Urban Informatics

Fall 2017

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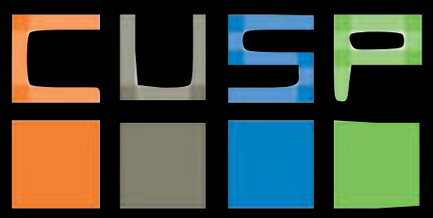
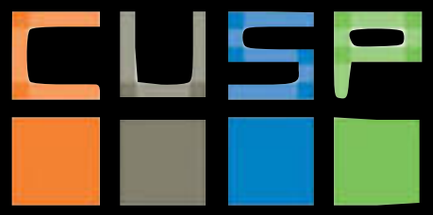
 @fedhere 



Recap:

- Good practices with data: falsifiability, reproducibility
- Basic data retrieving and munging: APIs, Data formats
- Basic statistics: distributions and their moments
- Hypothesis testing: *p*-value, statistical significance
- Statistical and Systematic errors
- Goodness of fit tests



Recap:

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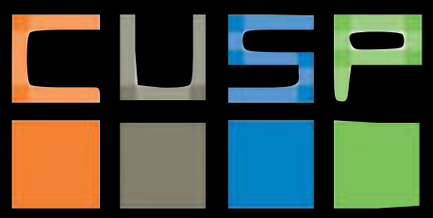
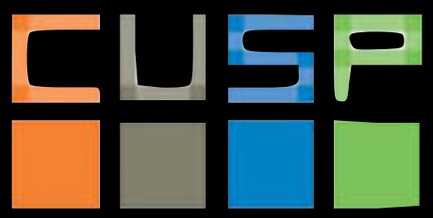
Today:

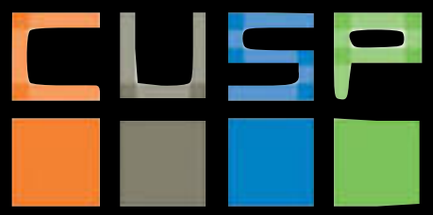
- Residuals minimization
- Likelihood
- model diagnostics
  Chi^2, R^2, and LR test
- Higher degree regression





V: Likelihood and Regression Models

Goodness of fit

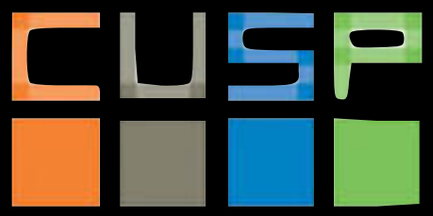


V: Likelihood and Regression Models

You have some data, and an idea of how it should look: a *model*

Is it a good model?

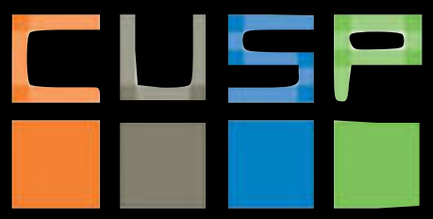
Goodness of fit



V: Likelihood and Regression Models

Tests Cheat Sheet:
goodness of fit

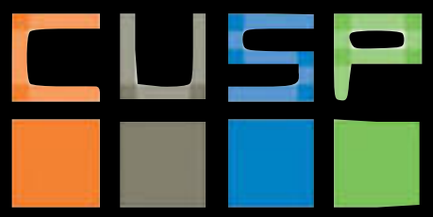
	metric (statistic)	compare to	
KS	$D_{n_1,n_2}(x)=max(\mid F_n(x)-F\ (x)\mid)$	$\frac{K_\alpha}{\sqrt{n}}$	power in the core only
Pearson's chi square	$\chi^2_{red}=\frac{\chi^2}{df}=\frac{1}{df}\sum\frac{(O-E)^2}{\sigma^2}$	scipy.stats.chisquare(f_obs, f_exp=None, ddof=0, axis=0)[	
Anderson-Darling	$A=n\int_{-\infty}^{\infty}\frac{(F_n(x)-F(x))^2}{F(x)\ (1-F(x))}\,dF(x)$	scipy.stats.anderson(x, dist='norm')	power in the tails
K-L divergence	$D_{KL}\ =\ -\int_x p(x)\ log(q(x))\ +\ p(x)\ log(p(x))$	scipy.stats.entropy(pk. qk=<not None>)	relates to information entropy
Likelihood ratio	$\frac{L\ (model\ 1\mid\ data)}{L\ (model\ 2\mid\ data)}$		suitable to bayesian analysis



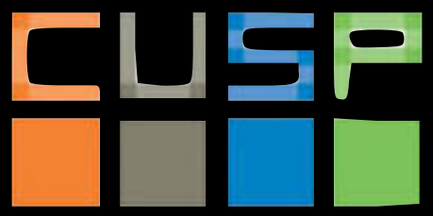
All models are wrong

Since *all models are wrong* the scientist cannot obtain a "correct" one by excessive elaboration. On the contrary following William of Occam he should seek an economical description of natural phenomena. Just as the ability to devise simple but evocative models is the signature of the great scientist so overelaboration and overparameterization is often the mark of mediocrity

George Box (1979),
"Robustness in the strategy of scientific model building",
in Launer, R. L.; Wilkinson, G. N., Robustness in Statistics,

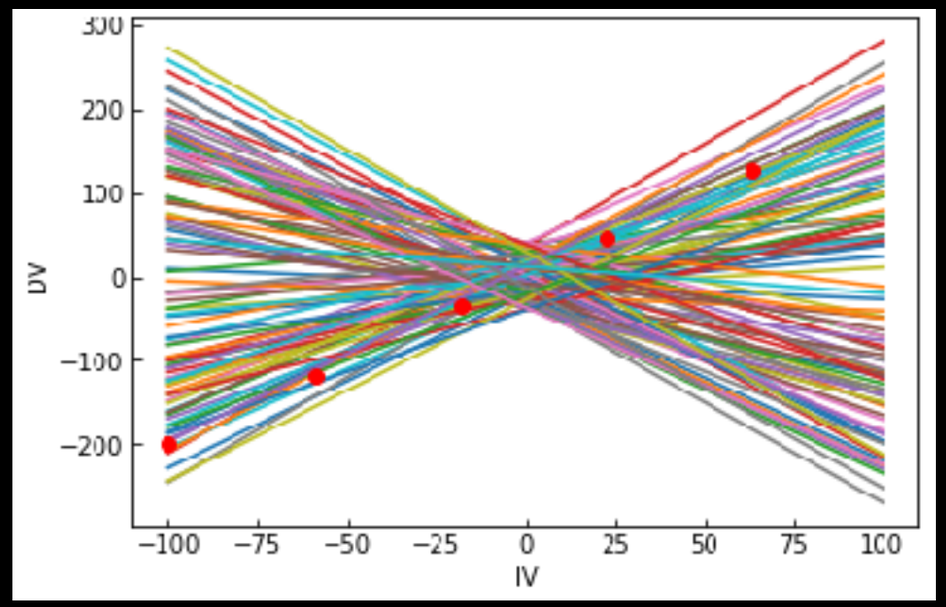


What's a model??



V: Likelihood and Regression Models

a formula that describes the data ⟶ *a family of models*

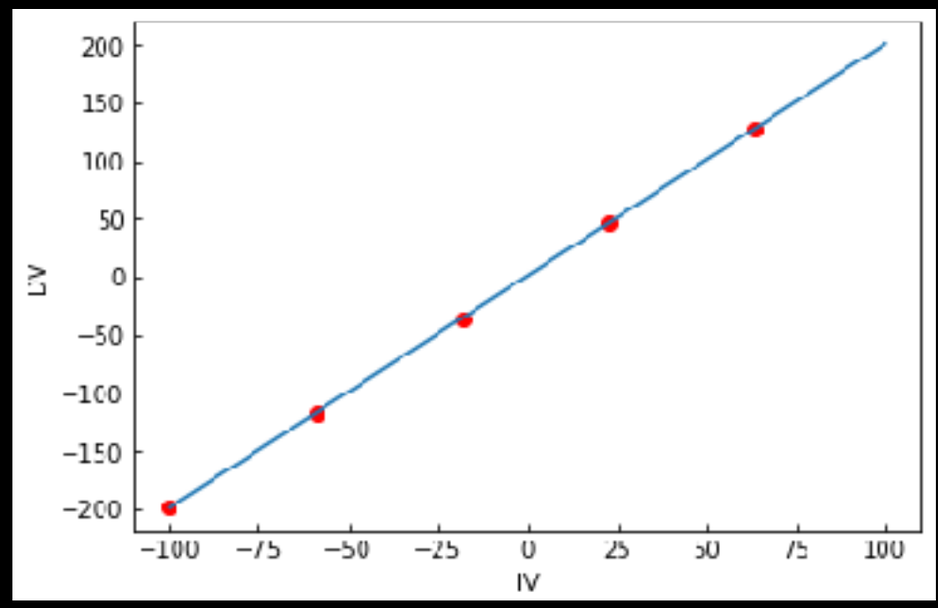


What's a model??

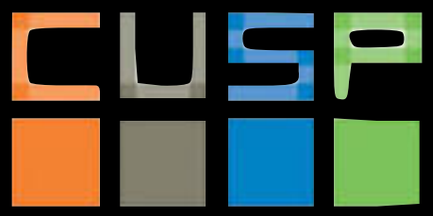
a formula that describes the data ⟶ *a family of models*

the best fit chooses within that family
the model that has the
best parameters

What's a model??



How do we fit a model to data?



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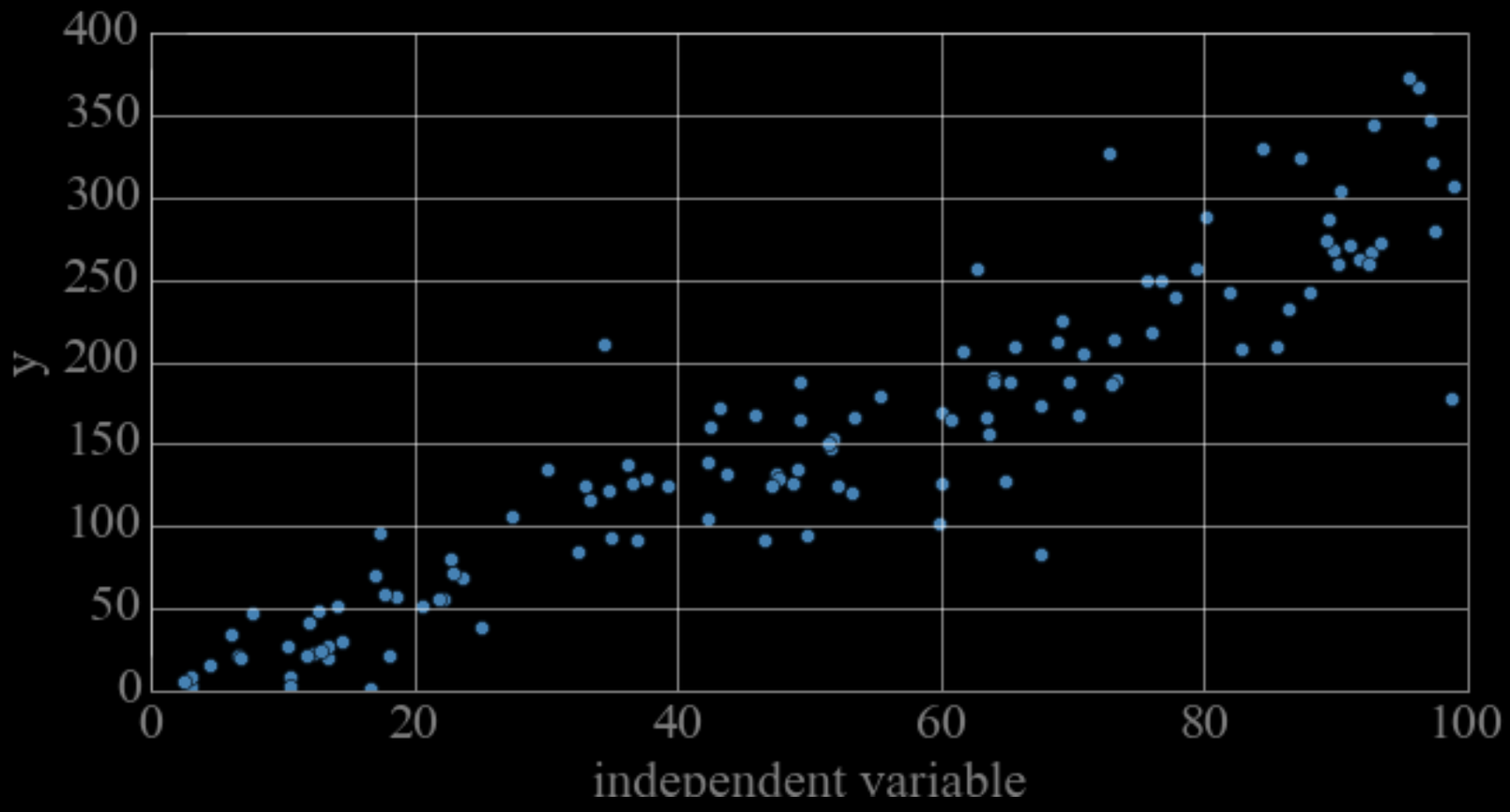

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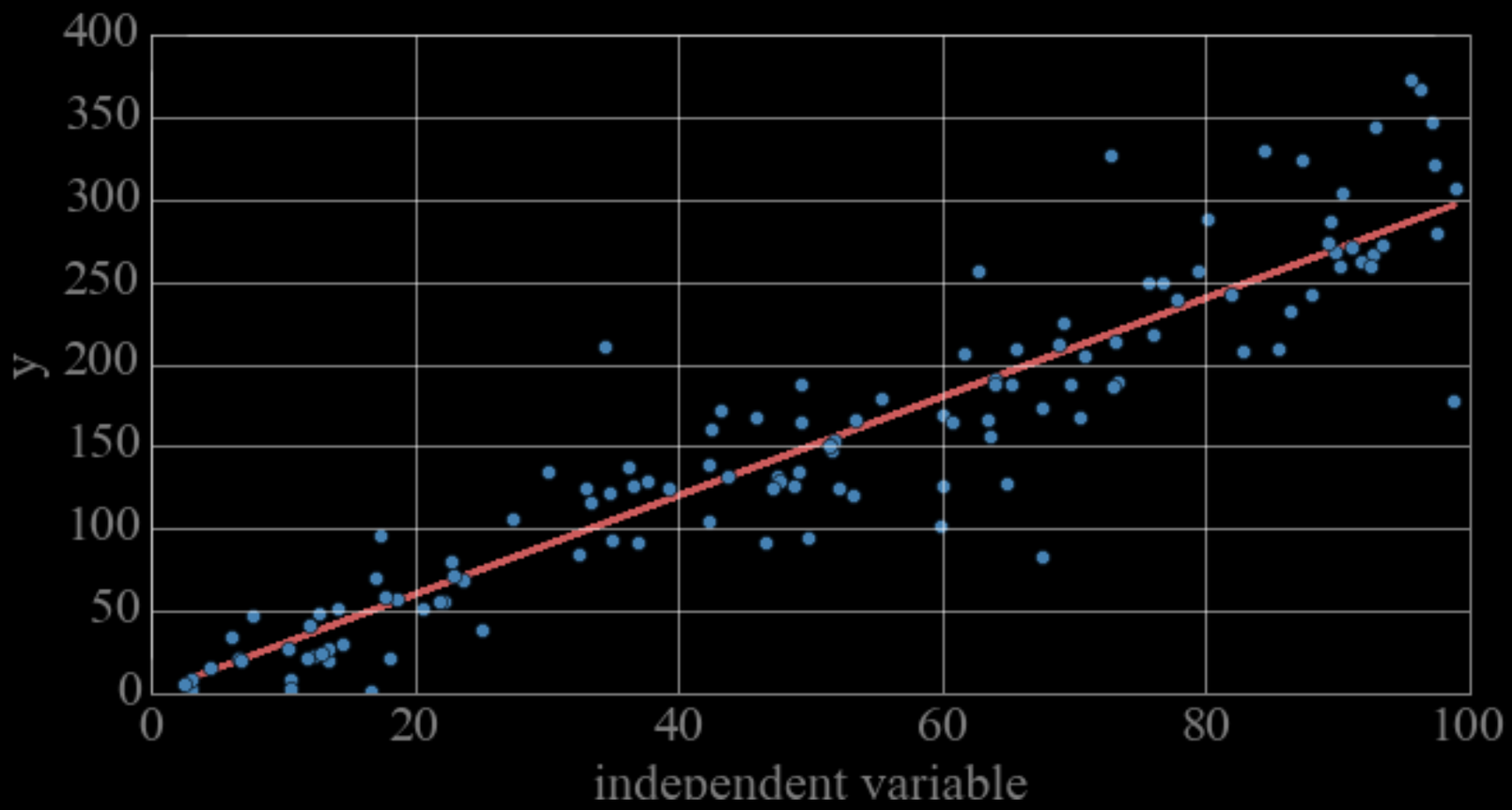

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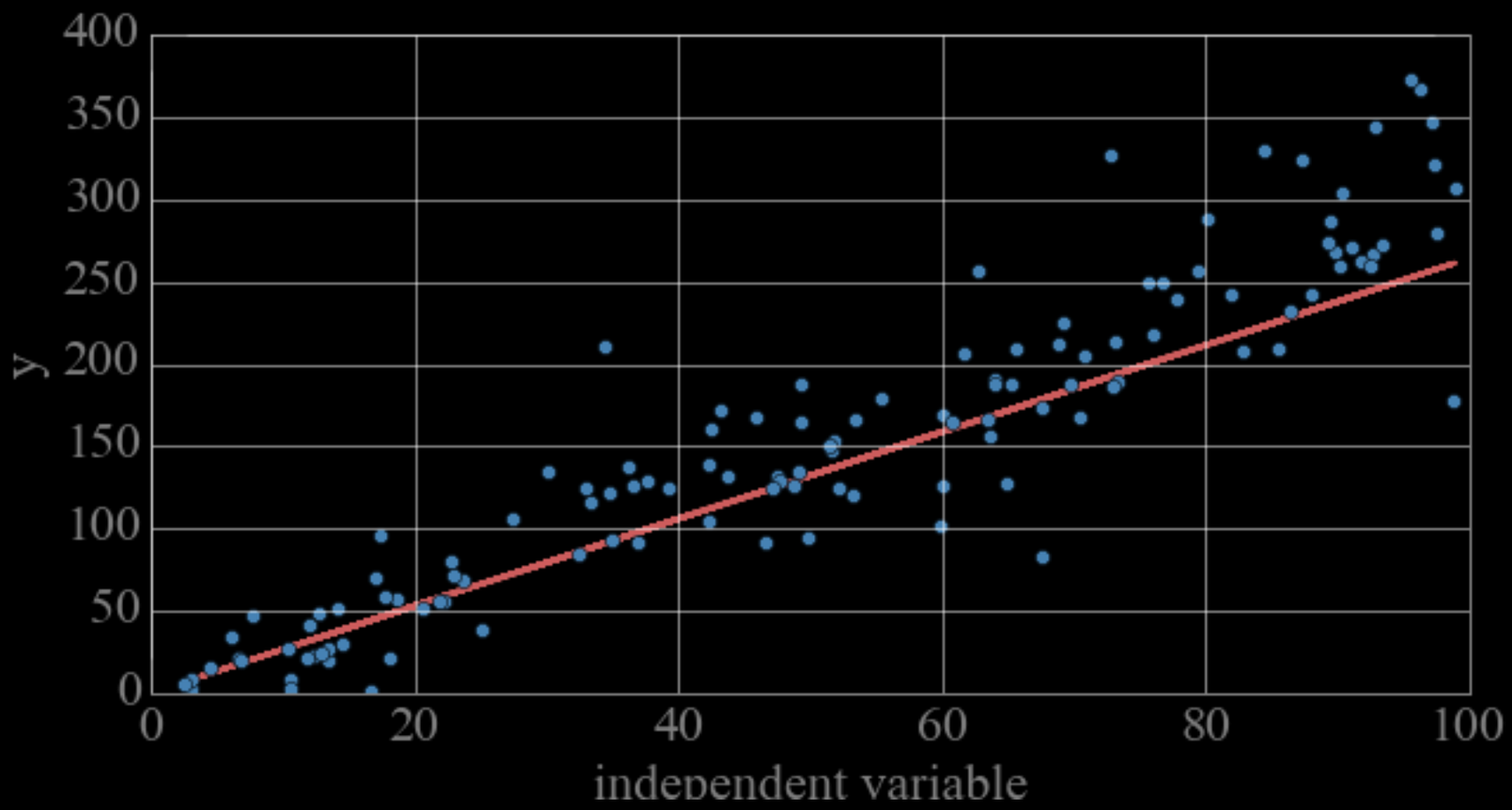

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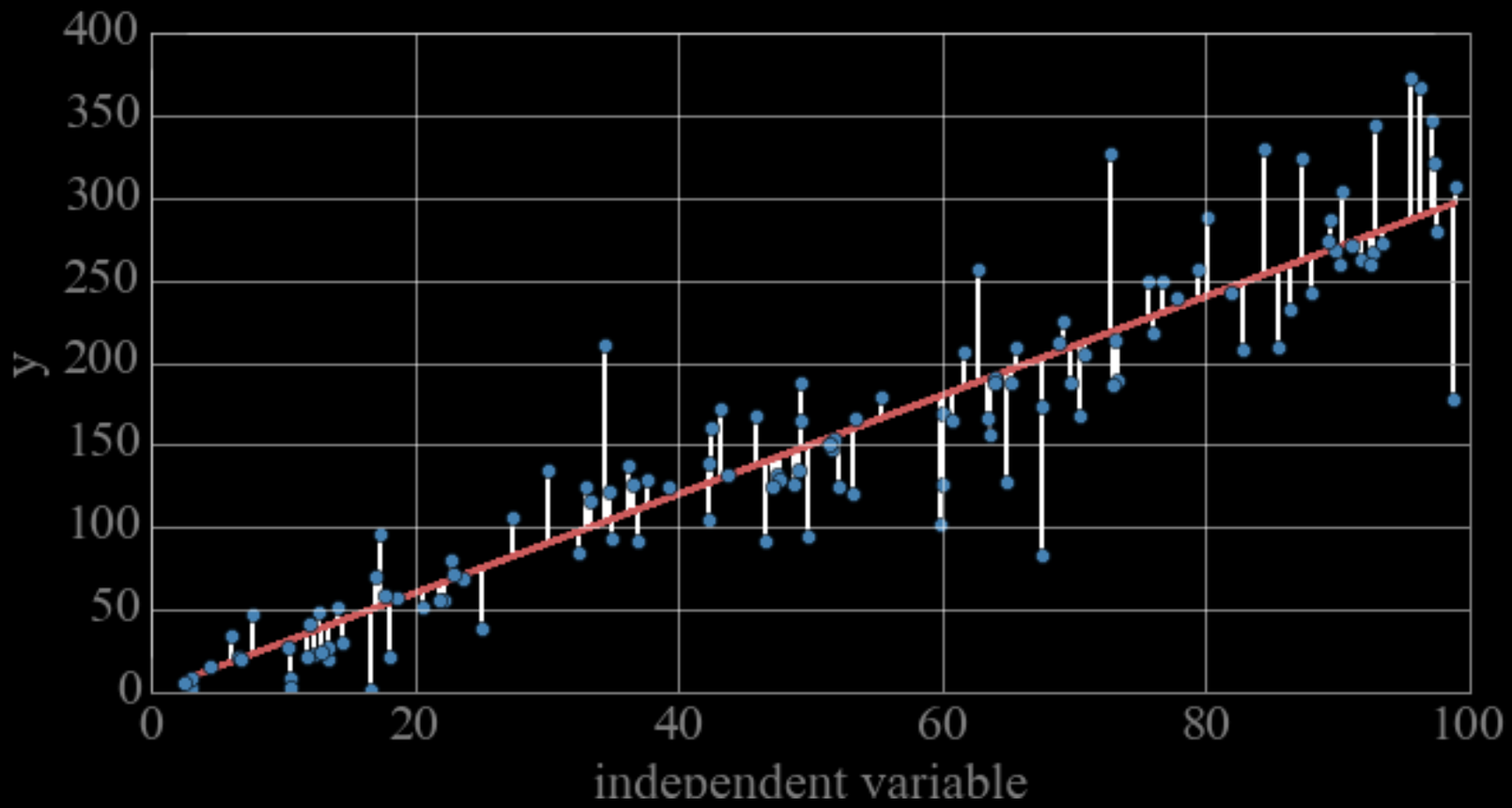

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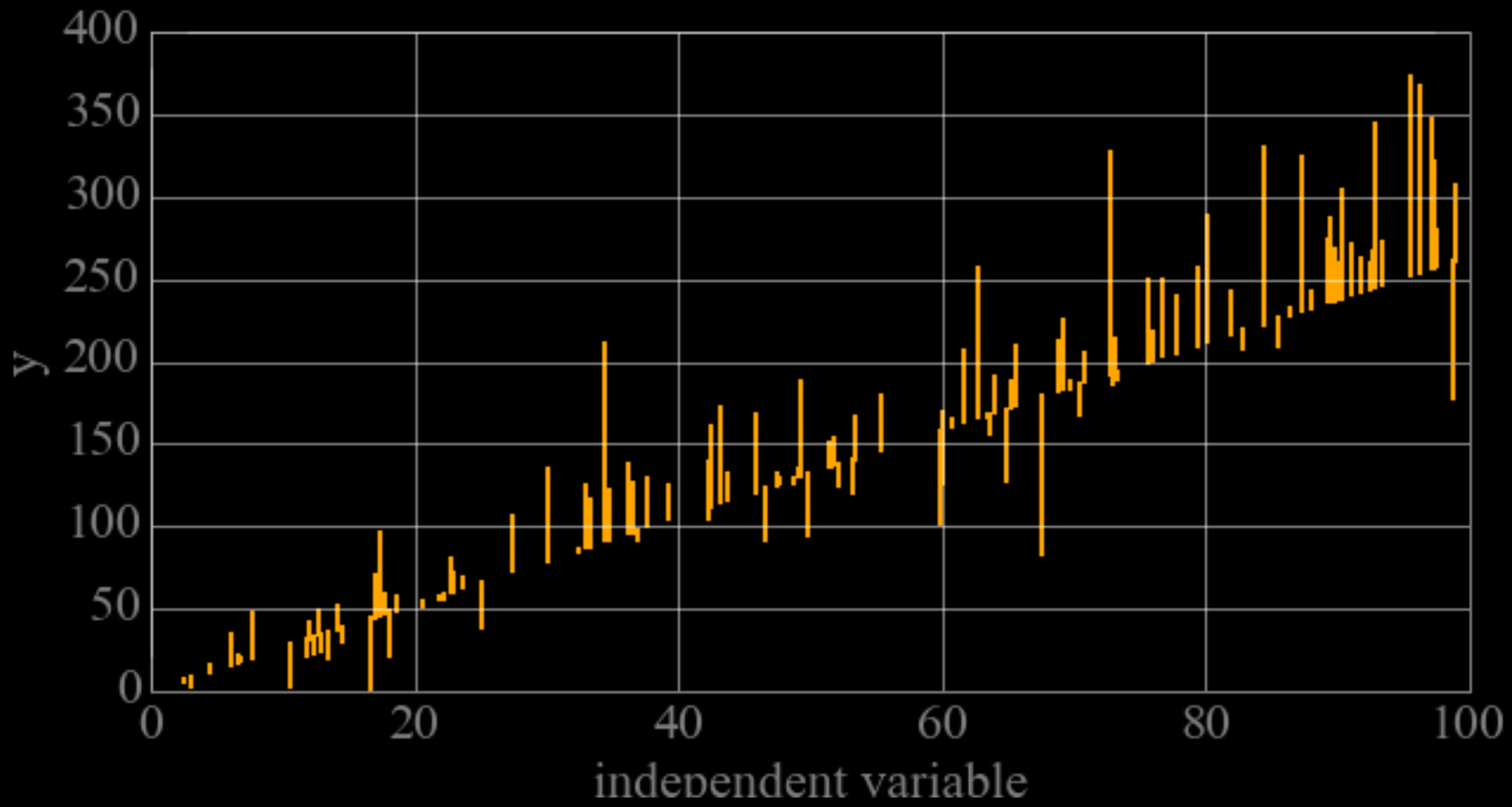

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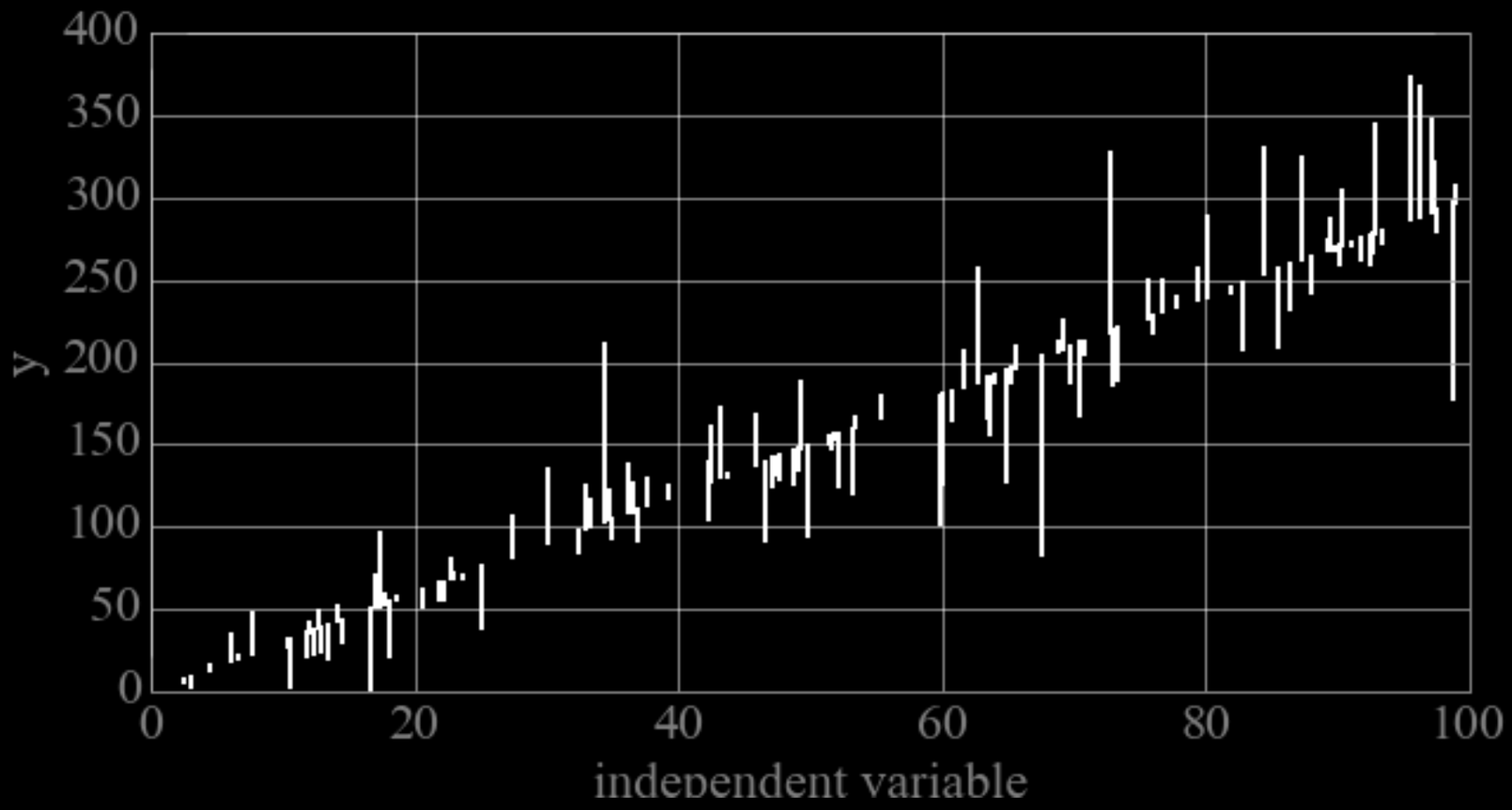
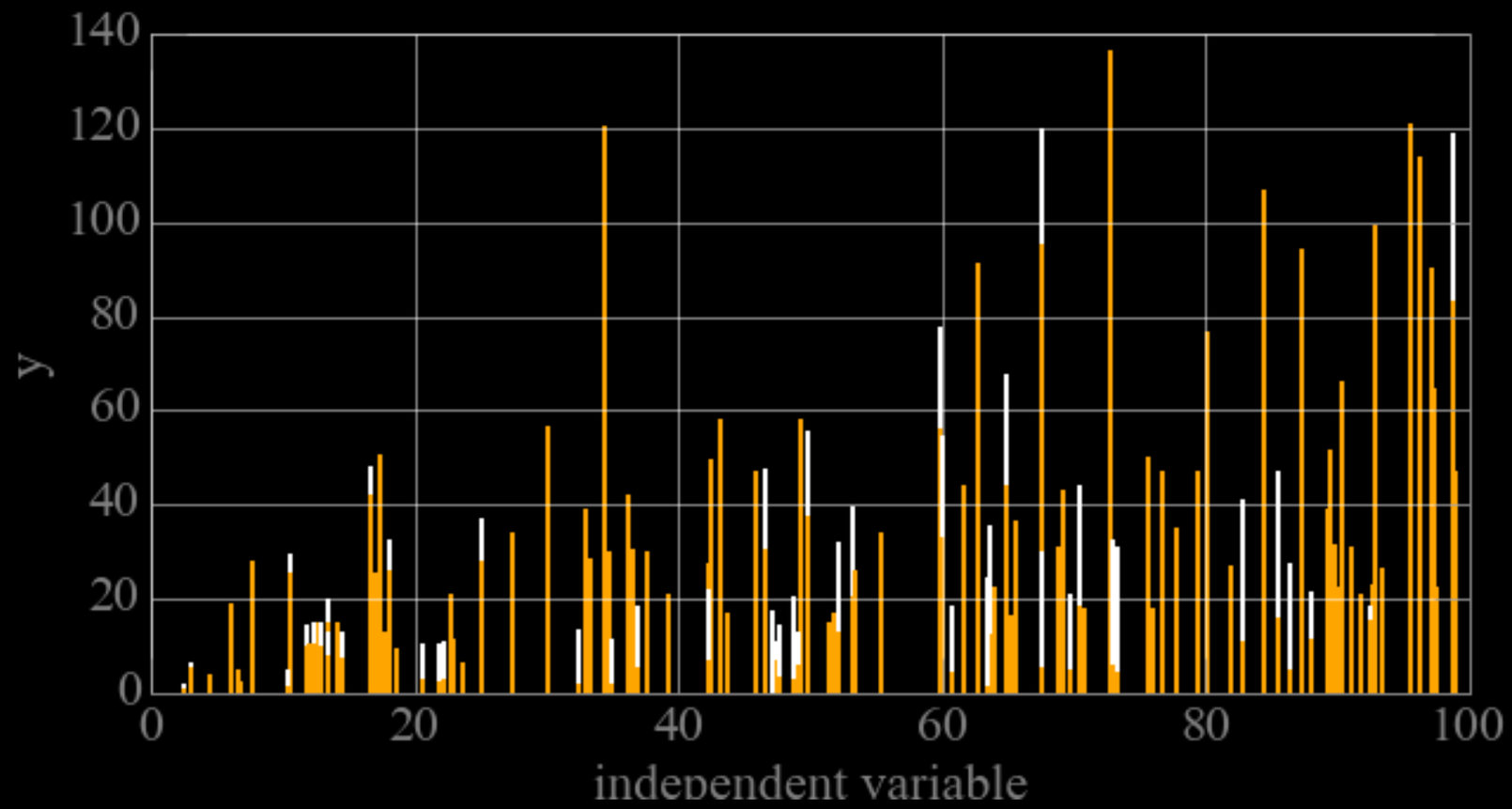


V: Likelihood and Regression Models

Fit model parameters =
minimize the
Sum of residuals squared $\sum_i (y_i - (ax_i + b))^2$

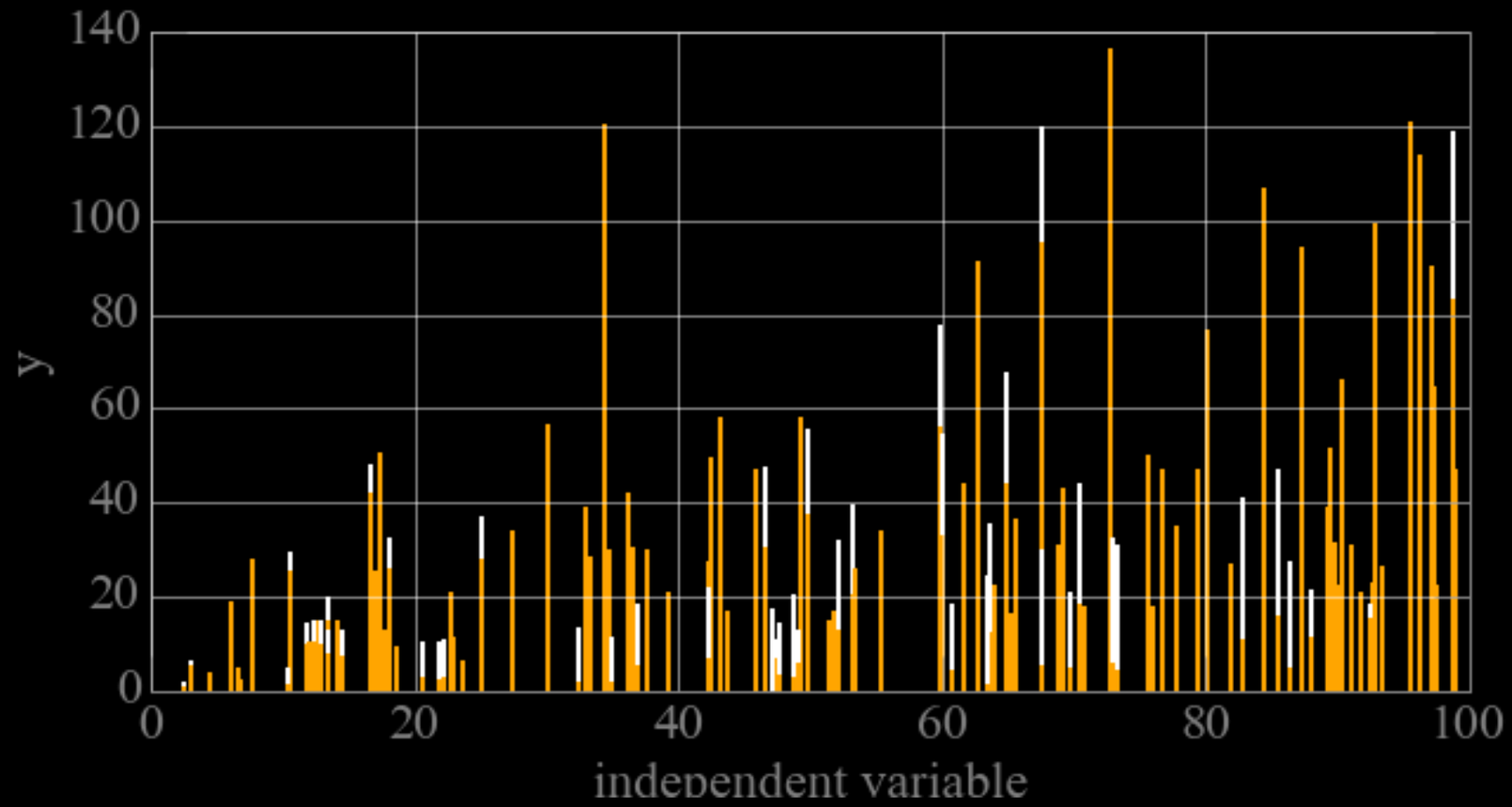
11655.34 < 12155.24



Fit model parameters =

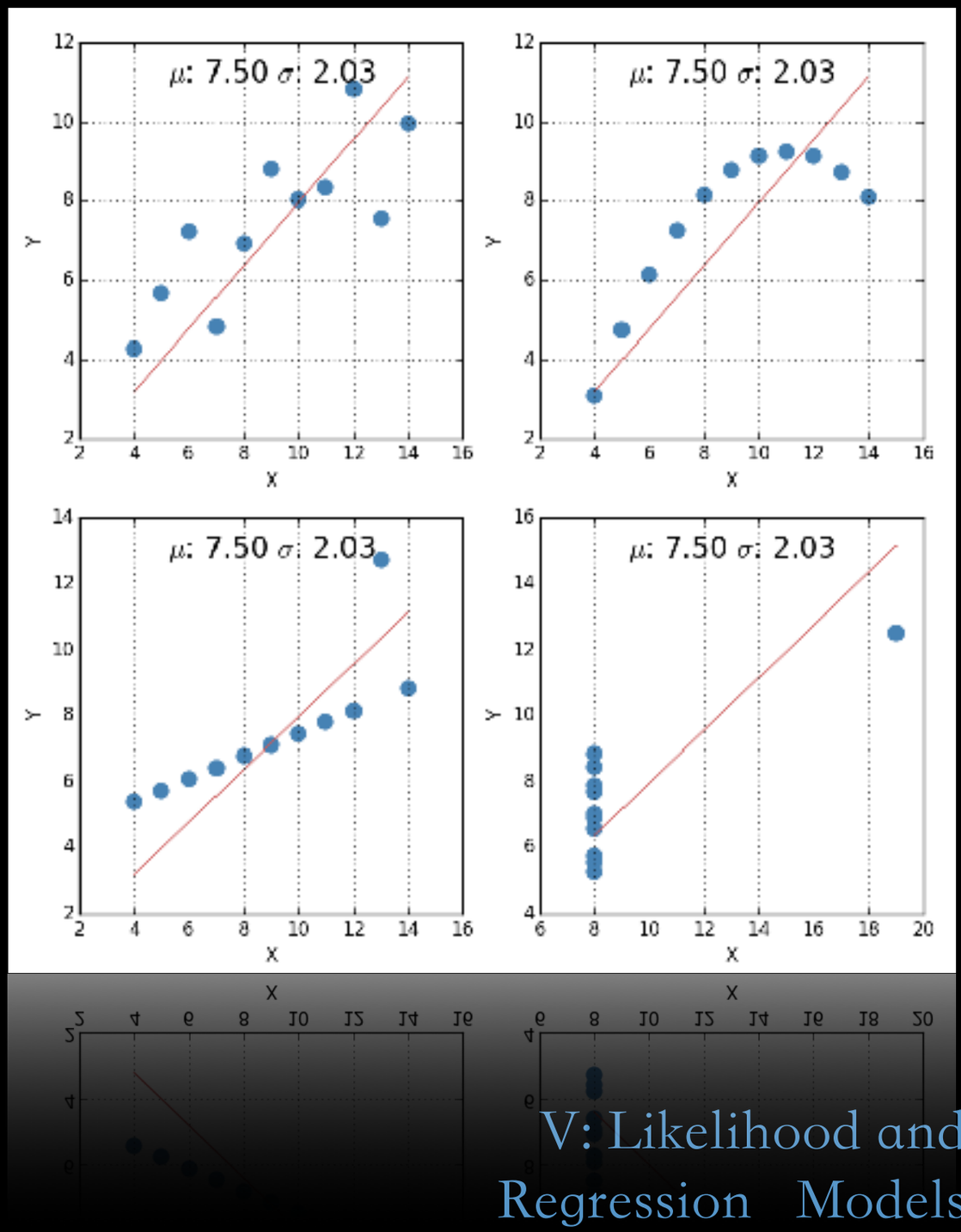
find *m* and *b* such that $\sum_i (y_i - (ax_i + b))^2$ *is minimal*

11655.34 < 12155.24



https://github.com/fedhere/UInotebooks/blob/master/Anscombe's%20Quartet.ipynb

Model residuals



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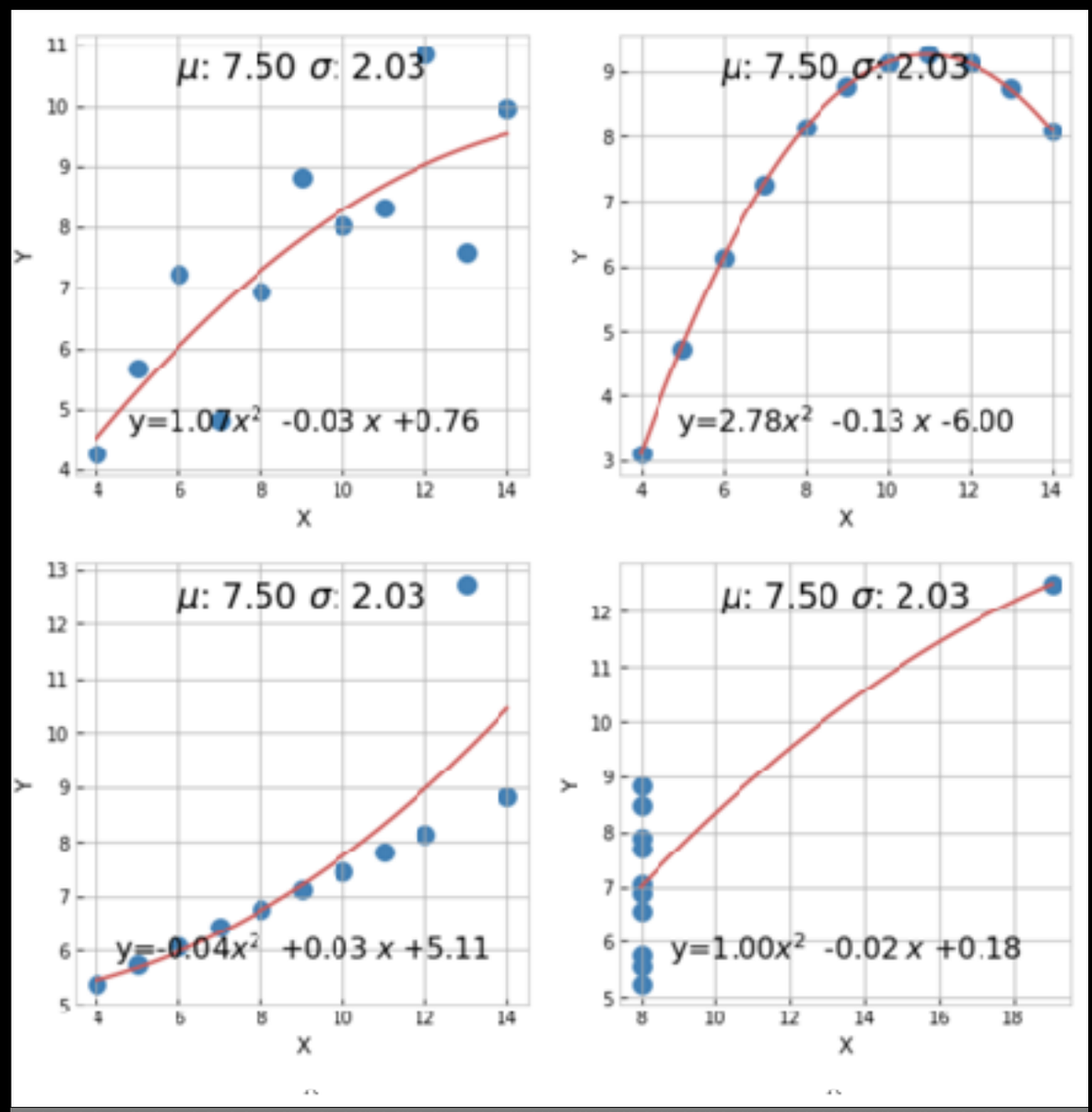


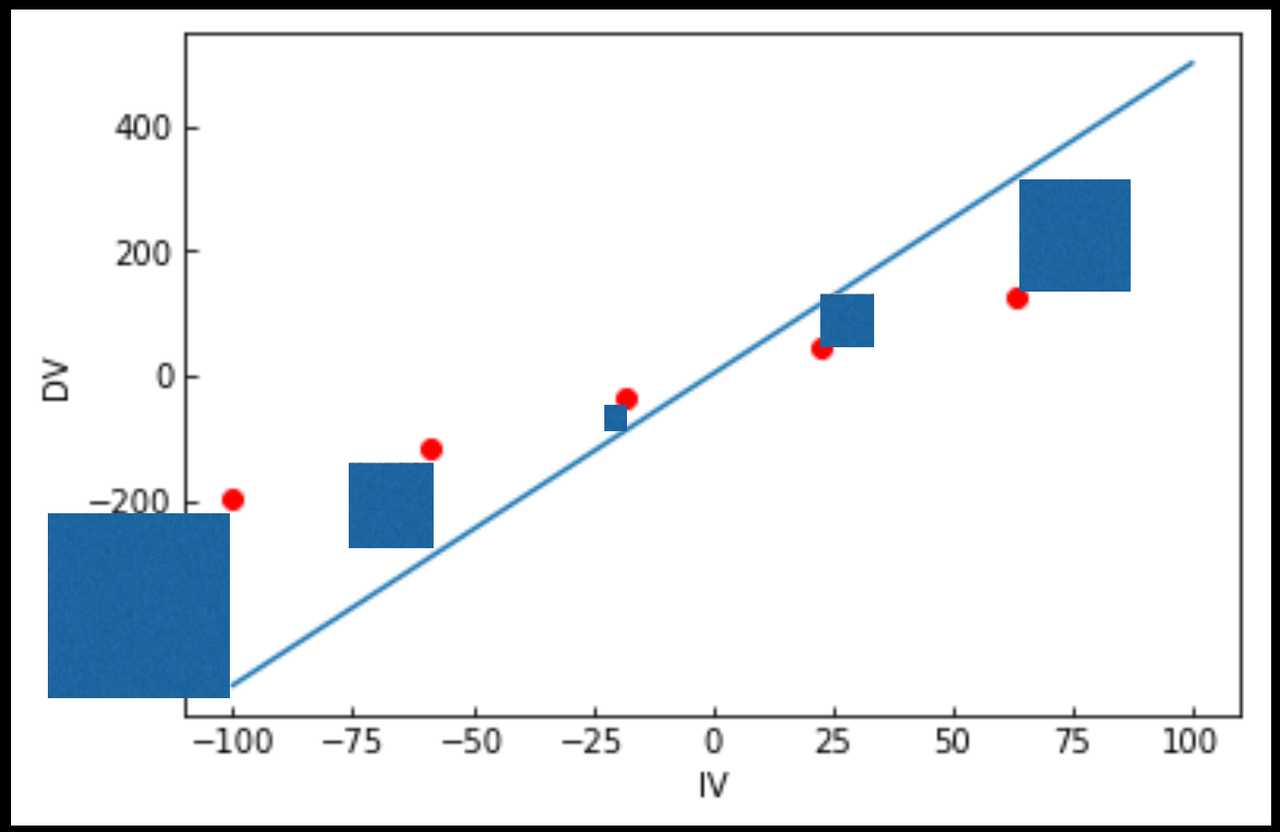
How good is a model?

Model diagnostics:

Chi^2, R^2, and LR test

R^2 $1-\sum_i (y_i-(ax_i+b))^2$

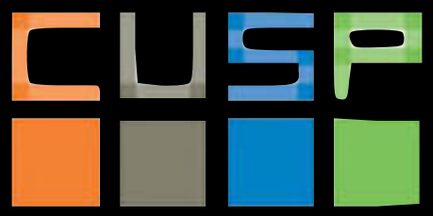
amount of variance in data explained by the model



Regression diagnostics

This example file shows how to use a few of the `statsmodels` regression diagnostic tests in a real-life context. You can learn about more tests and find out more information abou the tests here on the Regression Diagnostics page.

http://www.statsmodels.org/dev/examples/notebooks/generated/regression_diagnostics.html

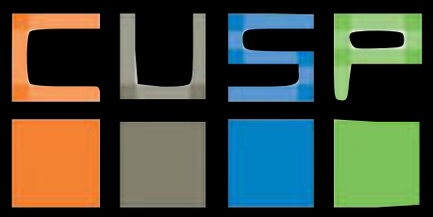


OLS Regression Results

Dep. Variable:	Y	R-squared:	0.687
Model:	OLS	Adj. R-squared:	0.609
Method:	Least Squares	F-statistic:	8.793
Date:	Tue, 11 Oct 2016	Prob (F-statistic):	0.00956
Time:	06:14:52	Log-Likelihood:	-16.487
No. Observations:	11	AIC:	38.97
Df Residuals:	8	BIC:	40.17
Df Model:	2		
Covariance Type:	nonrobust		

adjusted R^2 $\overline{R}^2 = R^2 - (1-R^2)\frac{p}{n-p-1}$

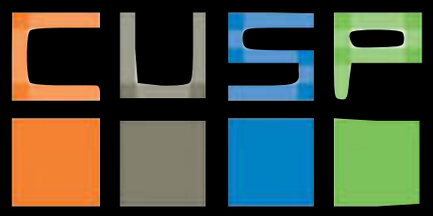
adjusts for the number of *explanatory terms* (parameters) in a model relative to the number of data points



χ^2 (chi^2)

$$\chi^2_F = \sum_i \frac{(m_i - x_i)^2}{\sigma_i^2}$$

how well model explains data *including uncertainties*

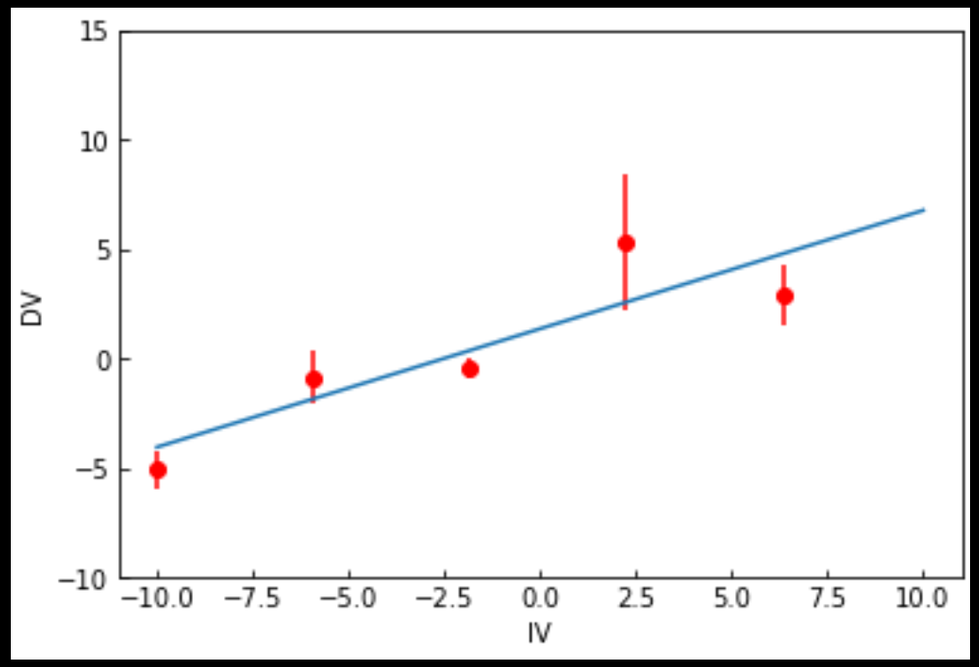


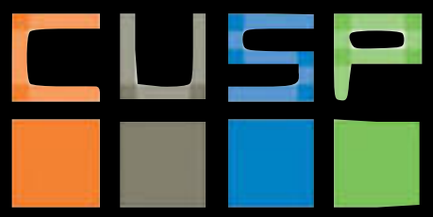
χ^2 (chi^2)

$$\chi^2_{/DOF} = \frac{1}{DOF} \sum_i \frac{(m_i - x_i)^2}{\sigma_i^2}$$

$R^2 = 0.8$

$\chi^2 = 2.5$



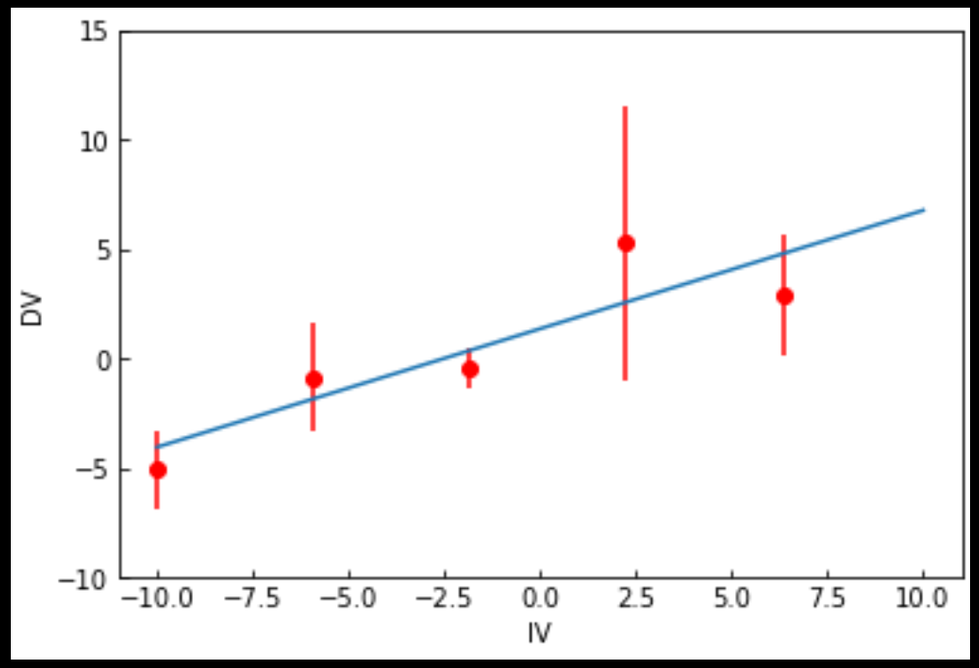


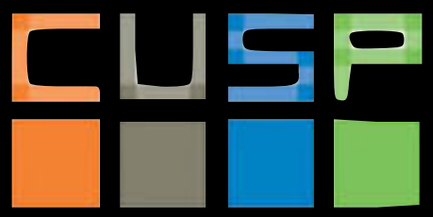
χ^2 (chi^2)

$$\chi^2_{/DOF} = \frac{1}{DOF} \sum_i \frac{(m_i - x_i)^2}{\sigma_i^2}$$

$R^2 = 0.8$

$\chi^2 = 0.6$



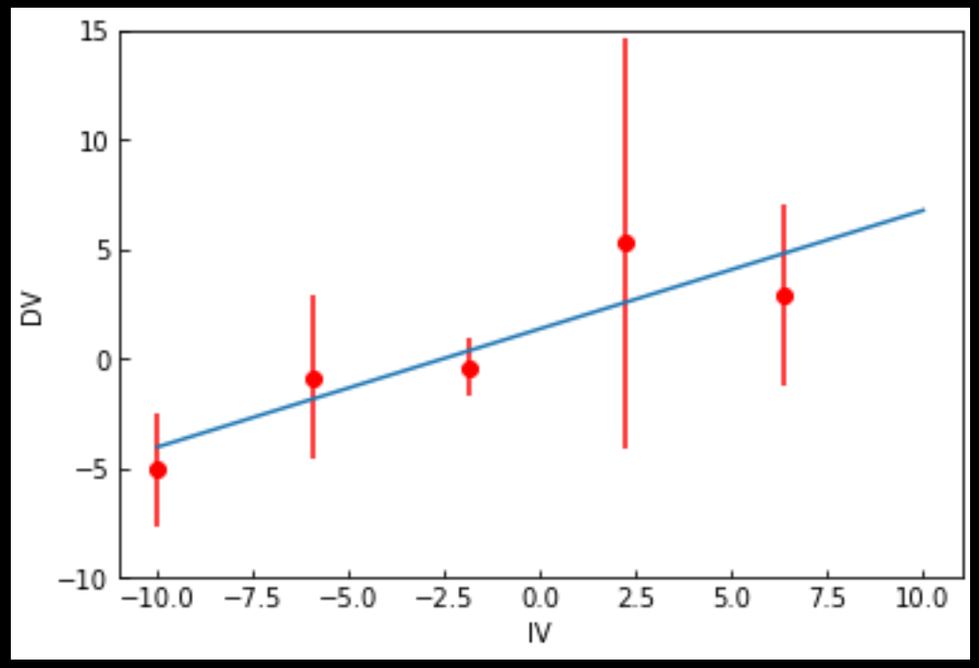


χ^2 (chi^2)

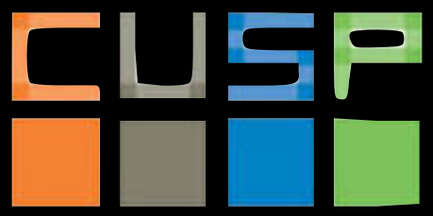
$$\chi^2_{/DOF} = \frac{1}{DOF} \sum_i \frac{(m_i - x_i)^2}{\sigma_i^2}$$

$R^2 = 0.8$

$\chi^2 = 0.3$



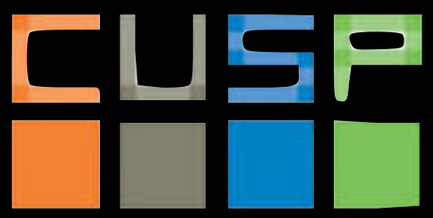
Likelihood



V: Likelihood and Regression Models

Probability $P(x \mid \theta)$

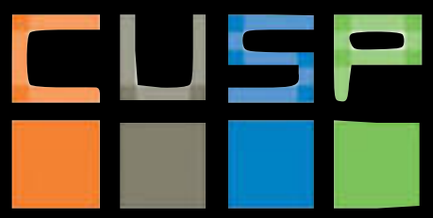
Likelihood



V: Likelihood and Regression Models

Probability $P(\vec{x} \mid \mu, \sigma)$

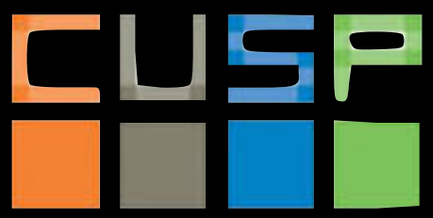
Likelihood



V: Likelihood and Regression Models

Probability $P(\vec{y} \mid \vec{x}, \mu, \sigma)$

Likelihood



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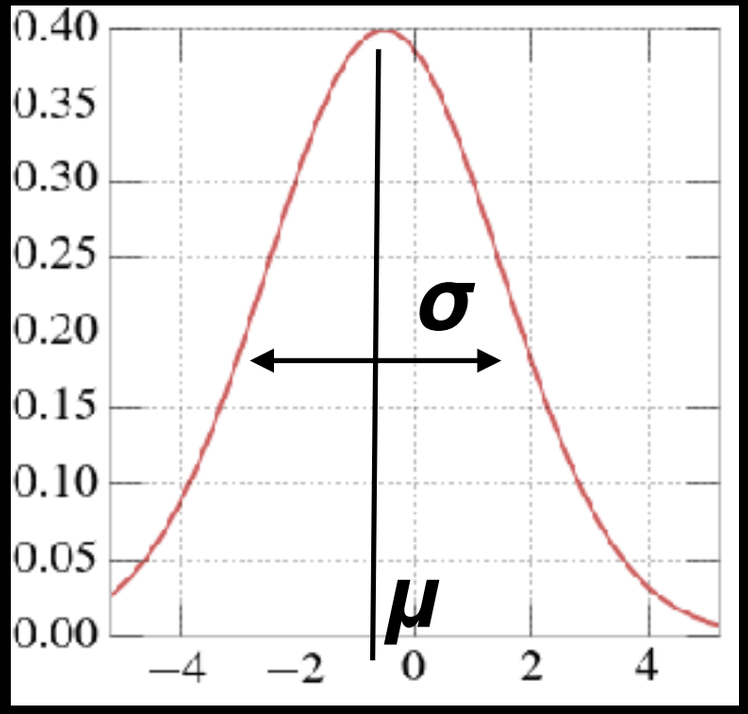
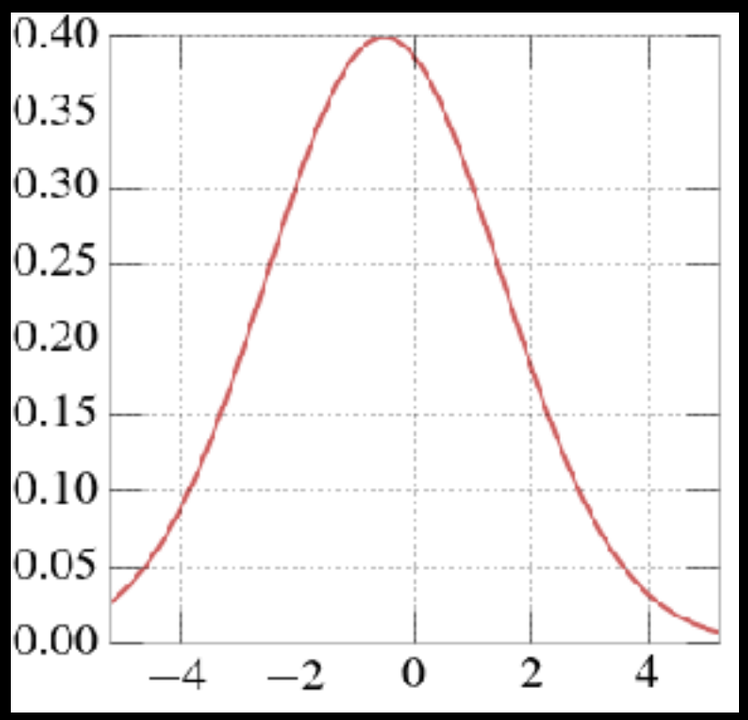
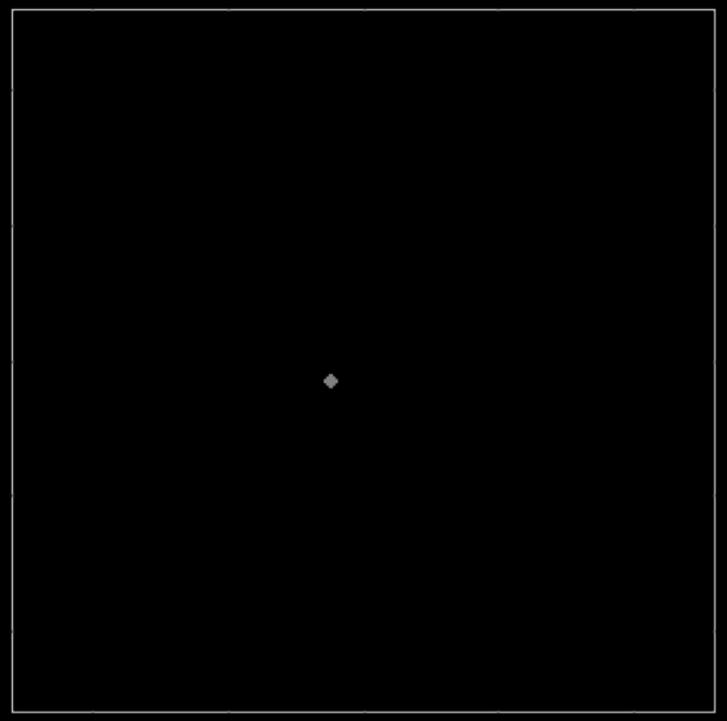
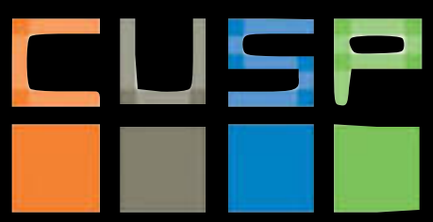
Probability $P(\vec{x} \mid \mu, \sigma)$



Probability $P(\vec{x} \mid \vec{\theta})$

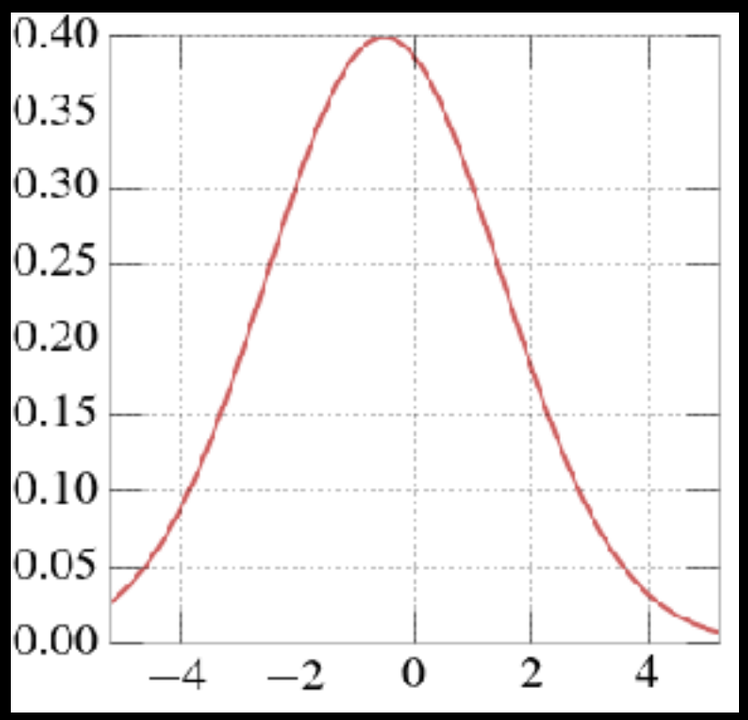


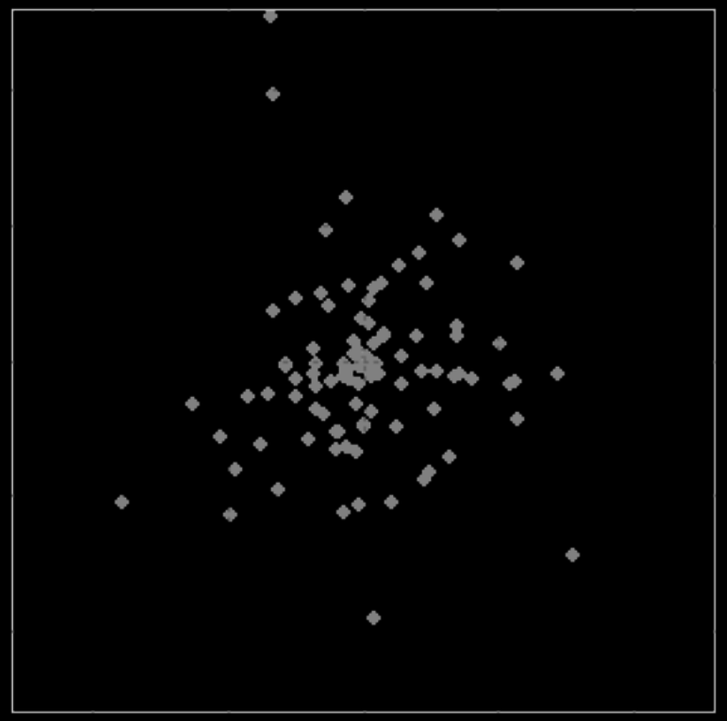


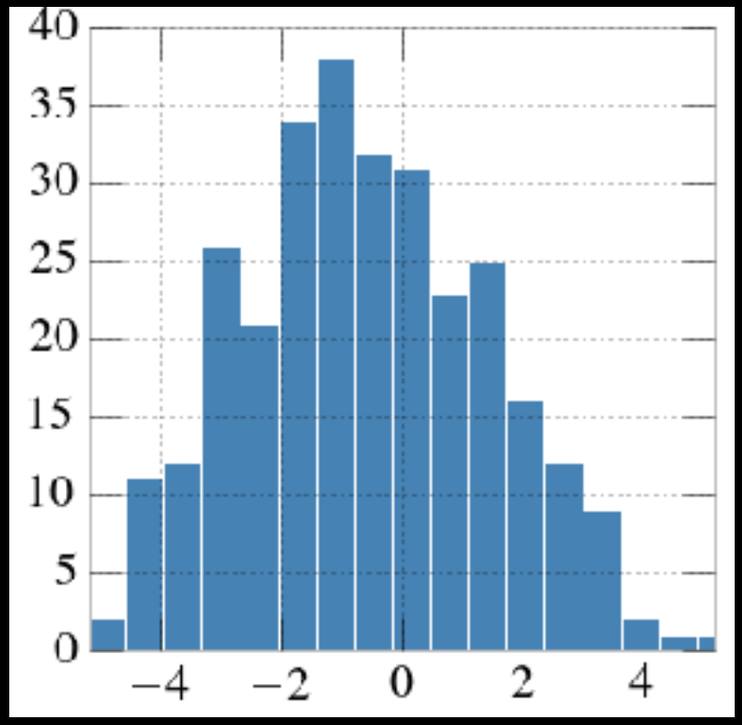


V: Likelihood and Regression Models

Probability $P(\vec{x} \mid \vec{\theta})$





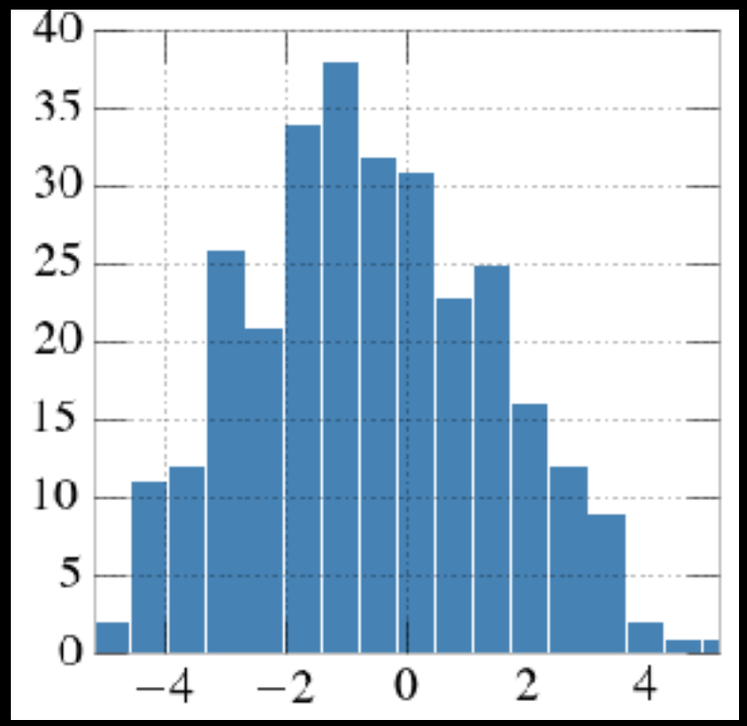


CUSP

V: Likelihood and Regression Models

Probability $P(\vec{x} \mid \vec{\theta})$

Likelihood $P(\vec{\theta} \mid \vec{x})$



Probability $P(\vec{x} \mid \vec{\theta})$

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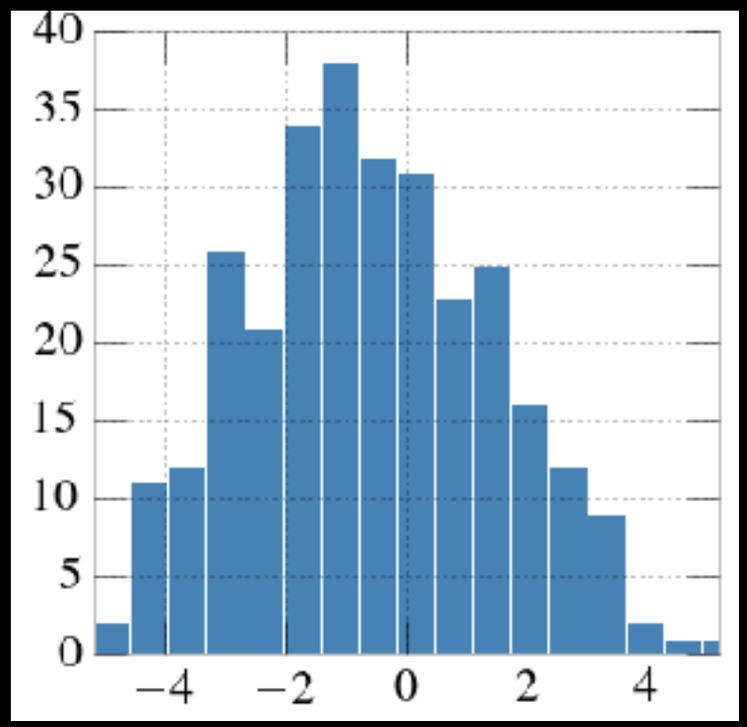
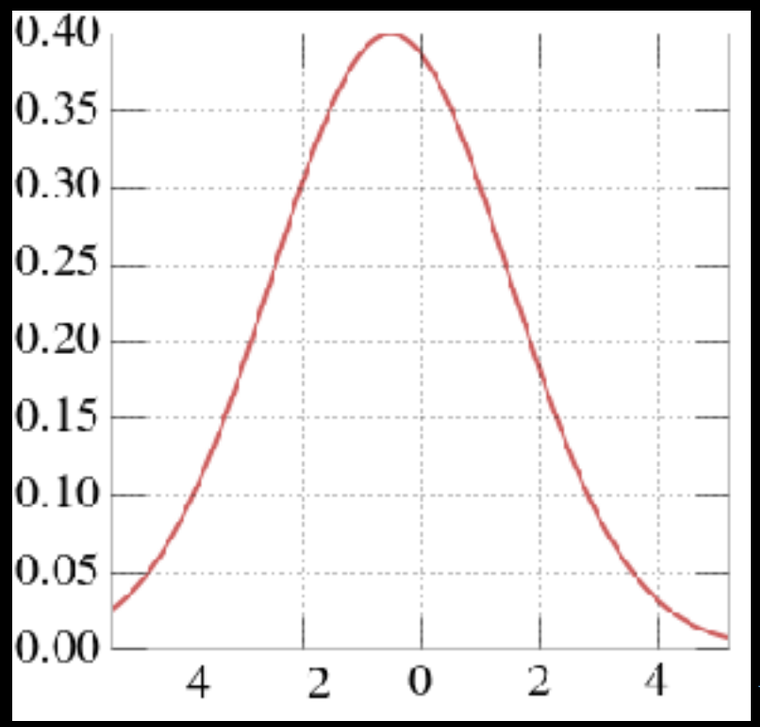
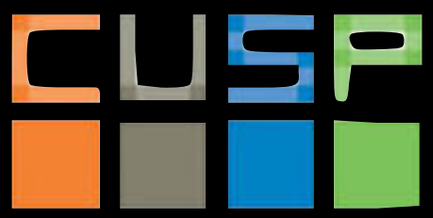




Probability $$N(\mu,\sigma) \sim \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

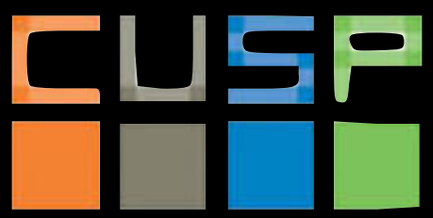
Likelihood



V: Likelihood and Regression Models

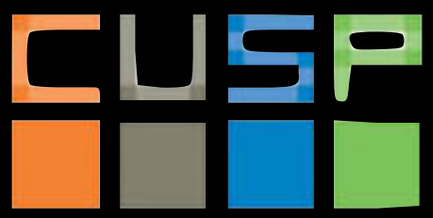
Probability $$N(\mu,\sigma) \sim \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Likelihood $$\mathscr{L}_{(\mu,\sigma)}(x) \sim \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Probability $$N(\mu,\sigma) \sim \Pi_i \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

Likelihood



V: Likelihood and Regression Models

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Likelihood $$\mathcal{L}_{(\mu,\sigma)}(\vec{x}) \sim \Pi_i \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

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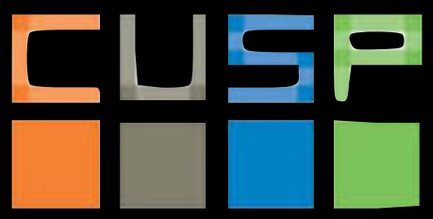
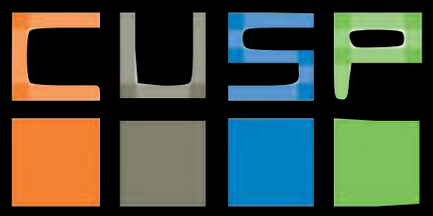
Likelihood $$\mathcal{L}_{(\mu,\sigma)}(\vec{x}) \sim \frac{1}{(\sigma\sqrt{2\pi})^n} \Pi_i e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$



Probability $$N(\mu,\sigma) \sim \Pi_i \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

Likelihood $$\mathcal{L}_{(\mu,\sigma)}(\vec{x}) \sim \frac{1}{(\sigma\sqrt{2\pi})^n} e^{-\frac{\Sigma_i(x_i-\mu)^2}{2\sigma^2}}$$

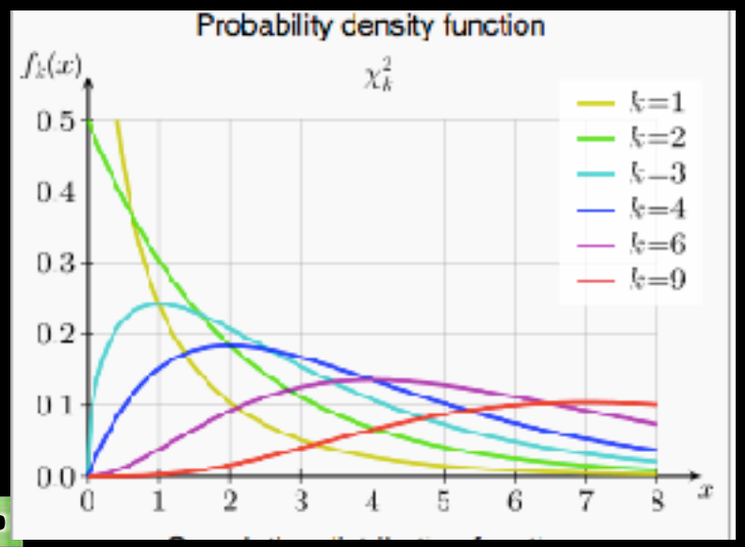
Likelihood-ratio tests

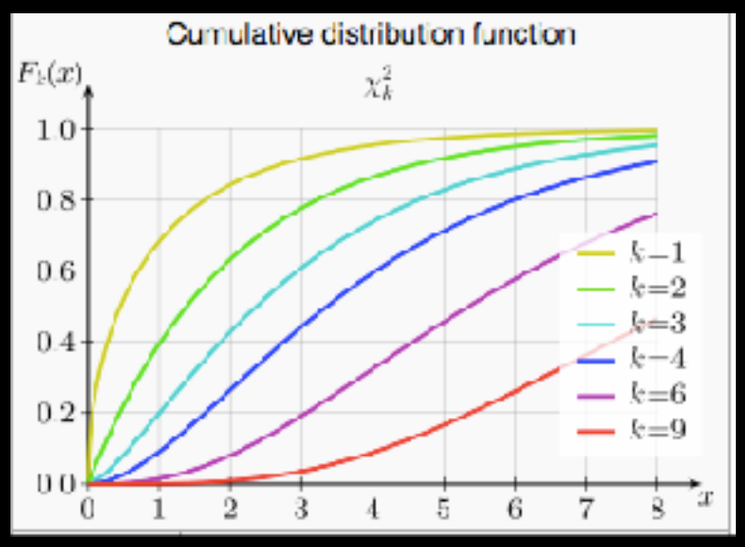


V: Likelihood and Regression Models

$$LR = -2 \log_e \frac{L(\text{model 1})}{L(\text{model 2})}$$

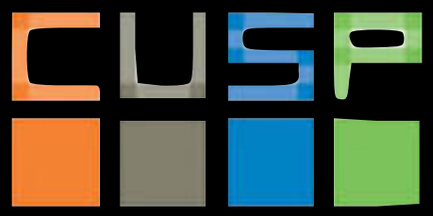
This statistic is chi-squared distributed



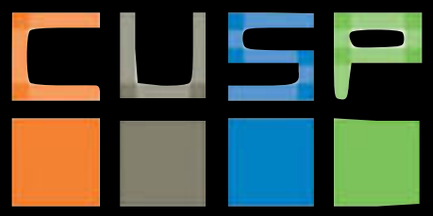


$$LR = -2 \log_e \frac{L(\text{model 1})}{L(\text{model 2})}$$

This statistic is chi-squared distributed with degrees of freedom equal to the difference in the number of degrees of freedom between the two models (i.e., the number of variables added to the model).



Maximizing Likelihood



V: Likelihood and Regression Models

Probability $$N(\mu,\sigma) \sim \Pi_i \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

Likelihood $$\mathcal{L}_{(\mu,\sigma)}(\vec{x}) \sim \frac{1}{(\sigma\sqrt{2\pi})^n} e^{-\frac{\Sigma_i(x_i-\mu)^2}{2\sigma^2}}$$

Given some observations $\vec{x}$ we want to model them with the best function: the one that is MAXIMALLY LIKELY.

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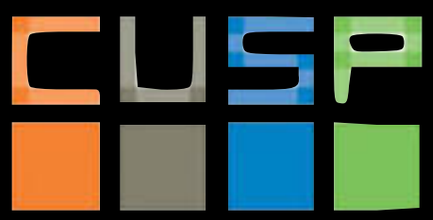
Given some observations $\vec{x}$ we want to model them with the best function: the one that is MAXIMALLY LIKELY. After we choose a functional form (N) for the model we want to choose the parameters (μ,σ) that maximiz $\mathcal{L}_{(\mu,\sigma)}(\vec{x})$

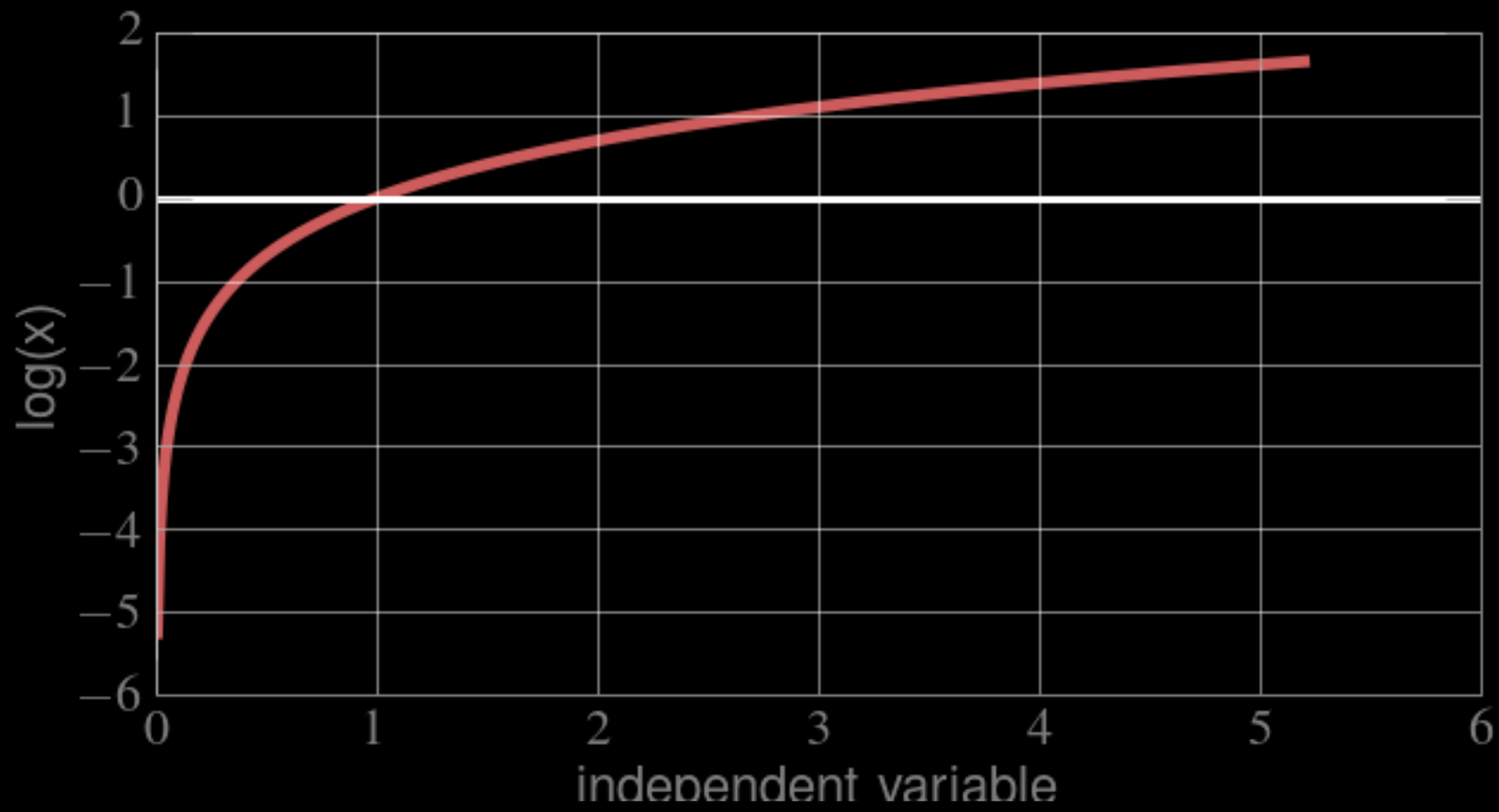
Probability $$N(\mu,\sigma) \sim \Pi_i \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

Likelihood $$\mathcal{L}_{(\mu,\sigma)}(\vec{x}) \sim \frac{1}{(\sigma\sqrt{2\pi})^n} e^{-\frac{\Sigma_i(x_i-\mu)^2}{2\sigma^2}}$$

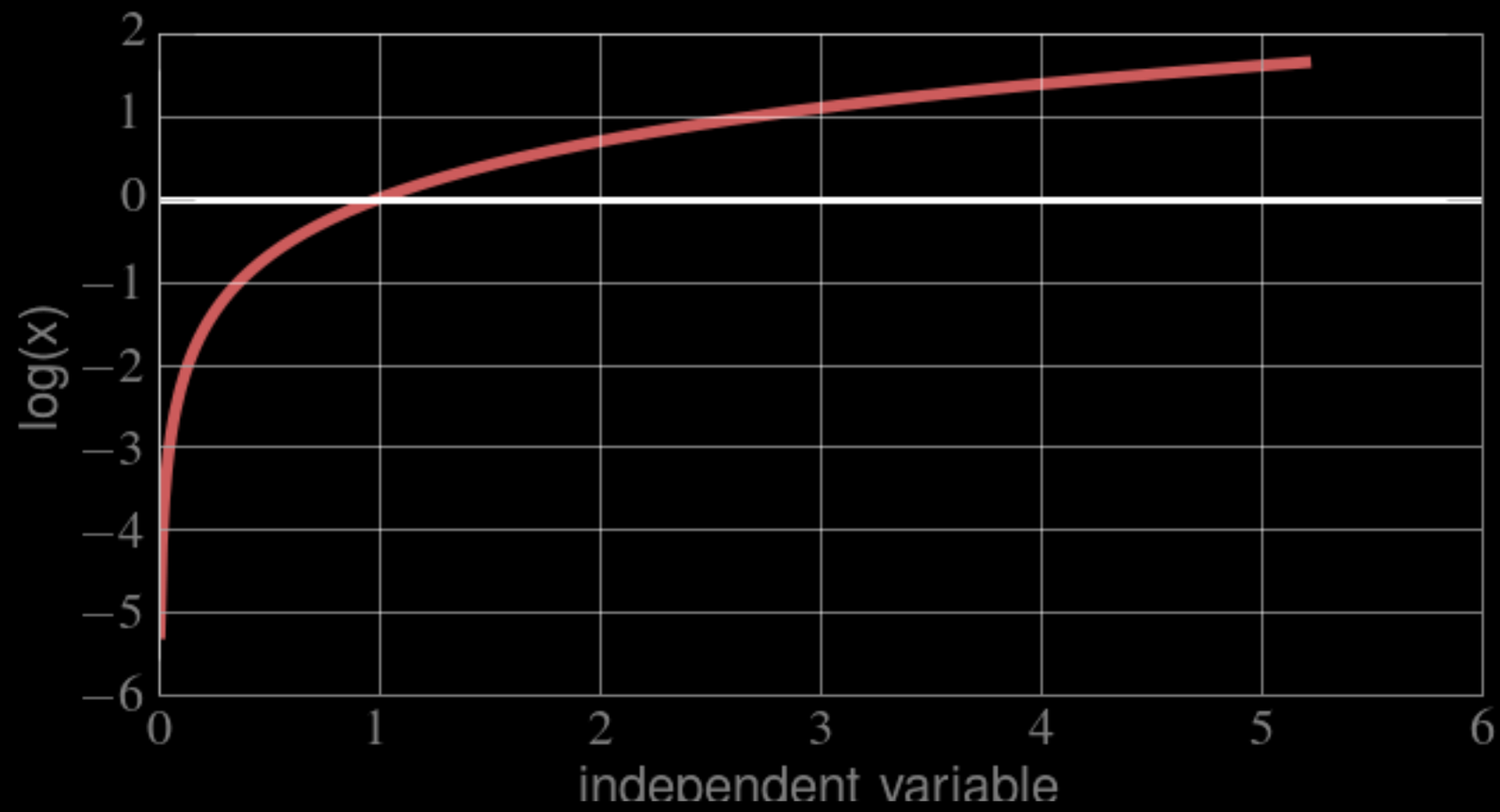
FIND $\mu^*, \sigma^* \mid \mathcal{L}_{(\mu^*,\sigma^*)} = \max(\mathcal{L}_{(\mu,\sigma)}(\vec{x}))$



Logarithm:



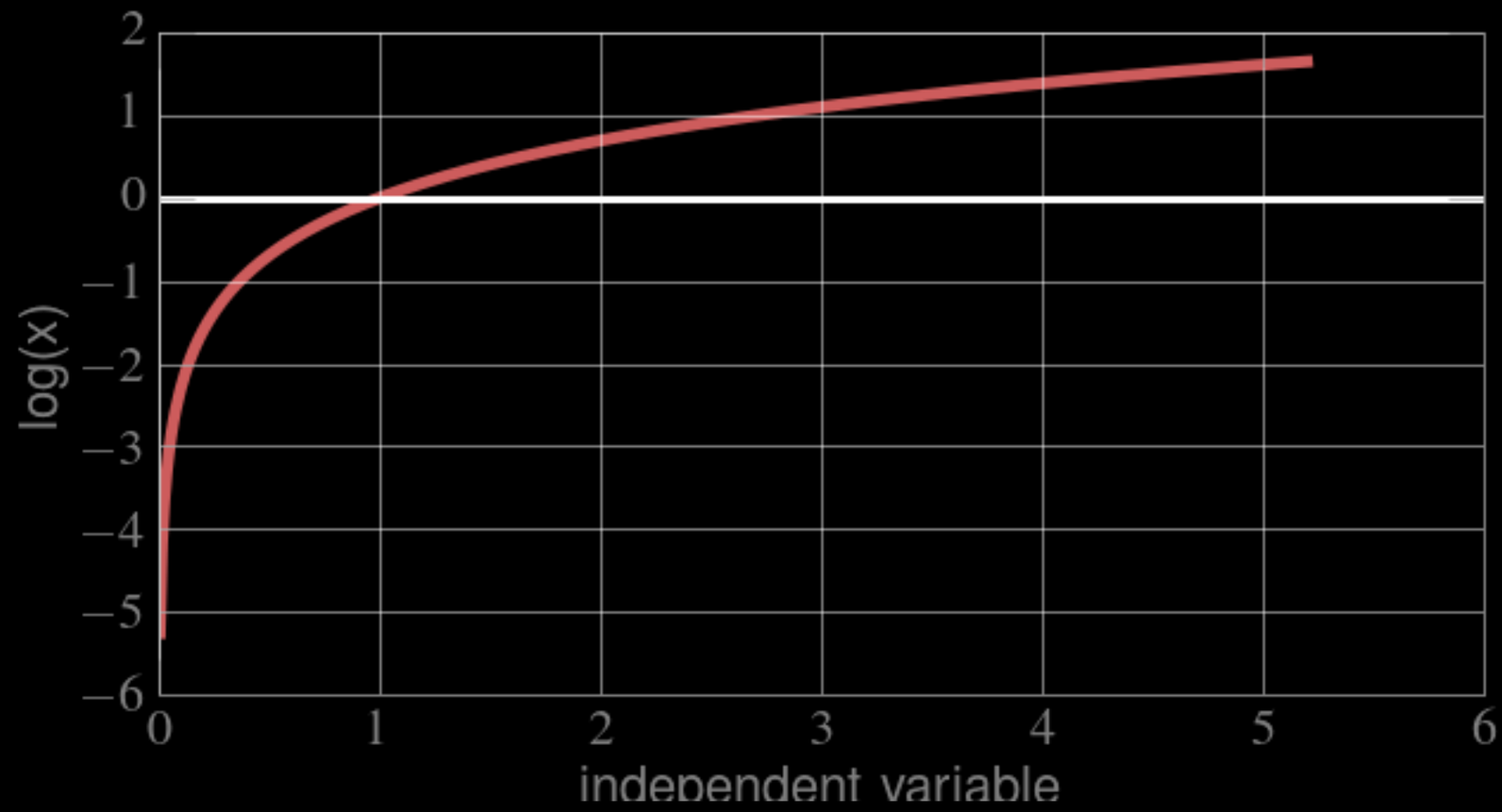
Logarithm: MONOTONICALLY INCREASING



Logarithm: MONOTONICALLY INCREASING

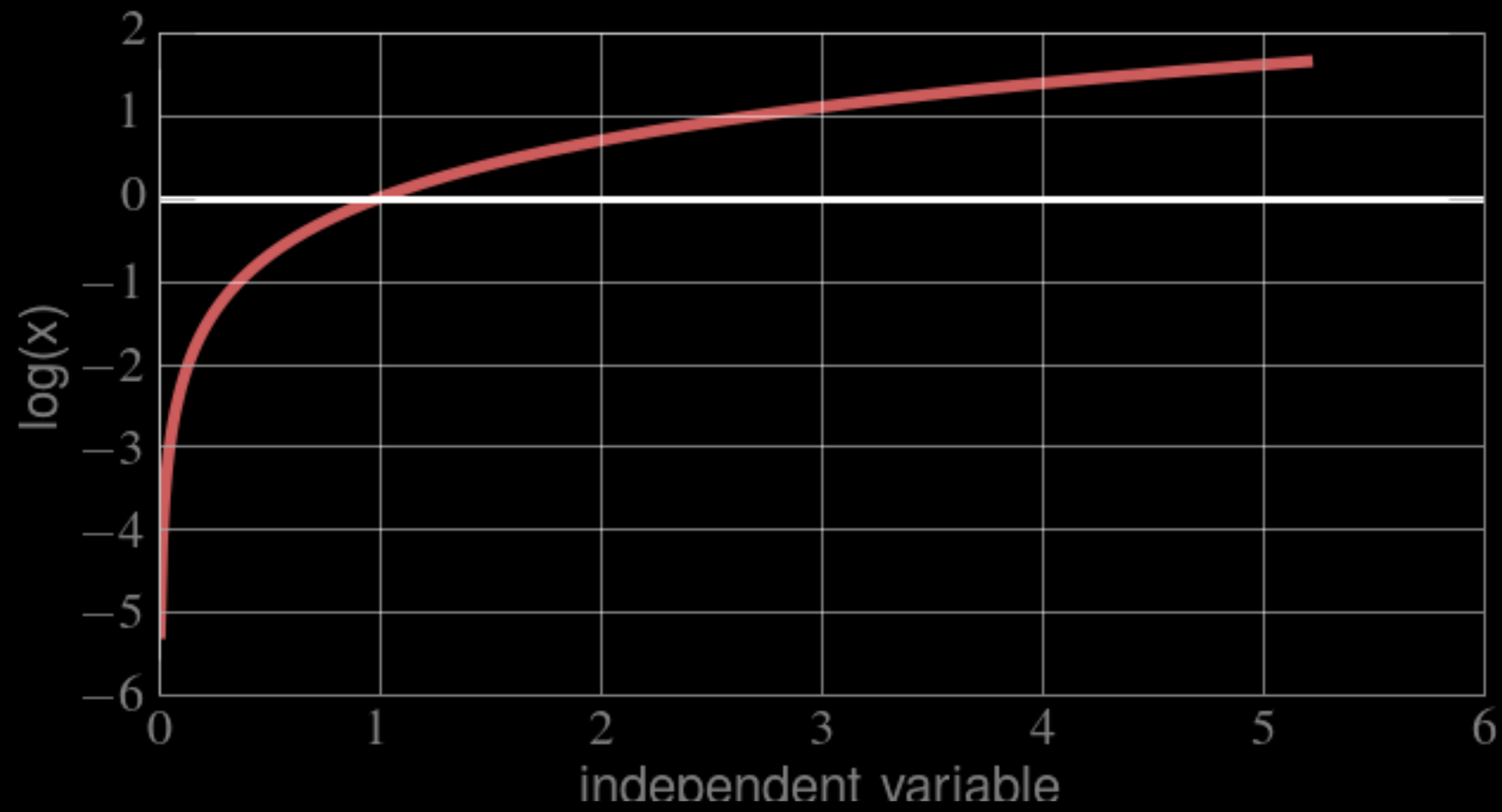
if x grows, log(x) grows, if x decreases, log(x) decreases

the location of the maximum is the same!



Logarithm: MONOTONICALLY INCREASING

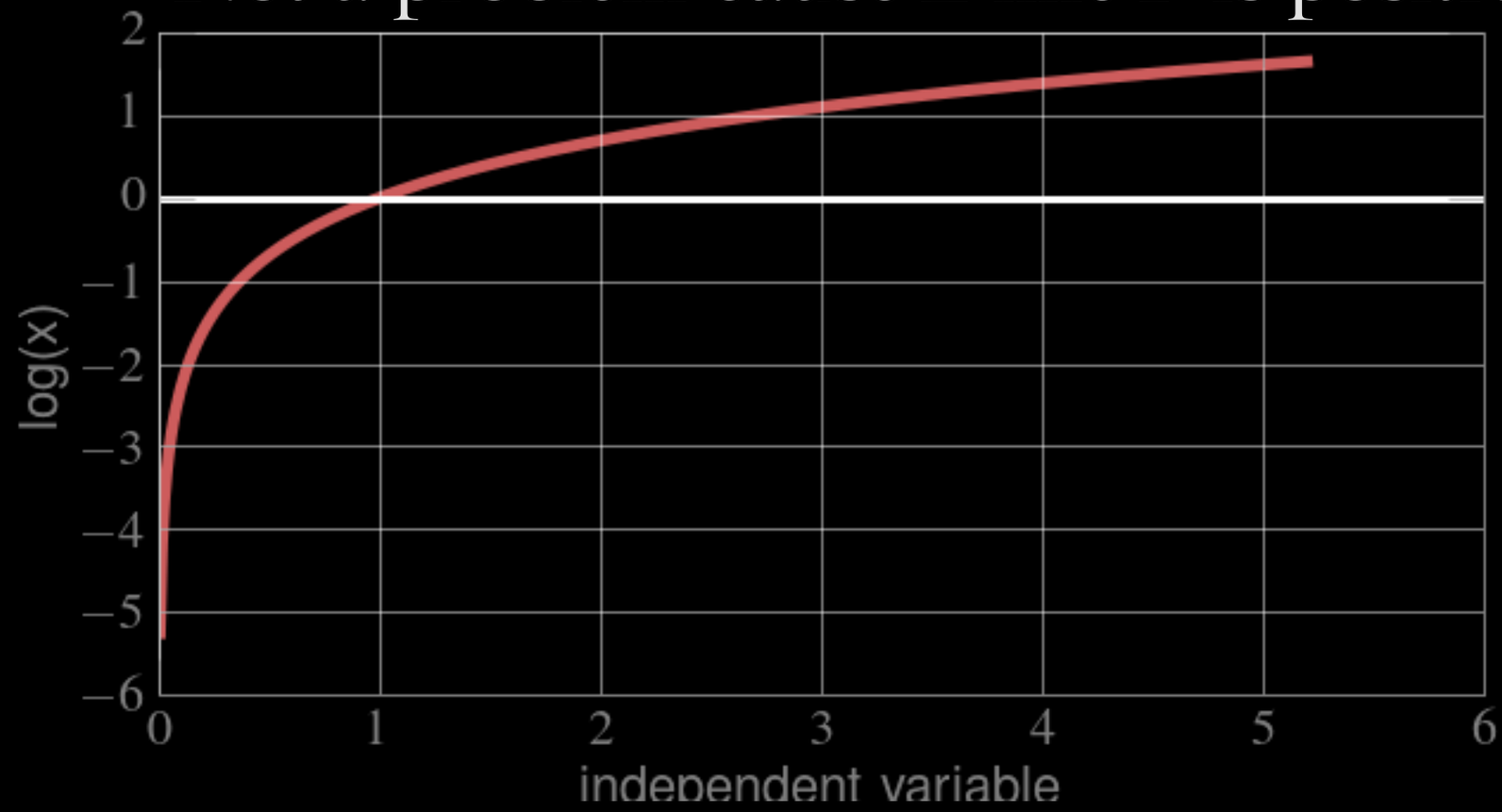
SUPPORT : (0: ∞]



Logarithm: MONOTONICALLY INCREASING

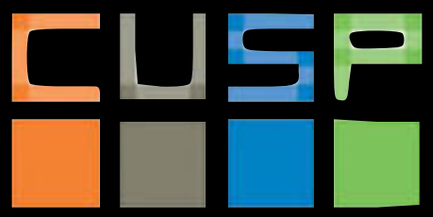
SUPPORT : (0: ∞]

Not a problem cause *L* like *P* is positive defined



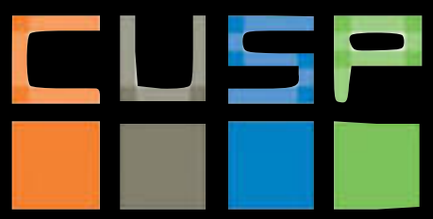
Probability $$N(\mu,\sigma) \sim \Pi_i \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

log Likelihood $$\log\left(\mathcal{L}_{(\mu,\sigma)}(\vec{x})\right) \sim \log\left(\frac{1}{(\sigma\sqrt{2\pi})^n} e^{-\frac{\Sigma_i (x_i-\mu)^2}{2\sigma^2}}\right)$$



Probability $$N(\mu,\sigma) \sim \Pi_i \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

log Likelihood $$\log\left(\mathcal{L}_{(\mu,\sigma)}(x)\right) \sim \log\left((2\pi\sigma^2)^{-\frac{n}{2}} exp\left(-\frac{1}{2\sigma^2}\sum_1^n (x_i-\mu)^2\right)\right)$$



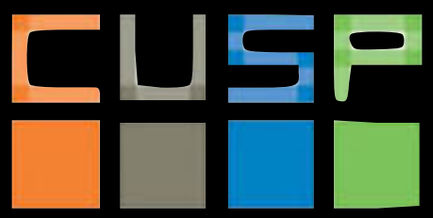
Probability

$$N(\mu,\sigma) \sim \Pi_i \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\rightarrow$$

log Likelihood

$$\ell(\mu,\sigma)(x) \sim -\frac{n}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_1^n (x_i-\mu)^2$$

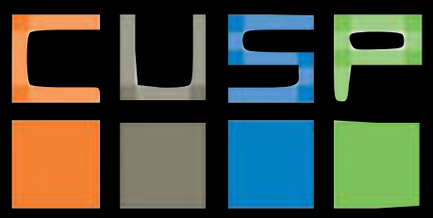


Probability

$$N(\mu,\sigma) \sim \Pi_i \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

log Likelihood

$$\ell(\mu,\sigma)(\vec{x}) \sim -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log(\sigma^2) - \frac{1}{2\sigma^2}\sum_1^n (x_i-\mu)^2$$



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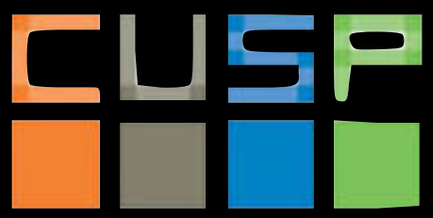
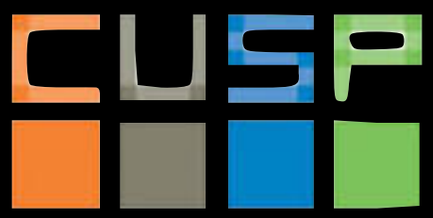


Probability $$N(\mu,\sigma) \sim \Pi_i \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\ell(\mu,\sigma)(\vec{x}) \sim$$

max log Likelihood $$-\frac{n}{2}\log(2\pi)-\frac{n}{2}\log(\sigma^2)-\frac{1}{2\sigma^2}\sum_1^n (x_i-\mu)^2$$



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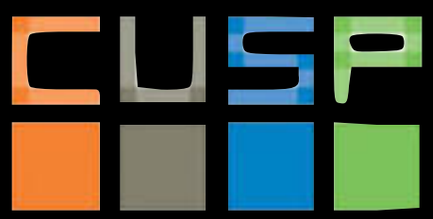
max log Likelihood $$\vec{\ell}_{(\mu^*,\sigma^*)}(\vec{x}) = max(\vec{\ell}_{(\mu,\sigma)}(\vec{x}))$$

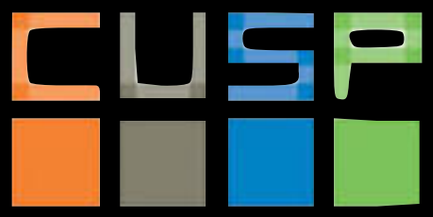


Probability

$$N(\mu,\sigma) \sim \Pi_i \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

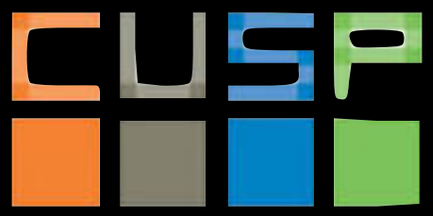
max log Likelihood

$$\frac{d\ell_{(\mu,\sigma)}(\vec{x})}{d(\mu,\sigma)} = 0$$



$$LR = -2 \log_e \frac{\max L(\text{model 1})}{\max L(\text{model 2})}$$

This statistic is chi-squared distributed



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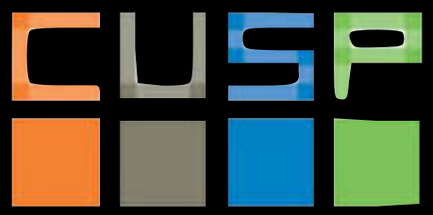
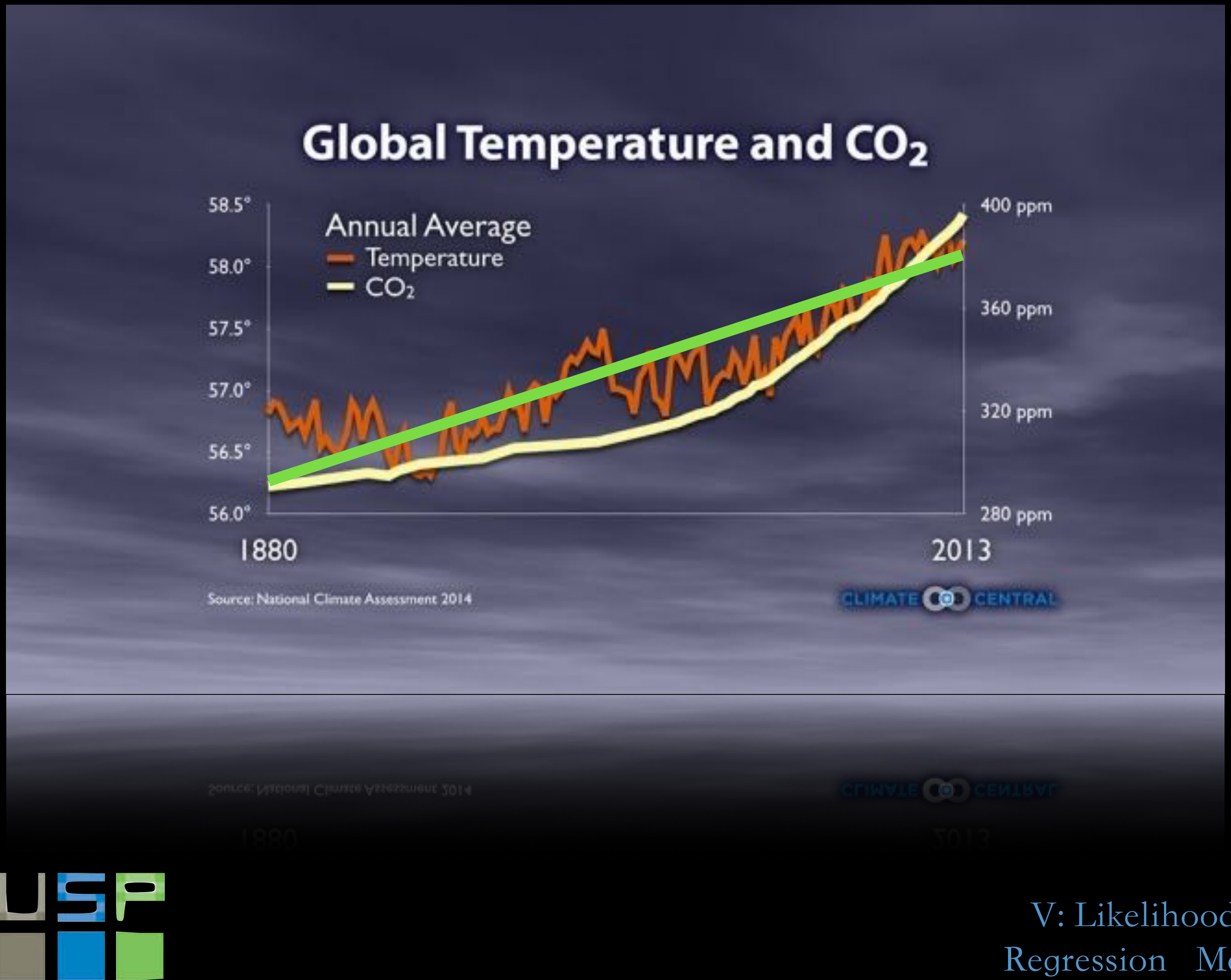
	H_0 is True	H_0 is False
H_0 is falsified	Type I error False Positive important message gets spammed	True Positive
H_0 is not falsified	True Negative	Type II error False negative Spam in your Inbox

Also called likelihood ratio…

$$LR = \frac{\text{False Negative}}{\text{True Negative}}$$





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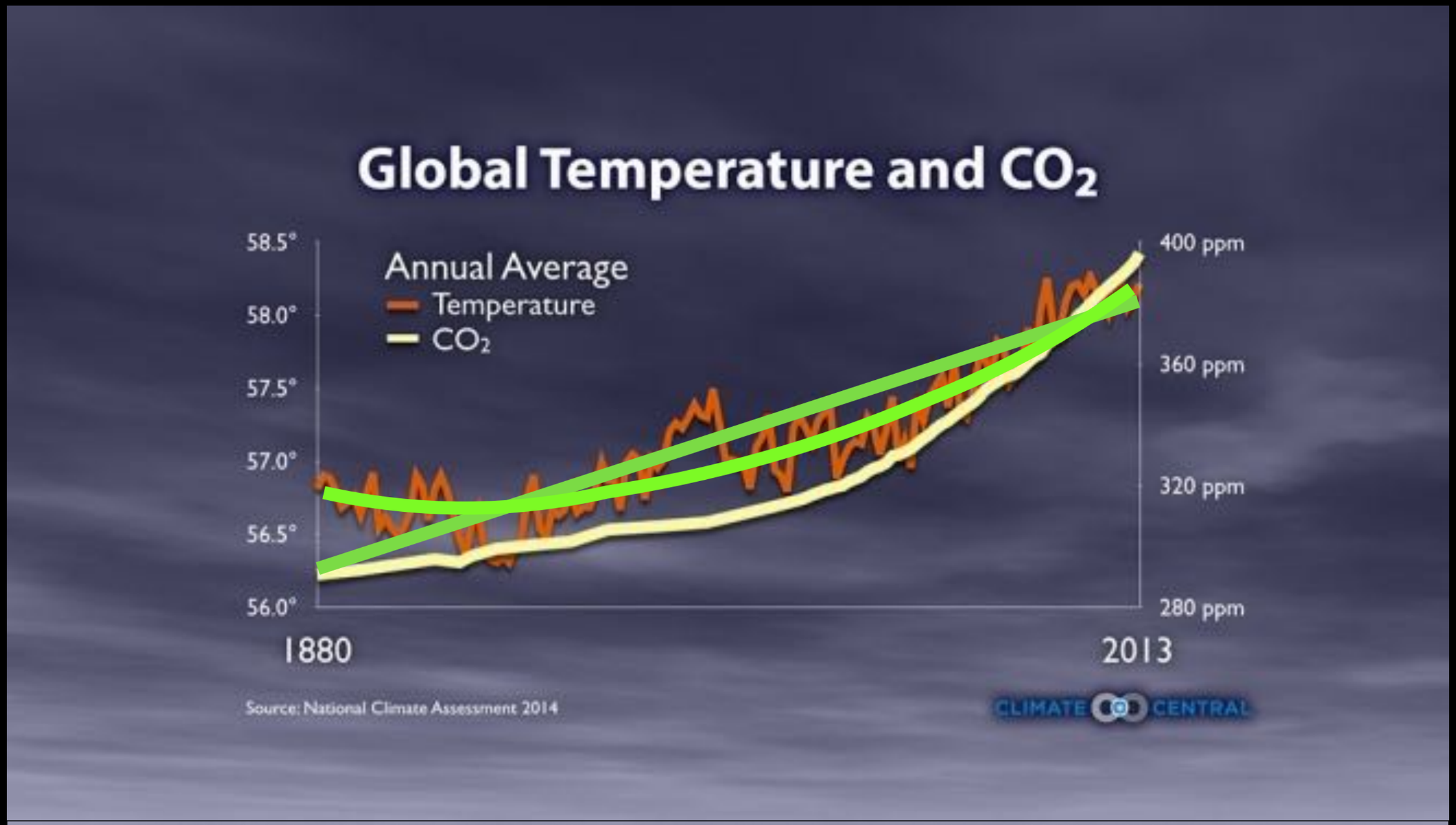
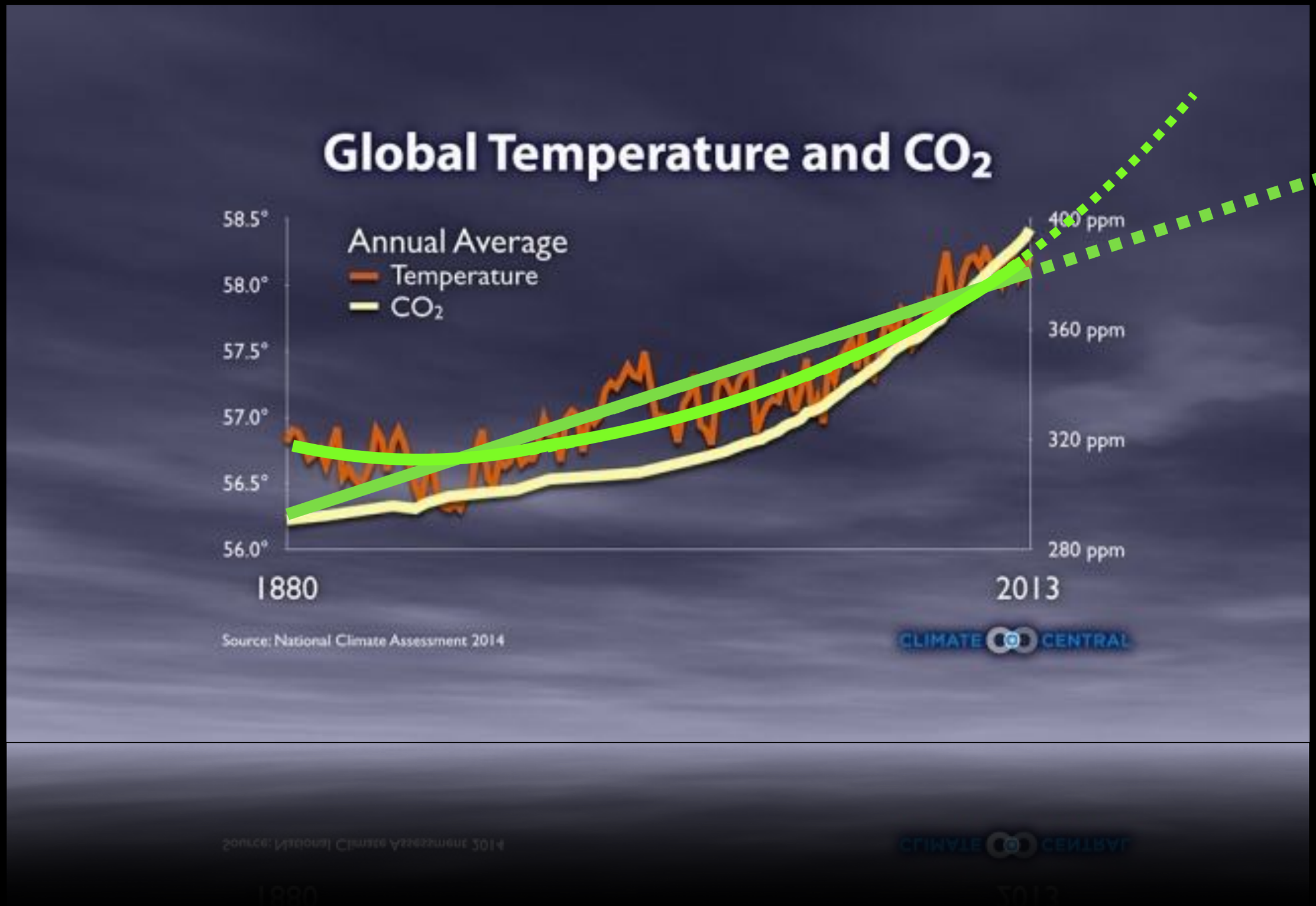
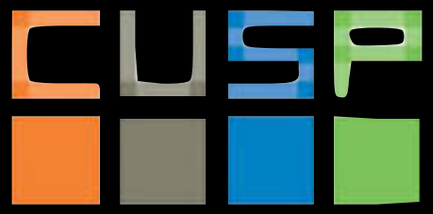


V: Likelihood and Regression Models


Global Temperature and CO_2
Annual Average
Temperature
CO_2
58.5°
58.0°
57.5°
57.0°
56.5°
56.0°
400 ppm
360 ppm
320 ppm
280 ppm
1880
2013
Source: National Climate Assessment 2014
CLIMATE CENTRAL

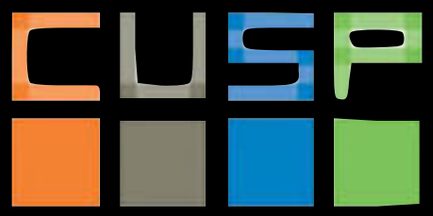
nrg buildings notebook



V: Likelihood and Regression Models

Homework:

ENERGY - SIZE building modeling:
follow in class instructions



MUST KNOWS:

- How to minimize fit parameters (OLS, WLS)
- goodness of fit tests
- R^2 , χ^2 , adjusted R^2 , reduced χ^2 , likelihood, Likelihood ratio test

Resources:

Sarah Boslaugh, Dr. Paul Andrew Watters, 2008
Introduction to General Linear Regression (Chap 12 in most versions)
https://books.google.com/books/about/Statistics_in_a_Nutshell.html?id=ZnhgO65Pyl4C

David M. Lane et al.
Introduction to Statistics (XVIII)
regression : Chapter 14
http://onlinestatbook.com/Online_Statistics_Education.epub
http://onlinestatbook.com/2/index.html