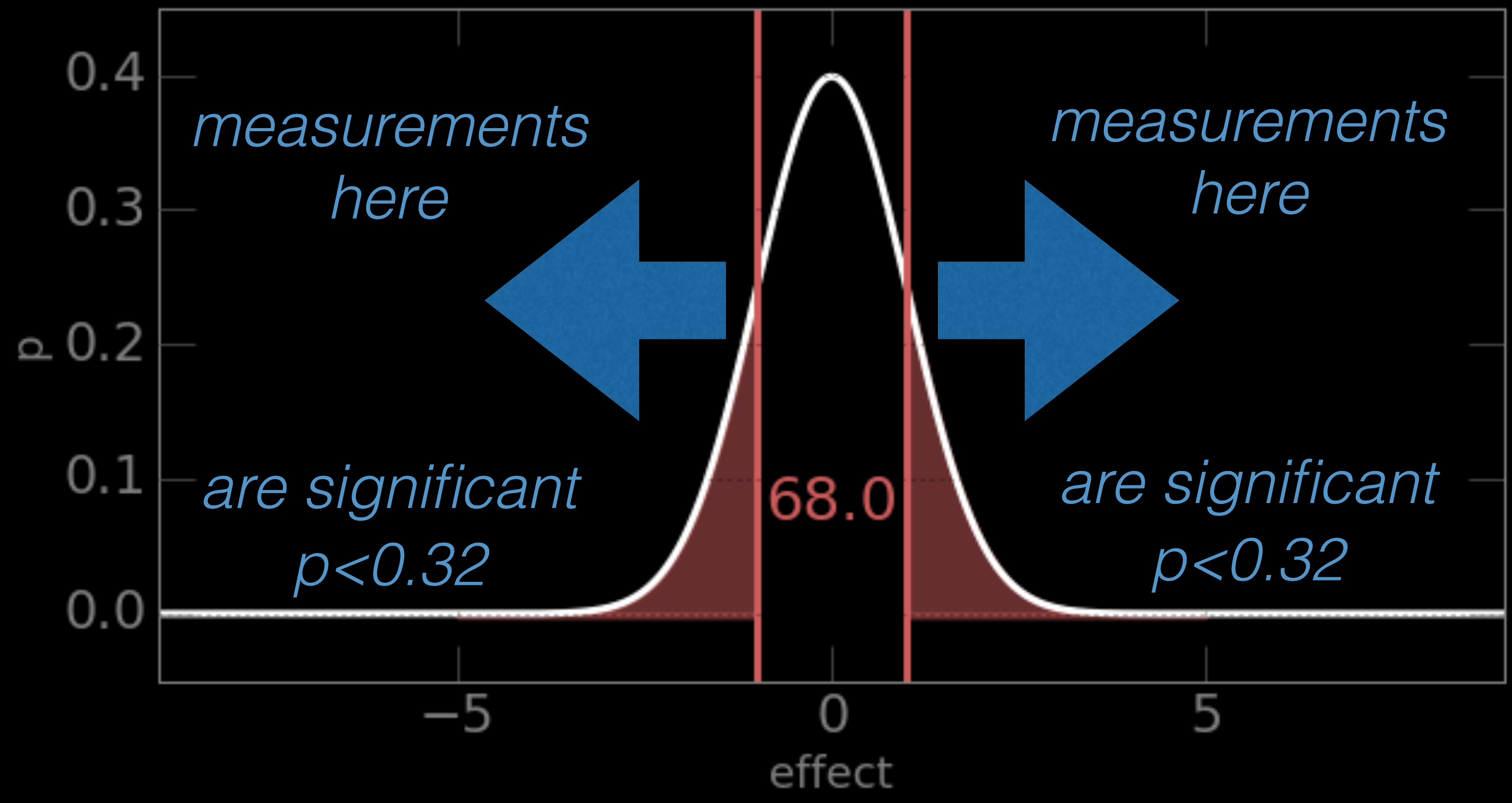
$\alpha = 0.05$

$1-0.05 = 0.95 \quad => 95\%$

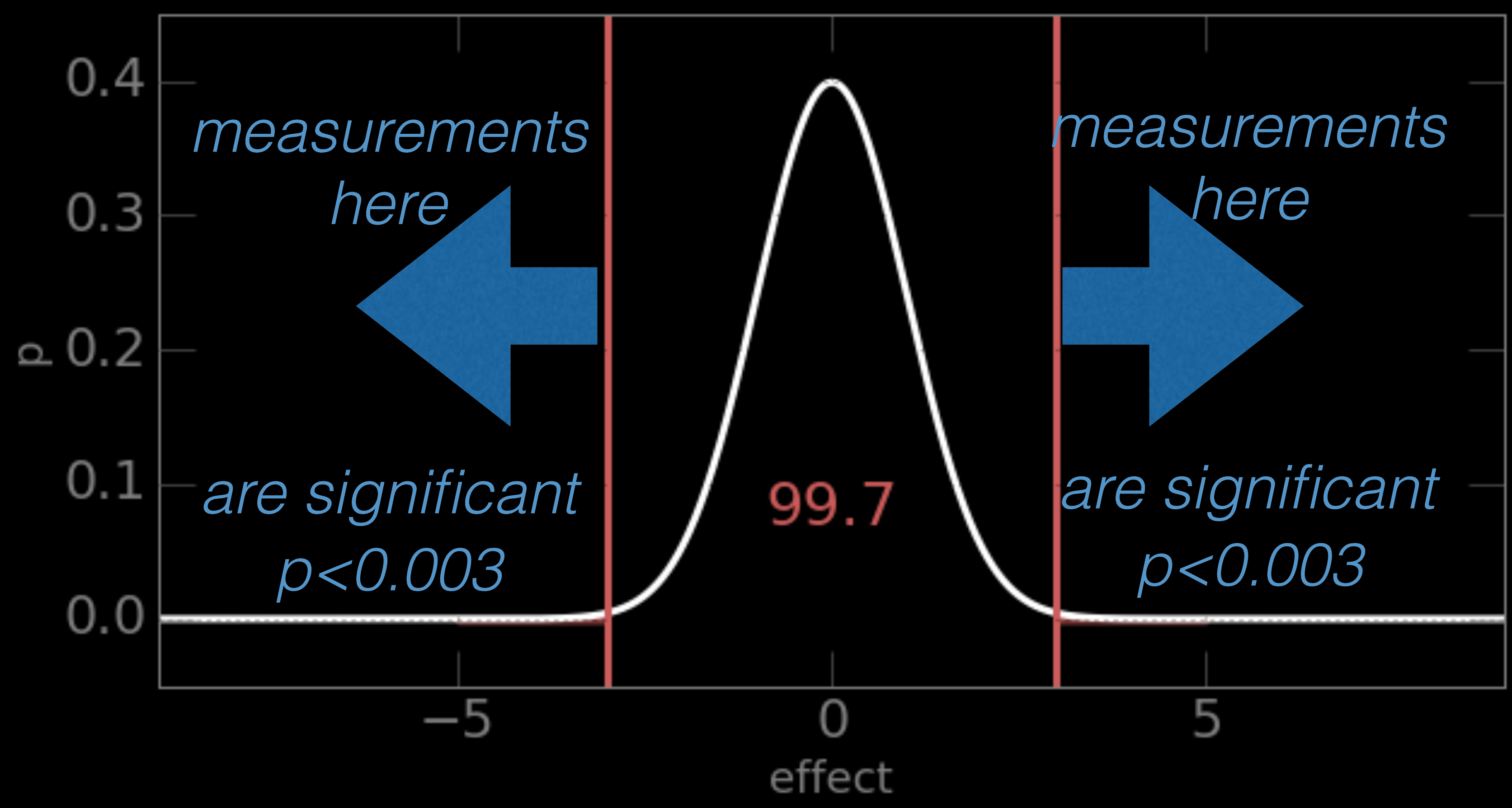


Z test

$\alpha = 0.32$


0.4
0.3
0.2
0.1
0.0
p
measurements here
are significant p<0.32
68.0
measurements here
are significant p<0.32
−5
0
5
effect

$\alpha = 0.003$



measurements here
are significant p<0.003
measurements here
are significant p<0.003
99.7
0.4
0.3
0.2
0.1
0.0
p
−5
0
5
effect

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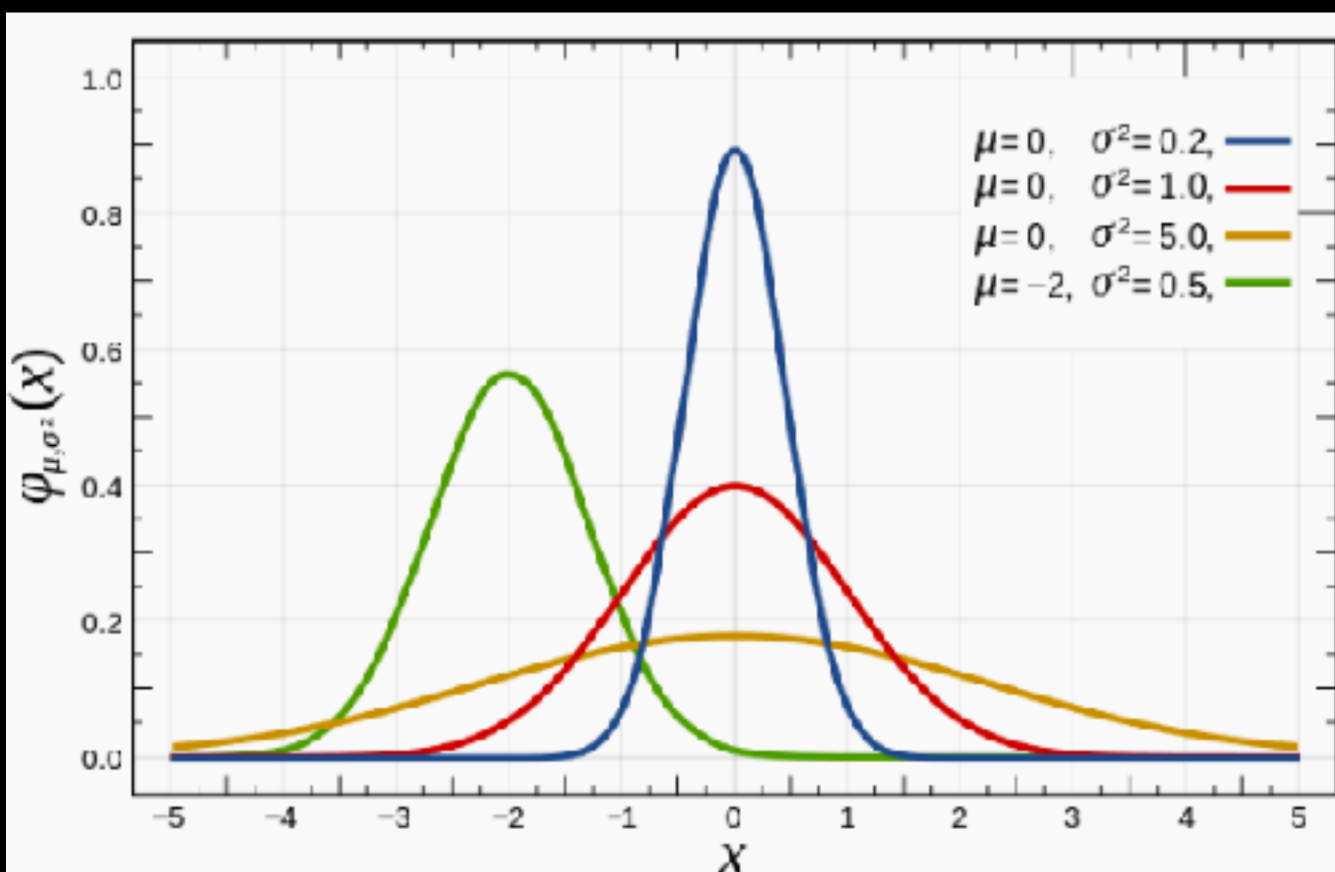
https://documents.software.dell.com/statistics/textbook/distribution-tables

Z statistics

Follows a Gaussian distribution $Z \sim \mathcal{N}$

$$Z = \frac{\mu - \overline{x}}{\sigma/\sqrt{n}}$$



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Quantile	$\mu + \sigma\sqrt{2}\, \operatorname{erf}^{-1}(2F-1)$
Mean	μ
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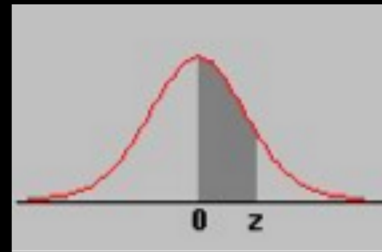
Mode	μ
Variance	σ^2

see
Statistics in a Nutshell
IV: Inferential Statistics

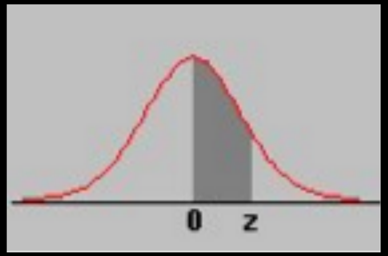


$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{n}} = 2.56$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md



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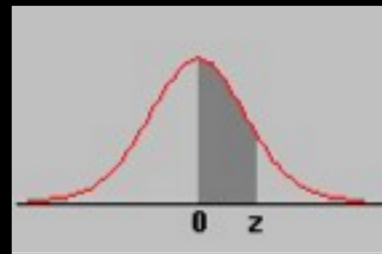
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md



$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{n}} = 2.56$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
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2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md

$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{n}} = 2.55$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
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0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
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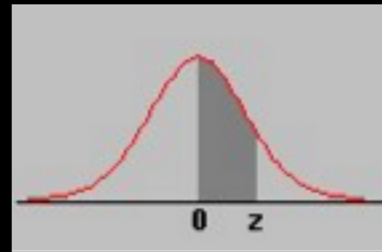
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md



$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{n}} = 2.56$$

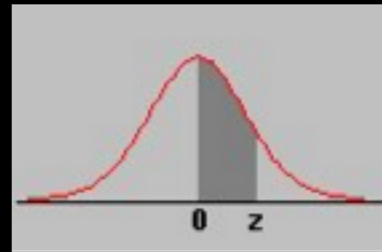
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md

2 sided test $1 - (0.4948 * 2) = 0.0104$

$p < 0.05$

H_0 IS REJECTED ($p<0.05$)



$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{n}} = 2.56$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

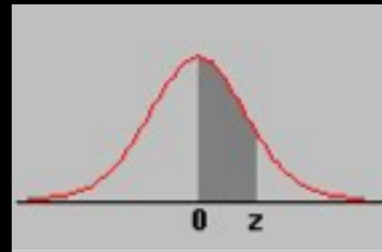
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md

1 sided test $1 - 0.4946 - 0.5 = 0.0054$

$p < 0.05$

H_0 IS REJECTED ($p<0.05$)



$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{n}} = 1.57$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

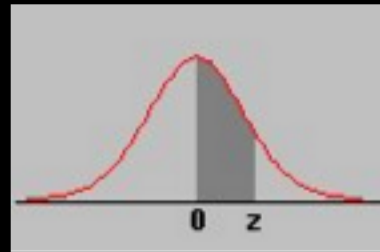
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md

2 sided test   1 - 0.4418 * 2 = 0.1164

$p > 0.05$

H_0 CANNOT BE REJECTED



$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{n}} = 1.57$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
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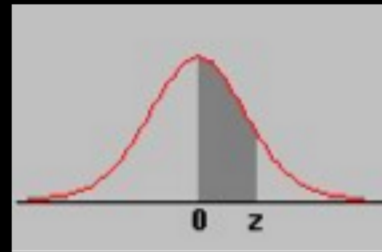
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md

2 sided test $1 - 0.4418*2 = 0.1164$

$p > 0.05$

H_0 CANNOT BE REJECTED



$$Z = \frac{\mu_{pop} - \mu_{sample}}{\sigma / \sqrt{n}} = 1.96$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md

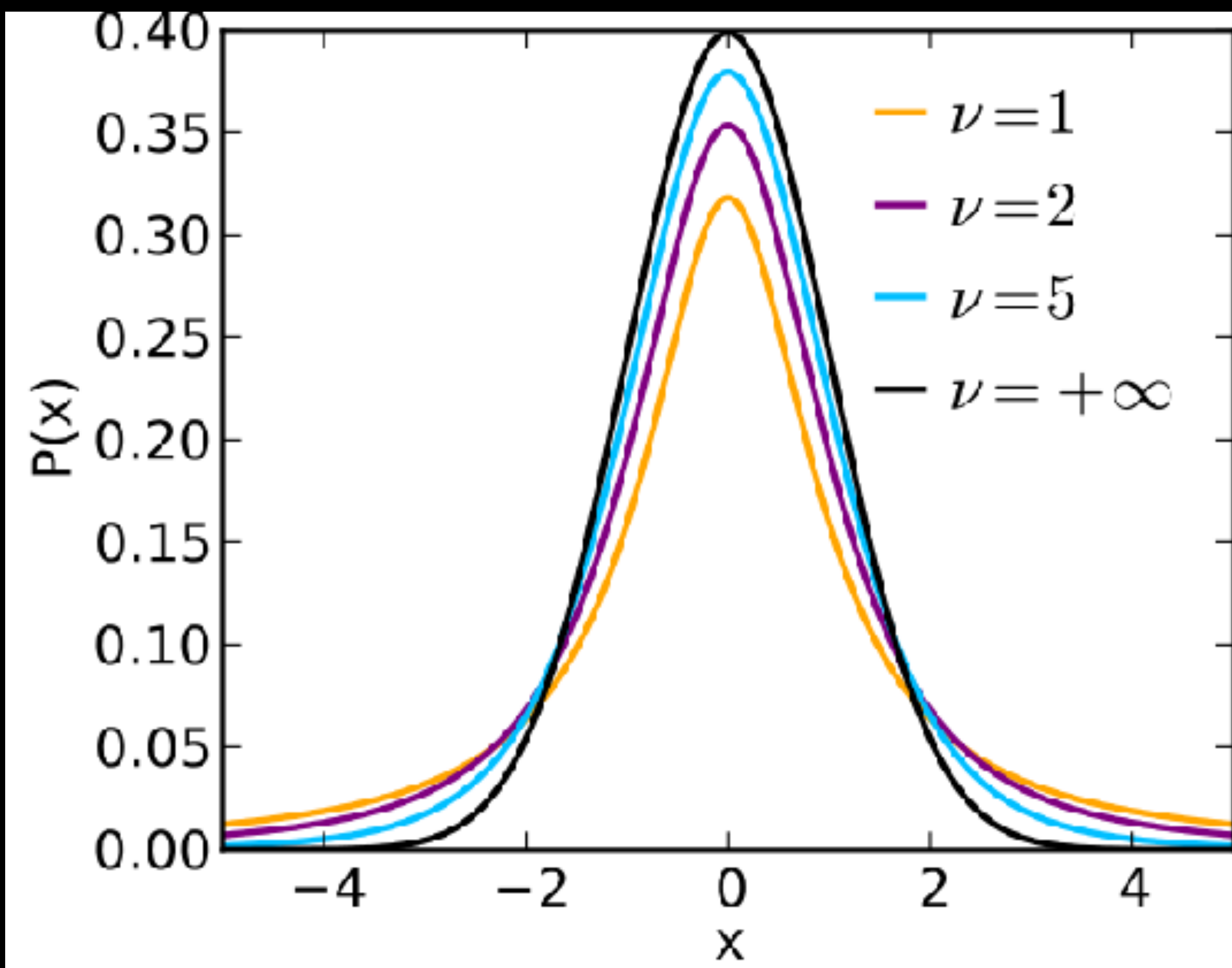
2 sided test $\quad 1 - 0.475*2 = 0.05$

$p = 0.05$

H_0 CANNOT BE REJECTED

Student's *t*

$$t = \frac{\mu - \bar{x}}{s/\sqrt{n}}$$

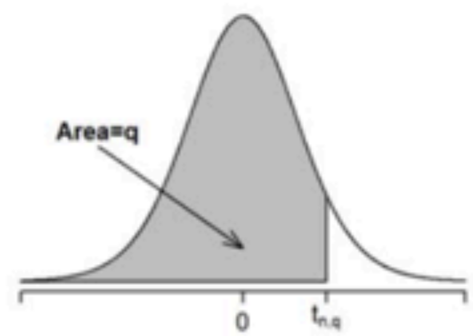


Parameters	$\nu > 0$ degrees of freedom (real)
Support	$x \in (-\infty; +\infty)$
PDF	$\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$
CDF	$\frac{1}{2} + x\Gamma\left(\frac{\nu+1}{2}\right) \times \frac{{}_2F_1\left(\frac{1}{2},\frac{\nu+1}{2};\frac{3}{2};-\frac{x^2}{\nu}\right)}{\sqrt{\pi\nu}\,\Gamma\left(\frac{\nu}{2}\right)}$ where ${}_2F_1$ is the hypergeometric function
Mean	0 for $\nu > 1$, otherwise undefined
Median	0
Mode	0
Variance	$\frac{\nu}{\nu-2}$ for $\nu > 2$, ∞ for $1 < \nu \le 2$, otherwise undefined

$$\chi^2_F = \sum_i \frac{(O_i - x_i)^2}{e_i}$$

see
Statistics in a Nutshell
V: Inferential Statistics

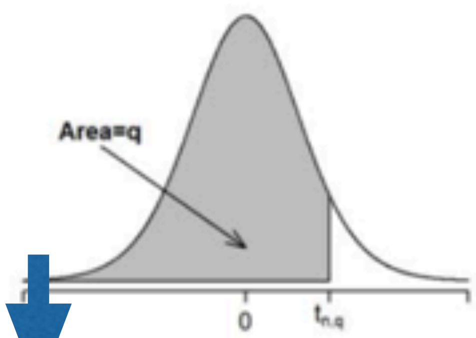
$$t_{n=13,\ q=.95} = \frac{\mu - \bar{x}}{s/\sqrt{n}} = 1.75$$



	q = 0.6	0.75	0.9	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
n = 1	0.3249	1.0000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.2887	0.8165	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.2767	0.7649	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.2707	0.7407	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.2672	0.7267	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.2648	0.7176	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.2632	0.7111	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.2619	0.7064	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.2610	0.7027	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.2602	0.6998	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.2596	0.6974	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.2590	0.6955	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.2586	0.6938	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.2582	0.6924	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140

$$t_{n=13,\ q=.95} = \frac{\mu - \overline{x}}{s/\sqrt{n}} = 1.75$$

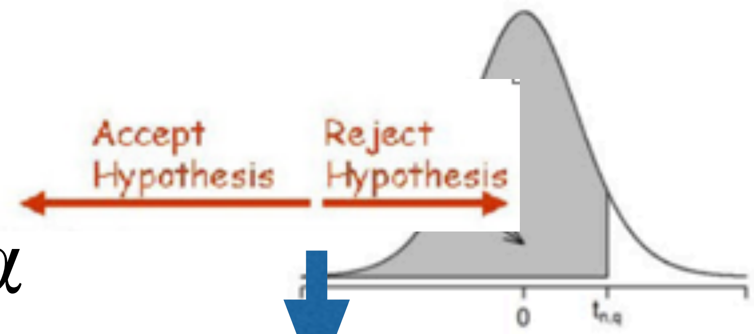
$q = 1 - \alpha$



	q = 0.6	0.75	0.9	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
n = 1	0.3249	1.0000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.2887	0.8165	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.2767	0.7649	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.2707	0.7407	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.2672	0.7267	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.2648	0.7176	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.2632	0.7111	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.2619	0.7064	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.2610	0.7027	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.2602	0.6998	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.2596	0.6974	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.2590	0.6955	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.2586	0.6938	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.2582	0.6924	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140

Is 1.75 > 1.771 ?

$$t_{n=13,\ q=.95} = \frac{\mu - \bar{x}}{s/\sqrt{n}} = 1.75$$



$q = 1 - \alpha$

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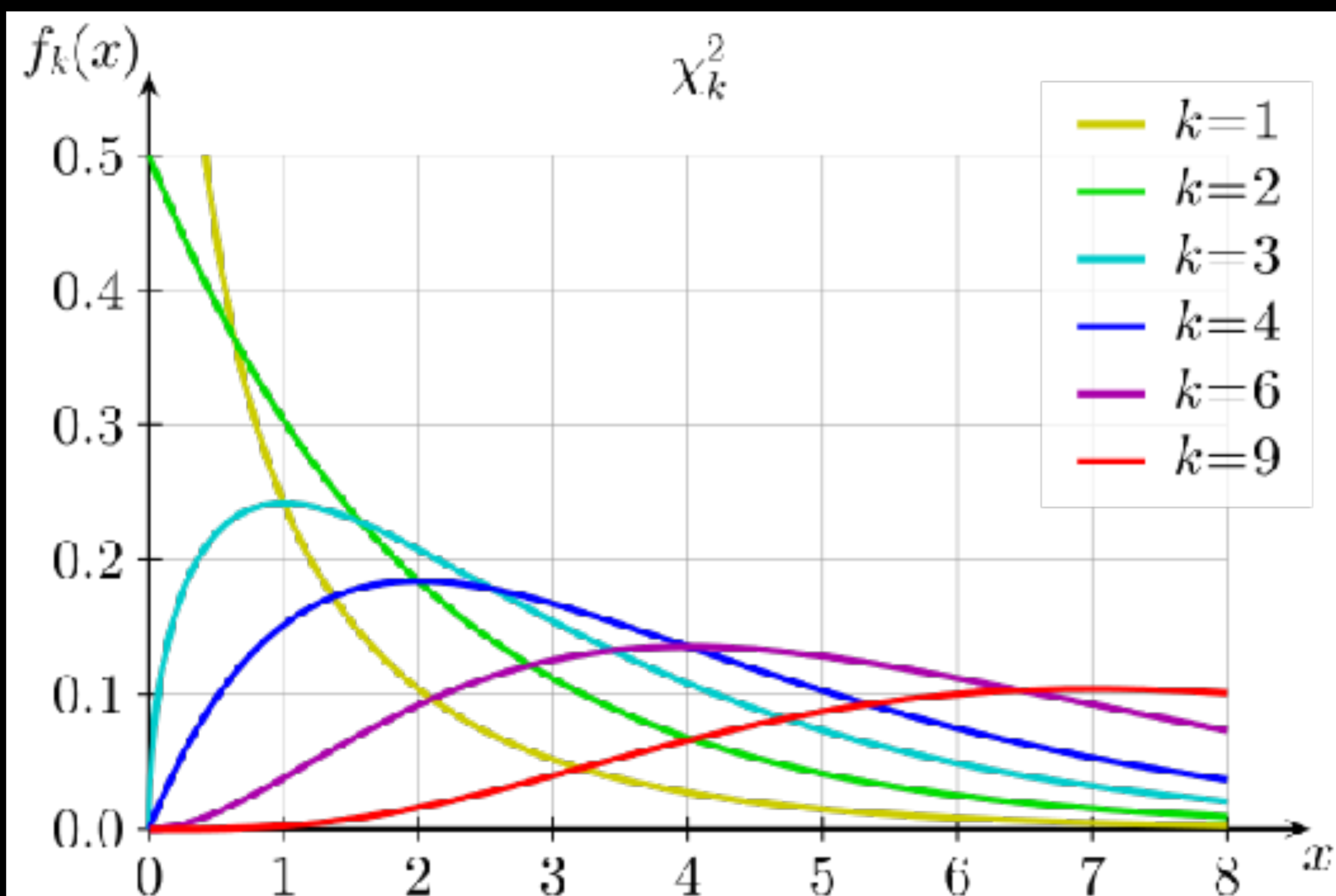
14	0.2582	0.6924	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140

Is 1.75 > 1.771 ? NO:

Cannot reject Null hypothesis

Pearson's χ^2

$$\chi^2_P = \sum_i \frac{(O_i - E_i)^2}{E_i}$$



Notation	$\chi^2(k)$ or χ^2_k
Parameters	$k \in \mathbb{N}_{>0}$ (known as "degrees of freedom")
Support	$x \in [0, +\infty)$
PDF	$\frac{1}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)} x^{\frac{k}{2}-1} e^{-\frac{x}{2}}$
CDF	$\frac{1}{\Gamma\left(\frac{k}{2}\right)} \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$
Mean	k
Median	$\approx k\left(1 - \frac{2}{9k}\right)^3$
Mode	$\max\{k-2, 0\}$
Variance	$2k$
Skewness	$\sqrt{8/k}$

goodness of fit χ^2 $\chi^2_F = \sum_i \frac{(m_i - x_i)^2}{e_i}$ -

see
Statistics in a Nutshell
V: Inferential Statistics

$$\chi^2_F = \sum_{i=1}^{4} \frac{(m_i - x_i)^2}{e_i} = 8.57$$

$\alpha = 0.05$

4 observations - 1 independent variable =
3 degreed of freedom

Accept Hypothesis ← | → Reject Hypothesis

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09

https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md

$p < 0.05$

H_0 REJECTED

$$\chi^2_F = \sum_{i=1}^{4} \frac{(m_i - x_i)^2}{e_i} = 7.11$$

$\alpha = 0.05$

4 observations - 1 independent variable = 3 degreed of freedom

Accept Hypothesis

Reject Hypothesis

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
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https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md

$p > 0.05$

H_0 CANNOT BE REJECTED

Example

Example

jupyter NHST and effect size

NULL **HYPOTHESIS**: the % of former prisoners employed 3 years after release is *the same or lower* for candidates who participated in the program as for the control group,

significance level p=0.05

What Strategies Work for the Hard-to-Employ?

Final Results of the Hard-to-Employ Demonstration and Evaluation Project and Selected Sites from the Employment Retention and Advancement Project

OPRE Report 2012-08

March 2012

http://www.mdrc.org/sites/default/files/What%20Strategies%20Work%20for%20the%20Hard%20FR.pdf

CUSP

The Hard-to-Employ evaluation was a 10-year study that used a rigorous random assignment research design in four sites to evaluate innovative strategies aimed at improving employment and other outcomes for groups who face serious barriers to employment. The

The Programs in the Hard-to-Employ Evaluation

Following discussions with HHS and extensive research about the implications of different targeting strategies, program models, and best practices for the evaluation design, the MDRC team recruited four sites to participate in the Hard-to-Employ study. Three of the four participating sites targeted discrete hard-to-employ populations, while the fourth (Kansas and Missouri Enhanced Early Head Start) served low-income parents with very young children, a population with more general barriers to finding and keeping jobs:

- **Center for Employment Opportunities, New York City.** Parolees were placed in temporary paid jobs at work sites around the city for several months and received a variety of other supports, along with job placement assistance.

Study Design

For each program presented in this report, the research teams studied the implementation of the programs and the programs' impacts. Additionally, the study of the Center for Employment Opportunities included a benefit-cost analysis, and the studies of the other three HtE programs included estimates of their financial costs.

Study participants at each site were assigned at random to either a program group, which had access to the program's services, or to a control group, which was not permitted to receive program services but could receive any public services that were normally available. The two research groups together make up the "research sample" or "study sample." A random assignment (experimental) design ensures that there are no systematic differences between the members of the two groups when they enter the study, so that any significant differences (that is, differences that are unlikely to arise by chance alone) that emerge over time between the groups can be reliably attributed to the fact that one group was exposed to the experimental program and the other was not. Such differences are known as impacts, or effects, of the program.

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The Enhanced Services for the Hard-to-Employ Demonstration and Evaluation Project

Table 2.1

Summary of Impacts, New York City Center for Employment Opportunities

Outcome	Program Group	Control Group	Difference (Impact)	P-Value
Employment (Years 1-3) (%)				
Ever employed	83.8	70.4	13.4 ***	0.000
Ever employed in a CEO transitional job[a]	70.1	3.5	66.6 ***	0.000
Ever employed in an unsubsidized job	63.7	69.0	-5.3 *	0.078
Postprogram unsubsidized employment (Years 2-3)				
Ever employed in an unsubsidized job (%)	53.3	52.1	1.2	0.713
Employed in an unsubsidized job, average per quarter (%)	28.2	27.2	1.1	0.618
Employed for six or more consecutive quarters (%)	14.7	11.9	2.8	0.195
Total UI-covered earnings[b] ($)	10,435	9,846	589	0.658
Sample size (total = 973)[c]	564	409		

SOURCES: MDRC earnings calculations from the National Directory of New Hires (NDNH) database and employment calculations from the unemployment insurance (UI) wage records from New York State, MDRC calculations using data from the New York State Division of Criminal Justice Services (DCJS) and the New York City Department of Correction (DOC).

NOTES: Statistical significance levels are indicated as: *** = 1 percent; ** = 5 percent; * = 10 percent.
The p-value indicates the likelihood that the difference between the program and control groups arose by chance.

http://www.mdrc.org/sites/default/files/What%20Strategies%20Work%20for%20the%20Hard%20FR.pdf

CUSP

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Employment (Years 1-3) (%)	P_1	P_0		
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$$H_0: P_0 - P_1 >= 0$$

$$H_a: P_0 - P_1 < 0$$

$$\alpha = 0.05$$

http://www.mdrc.org/sites/default/files/What%20Strategies%20Work%20for%20the%20Hard%20FR.pdf

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H_0: $P_0 - P_1 >= 0$

H_a: $P_0 - P_1 < 0$

$\alpha = 0.05$

http://www.mdrc.org/sites/default/files/What%20Strategies%20Work%20for%20the%20Hard%20FR.pdf

CUSP

Beyond Significance Based Hypothesis Rejection

BEYOND SIGNIFICANCE TESTING

REFORMING DATA ANALYSIS METHODS IN BEHAVIORAL RESEARCH

REX B. KLINE

http://edhar.cas.msu.edu/readings/Kline%20Significance%20-%20ch%203%20-%20NHST%20-%20hires.pdf

Recommendations

Specific suggestions are listed and then discussed afterward:

1. Only in very exploratory research where it is unknown whether effects exist may a primary role for NHST be appropriate.
2. If statistical tests are used, (a) information about power must be reported, and (b) the null hypothesis must be plausible.
3. In any kind of behavioral research, it is not acceptable anymore to describe results solely in terms of NHST outcomes.
4. Drop the word "significant" from our data analysis vocabulary. Use it only in its everyday sense to describe something actually noteworthy or important.
5. It is the researcher's responsibility to report and interpret, whenever possible, effect size estimates and confidence intervals for primary results. This does *not* mean to report effect sizes only for H_0 rejections.
6. It is also the researcher's responsibility to demonstrate the substantive (theoretical, clinical, or practical) significance of the results. Statistical tests are inadequate for this purpose.
7. Replication is the best way to deal with sampling error.
8. Education in statistical methods needs to be reformed, too. The role of NHST should be greatly deemphasized so that more time can be spent showing students how to determine whether a result has substantive significance and how to replicate it.
9. Researchers need more help from their statistical software to compute effect sizes and confidence intervals.

Arguments against NHST

1. $P(H_0 \mid D) = \dfrac{P(H_0)\,P(D \mid H_0)}{P(D)}$ $\quad P(H_0 \mid D) \neq P(D \mid H_0)$

2. *Significance depends on sample size, e.g.:*

$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}}$$

If you measure the same effect in two experiments with 2 samples of different size, the significance of the effect will be different

Effect size to measure the strength of a phenomenon

Estimate the strength of, for example, an apparent relationship, rather than assigning a significance level. The effect size does not directly determine the significance level, or vice versa.

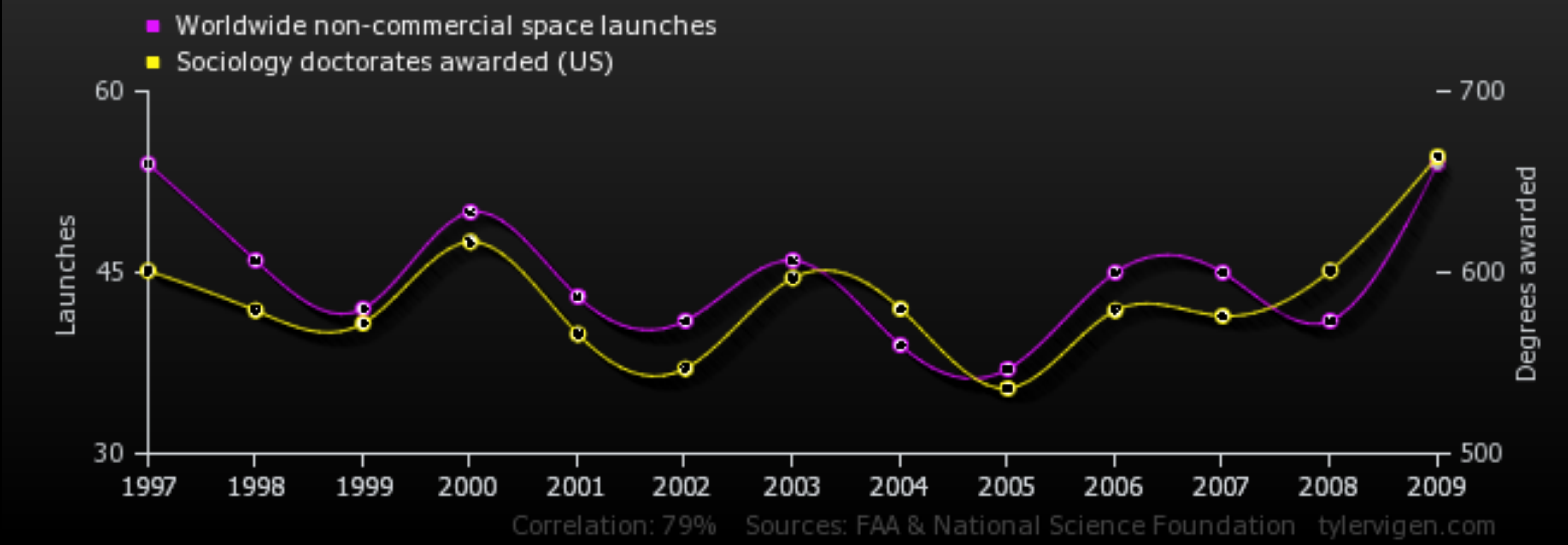
Jacob Cohen (1960), *"A coefficient of agreement for nominal scales"*, Educational and Psychological Measurement, 20 (1): 37–46,

https://github.com/fedhere/PUI2016_fb55/blob/master/HW3_fb55/citibikes_gender.ipynb small,medium,large

Effect size to measure the strength of a phenomenon

Cohen's *d*	$d = \dfrac{\overline{x}_1 - \overline{x}_2}{s}$ $\quad s = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$	for ***Z*** and ***t***	.20 .50 .80
Cohen's *w*	$w = \sqrt{\sum_{i=1}^{N} \dfrac{(p_{0i}-p_{1i})^2}{p_{0i}}}$	for χ^2	.10 .30 .50
Cohen's *h*	$h = 2(\arcsin\sqrt{p_1} - \arcsin\sqrt{p_2})$	for proportions	.20 .50 .80
f-squared	$\hat{f}_{effect} = \sqrt{(df_{effect}/N)(F_{effect}-1)}$.	for F-test/ ANOVA	.10 .25 .40
R^2 coefficient of determination		for regression	.02 .15 .35

http://core.ecu.edu/psyc/wuenschk/docs30/EffectSizeConventions.pdf jupyter NHST and effect size

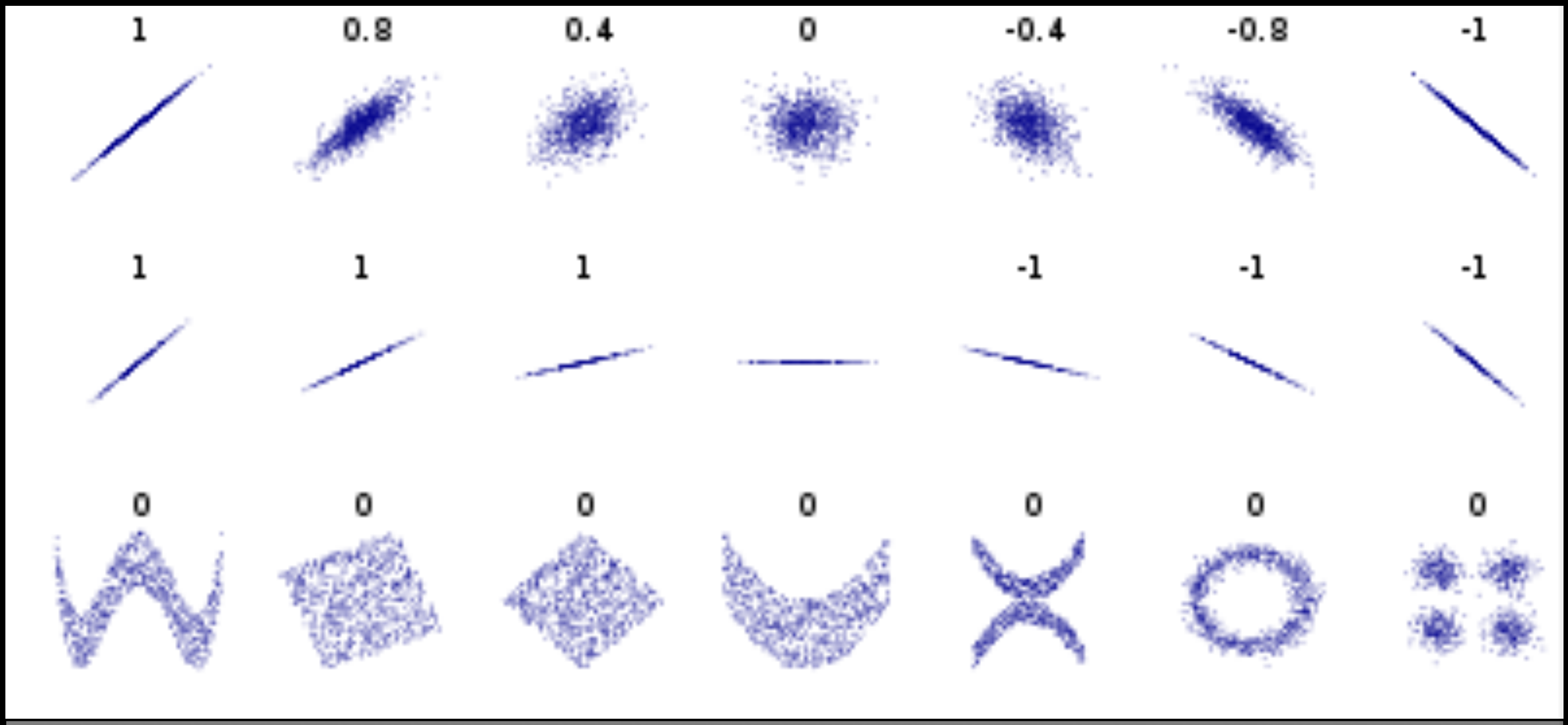


Correlation

Pearson's test:

$$r_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

$$s_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$



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Spearman's test:
(Pearson's for ranks)

$$\rho = 1 - \frac{6 \sum (x_i - y_i)^2}{n(n^2 - 1)}$$

Choosing the test

Use the table below to choose the test. See below for further details.

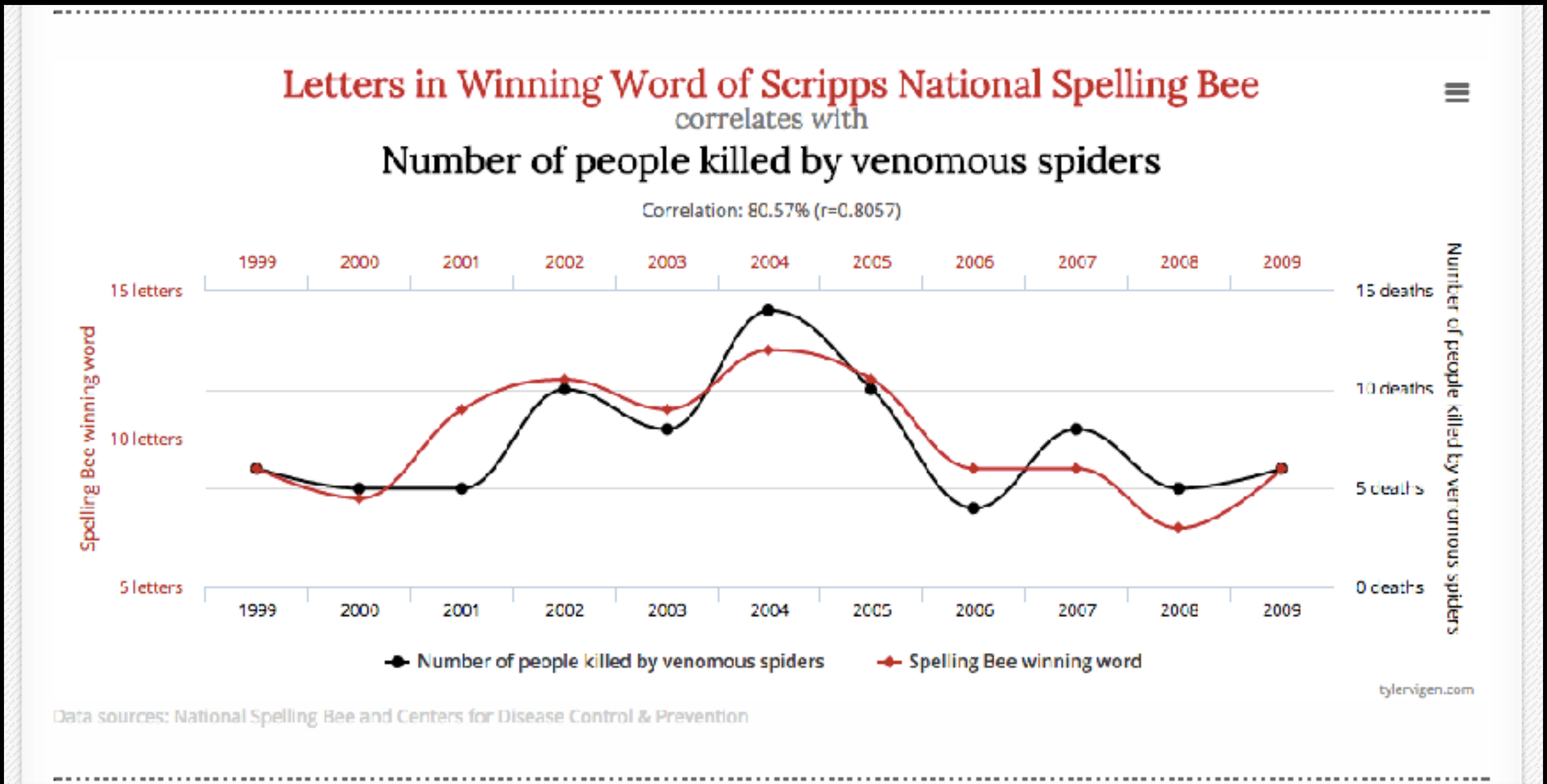
How many dichotomous† (binary) variables?				
0	Both variables interval or ratio?			
	Y	Measures are linear? (No = monotonic*)		
		Y	Pearson correlation	
		N	Spearman correlation	
	N	Both variables are ordinal?		
		Y	Kendall correlation	
		N	Both variables can be ranked?	
			Y	Kendall correlation
			N	Convert to frequency data and use Chi-square test for independence
1	Biserial Correlation Coefficient			
2	2 x 2 table?			
	Y	Phi		
	N	Cramer's V		

Data has frequency values for each category?	
Y	Chi-square test for independence

†dichotomous = 'can have only two values' (eg. yes/no or 0/1).
*monotonic = constantly increasing or decreasing.

http://changingminds.org/explanations/research/analysis/choose_correlation.htm

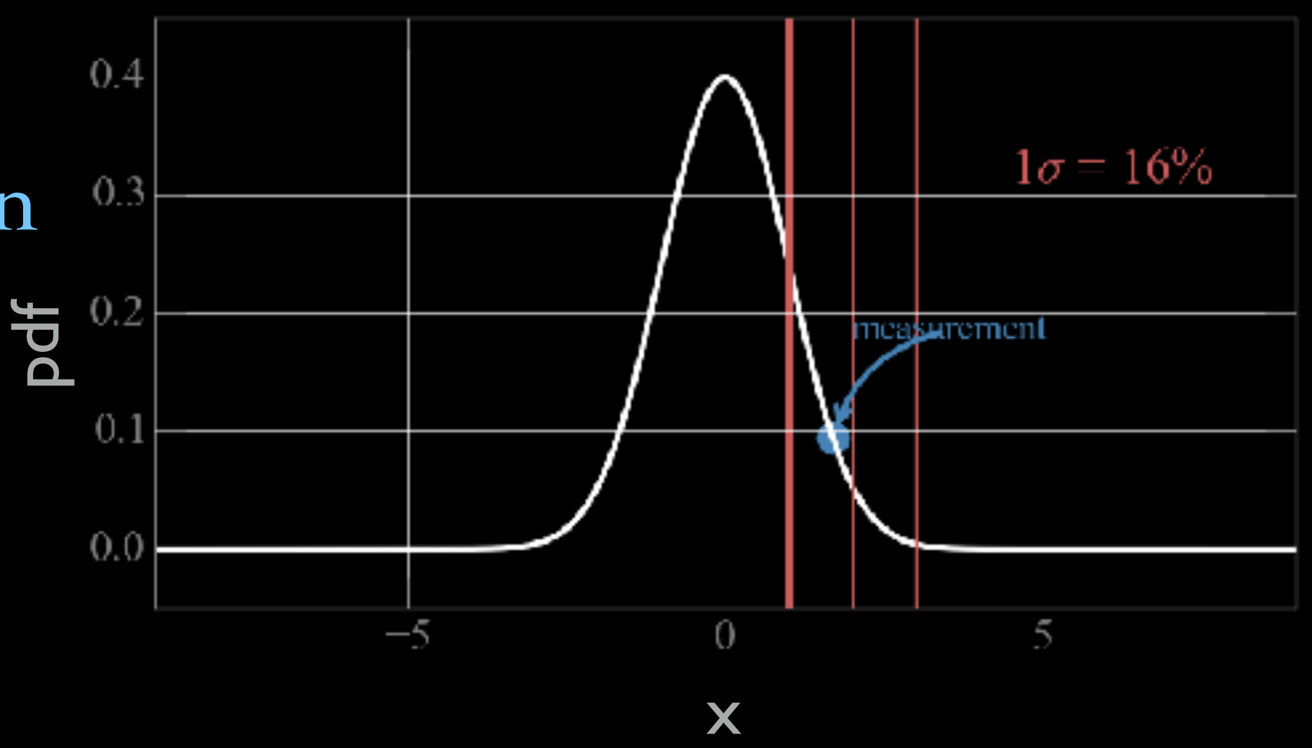
WARNING: Correlation is not causation!



http://www.tylervigen.com/spurious-correlations

Tests for correlation and independence
(continuous variables)

Probability Distribution Function



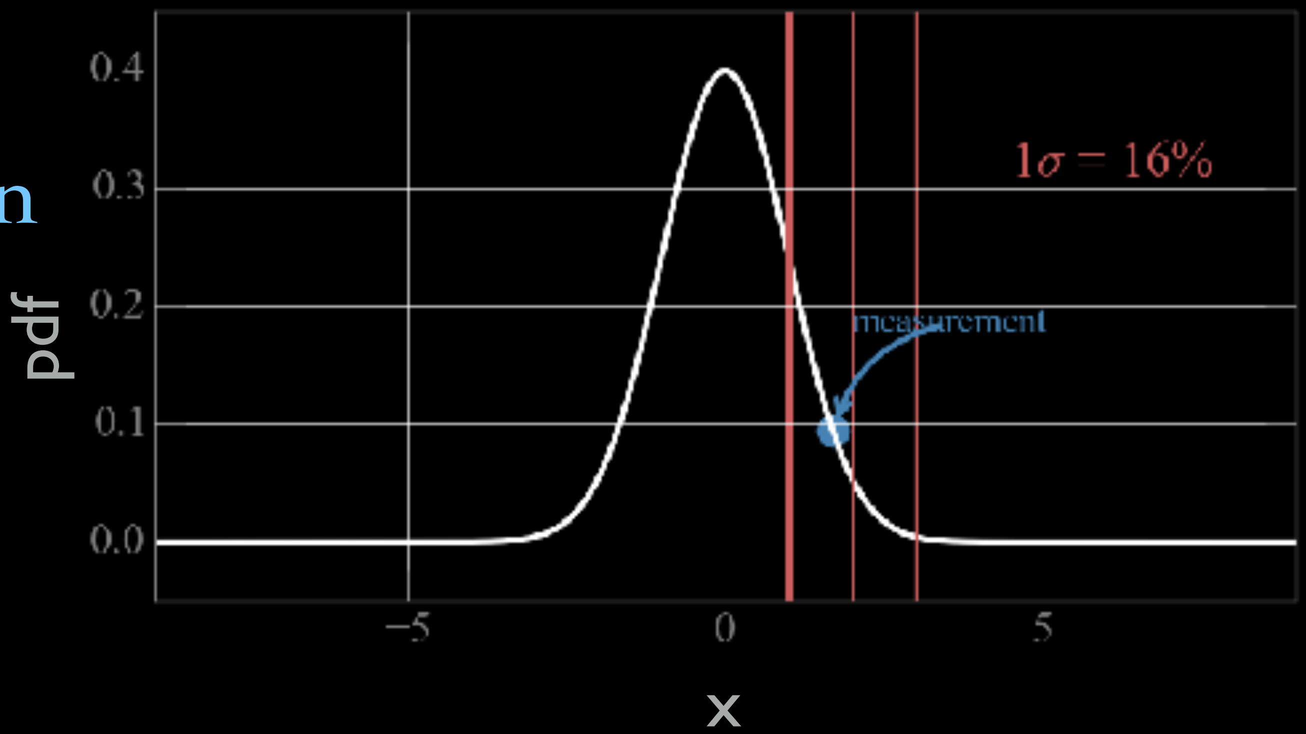
$$f_{x_0}(x) \sim \; p(x = x_0)$$

$$f_{x_0}(x) \sim \; p(x > x_0 - dx) \bigcap p(x < x_0 + dx)$$

Probability Distribution Function

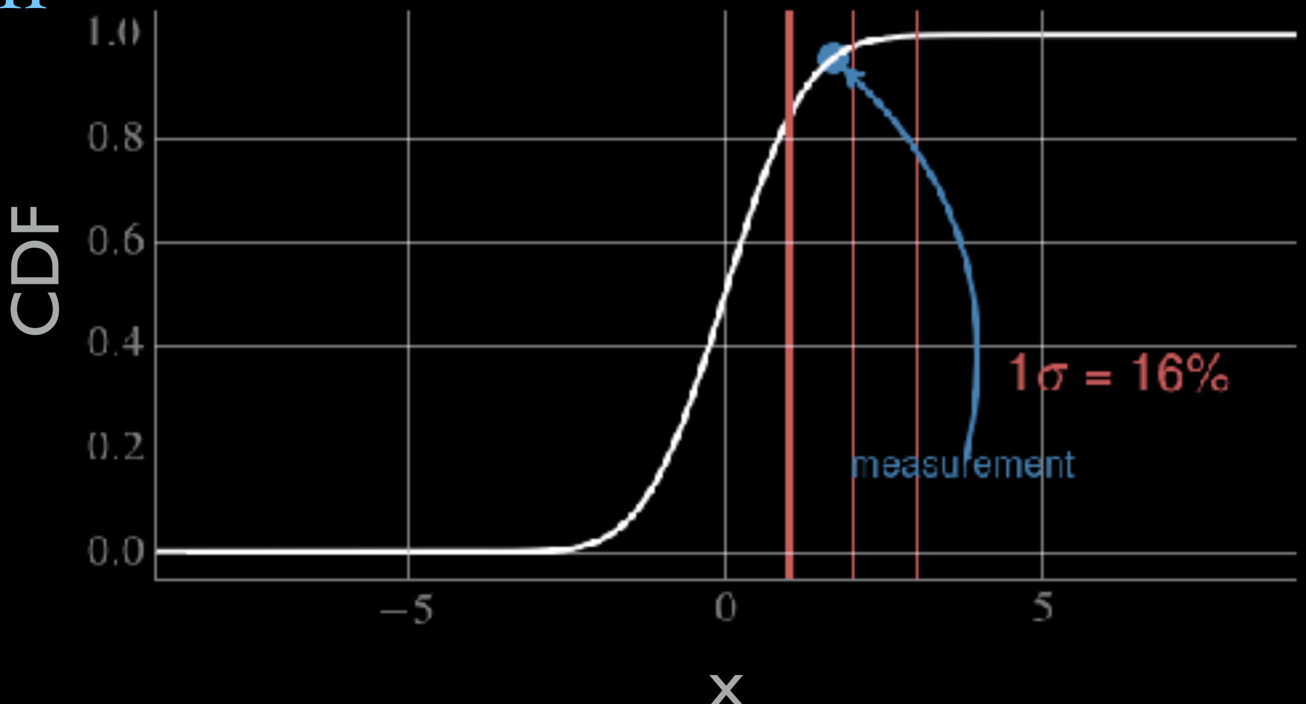
$$f_{x_0}(x) \sim p(x=x_0)$$

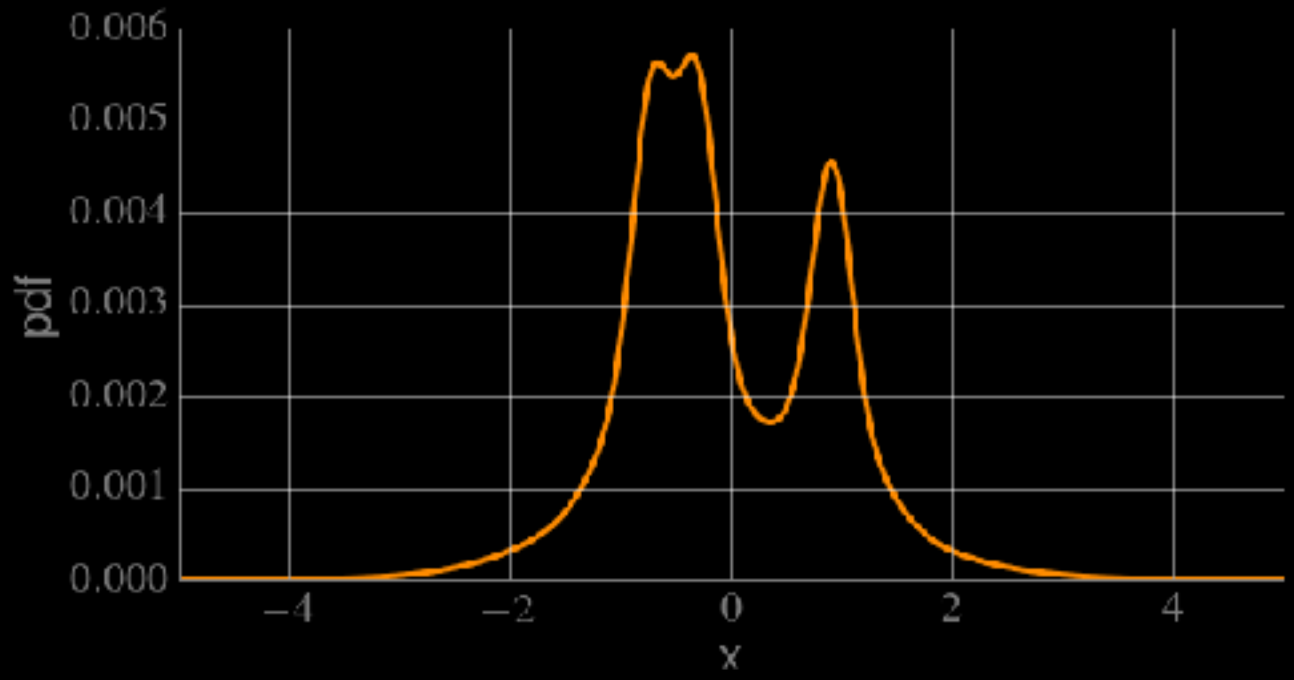
$$f_{x_0}(x) \sim p(x>x_0-dx) \bigcap p(x<x_0+dx)$$



Cumulative Distribution Function

$$F_{x_0}(x) = P(x<x_0)$$

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0.003
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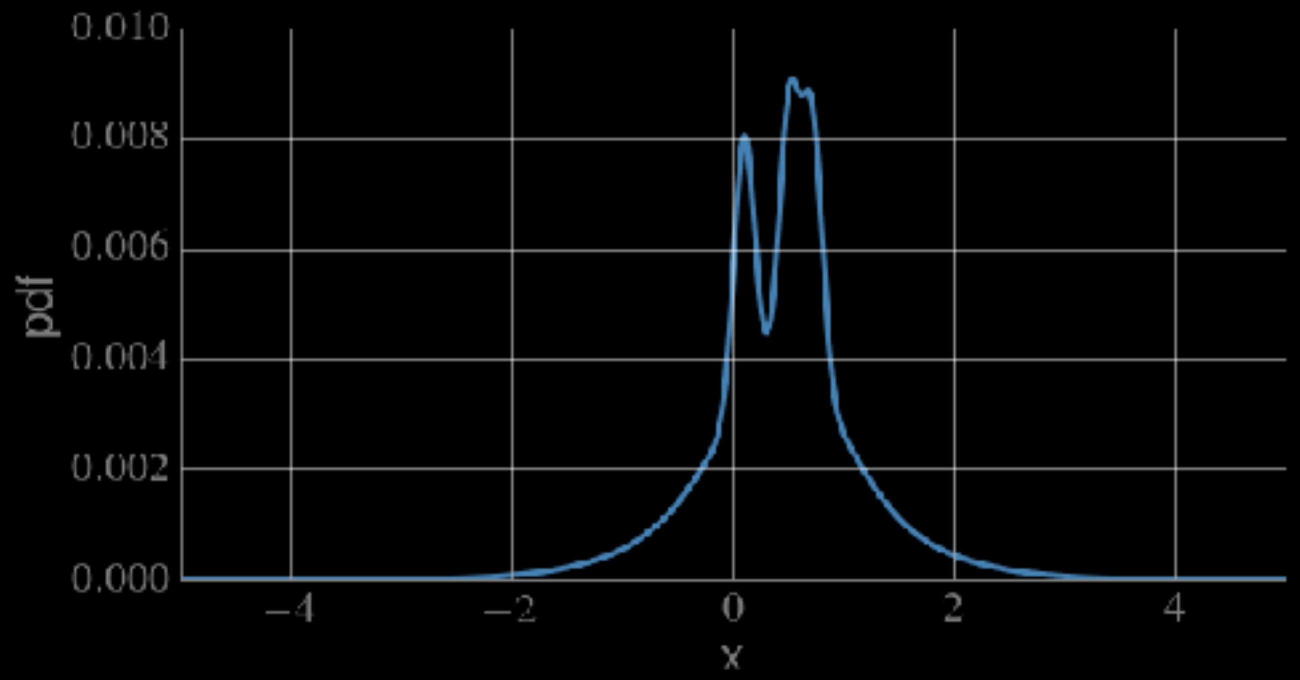
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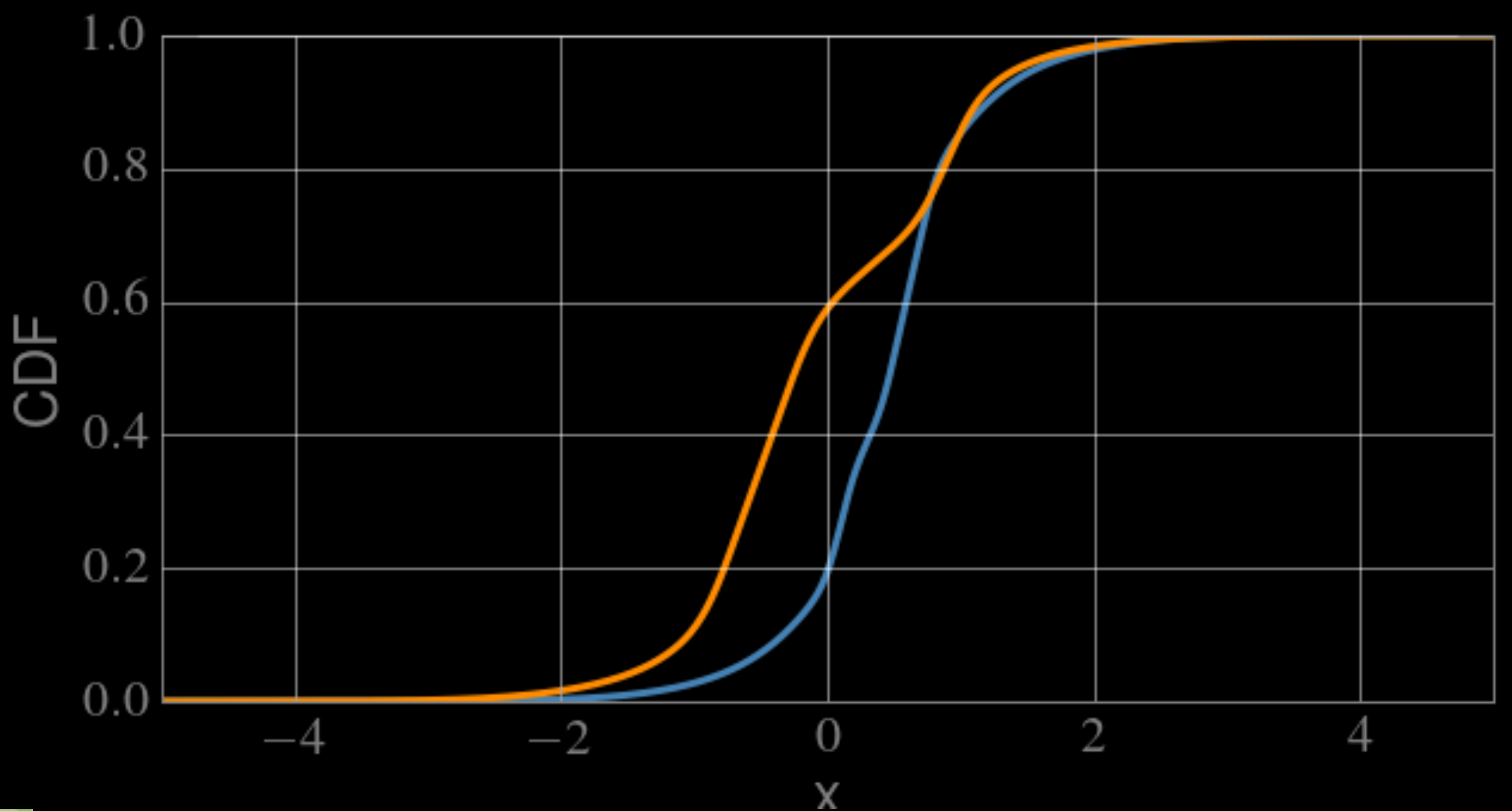
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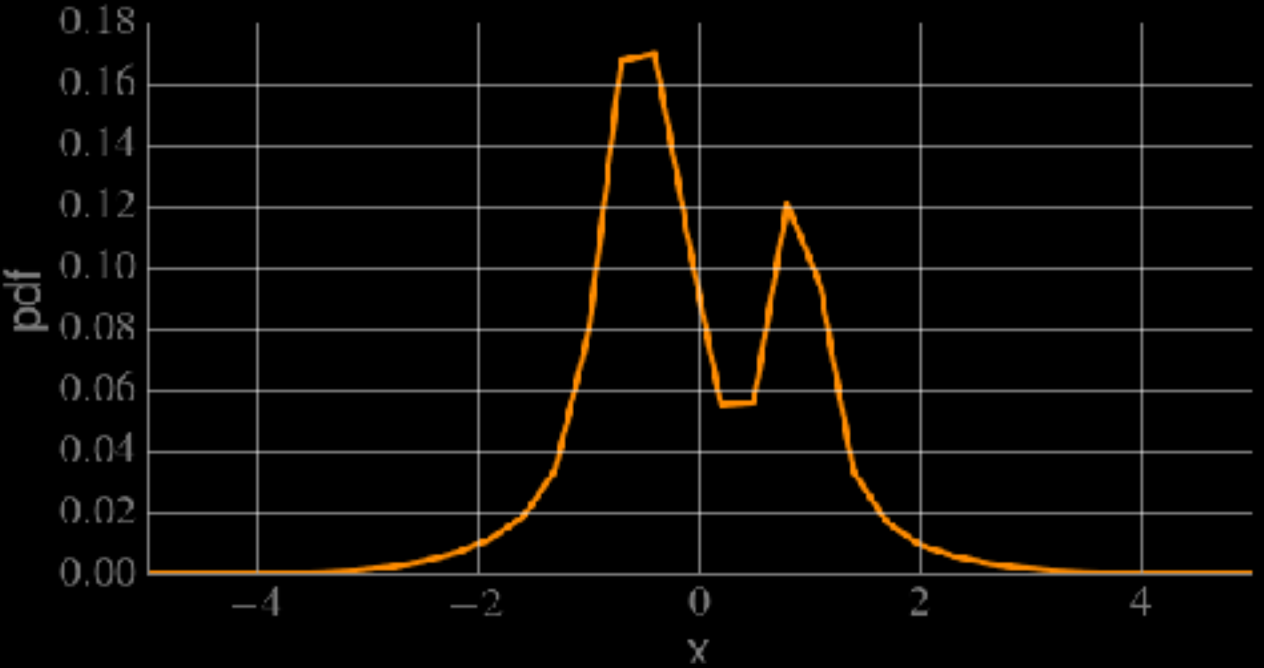
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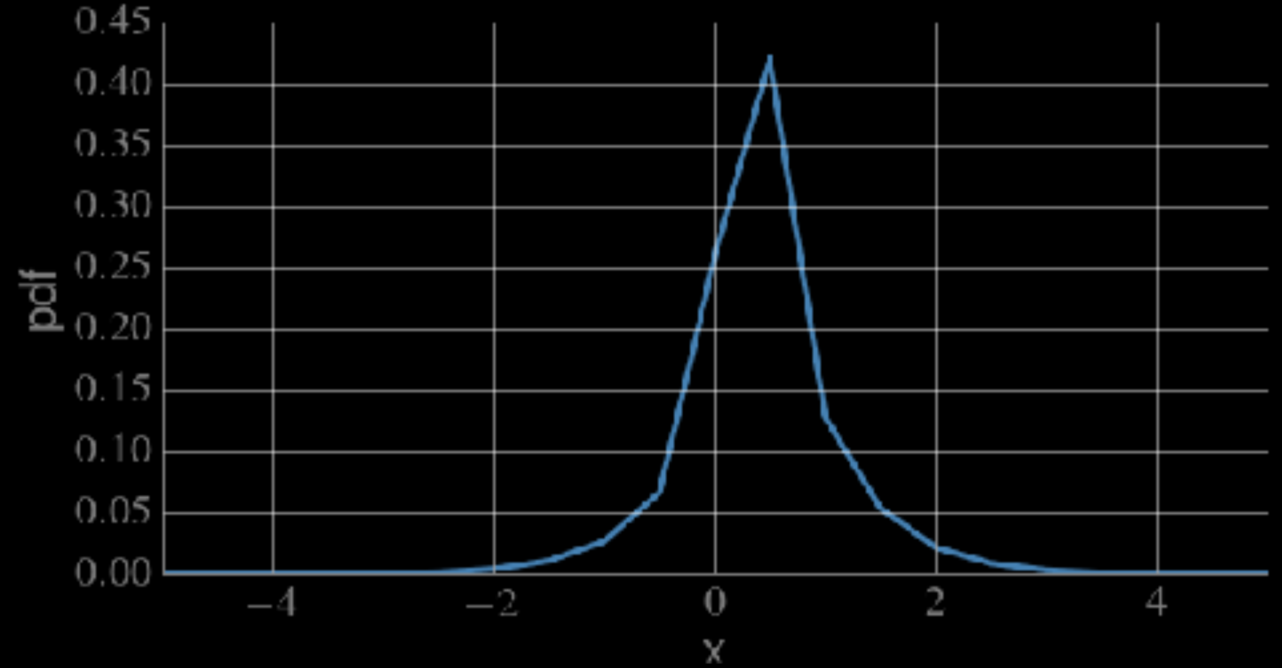
4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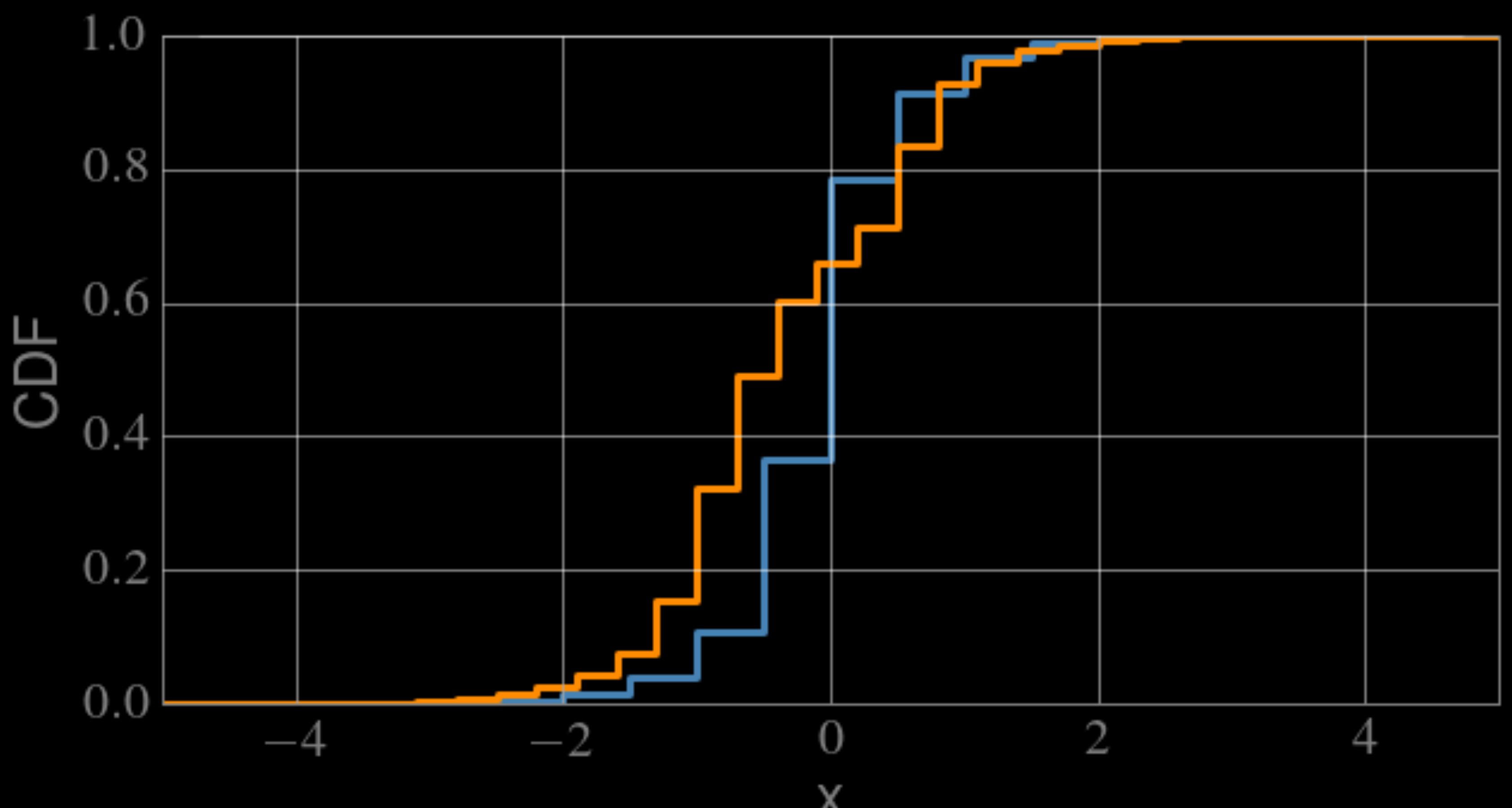
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−4
−2
0
2
4


CDF
x
1.0
0.8
0.6
0.4
0.2
0.0
−4
−2
0
2
4

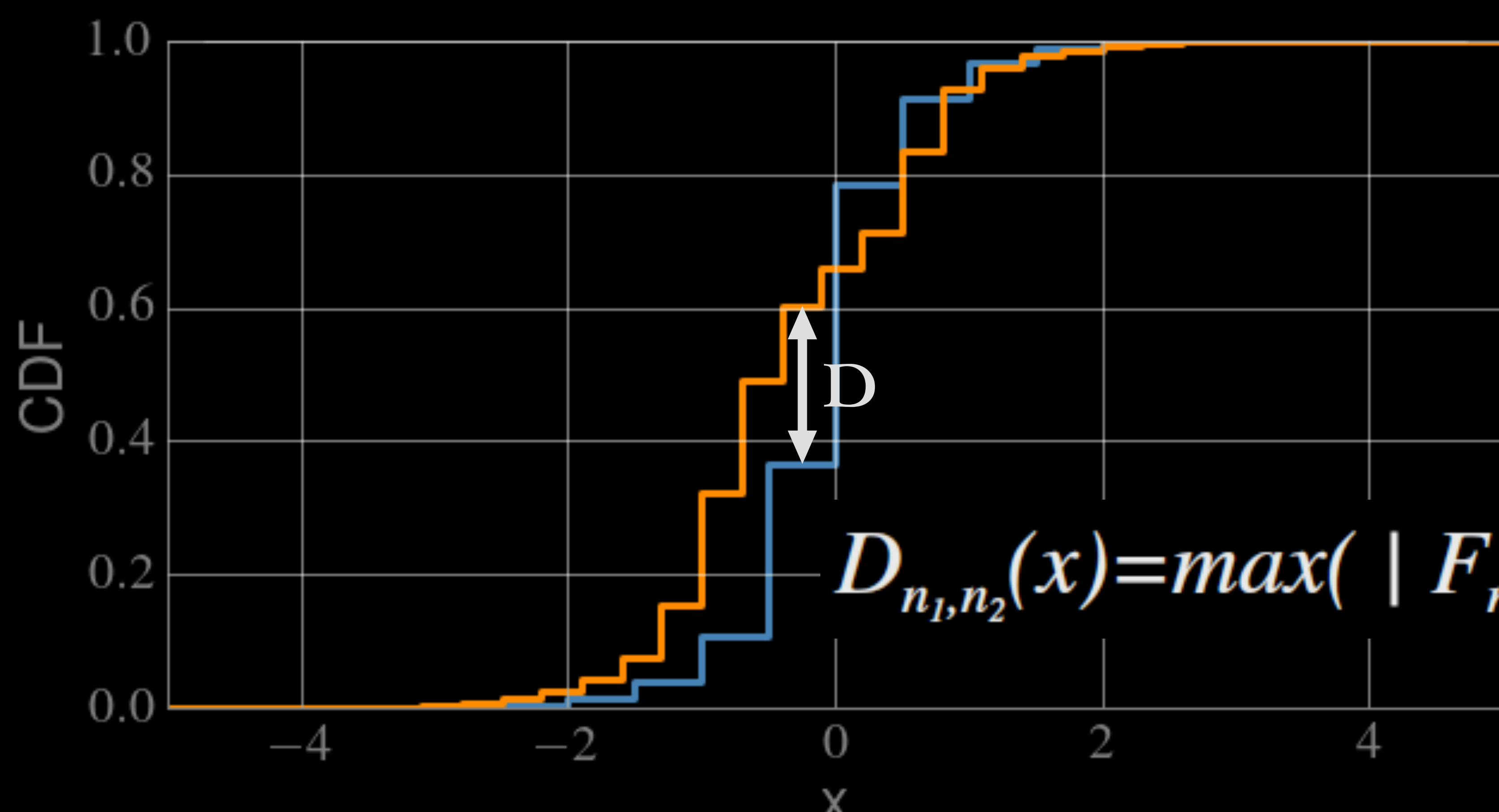
Two sample Kolmogorov Smirnoff test:

null hypothesis H_0: the samples come from the same parent distribution

H_0 is rejected at level α if $D(n_1,n_2) > c(\alpha)\sqrt{\frac{n_1+n_2}{n_1 n_2}}$

with c(α) given by a table

NOTE: it ONLY woks in 2D where the Euclidian distance is uniquely defined!

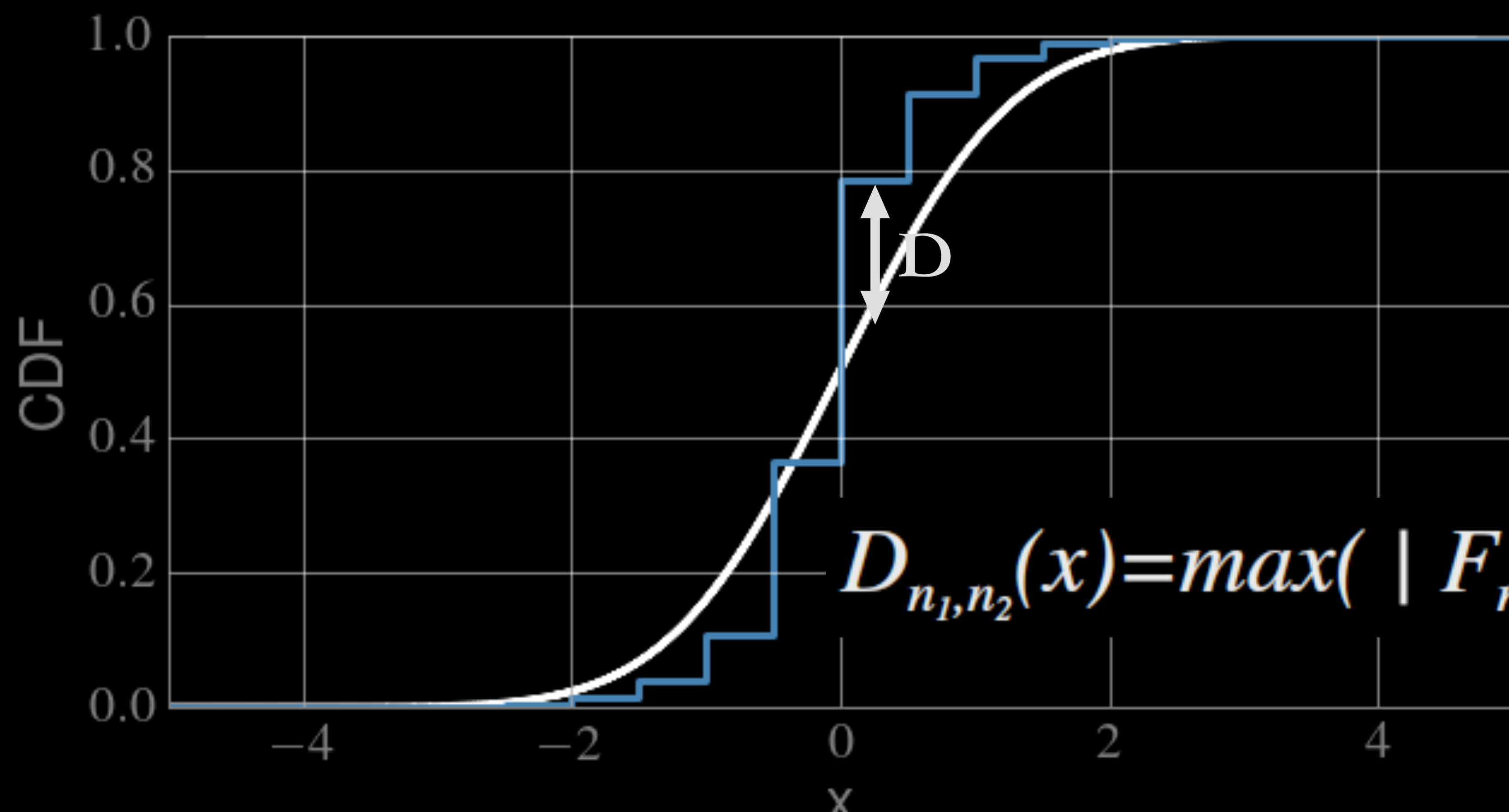


$$D_{n_1,n_2}(x) = max(\;|\;F_{n_1}(x) - F_{n_2}(x)\;|\;)$$

Goodness-of-fit Kolmogorov Smirnoff test:

null hypothesis H_0: the sample does comes from the *model* distribution

H_0 is rejected at level α if $\sqrt{n}\, D_n > K_\alpha$ where $P(K \leq K_\alpha) = 1 - \alpha$



$$D_{n_1,n_2}(x) = max(\mid F_{n_1}(x) - F_{n_2}(x) \mid)$$

Tests Cheat Sheet:
2 (+) samples comparison

	metric (statistic)	compare to	
KS	$D_{n_1,n_2}(x)=max(\mid F_{n_1}(x)-F_{n_2}(x)\mid)$	$c(\alpha)\sqrt{\frac{n_1+n_2}{n_1 n_2}}$	*Non parametric* *2 samples only*
K-sample Anderson-Darling	$ADK=\frac{n-1}{n^2(k-1)}\sum_{i=1}^{k}\frac{1}{n(i)}\left(\sum_{j=1}^{L}h_j\frac{(nF_{ij}-n_iH_j)^2}{H_j(n-H_j)-nh_j/4}\right)$	•AK table	*Non parametric,* *N samples*
Pearson's	$r_{xy}=\frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i-\overline{x}}{s_x}\right)\left(\frac{y_i-\overline{y}}{s_y}\right)$	The interpretation of a correlation coefficient depends on the context and purpose	-1 anticorrelated 0 uncorrelated 1 correlated .
Spearman's	$\rho=1-\frac{6\sum(x_i-y_i)^2}{n(n^2-1)}$	t test $t=r\sqrt{\frac{n-2}{1-r^2}}$	*ranked data only* p-value from t-test, Fisher's transformation +z score, permutation test

assignment 3: Z-test and chi sq test

- Reproduce the analysis of the Hard to Employ program. Reproduce the results in cell 2 and 10. Follow the notebook in the HW directory (turn in the python notebook in the HW4_<netID> directory

assignment 4. Compare Tests for Correlation

The following are 3 tests that assess correlation between 2 samples of citibike data:

- **Pearson's test** (answer: are the 2 samples correlated?)
- **Spearman's test** (answer: are the 2 samples correlated?)
- **K-S test** (answer: are the samples likely to come from the same parent distribution?)

Use: age of bikers for 2 genders. State your result in words in terms of the Null Hypothesis

Extra Credit: Use the age of bikers in day vs night and assess the correlation/independence of the 2 samples in each case..

CUSP