

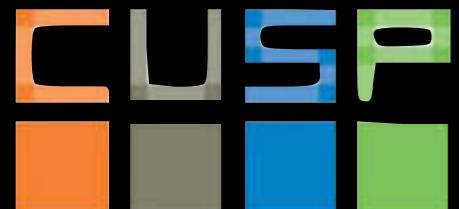
# Urban Informatics

Fall 2017

dr. federica bianco [fbianco@nyu.edu](mailto:fbianco@nyu.edu)



@fedhere

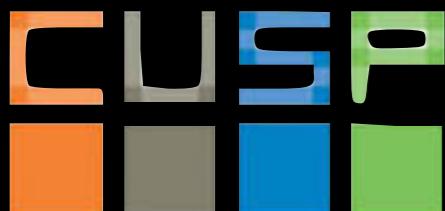


## Recap:

- Good practices with data: falsifiability, reproducibility
- Basic data retrieving and munging: APIs, Data formats
- Basic statistics: distributions and their moments
- Hypothesis testing:  $p$ -value, statistical significance

## Today:

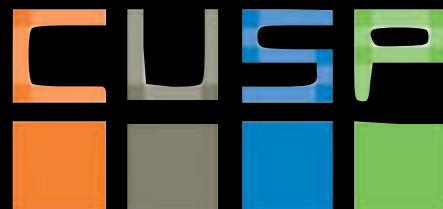
- How to choose the right statistical test
- Z, t, F test and tests for correlation
- Correlation vs Causation



# hypothesis testing

*null hypothesis 2 tailed:* no relationship between two measured phenomena,  
or no difference among groups  
if you have a test control sample: test sample and control sample are the same - no effect

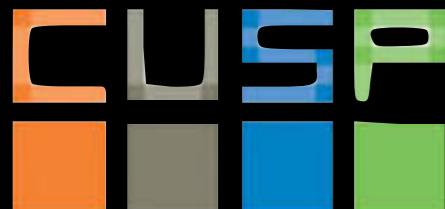
*falsify the null hypothesis:* do you see an effect?  
do you see a difference b/w samples?



# hypothesis testing

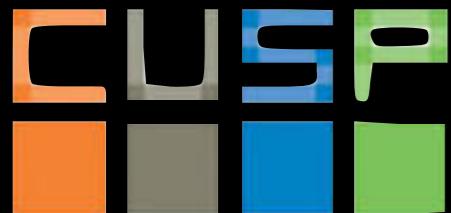
*null hypothesis 1 tailed:* measured phenomenon larger/smaller for one group than the other

*falsify the null hypothesis:* do you see an effect in a specific direction?  
do you see a difference b/w samples with the correct sign?



## A simple (too simple?) answer

*p-value* a measure of the probability that the result you observed could have been observed by chance under the *Null hypothesis*



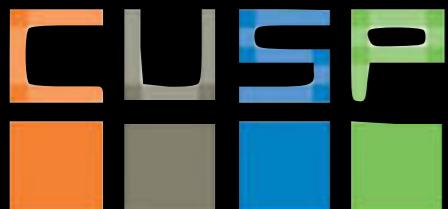
# Steps in Hypothesis Testing

1. Formulate Null (and alternative) Hypothesis
2. Choose a significance level  $\alpha$

OR

Measure a statistics for *two or more samples* to be compared to *each other*

4. Assess if your statistics is significant or not. In practice: compare the statistics (Z, t, F, chisq) with a distribution table



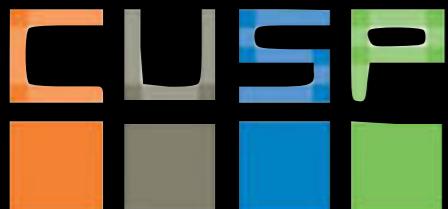
# Steps in Hypothesis Testing

1. Formulate Null (and alternative) Hypothesis
2. Choose a significance level  $\alpha$

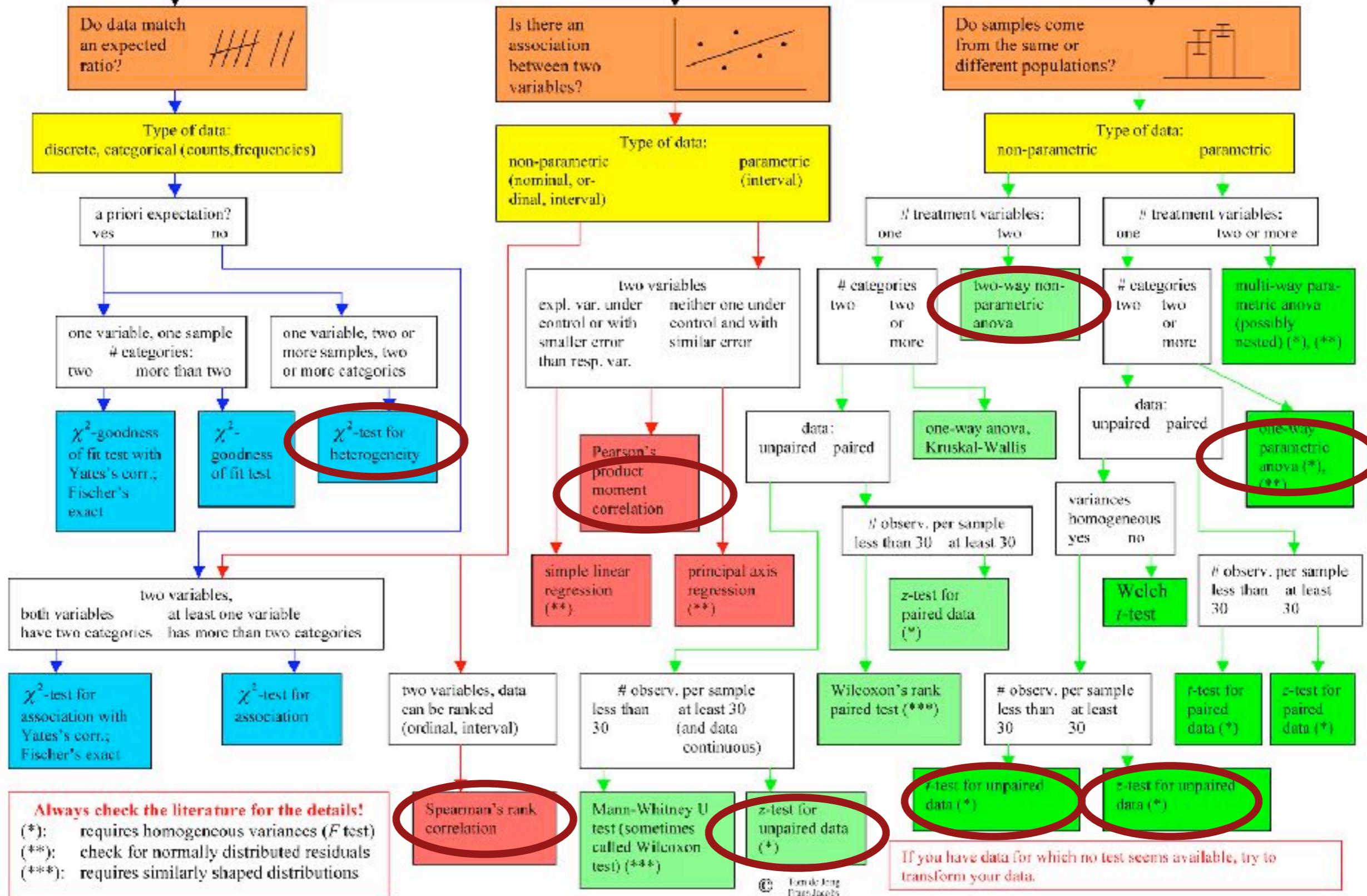
OR

Measure a statistics for *two or more samples* to be compared to *each other*

4. Assess if your statistics is significant or not. In practice: compare the statistics (Z, t, F, chisq) with a distribution table



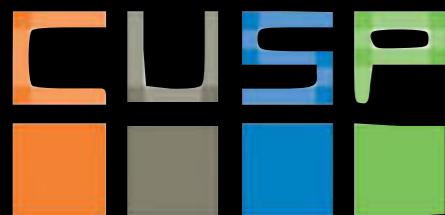
## Type of question:



# When to Use a Particular Statistical Test

[http://www.csun.edu/~amarenco/Fcs%20682/  
When%20to%20use%20what%20test.pdf](http://www.csun.edu/~amarenco/Fcs%20682/When%20to%20use%20what%20test.pdf)

Statistical Analyses	Independent Variables		Dependent Variables		Control Variables	Question Answered by the Statistic
	# of IVs	Data Type	# of DVs	Type of Data		
<b>Chi square</b>	1	categorical	1	categorical	0	Do differences exist between groups?
<b>t-Test</b>	1	dichotomous	1	continuous	0	Do differences exist between 2 groups on one DV?
<b>ANOVA</b>	1 +	categorical	1	continuous	0	Do differences exist between 2 or more groups on one DV?
<b>ANCOVA</b>	1 +	categorical	1	continuous	1 +	Do differences exist between 2 or more groups after controlling for CVs on one DV?
<b>MANOVA</b>	1 +	categorical	2 +	continuous	0	Do differences exist between 2 or more groups on multiple DVs?
<b>MANCOVA</b>	1 +	categorical	2 +	continuous	1 +	Do differences exist between 2 or more groups after controlling for CVs on multiple DVs?
<b>Correlation</b>	1	dichotomous or continuous	1	continuous	0	How strongly and in what direction (i.e., +, -) are the IV and DV related?
<b>Multiple regression</b>	2 +	dichotomous or continuous	1	continuous	0	How much variance in the DV is accounted for by linear combination of the IVs? Also, how strongly related to the DV is the beta coefficient for each IV?
<b>Path analysis</b>	2 +	continuous	1 +	continuous	0	What are the direct and indirect effects of predictor variables on the DV?
<b>Logistic Regression</b>	1 +	categorical or continuous	1	dichotomous	0	What is the odds probability of the DV occurring as the values of the IVs change?



## assignment 1:

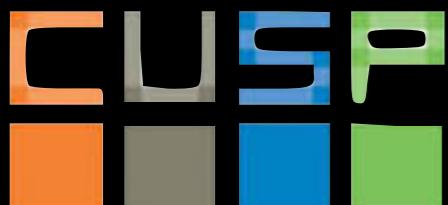
Brows Plos One for papers that uses each of the 4 tests

[http://www.csun.edu/~amarenco/Fcs%20682/  
When%20to%20use%20what%20test.pdf](http://www.csun.edu/~amarenco/Fcs%20682/When%20to%20use%20what%20test.pdf)

choose 1 choose 1

choose 1 →

Statistical Analyses	Independent Variables		Dependent Variables		Control Variables	Question Answered by the Statistic
	# of IVs	Data Type	# of DVs	Type of Data		
<b>Chi square</b>	1	categorical	1	categorical	0	Do differences exist between groups?
<b>t-Test</b>	1	dichotomous	1	continuous	0	Do differences exist between 2 groups on one DV?
<b>ANOVA</b>	1 +	categorical	1	continuous	0	Do differences exist between 2 or more groups on one DV?
<b>ANCOVA</b>	1 +	categorical	1	continuous	1 +	Do differences exist between 2 or more groups after controlling for CVs on one DV?
<b>MANOVA</b>	1 +	categorical	2 +	continuous	0	Do differences exist between 2 or more groups on multiple DVs?
<b>MANCOVA</b>	1 +	categorical	2 +	continuous	1 +	Do differences exist between 2 or more groups after controlling for CVs on multiple Dvs?
<b>Correlation</b>	1	dichotomous or continuous	1	continuous	0	How strongly and in what direction (i.e., +, -) are the IV and DV related?
<b>Multiple regression</b>	2 +	dichotomous or continuous	1	continuous	0	How much variance in the DV is accounted for by linear combination of the IVs? Also, how strongly related to the DV is the beta coefficient for each IV?
<b>Path analysis</b>	2 +	continuous	1 +	continuous	0	What are the direct and indirect effects of predictor variables on the DV?
<b>Logistic Regression</b>	1 +	categorical or continuous	1	dichotomous	0	What is the odds probability of the DV occurring as the values of the IVs change?



IV: Statistical analysis

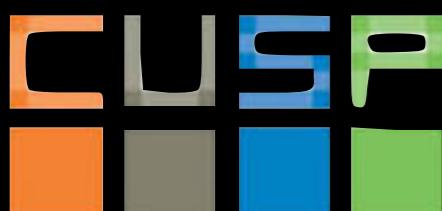
## assignment 2:

Prepare a markdown file containing one table as below for each of the 3 tests you select describing the use of the test in a PlosOne publication of your choice

Example: ANCOVA

<http://journals.plos.org/plosone/article?id=10.1371/journal.pone.0062593>

Statistical Analyses	Dependent Variables	Independent Variables	Control Variables	Question Answered
ANCOVA	Ratings about their values (ordinal)	Did Self Affirmation or not (category)	age	self-affirmation group rates the value significantly more than control group

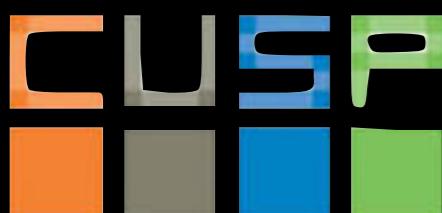


# Comparing to population

## Standard DEVIATION of Sample Estimates

Sample mean, $\bar{x}$	$Z = \frac{ \mu_{\text{pop}} - \mu_{\text{sample}} }{\sigma / \sqrt{N}}$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
Sample proportion, $p$	$z = \frac{ p - p_0 }{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$\sigma_p = \sqrt{\frac{P(1-P)}{n}}$
Difference between proportions, $p_1 - p_2$	$z = \frac{(p_2 - p_1)}{\sqrt{p(1-p)(\frac{1}{n_2} + \frac{1}{n_1})}}, p = \frac{p_2 n_2 + p_1 n_1}{n_2 + n_1}$	$\sigma_{p_1 - p_2} = \sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}$

use if you know the *parameters* of the *population*:  
e.g. in the Z test



# Comparing sample to population or between samples

## Standard ERROR of Sample Estimates

Sample mean,  $\bar{x}$

$$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$$

Sample proportion, p

$$SE_p = \sqrt{\frac{p(1-p)}{n}}$$

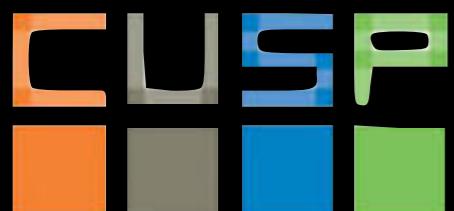
Difference between means,  
 $x_1 - x_2$

$$SE_{x_1 - x_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Difference between proportions,  
 $p_1 - p_2$

$$SE_{p_1 - p_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

use if you DO NOT know the parameters of the population:  
e.g. in the t-test



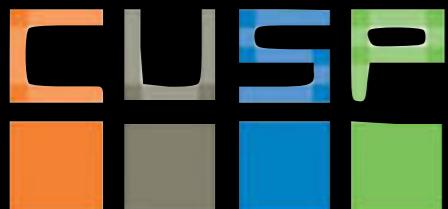
# Steps in Hypothesis Testing

1. Formulate Null (and alternative) Hypothesis
2. Choose a significance level  $\alpha$

OR

Measure a statistics for *two or more samples* to be compared to *each other*

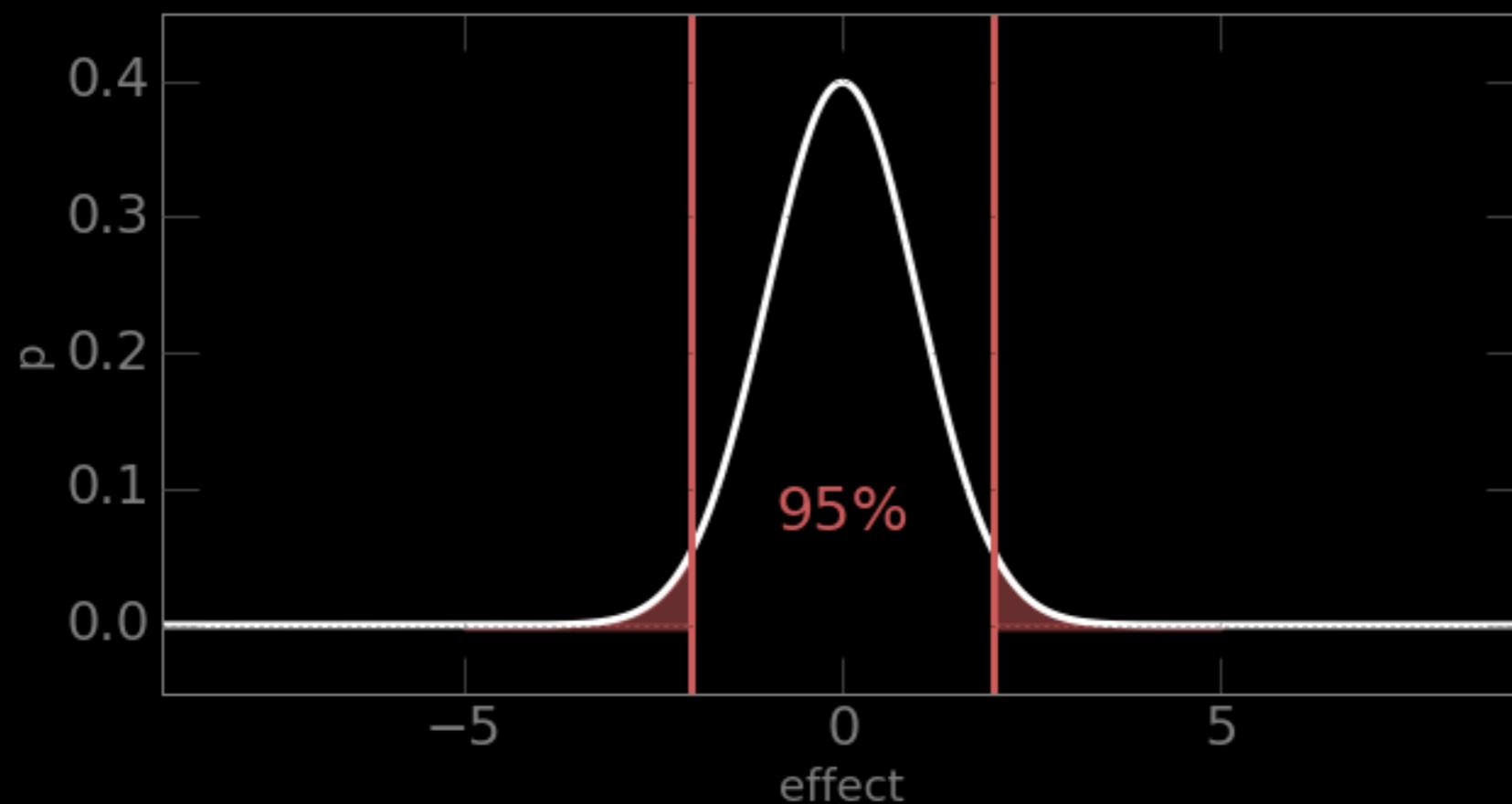
4. Assess if your statistics is significant or not. In practice: compare the statistics (Z, t, F, chisq) with a distribution table



*if you knew how your statistics should be distributed...*

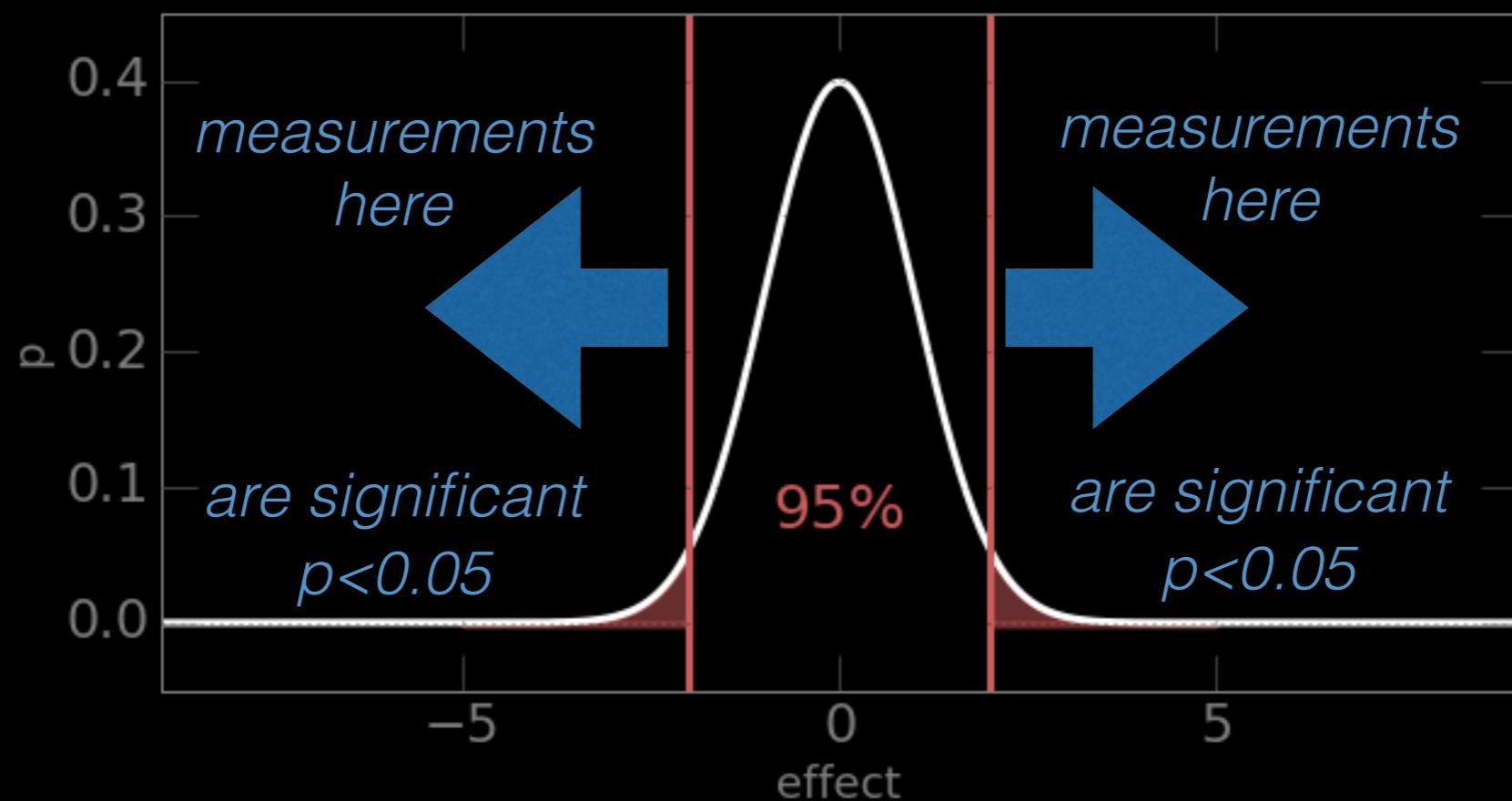
$$\alpha = 0.05$$

$$1 - 0.05 = 0.95 \Rightarrow 95\%$$



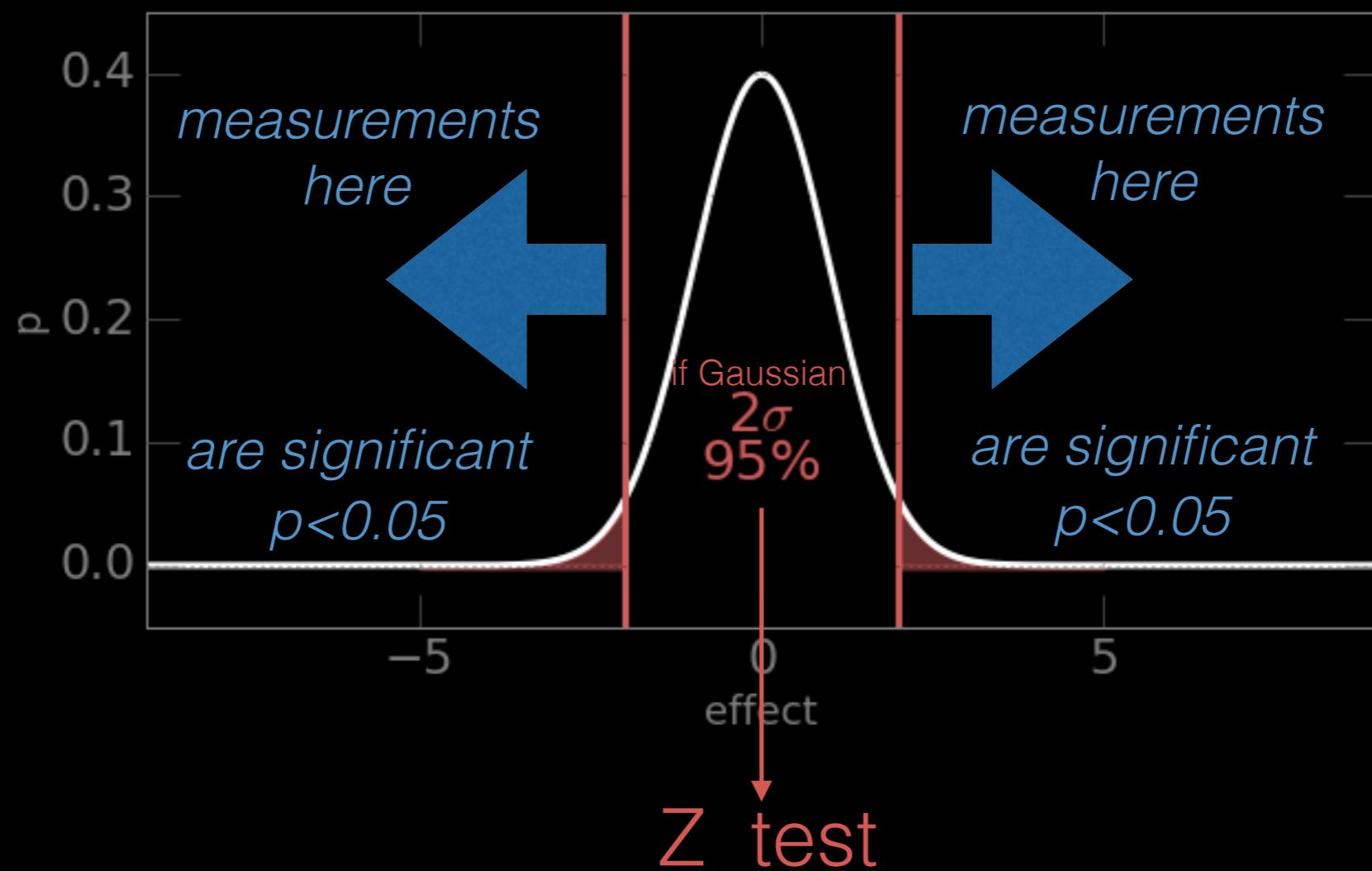
$$\alpha = 0.05$$

$$1 - 0.05 = 0.95 \Rightarrow 95\%$$

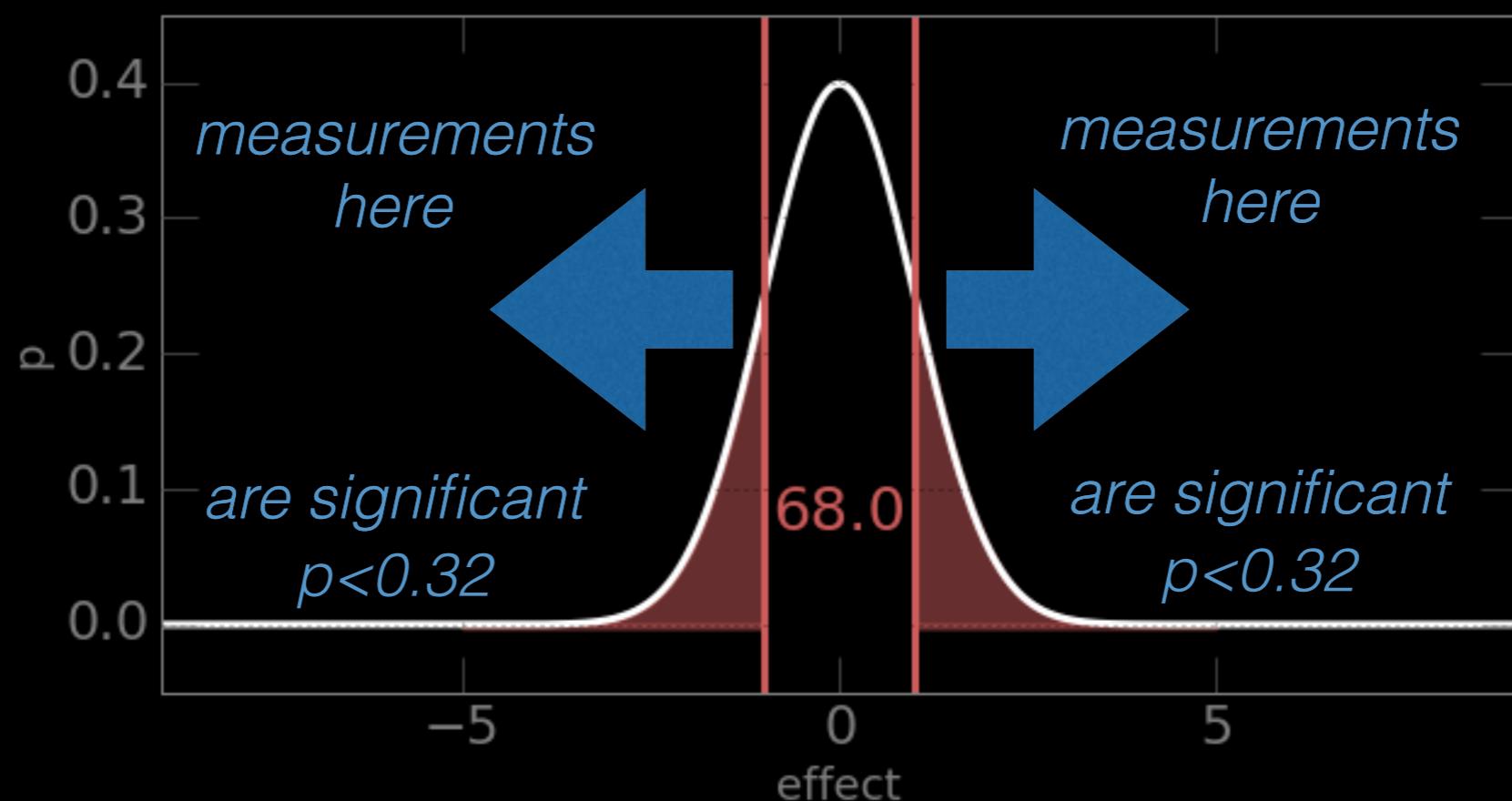


$$\alpha = 0.05$$

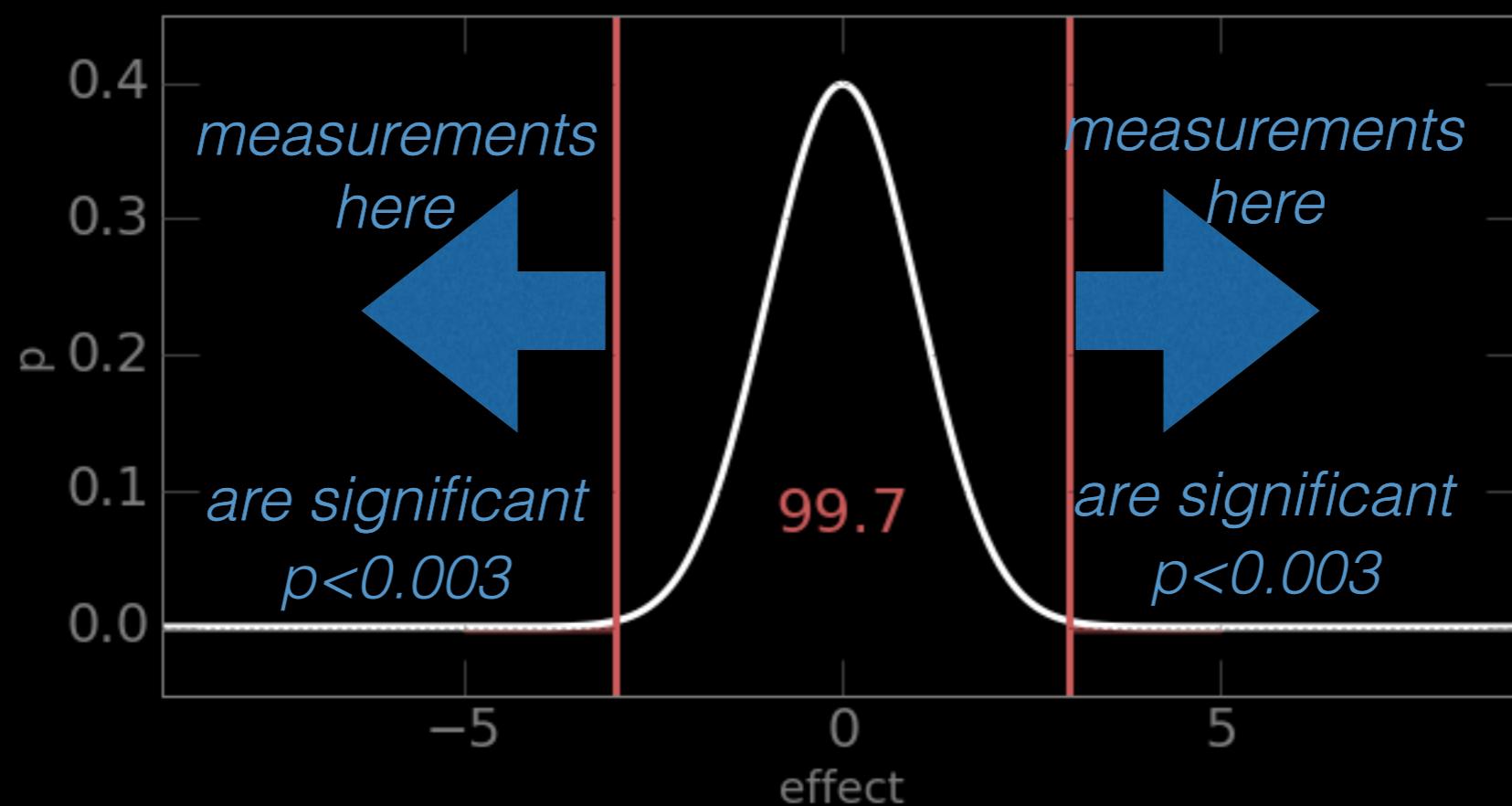
$$1 - \alpha = 0.95 \Rightarrow 95\%$$



$$a = 0.32$$



$$\alpha = 0.003$$



## What is the distribution of a statistics?

To measure the probability of the measured value of a statistic I need to know *how the statistics is distributed* (under the alternative hypothesis)  
Each statistics follows some distribution, which though?

# Steps in Hypothesis Testing

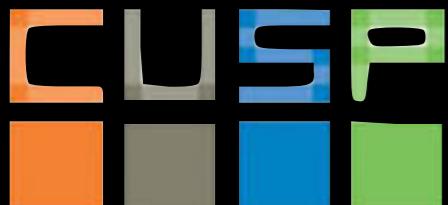
1. Formulate Null (and alternative) Hypothesis
2. Choose a significance level  $\alpha$

OR

Measure a statistics for *two or more samples* to be compared to *each other*

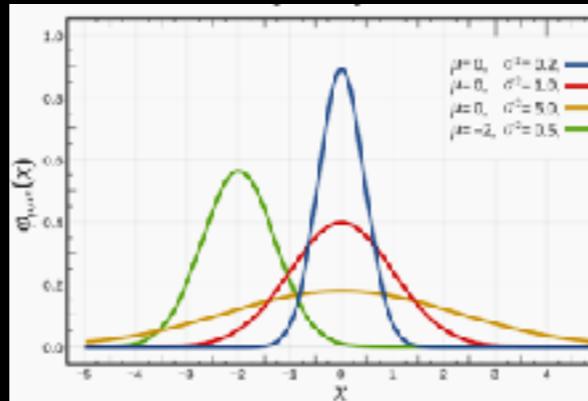
4. Assess if your statistics is significant or not. In practice: compare the statistics (Z, t, F, chisq) with a distribution table

<https://documents.software.dell.com/statistics/textbook/distribution-tables>



# Z statistics Gaussian

$$Z = \frac{\mu - \bar{x}}{\sigma / \sqrt{n}}$$

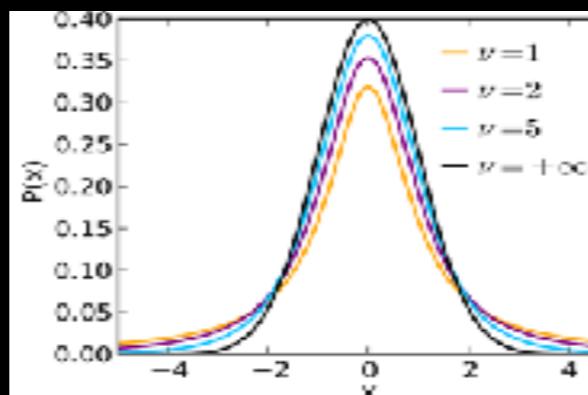


Notation	$N(\mu, \sigma^2)$
Parameters	$\mu \in \mathbb{R}$ — mean (location) $\sigma^2 > 0$ — variance (squared scale)
Support	$x \in \mathbb{R}$
PDF	$\frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
CDF	$\frac{1}{2} \left[ 1 + \operatorname{erf}\left( \frac{x-\mu}{\sigma\sqrt{2}} \right) \right]$
Quantile	$\mu + \sigma\sqrt{2} \operatorname{erf}^{-1}(2F - 1)$
Mean	$\mu$
Median	$\mu$
Mode	$\mu$
Variance	$\sigma^2$

Quantile	$\mu + \sigma\sqrt{2} \operatorname{erf}^{-1}(2F - 1)$
Mean	$\mu$
Median	$\mu$
Mode	$\mu$
Variance	$\sigma^2$

# Student's t

$$t = \frac{\mu - \bar{x}}{s / \sqrt{n}}$$

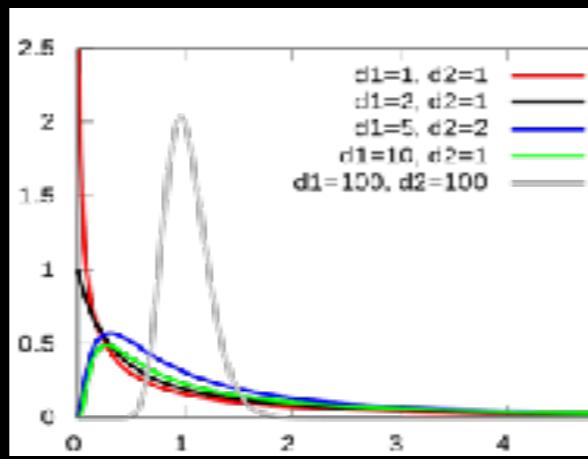


Parameters	$\nu > 0$ degrees of freedom (real)
Support	$x \in (-\infty; +\infty)$
PDF	$\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$
CDF	$\frac{1}{2} + x\Gamma\left(\frac{\nu+1}{2}\right) \times \\ \frac{{}_2F_1\left(\frac{1}{2}, \frac{\nu+1}{2}; \frac{3}{2}; -\frac{x^2}{\nu}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)}$
where ${}_2F_1$ is the hypergeometric function	

Mean	0 for $\nu > 1$ , otherwise undefined
Median	0
Mode	0
Variance	$\frac{\nu}{\nu-2}$ for $\nu > 2$ , = for $1 < \nu \leq 2$ , otherwise undefined

# F statistics

$$F = \frac{\sum_i n_i (\bar{x}_i - \bar{\bar{x}})^2 / (K-1)}{\sum_{ij} (x_{ij} - \bar{x}_i)^2 / (N-K)}$$

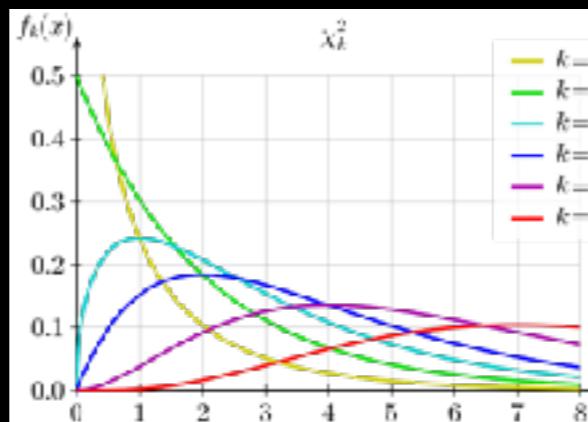


Parameters	$d_1, d_2 > 0$ deg. of freedom
Support	$x \in [0, +\infty)$
PDF	$\frac{\sqrt{(d_1 x)^{d_1} d_2^{d_2}}}{\sqrt{(d_1 + d_2)^{d_1+d_2}}} x \operatorname{B}\left(\frac{d_1}{2}, \frac{d_2}{2}\right)$
CDF	$I \frac{d_1 x}{d_1 x + d_2} \left(\frac{d_1}{2}, \frac{d_2}{2}\right)$

Mean	$\frac{d_2}{d_2 - 2}$ for $d_2 > 2$
Mode	$\frac{d_1 - 2}{d_1 - 4} \frac{d_2}{d_2 + 2}$ for $d_1 > 2$
Variance	$\frac{2 d_2^2 (d_1 + d_2 - 2)}{d_1 (d_2 - 2)^2 (d_2 - 4)}$ for $d_2 > 4$
Skewness	$\frac{(2d_1 + d_2 - 2)\sqrt{8(d_2 - 4)}}{(d_2 - 6)\sqrt{d_1(d_1 + d_2 - 2)}}$ for $d_2 > 6$

# Pearson's $\chi^2$

$$\chi_P^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$



Notation	$\chi^2(k)$ or $\chi_k^2$
Parameters	$k \in \mathbb{N}_{>0}$ (known as "degrees of freedom")
Support	$x \in [0, +\infty)$
PDF	$\frac{1}{2^{\frac{k}{2}} \Gamma\left(\frac{k}{2}\right)} x^{\frac{k}{2}-1} e^{-\frac{x}{2}}$
CDF	$\frac{1}{\Gamma\left(\frac{k}{2}\right)} \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$

Mean	$k$
Median	$\approx k \left(1 - \frac{2}{9k}\right)^3$
Mode	$\max\{k-2, 0\}$
Variance	$2k$
Skewness	$\sqrt{\frac{8}{k}}$

see

**CUSP** goodness of fit  $\chi^2$   $\chi_F^2 = \sum_i \frac{(m_i - x_i)^2}{e_i}$  - Statistics in a Nutshell  
IV: Statistical analysis

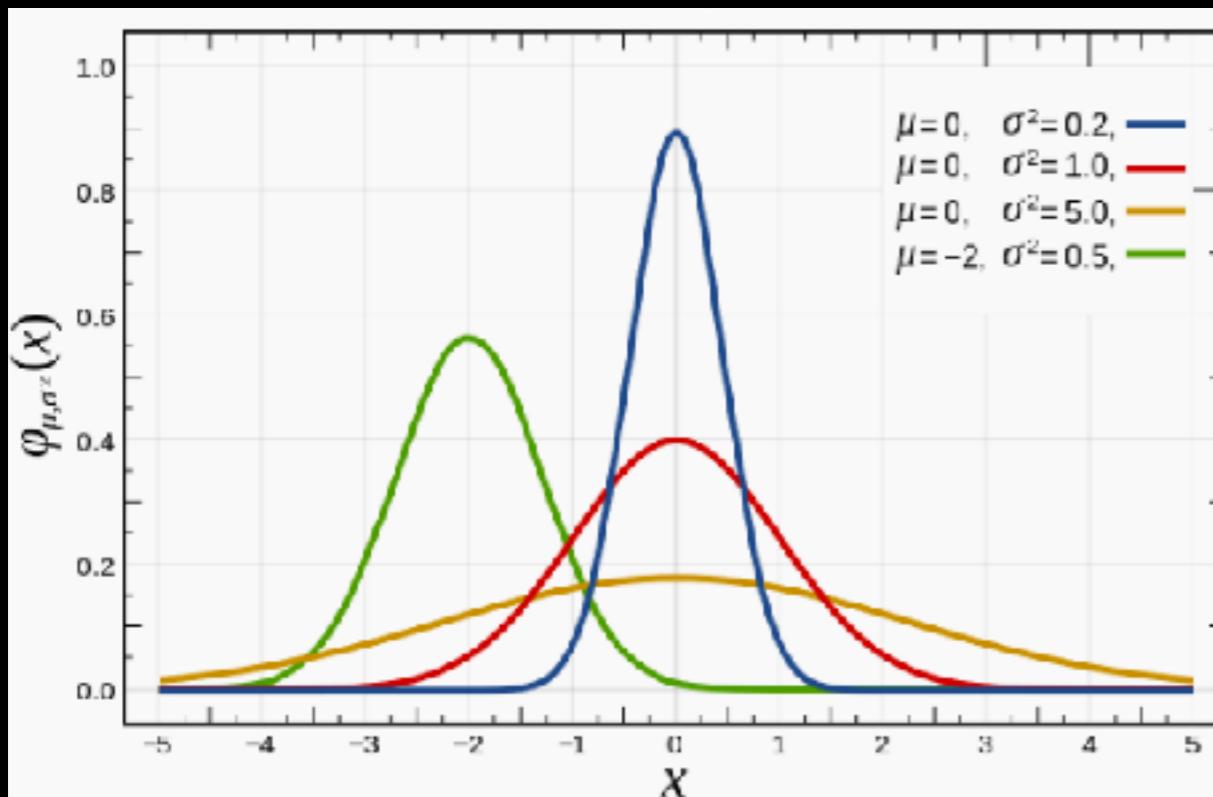
Is there a difference between means or population and sample,  
difference between proportion in 2 samples?

**Z statistics**

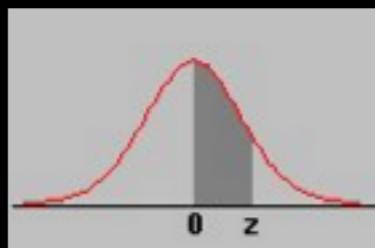
$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

In absence of effect (i.e. under the Null)

Z is distributed according to a Gaussian  $\mathcal{N}(\mu=0, \sigma=1)$



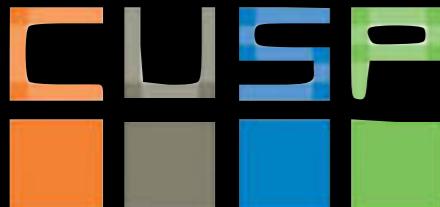
<b>Notation</b>	$\mathcal{N}(\mu, \sigma^2)$
<b>Parameters</b>	$\mu \in \mathbb{R}$ – mean ( <b>location</b> ) $\sigma^2 > 0$ – variance (squared <b>scale</b> )
<b>Support</b>	$x \in \mathbb{R}$
<b>PDF</b>	$\frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
<b>CDF</b>	$\frac{1}{2} \left[ 1 + \text{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right) \right]$
<b>Quantile</b>	$\mu + \sigma\sqrt{2} \text{erf}^{-1}(2F - 1)$
<b>Mean</b>	$\mu$
<b>Median</b>	$\mu$
<b>Mode</b>	$\mu$
<b>Variance</b>	$\sigma^2$

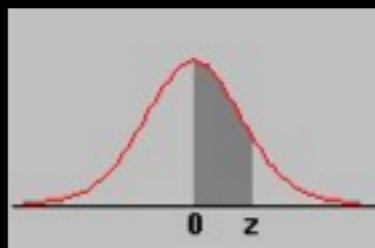


$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 2.55$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
											3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990

<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

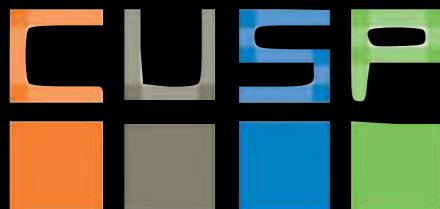


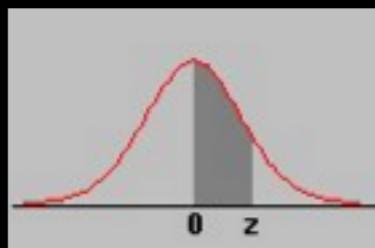


$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 2.55$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
											3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990

<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

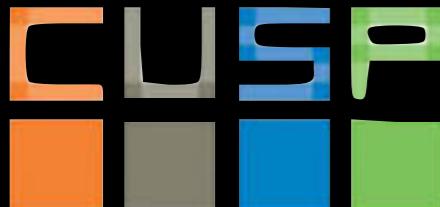


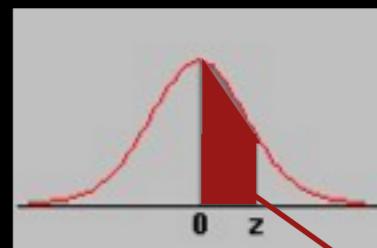


$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 2.55$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
											3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4990	0.4990

<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>



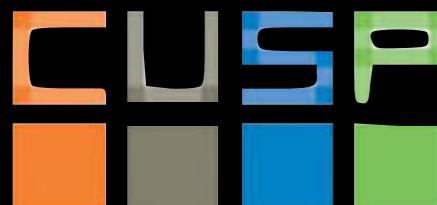


$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 2.55$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359		1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753		1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141		1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517		1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879		1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224		2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549		2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852		2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133		2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389		2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621		2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830		2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015		2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177		2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319		2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
												3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4990	0.4990

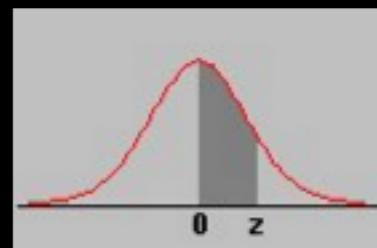
<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

2 sided test  $1 - (0.4946 * 2) = 0.0108$   
 $p < 0.05$



$H_0$  IS REJECTED ( $p < 0.05$ )

IV: Statistical analysis

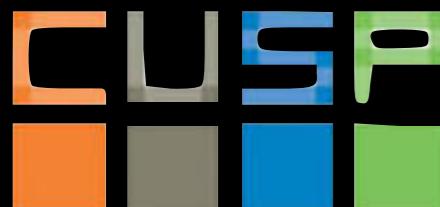


$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 2.55$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
											3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4990	0.4990

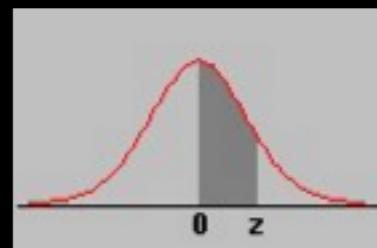
<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

1 sided test     $1 - 0.4946 - 0.5 = 0.0054$   
                              $p < 0.05$



$H_0$  IS REJECTED ( $p < 0.05$ )

IV: Statistical analysis

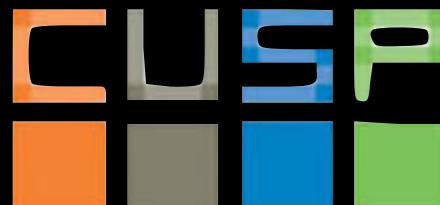


$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 1.57$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
											3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990

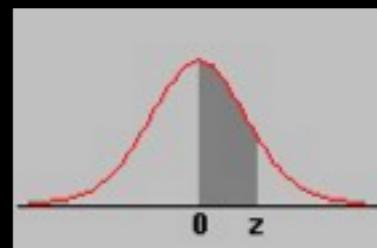
<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

2 sided test       $1 - 0.4418^2 = 0.1164$   
 $p > 0.05$



$H_0$  CANNOT BE REJECTED

IV: Statistical analysis

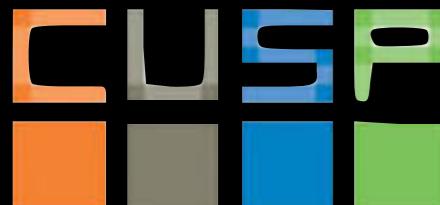


$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 1.57$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
											3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990

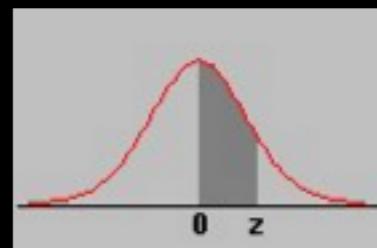
<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

2 sided test       $1 - 0.4418^2 = 0.1164$   
 $p > 0.05$



$H_0$  CANNOT BE REJECTED

IV: Statistical analysis



$$Z = \frac{\mu_{\text{pop}} - \mu_{\text{sample}}}{\sigma / \sqrt{N}} = 1.96$$

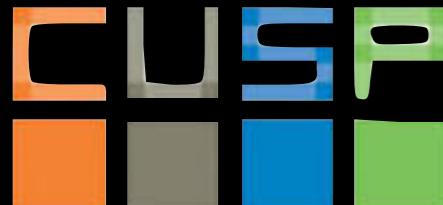
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4750	0.4418	0.4429	0.4441
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753	1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1871	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
											3.0	0.4987	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990

<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

2 sided test

$$1 - 0.4750^2 = 0.05$$

$$p = 0.05$$



$H_0$  CANNOT BE REJECTED

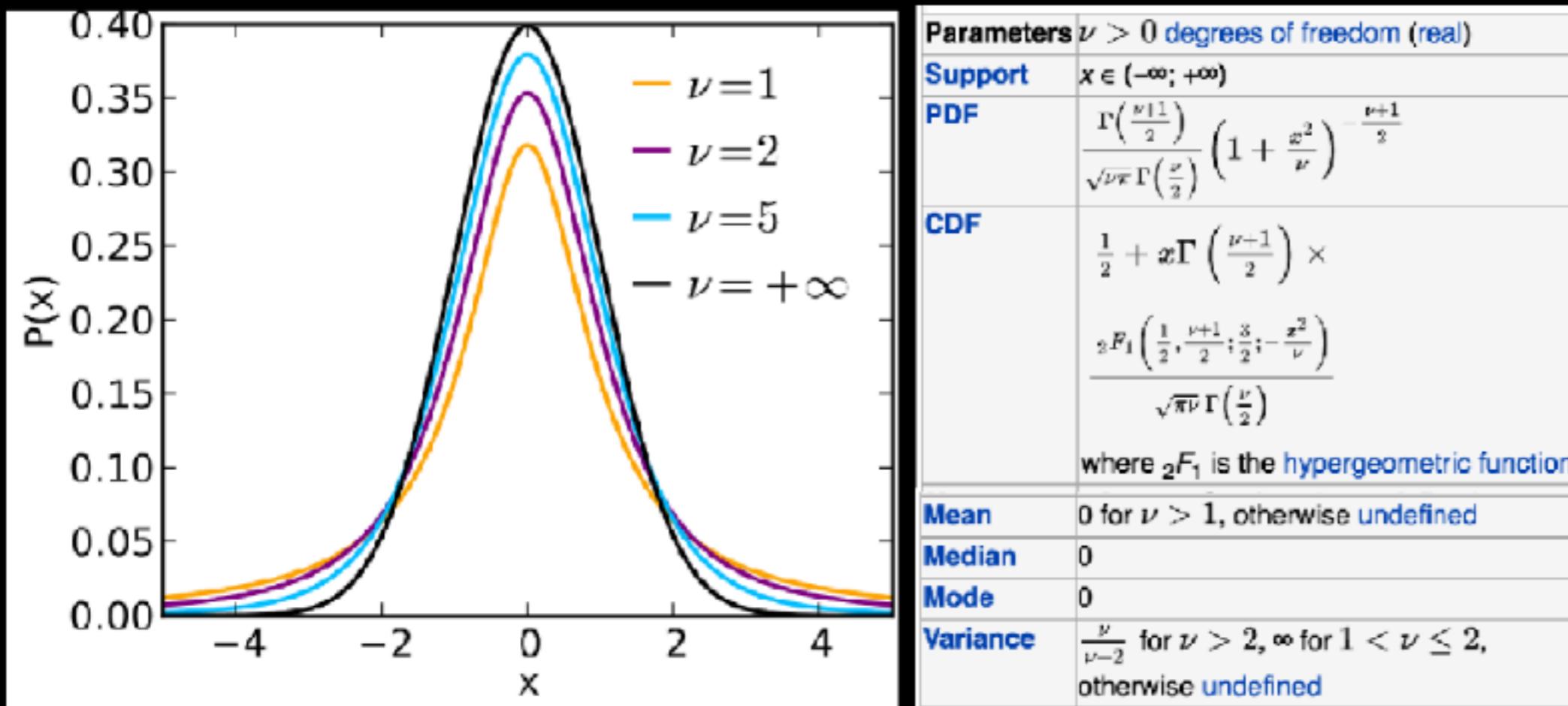
IV: Statistical analysis

Is there a difference between means of 2 sample?

### **Student's $t$**

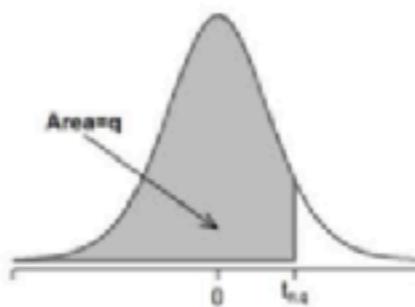
$$t = \frac{\mu - x}{s/\sqrt{n}}$$

In absence of effect (i.e. under the Null)  
 $t$  is distributed according to a *student's t* distribution with  $\nu$  number of degrees of freedom



$$t = \frac{\mu - \bar{x}}{s/\sqrt{n}} = 1.75$$

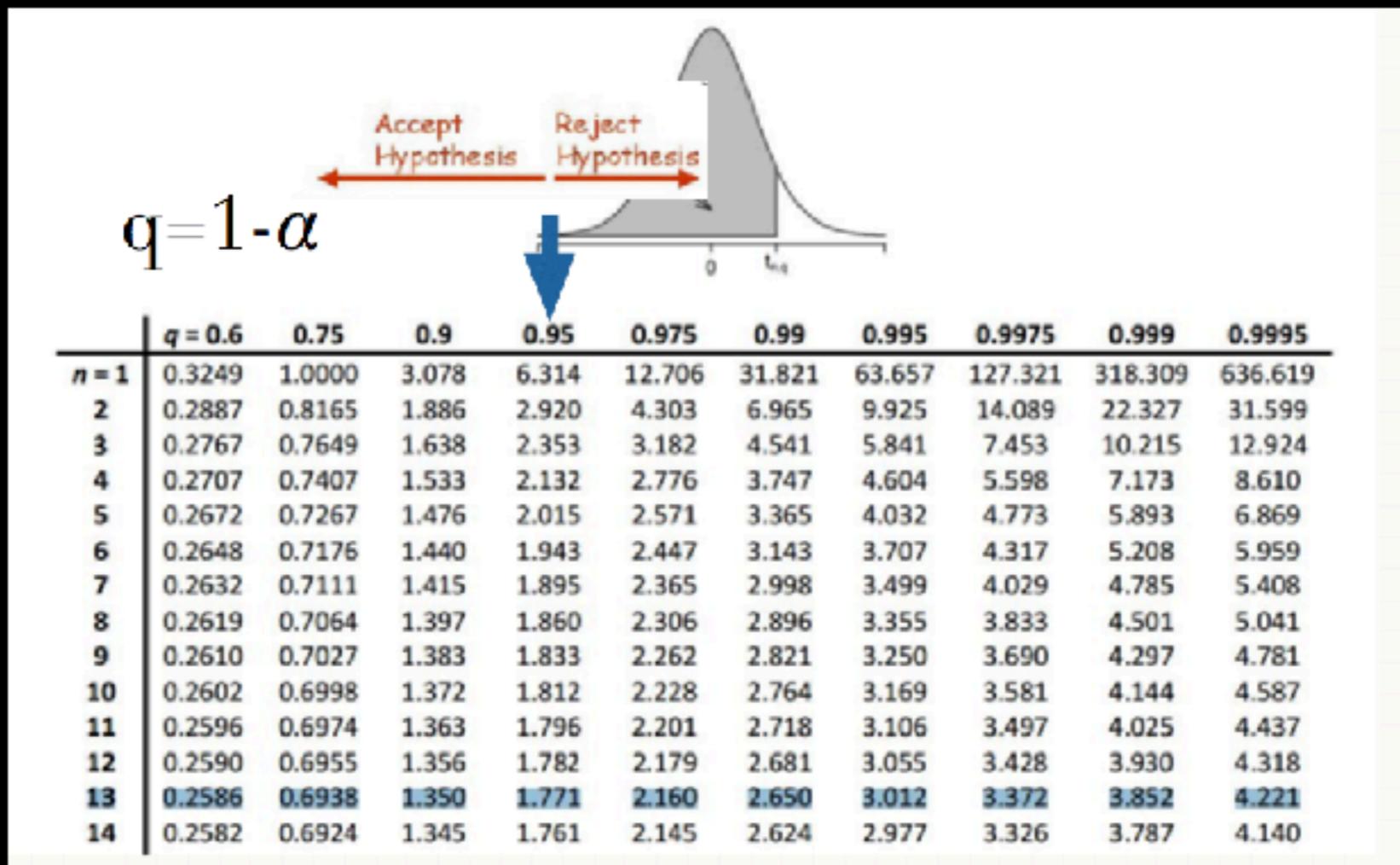
$n=13$ ,  $q=.95$



	$q = 0.6$	$0.75$	$0.9$	$0.95$	$0.975$	$0.99$	$0.995$	$0.9975$	$0.999$	$0.9995$
<b><math>n = 1</math></b>	0.3249	1.0000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
<b>2</b>	0.2887	0.8165	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
<b>3</b>	0.2767	0.7649	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
<b>4</b>	0.2707	0.7407	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
<b>5</b>	0.2672	0.7267	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
<b>6</b>	0.2648	0.7176	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
<b>7</b>	0.2632	0.7111	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
<b>8</b>	0.2619	0.7064	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
<b>9</b>	0.2610	0.7027	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
<b>10</b>	0.2602	0.6998	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
<b>11</b>	0.2596	0.6974	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
<b>12</b>	0.2590	0.6955	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
<b>13</b>	<b>0.2586</b>	<b>0.6938</b>	<b>1.350</b>	<b>1.771</b>	<b>2.160</b>	<b>2.650</b>	<b>3.012</b>	<b>3.372</b>	<b>3.852</b>	<b>4.221</b>
<b>14</b>	0.2582	0.6924	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140

$$t = \frac{\mu - \bar{x}}{s/\sqrt{n}} = 1.75$$

$n=13$ ,  $\alpha=.95$



Is  $1.75 > 1.771$ ? NO:

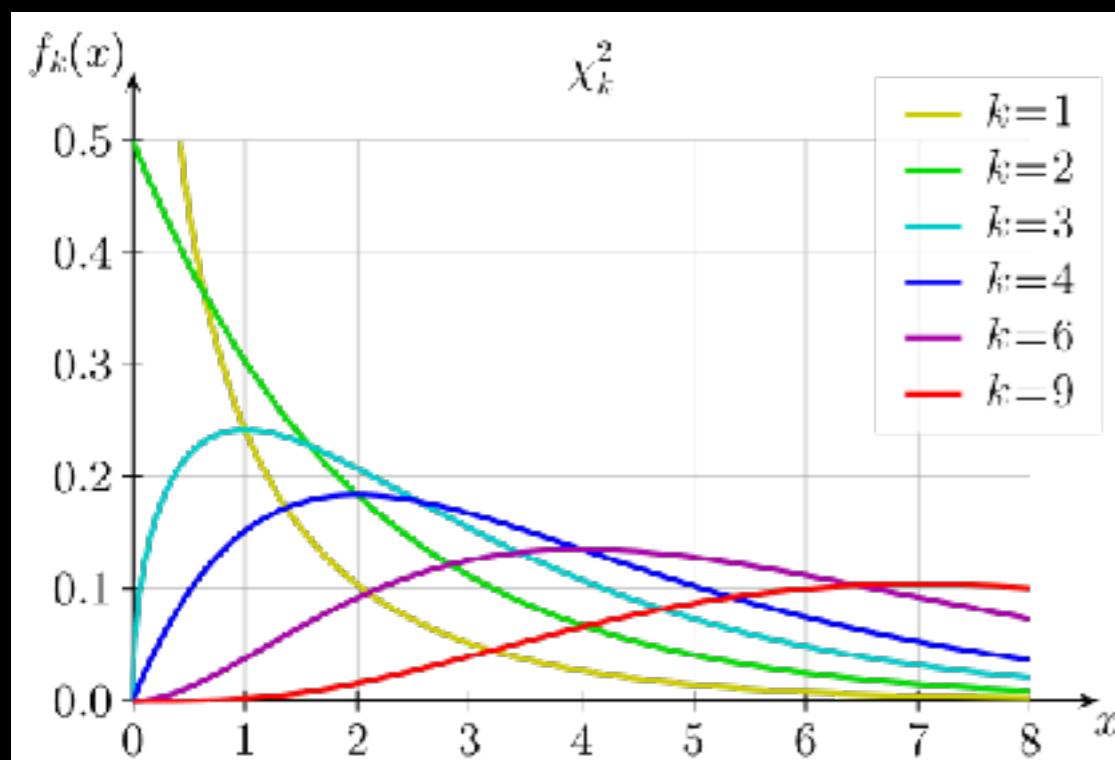
Cannot reject the Null Hypothesis

Is there a difference between means or population and sample,  
difference between proportion in 2 samples?

## $\chi^2$ statistics

$$\chi_P^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

In absence of effect (i.e. under the Null)  
 Z is distributed according to a  $\chi^2$  distribution  
 with k=number of degrees of freedom



<b>Notation</b>	$\chi^2(k)$ or $\chi_k^2$
<b>Parameters</b>	$k \in \mathbb{N}_{>0}$ (known as "degrees of freedom")
<b>Support</b>	$x \in [0, +\infty)$
<b>PDF</b>	$\frac{1}{2^{\frac{k}{2}} \Gamma\left(\frac{k}{2}\right)} x^{\frac{k}{2}-1} e^{-\frac{x}{2}}$
<b>CDF</b>	$\frac{1}{\Gamma\left(\frac{k}{2}\right)} \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$
<b>Mean</b>	$k$
<b>Median</b>	$\approx k \left(1 - \frac{2}{9k}\right)^3$
<b>Mode</b>	$\max\{k-2, 0\}$
<b>Variance</b>	$2k$
<b>Skewness</b>	$\sqrt{8/k}$

$$\chi^2_F = \sum_{i=1}^4 \frac{(m_i - x_i)^2}{e_i} = 8.57$$

$$\alpha = 0.05$$

4 observations - 1 independent variable =  
3 degrees of freedom

**Percentage Points of the Chi-Square Distribution**

Degrees of Freedom	Probability of a larger value of $\chi^2$								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09

<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

$$p < 0.05$$

$H_0$  REJECTED

$$\chi^2_F = \sum_{i=1}^4 \frac{(m_i - x_i)^2}{e_i} = 7.11$$

$$\alpha = 0.05$$

4 observations - 1 independent variable =  
3 degrees of freedom

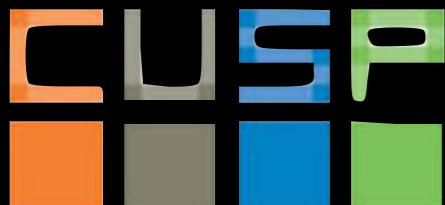
Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of $\chi^2$								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09

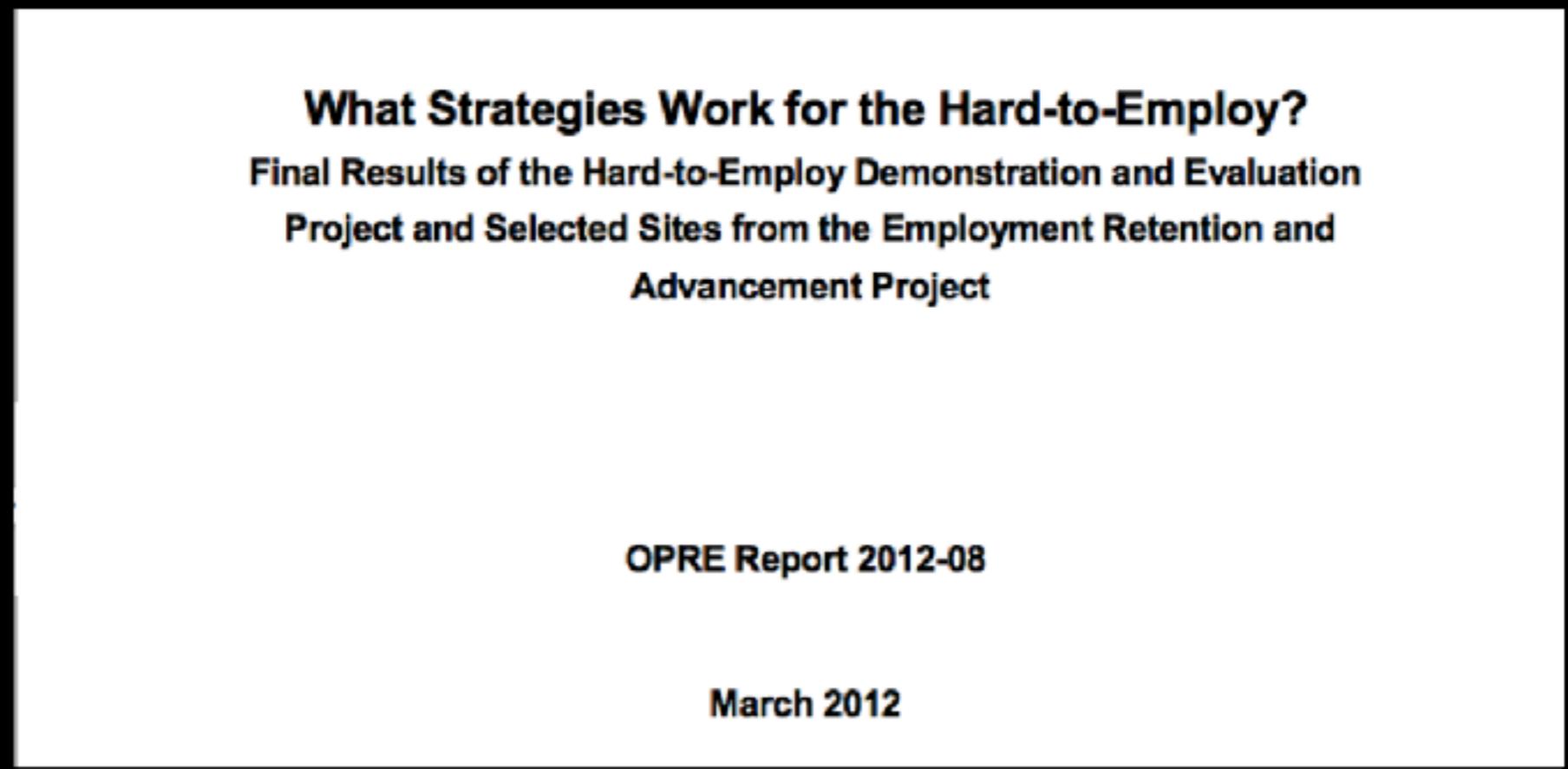
<https://github.com/fedhere/UInotebooks/blob/master/HowToReadZandChisqTables.md>

$$p > 0.05$$

$H_0$  CANNOT BE REJECTED



Example: **NULL HYPOTHESIS:** the % of former prisoners employed 3 years after release is *the same or lower* for candidates who participated in the program as for the control group,  
*significance level p=0.05*



<http://www.mdrc.org/sites/default/files/What%20Strategies%20Work%20for%20the%20Hard%20FR.pdf>

[https://github.com/fedhere/PUI2017\\_fb55/blob/master/Lab4\\_fb55/effectiveness%20of%20NYC%20Post-Prison%20Employment%20Programs.ipynb](https://github.com/fedhere/PUI2017_fb55/blob/master/Lab4_fb55/effectiveness%20of%20NYC%20Post-Prison%20Employment%20Programs.ipynb)

**NULL HYPOTHESIS:** the % of former prisoners employed 3 years after release is *the same or lower* for candidates who participated in the program as for the control group,  
*significance level p=0.05*

The Enhanced Services for the Hard-to-Employ Demonstration and Evaluation Project

Table 2.1

Summary of Impacts, New York City Center for Employment Opportunities

Outcome	Program Group	Control Group	Difference (Impact)	P-Value
<u>Employment (Years 1-3) (%)</u>	<b>P<sub>1</sub></b>	<b>P<sub>0</sub></b>		
Ever employed	83.8	70.4	13.4 ***	0.000
Ever employed in a CEO transitional job <sup>a</sup>	70.1	3.5	66.6 ***	0.000
Ever employed in an unsubsidized job	63.7	69.0	-5.3 *	0.078
<u>Postprogram unsubsidized employment (Years 2-3)</u>				
Ever employed in an unsubsidized job (%)	53.3	52.1	1.2	0.713
Employed in an unsubsidized job, average per quarter (%)	28.2	27.2	1.1	0.618
Employed for six or more consecutive quarters (%)	14.7	11.9	2.8	0.195
Total UI-covered earnings <sup>b</sup> (\$)	10,435	9,846	589	0.658
Sample size (total = 973) <sup>c</sup>	564	409		

$$H_0: P_0 - P_1 \geq 0$$

$$H_a: P_0 - P_1 < 0$$

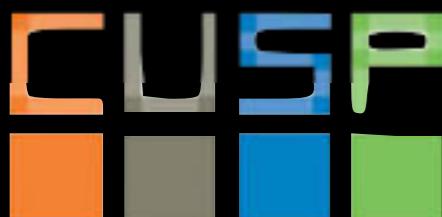
$$\alpha = 0.05$$

SOURCES: MDRC earnings calculations from the National Directory of New Hires (NDNH) database and employment calculations from the unemployment insurance (UI) wage records from New York State, MDRC calculations using data from the New York State Division of Criminal Justice Services (DCJS) and the New York City Department of Correction (DOC).

NOTES: Statistical significance levels are indicated as: \*\*\* = 1 percent; \*\* = 5 percent; \* = 10 percent.

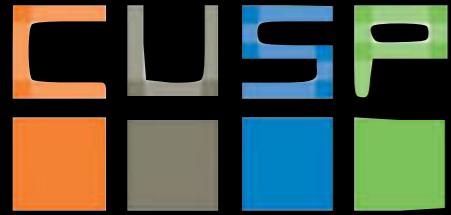
The p-value indicates the likelihood that the difference between the program and control groups arose by chance.

<http://www.mdrc.org/sites/default/files/What%20Strategies%20Work%20for%20the%20Hard%20FR.pdf>





[https://github.com/fedhere/PUI2017\\_fb55/blob/master/Lab4\\_fb55/effectiveness%20of%20NYC%20Post-Prison%20Employment%20Programs.ipynb](https://github.com/fedhere/PUI2017_fb55/blob/master/Lab4_fb55/effectiveness%20of%20NYC%20Post-Prison%20Employment%20Programs.ipynb)



IV: Statistical analysis

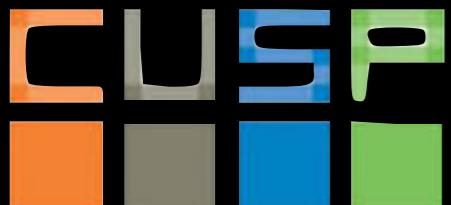
# BEYOND SIGNIFICANCE TESTING

**REFORMING DATA ANALYSIS METHODS  
IN BEHAVIORAL RESEARCH**

REX B. KLINE

РЕХ В. КЛІНЕ

[https://www.dropbox.com/s/0z74eh210fgucl5/  
Screenshot%202017-10-05%2000.23.12.png?dl=0](https://www.dropbox.com/s/0z74eh210fgucl5/Screenshot%202017-10-05%2000.23.12.png?dl=0)



## Recommendations

Specific suggestions are listed and then discussed afterward:

1. Only in very exploratory research where it is unknown whether effects exist may a primary role for NHST be appropriate.
2. If statistical tests are used, (a) information about power must be reported, and (b) the null hypothesis must be plausible.
3. In any kind of behavioral research, it is not acceptable anymore to describe results solely in terms of NHST outcomes.
4. Drop the word “significant” from our data analysis vocabulary. Use it only in its everyday sense to describe something actually noteworthy or important.
5. It is the researcher’s responsibility to report and interpret, whenever possible, effect size estimates and confidence intervals for primary results. This does not mean to report effect sizes only for  $H_0$  rejections.
6. It is also the researcher’s responsibility to demonstrate the substantive (theoretical, clinical, or practical) significance of the results. Statistical tests are inadequate for this purpose.
7. Replication is the best way to deal with sampling error.
8. Education in statistical methods needs to be reformed, too. The role of NHST should be greatly deemphasized so that more time can be spent showing students how to determine whether a result has substantive significance and how to replicate it.
9. Researchers need more help from their statistical software to compute effect sizes and confidence intervals.

compute effect sizes and confidence intervals.  
Researchers need more help from their statistical software to  
compute effect sizes and confidence intervals.

whether a result has substantive significance and how to replicate it.  
more time can be spent showing students how to determine  
the role of NHST in research. Finally, researchers need help from  
their statistical software to compute effect sizes and confidence intervals.

## Arguments against NHST

1.  $P(H_0 | D) = \frac{P(H_0) P(D | H_0)}{P(D)}$        $P(H_0 | D) \neq P(D | H_0)$
2. *Significance depends on sample size, e.g.:*

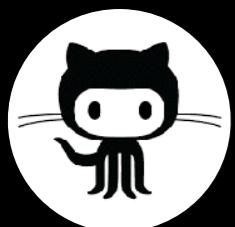
$$Z = \frac{|\mu_{\text{pop}} - \mu_{\text{sample}}|}{\sigma / \sqrt{N}}$$

If you measure the same effect in two experiments with 2 samples of different size, the significance of the effect will be different

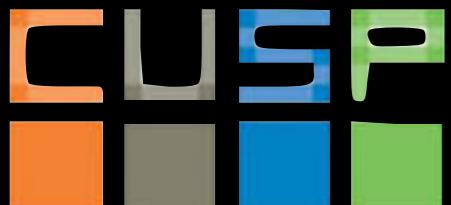
## Effect size to measure the strength of a phenomenon

Estimate the strength of, for example, an apparent relationship, rather than assigning a significance level. The effect size does not directly determine the significance level, or vice versa.

Jacob Cohen (1960), "*A coefficient of agreement for nominal scales*", Educational and Psychological Measurement, 20 (1): 37–46,



[https://github.com/fedhere/PUI2016\\_fb55/blob/master/  
HW3\\_fb55/citibikes\\_gender.ipynb](https://github.com/fedhere/PUI2016_fb55/blob/master/ HW3_fb55/citibikes_gender.ipynb)

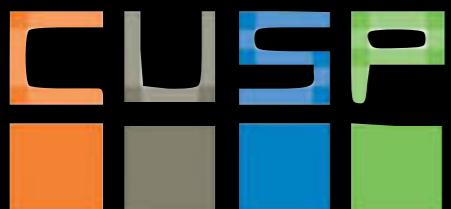


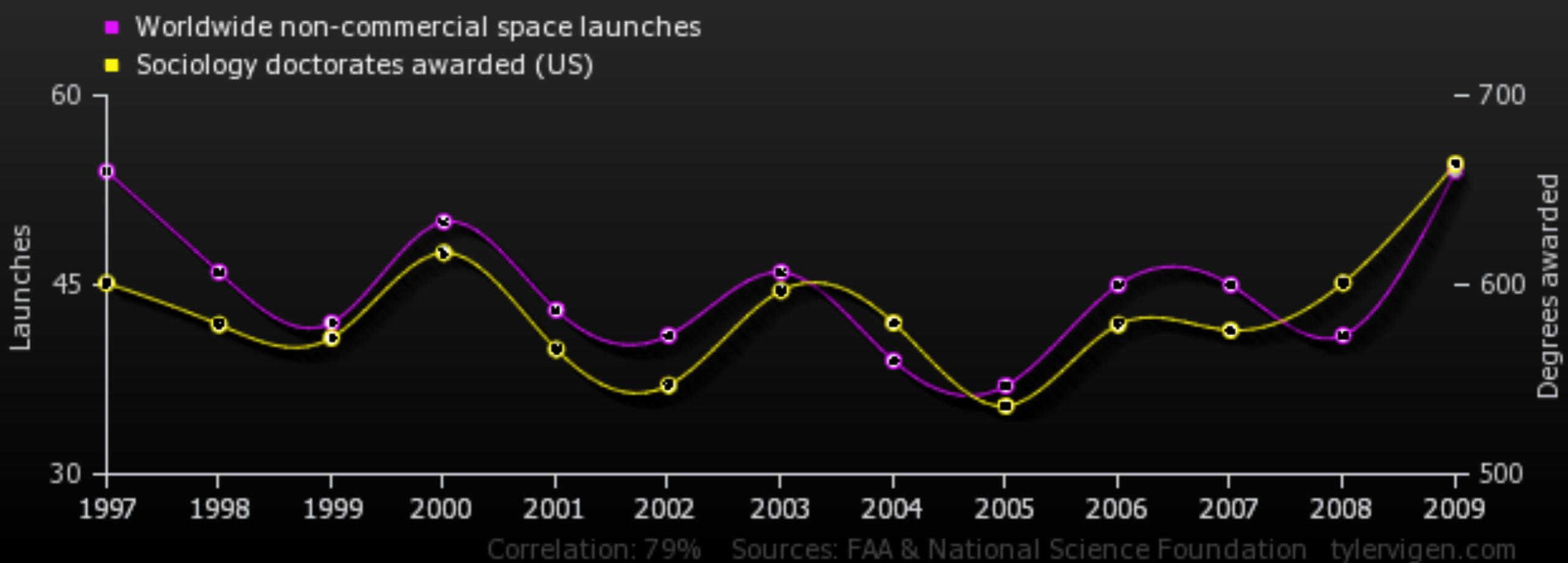
small,medium,large

# Effect size to measure the strength of a phenomenon



Cohen's <i>d</i>	$d = \frac{ \bar{x}_1 - \bar{x}_2 }{s} \quad s = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$	for <b>Z</b> and <b>t</b>	.20 .50 .80
Cohen's <i>w</i>	$w = \sqrt{\sum_{i=1}^N \frac{(p_{0i} - p_{1i})^2}{p_{0i}}}$	for <b>X<sup>2</sup></b>	.10 .30 .50
Cohen's <i>h</i>	$h = 2(\arcsin \sqrt{p_1} - \arcsin \sqrt{p_2})$	for proportions	.20 .50 .80
<i>f</i> -squared	$\hat{f}_{effect} = \sqrt{(df_{effect}/N)(F_{effect}-1)}$	for F-test/ ANOVA	.10 .25 .40
R <sup>2</sup> coefficient of determination		for regression	.02 .15 .35

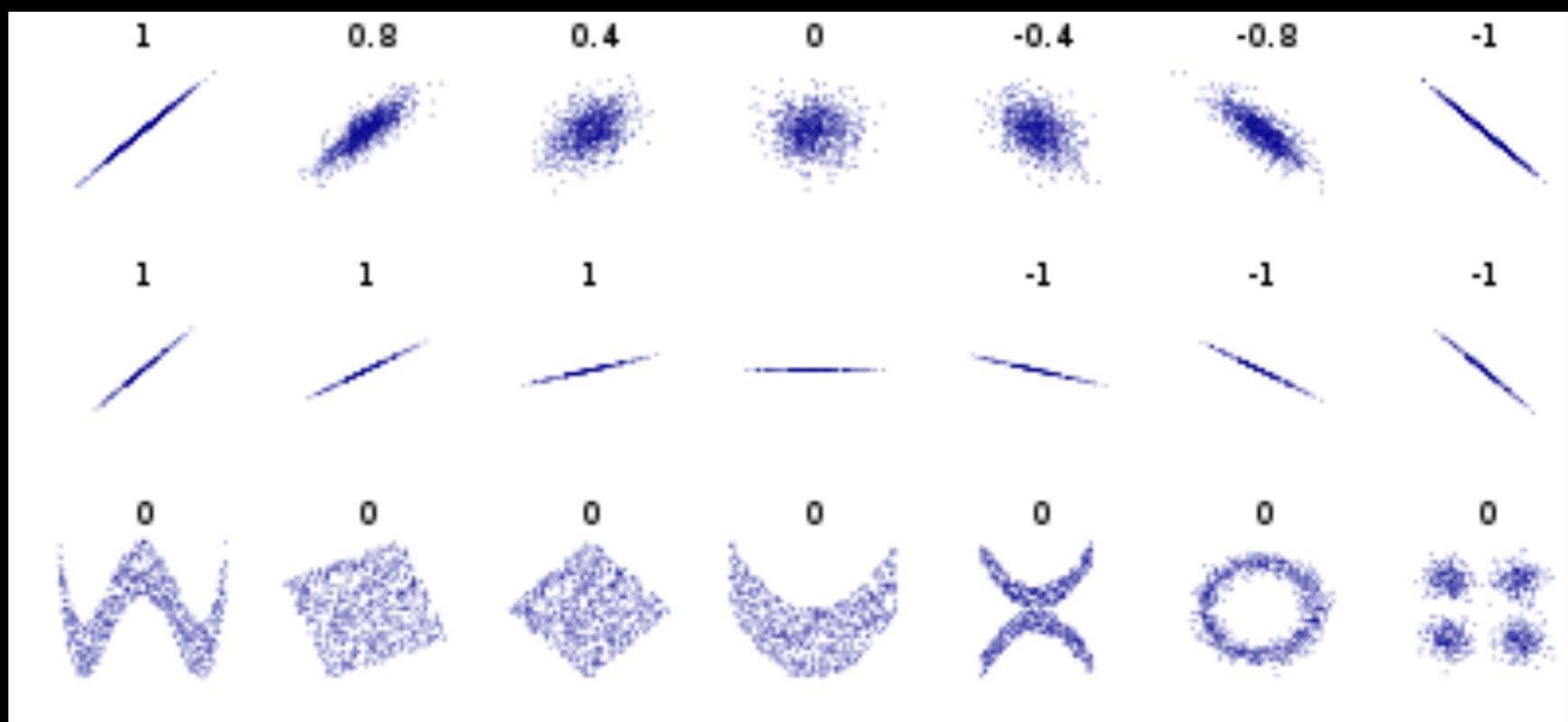




## Correlation

Pearson's test:

$$r_{xy} = \frac{1}{n-1} \sum_{i=1}^n \left( \frac{x_i - \bar{x}}{s_x} \right) \left( \frac{y_i - \bar{y}}{s_y} \right)$$
$$s_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$



Pearson's test:

$$r_{xy} = \frac{1}{n-1} \sum_{i=1}^n \left( \frac{x_i - \bar{x}}{s_x} \right) \left( \frac{y_i - \bar{y}}{s_y} \right)$$

Spearman's test:  
(Pearson's for ranks)

$$\rho = 1 - \frac{6 \sum (x_i - y_i)^2}{n(n^2 - 1)}$$

### Choosing the test

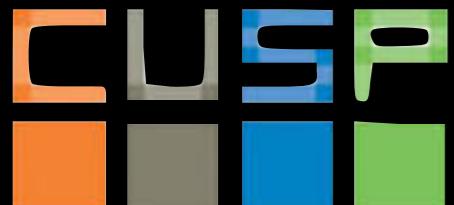
Use the table below to choose the test. See below for further details.

How many dichotomous* (binary) variables?				
Both variables interval or ratio?				
0	Y	Measures are linear? (No = monotonic*)		
		Y Pearson correlation		
0	N	N Spearman correlation		
		Both variables are ordinal?		
1	Y	Kendall correlation		
		Both variables can be ranked?		
1	N	Y Kendall correlation		
		N Convert to frequency data and use Chi-square test for independence		
1 serial Correlation Coefficient				
2 x 2 table?				
2	Y	Phi		
		N Cramer's V		
Data has frequency values for each category?				
Y	Chi-square test for independence			

\*dichotomous = 'can have only two values' (eg. yes/no or 0/1).

†monotonic = constantly increasing or decreasing.

[http://changingminds.org/explanations/research/analysis/choose\\_correlation.htm](http://changingminds.org/explanations/research/analysis/choose_correlation.htm)



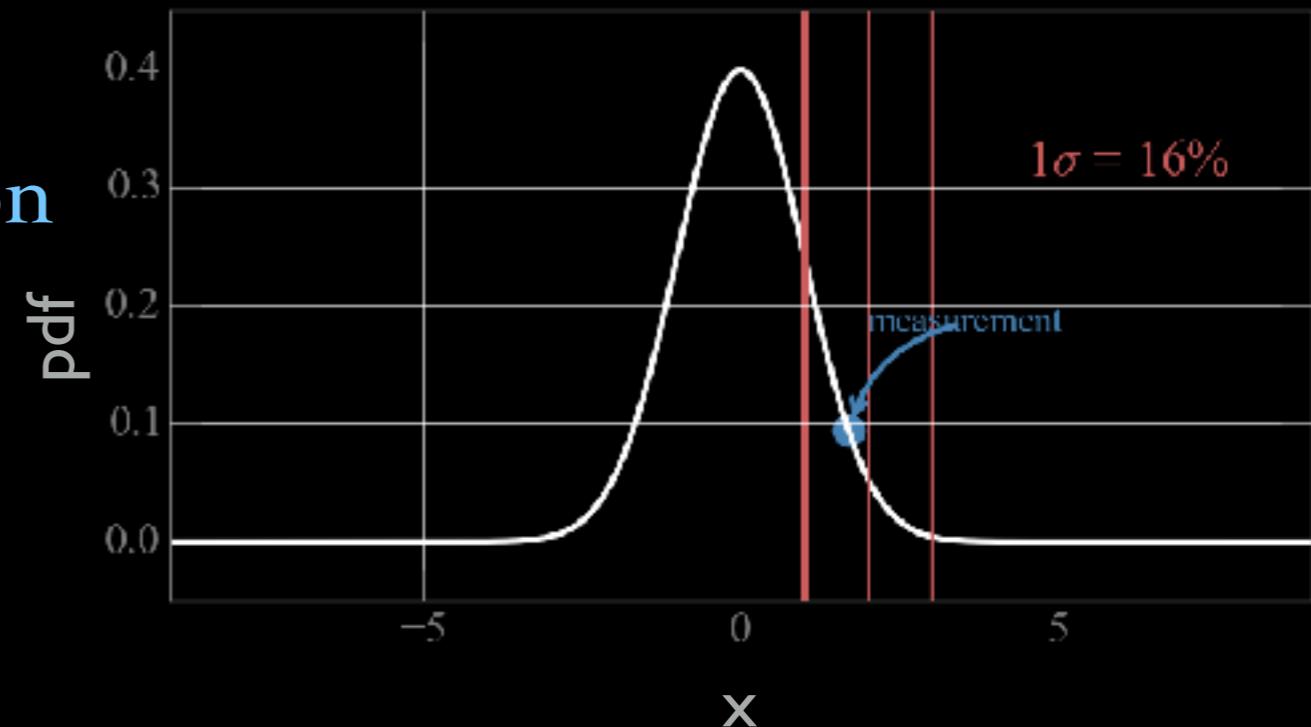
IV: Statistical analysis

# Tests for correlation and independence (continuous variables)

## Probability Distribution Function

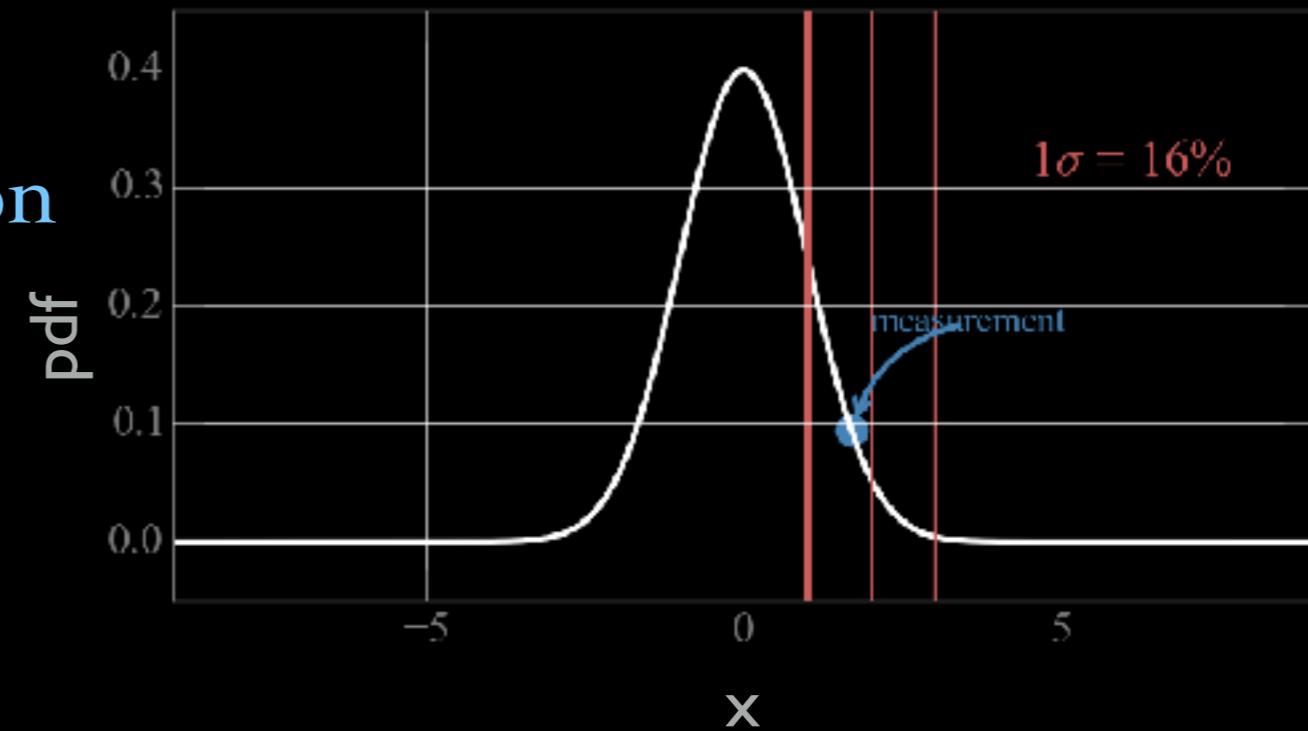
$$f_{x_0}(x) \sim p(x=x_0)$$

$$f_{x_0}(x) \sim p(x > x_0 - dx) \cap p(x < x_0 + dx)$$



## Probability Distribution Function

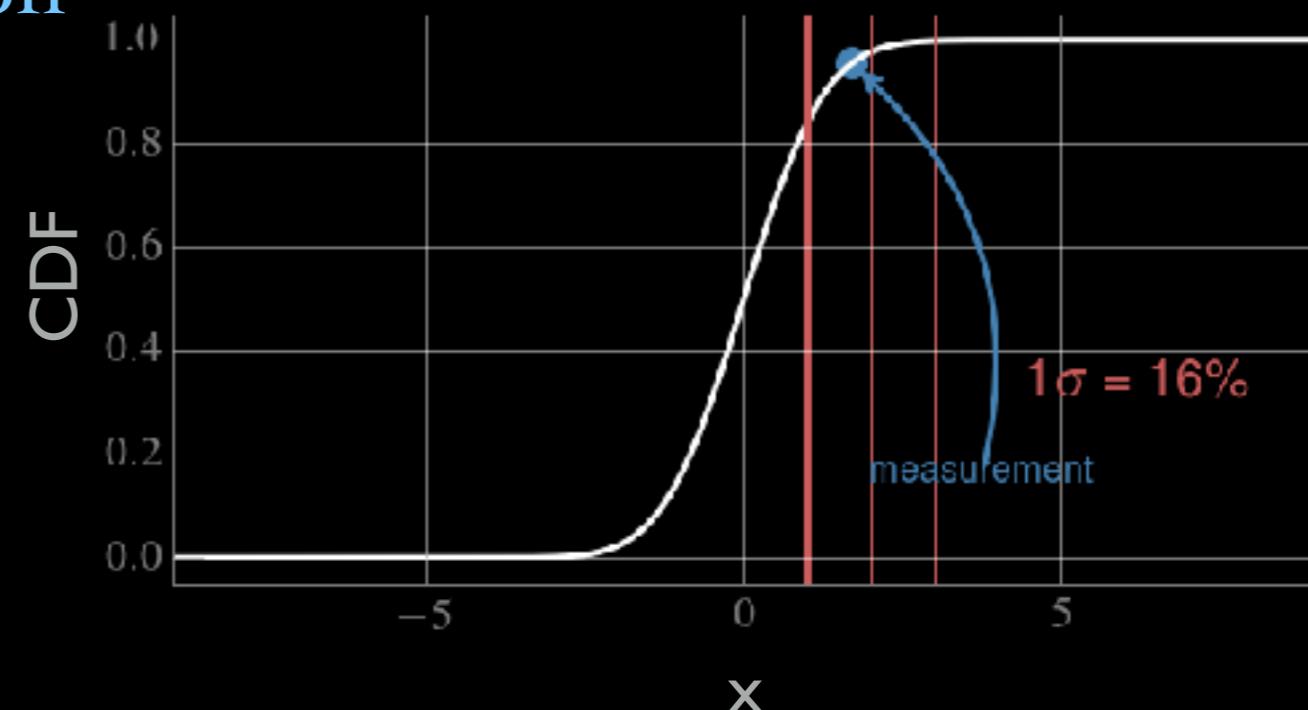
$$f_{x_0}(x) \sim p(x=x_0)$$

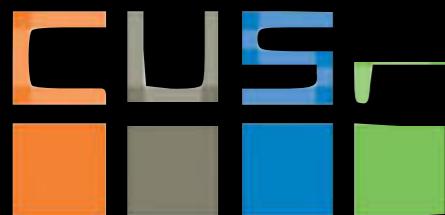
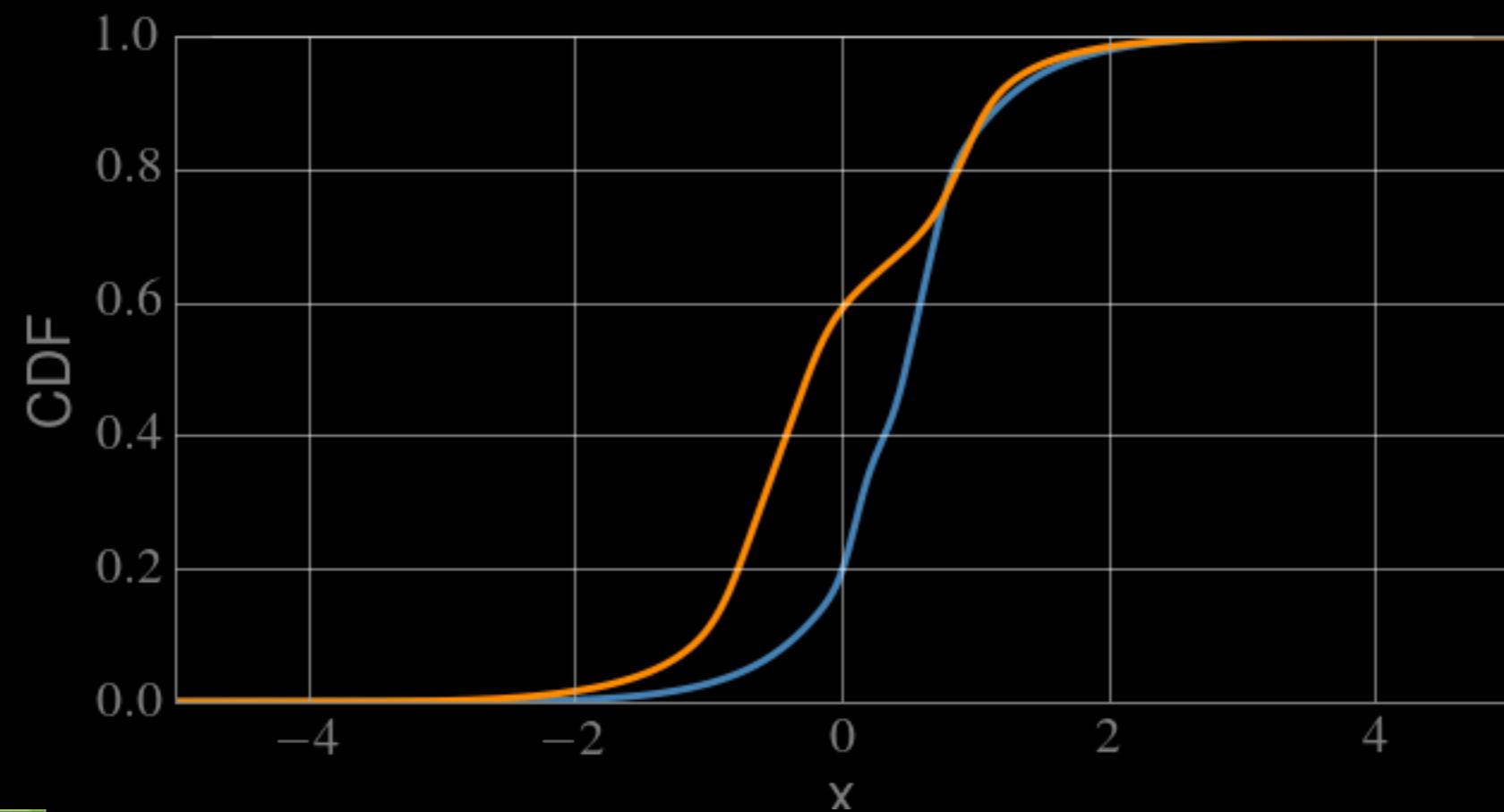
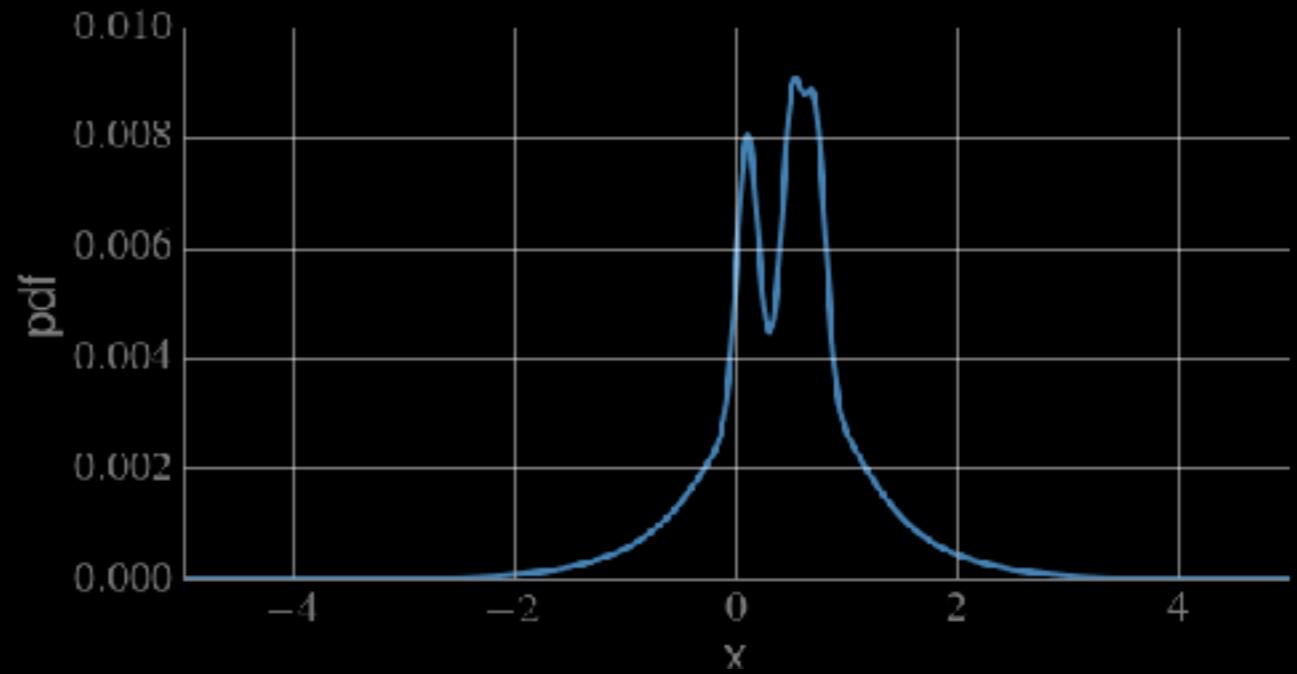
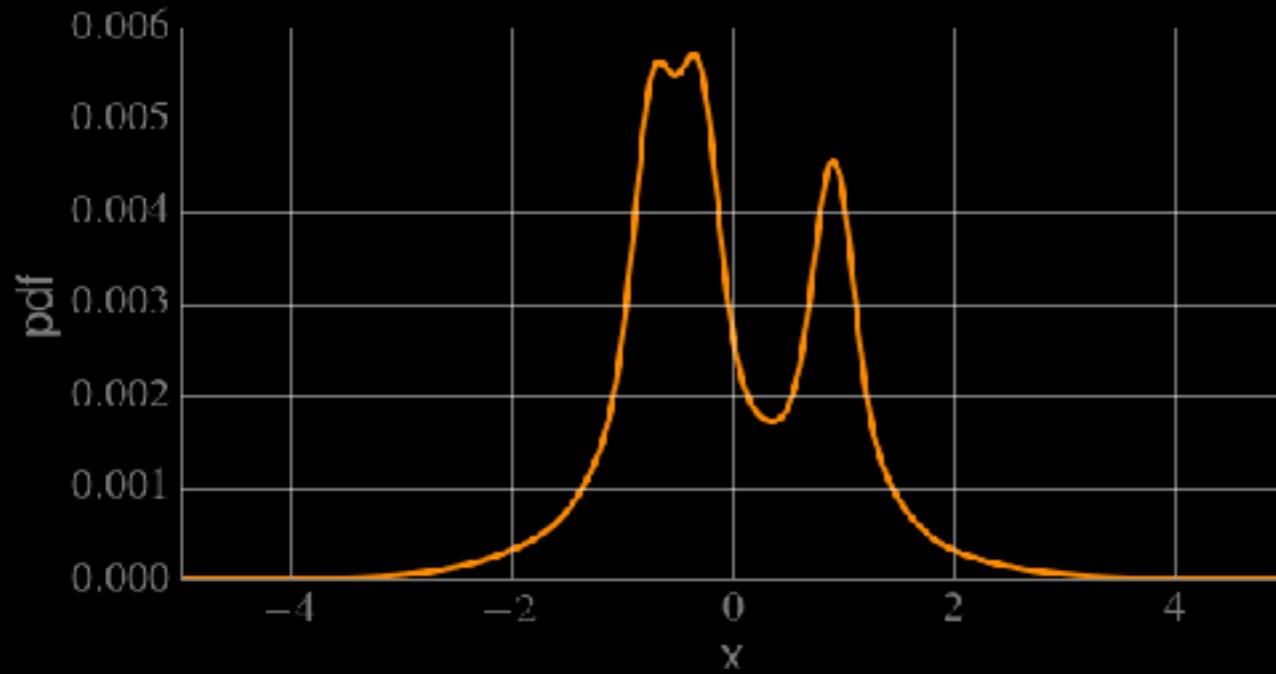


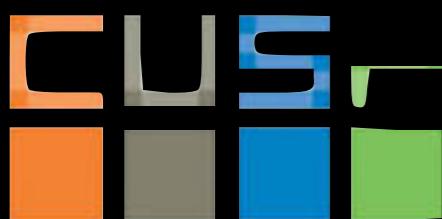
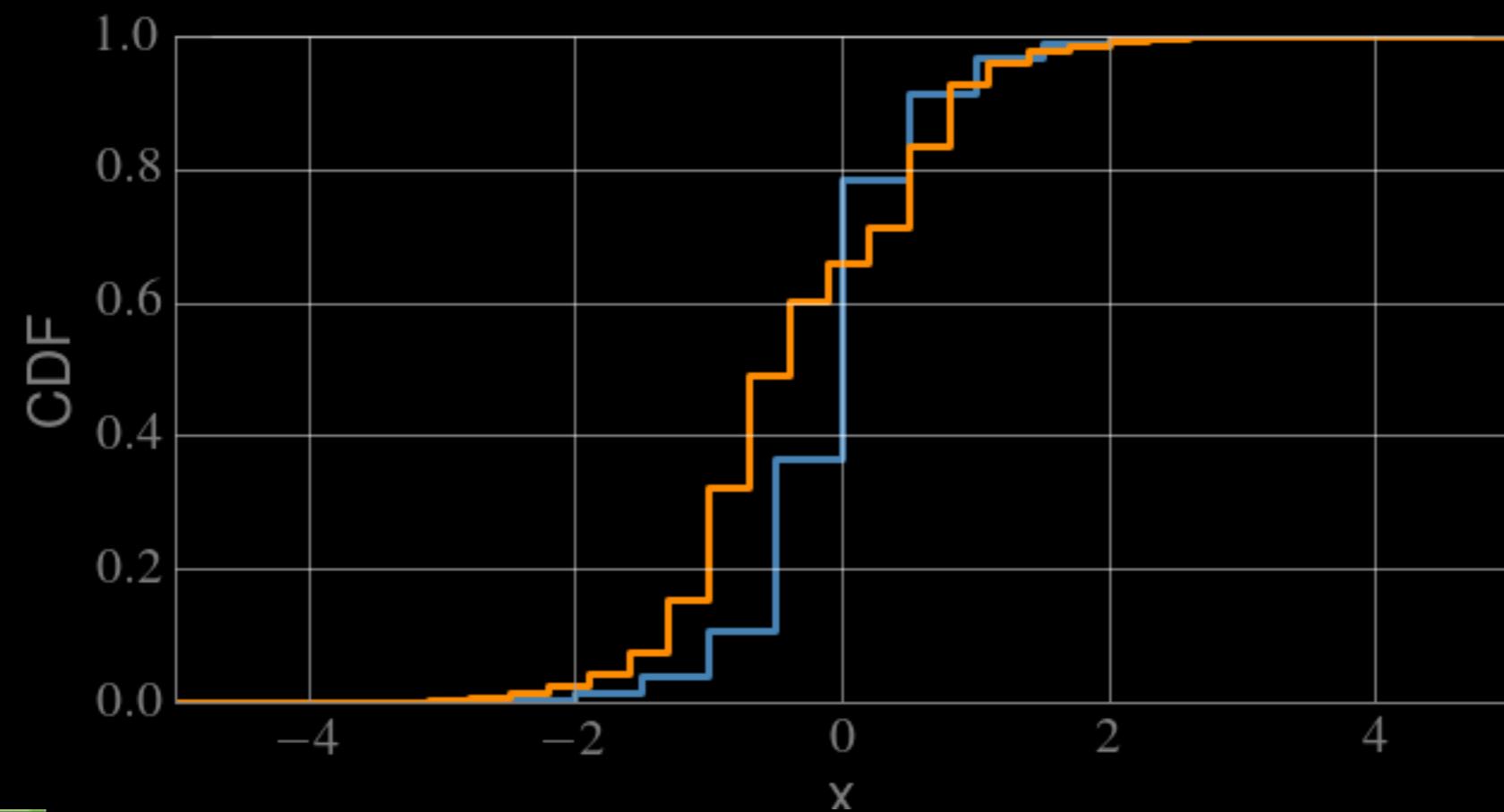
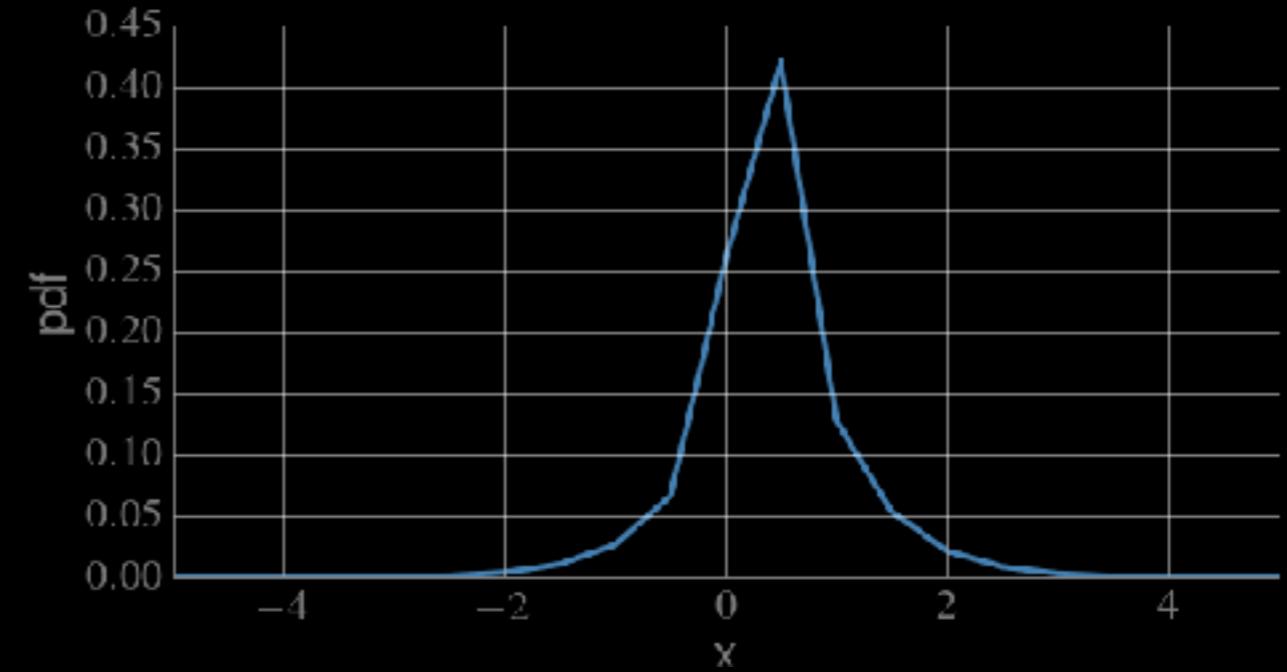
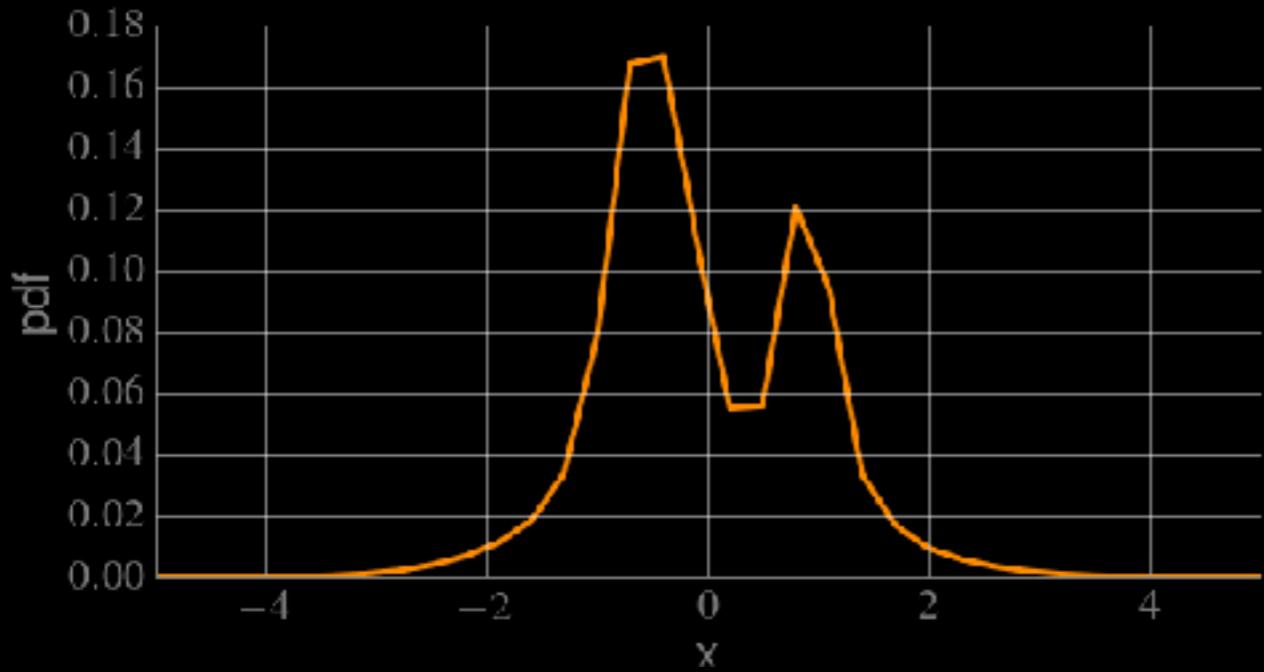
$$f_{x_0}(x) \sim p(x > x_0 - dx) \cap p(x < x_0 + dx)$$

## Cumulative Distribution Function

$$F_{x_0}(x) = P(x < x_0)$$







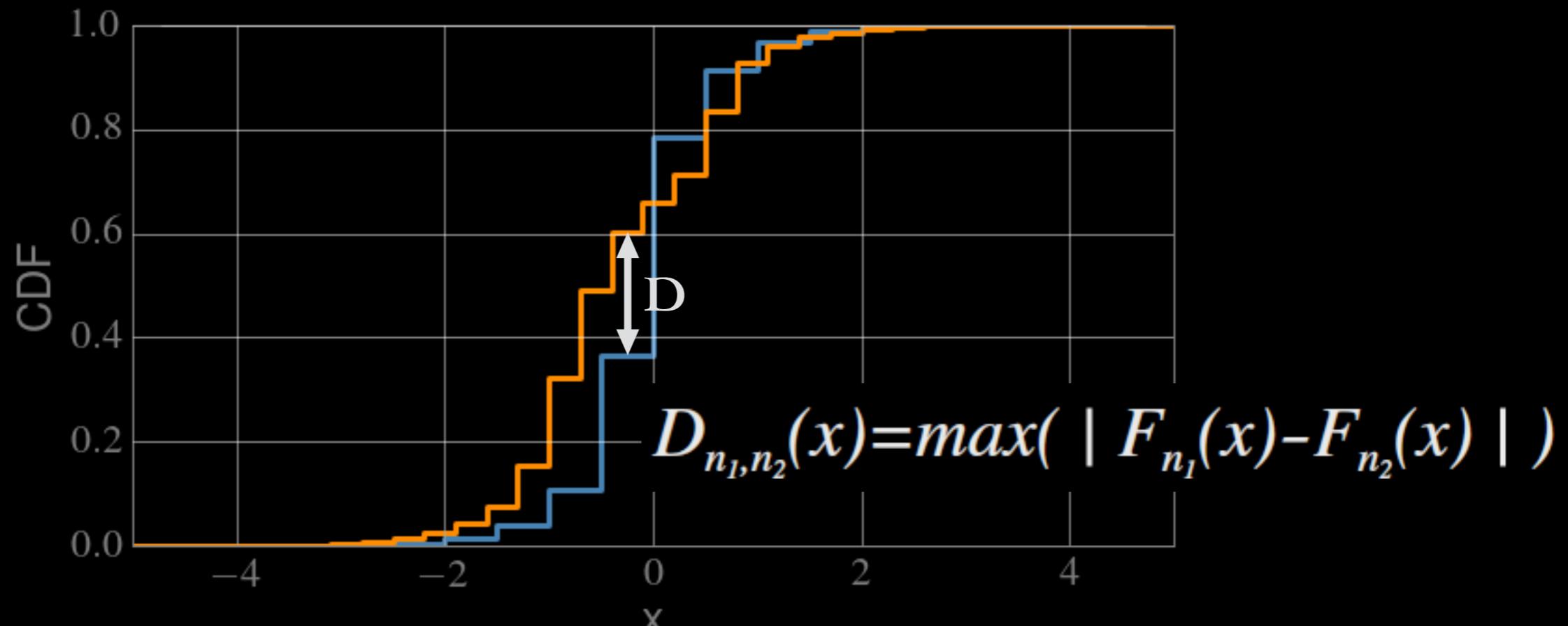
Two sample Kolmogorov Smirnoff test:

*null hypothesis*  $H_0$ : the samples come from the same parent distribution

$H_0$  is rejected at level  $\alpha$  if  $D(n_1, n_2) > c(\alpha) \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$

with  $c(\alpha)$  given by a table

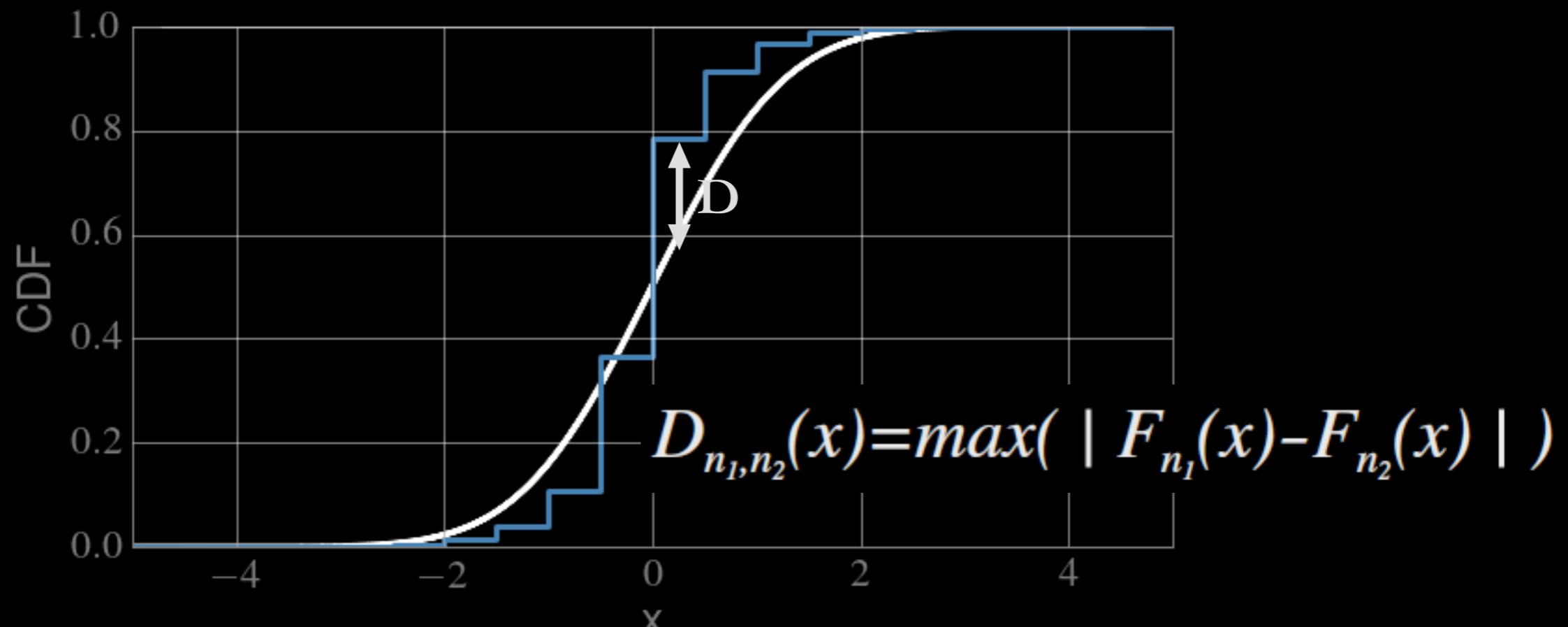
NOTE: it ONLY works in 2D where the Euclidian distance is uniquely defined!



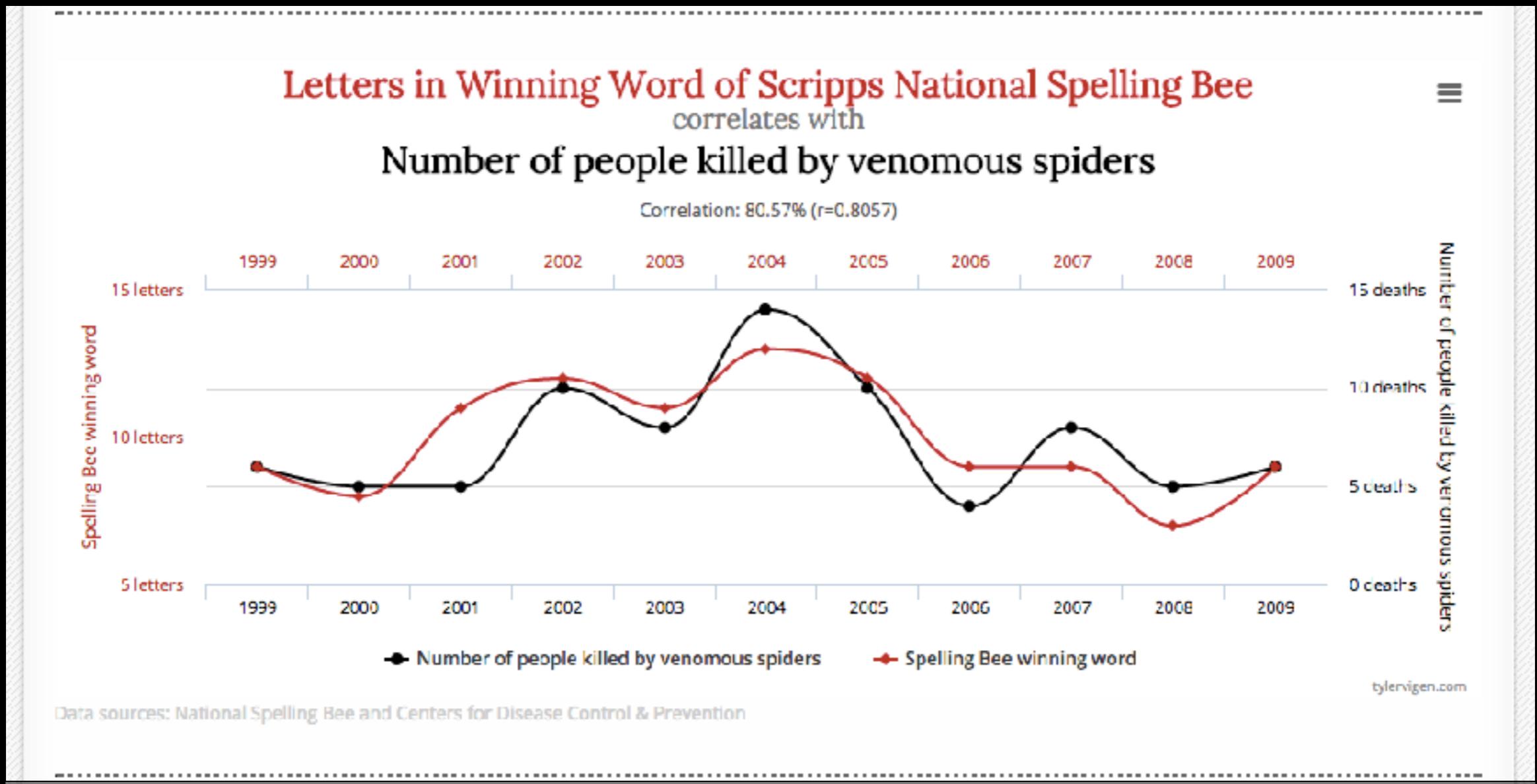
Goodness-of-fit Kolmogorov Smirnoff test:

*null hypothesis*  $H_0$ : the sample does comes from the model distribution

$H_0$  is rejected at level  $\alpha$  if  $\sqrt{n} D_n > K_\alpha$  where  $P(K \leq K_\alpha) = 1 - \alpha$



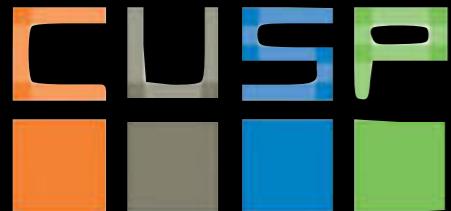
# WARNING: Correlation is not causation!



<http://www.tylervigen.com/spurious-correlations>

# ERRORS

What if our conclusion is after all wrong??  
we have not ruled out the null, just decided that is it unlikely



# ERRORS

<i>reality</i> → <i>result of analysis</i> ↓	$H_0$ is True	$H_0$ is False
$H_0$ is falsified	Type I error False Positive	True Positive
$H_0$ is not falsified	True Negative	Type II error False negative

# ERRORS

	$H_0$ is True	$H_0$ is False
$H_0$ is falsified	Type I error False Positive important message gets spammed	True Positive
$H_0$ is not falsified	True Negative	Type II error False negative Spam in your Inbox

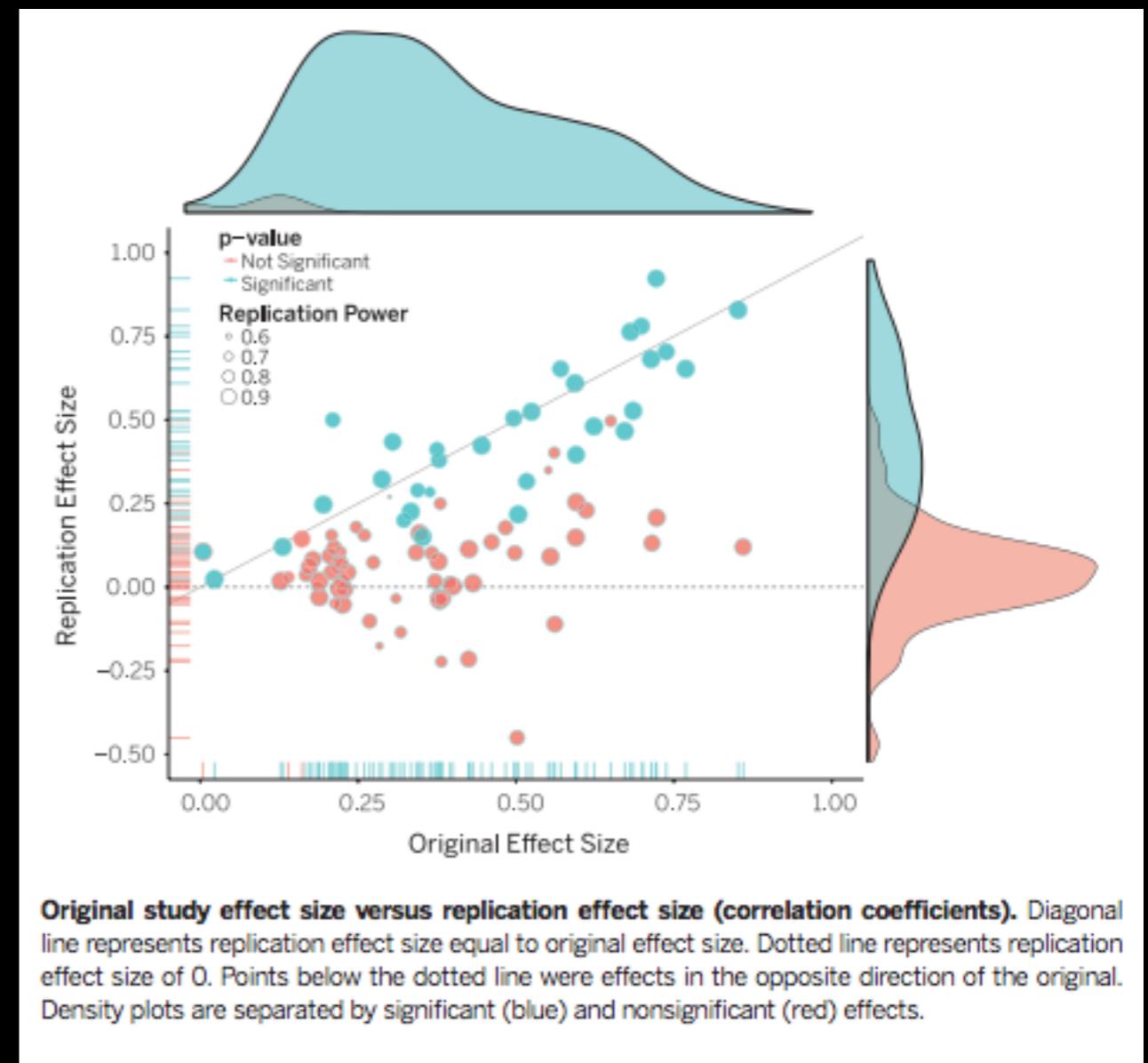
# Homework: READING

## RESEARCH ARTICLE SUMMARY

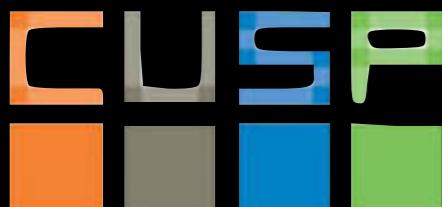
PSYCHOLOGY

### Estimating the reproducibility of psychological science

Open Science Collaboration\*



<http://www.sciencemag.org/content/349/6251/aac4716.full.pdf>



IV: Statistical analysis

## Homework: 3. Compare Tests for Correlation

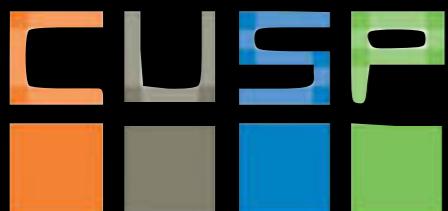
The following are 3 tests that assess correlation between 2 samples of citibike data:

- **Pearson's test** (answer: are the 2 samples correlated?)
- **Spearman's test** (answer: are the 2 samples correlated?)
- **K-S test** (answer: are the samples likely to come from the same parent distribution?)

Use: age of bikers for 2 genders, age of bikers in day vs night and assess the correlation/independence of the 2 samples in each case. State your result in words.

## MUST KNOWS:

- p-values pros and cons
- How to chose a statistical test
- How to get from a statistics to a p-value
- Effect Size
- PDF vs CDF
- correlation vs causation
- KS test for 2 samples, Pearson's, Spareman
- Type I & II errors and power of a test



# Resources:

Sarah Boslaugh, Dr. Paul Andrew Watters, 2008

**Statistics in a Nutshell (Chapters 3,4,5)**

[https://books.google.com/books/about/Statistics\\_in\\_a\\_Nutshell.html?id=ZnhgO65Pyl4C](https://books.google.com/books/about/Statistics_in_a_Nutshell.html?id=ZnhgO65Pyl4C)

David M. Lane et al.

**Introduction to Statistics (XVIII)**

[http://onlinestatbook.com/Online\\_Statistics\\_Education.epub](http://onlinestatbook.com/Online_Statistics_Education.epub)

<http://onlinestatbook.com/2/index.html>

Barun K Nayak and Avijit Hazral

**How to choose the right statistical test?**

<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3116565/>

