CUSP UCSL Summer 2016: Introduction to Linear Algebra

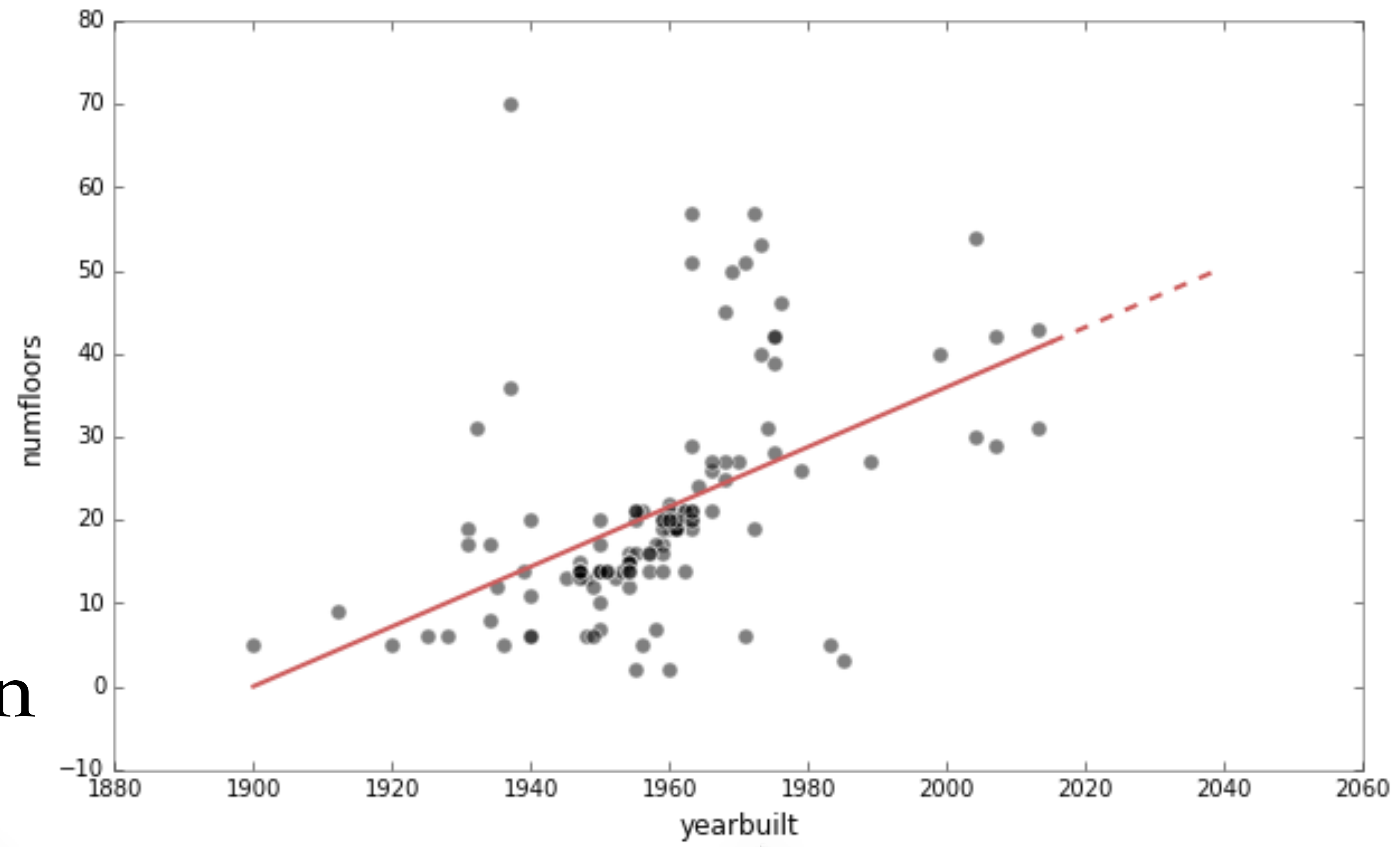
dr. federica bianco
fbianco@nyu.edu
@fedhere

Material related to this lecture can be found at
https://github.com/fedhere/UInotebooks/tree/master/UCSL2016

why do we bother with linear algebra?

LINEAR ALGEBRA ALLOWS US TO SOLVE COMPLEX SYSTEMS OF EQUATIONS EFFICIENTLY. Most Data Science and Computational problems can be describes by sets of linear equations.

- Finding similarity patterns in data:e.g. network analysis
- Processing an image: e.g. rotation
- Regression: predicting trends, understanding correlation

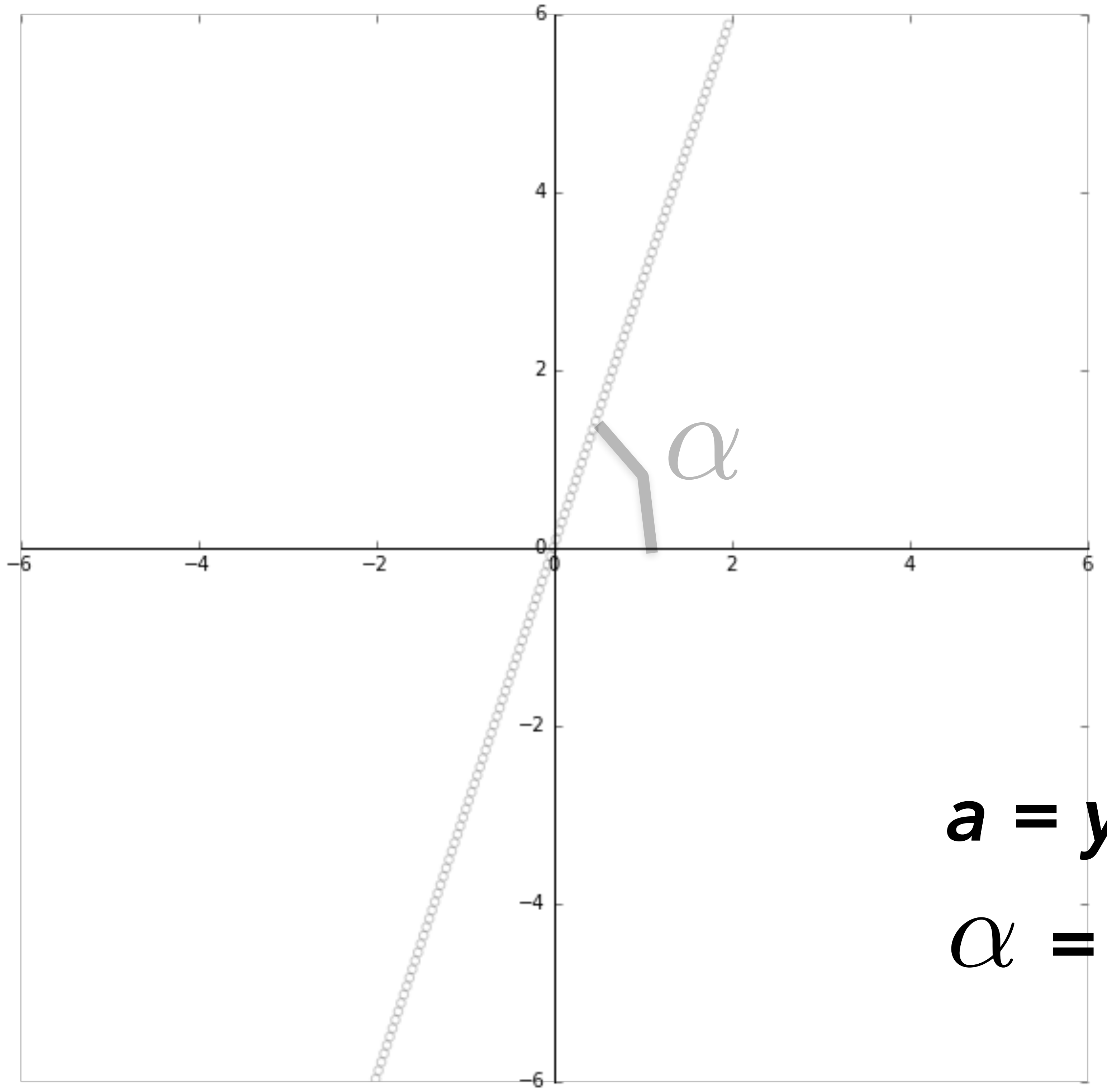


outline:

- What is a vector
- What is a matrix
- vector & matrix transformations
- Solving systems of linear equations
- Eigenvectors/Eigenvalues

Scalar multiplication

equation of a line



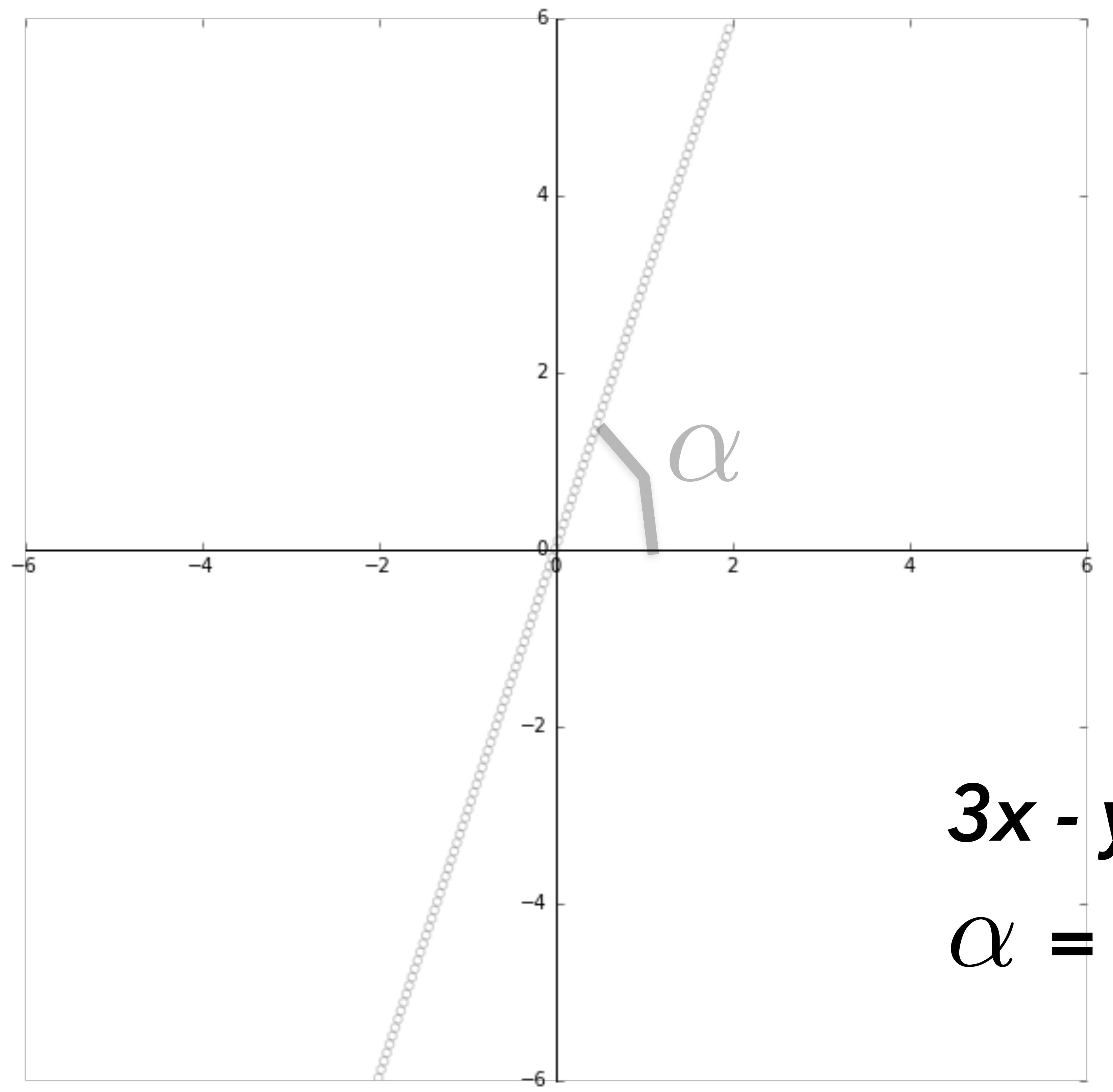
constant direction : constant y/x

a = y/x = 3

α ***= tan^{-1}(y/x)***

Scalar multiplication

equation of a line



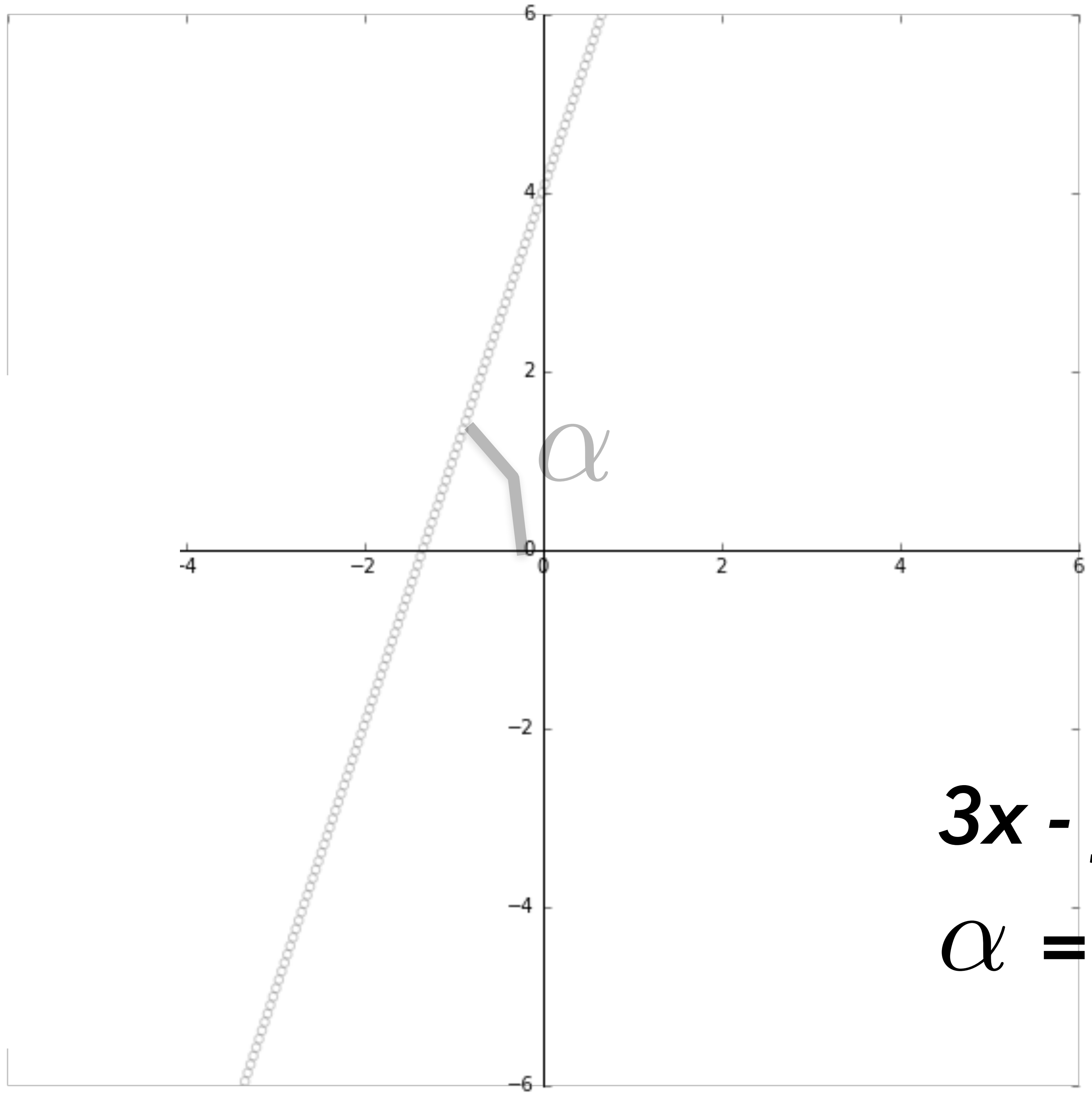
constant direction : constant y/x

$3x - y = 0$

$\alpha = \tan^{-1}(y/x)$

Scalar multiplication

equation of a line



constant direction : constant y/x

$3x - y = -6$

$\alpha = \tan^{-1}(y/x)$

Systems of linear equations

$$2x - 3y = 34$$
$$4x + y = 12$$

Systems of **linear** equations

$$2x - 3y = 34$$
$$4x + y = 12$$

Systems of **linear** equations

the unknowns (x, y) only appear in linear format

(power of 1, no squares, square root, exponent…)

$$2x - 3y = 34$$
$$4x + y = 12$$

$$\left.\begin{array}{l} N_{eq} \text{ equations,} \\ \mathbf{N_{un} > N_{eq} \text{ unknowns}} \end{array}\right\} >1 \text{ solution}$$

$$2x - 3y = 34$$
$$4x + y = 12$$

$$\left.\begin{array}{l}N_{eq}\text{ equations,}\\ \mathbf{N_{un}<N_{eq}\ unknowns}\end{array}\right\}\text{ overconstrained}$$

$$2x - 3y = 34$$
$$4x + y = 12$$

$N_{un=}N_{eq}$ 1 solution

$$2x - 3y = 34$$
$$4x + y = 12$$

solving by substitution

$$2x - 3y = 34$$
$$4x + y = 12$$

$$\boxed{\begin{array}{c} 2x - 3y = 34 \\ y = 12 - 4x \end{array}}$$

solving by substitution

$$2x - 3y = 34$$
$$4x + y = 12$$

$$\boxed{\begin{array}{c} 2x - 3y = 34 \\ y = 12 - 4x \end{array}} \quad \boxed{\begin{array}{c} 2x - 3(12-4x) = 34 \\ y = 12 - 4x \end{array}}$$

solving by substitution

$$2x - 3y = 34$$
$$4x + y = 12$$

$$\boxed{\begin{array}{c} 2x - 3y = 34 \\ y = 12 - 4x \end{array}} \quad \boxed{\begin{array}{c} 2x - 3(12-4x) = 34 \\ y = 12 - 4x \end{array}} \quad \boxed{\begin{array}{c} -36 + 14x = 34 \\ y = 12 - 4x \end{array}}$$

solving by substitution

$$2x - 3y = 34$$
$$4x + y = 12$$

$2x - 3y = 34$ $y = 12 - 4x$	$2x - 3(12-4x) = 34$ $y = 12 - 4x$	$-36 + 14x = 34$ $y = 12 - 4x$	$x = 70/14$ $y = 12 - 4x$

solving by substitution

$$2x - 3y = 34$$
$$4x + y = 12$$

$2x - 3y = 34$ $y = 12 - 4x$	$2x - 3(12 - 4x) = 34$ $y = 12 - 4x$	$-36 + 14x = 34$ $y = 12 - 4x$	$x = 70/14$ $y = 12 - 4x$	$x = 5$ $y = 12 - 20$

solving by substitution

$$2x - 3y = 34$$
$$4x + y = 12$$

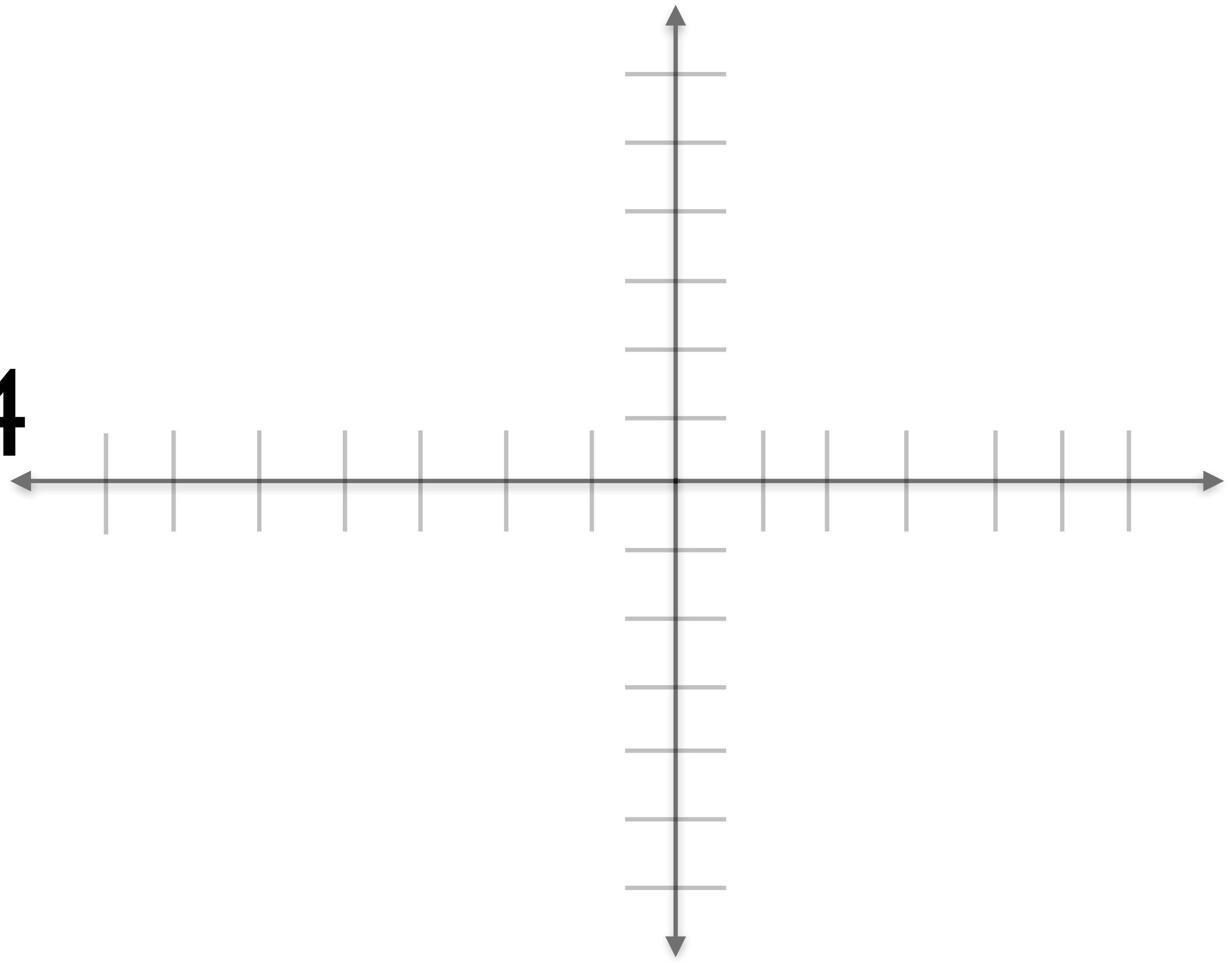
$2x - 3y = 34$ $y = 12 - 4x$	$2x - 3(12-4x) = 34$ $y = 12 - 4x$	$-36 + 14x = 34$ $y = 12 - 4x$	$x = 70/14$ $y = 12 - 4x$	$x = 5$ $y = 12 - 20$

$$\boxed{\begin{aligned} x &= 5 \\ y &= -8 \end{aligned}}$$

solved in 6 operations

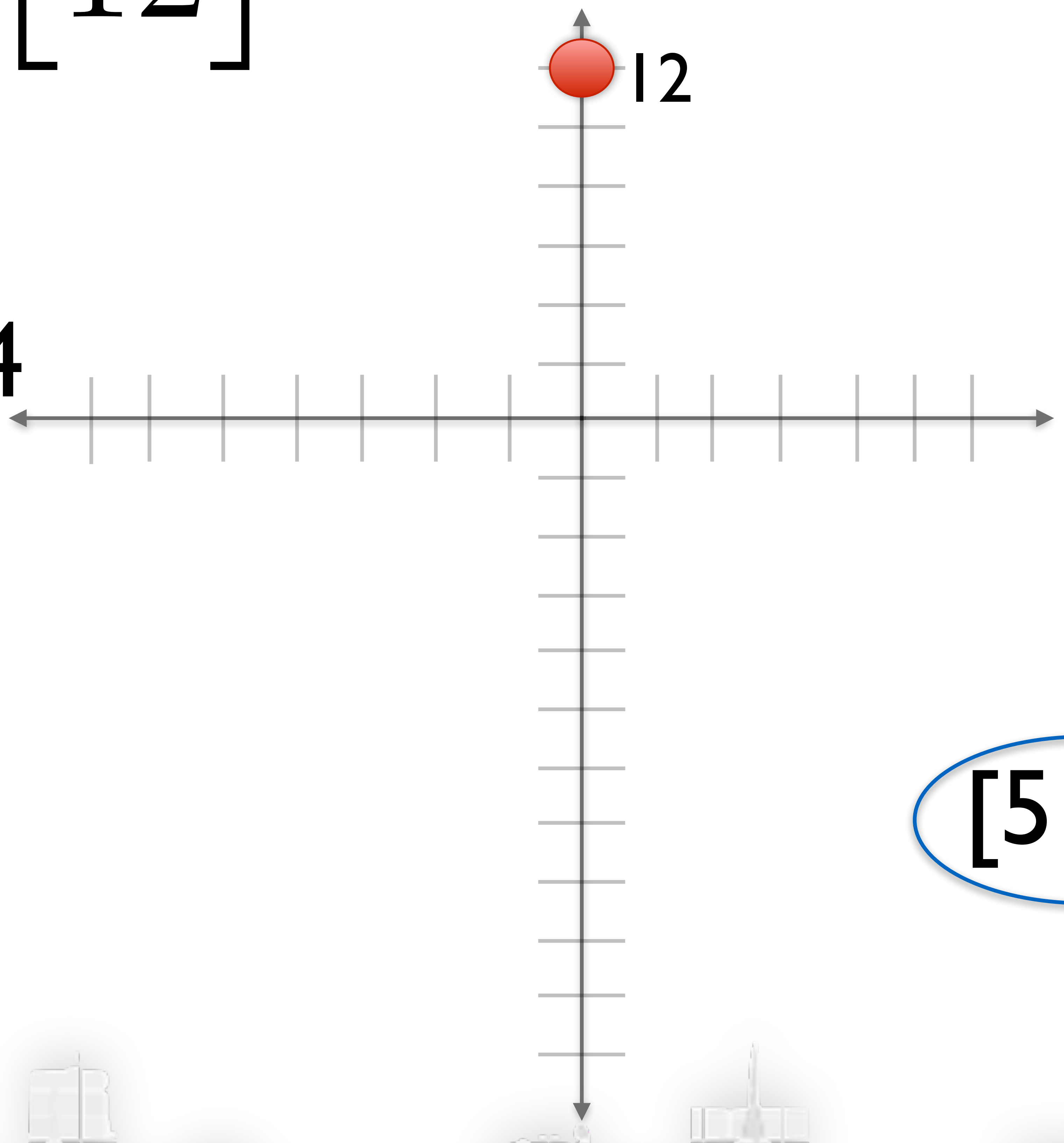
Systems of linear equations

$$2x - 3y = 34$$
$$4x + y = 12$$



$$\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 34 \\ 12 \end{bmatrix}$$

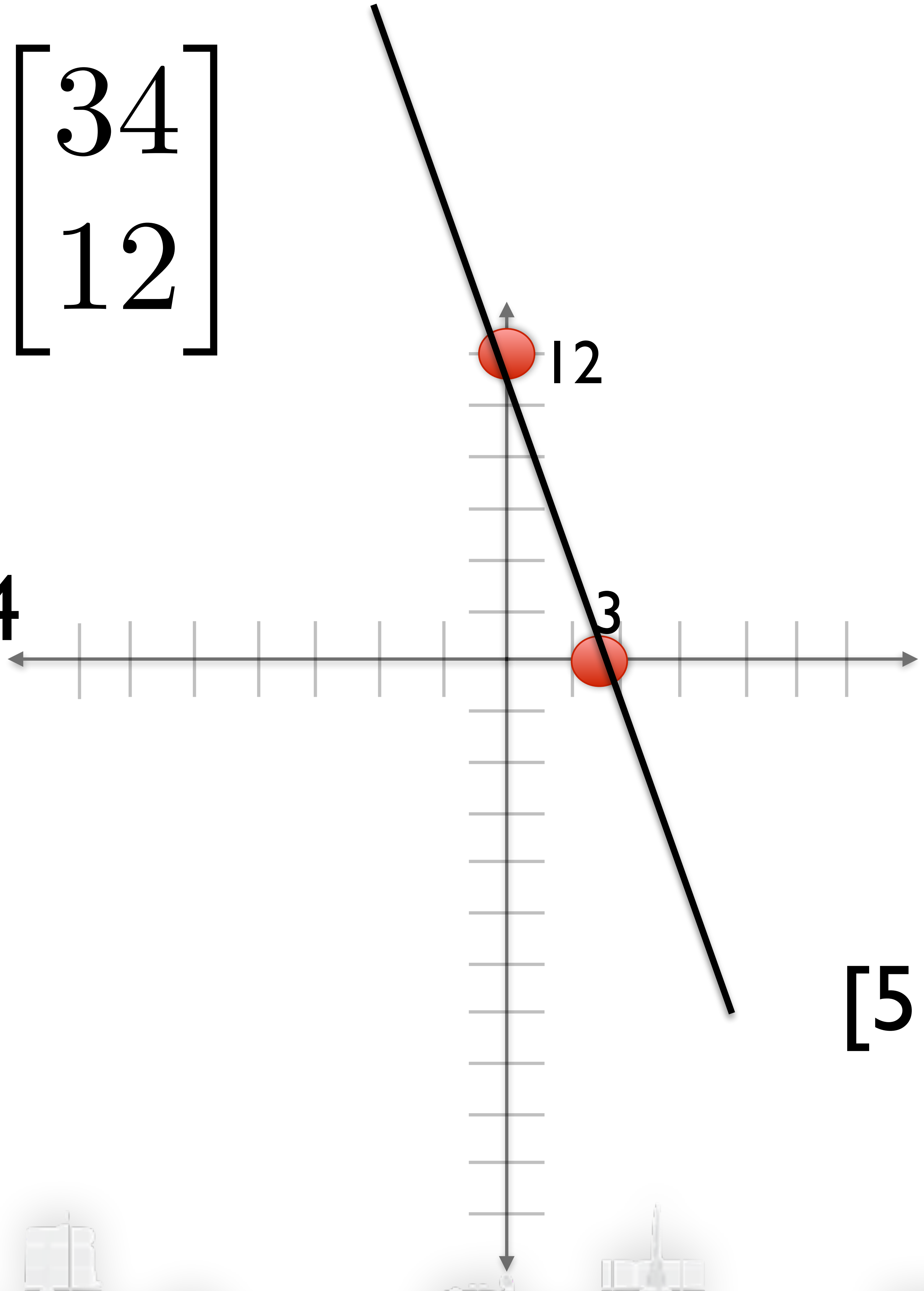
$2x - 3y = 34$
$4x + y = 12$



[5 -8]
solution

$$\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 34 \\ 12 \end{bmatrix}$$

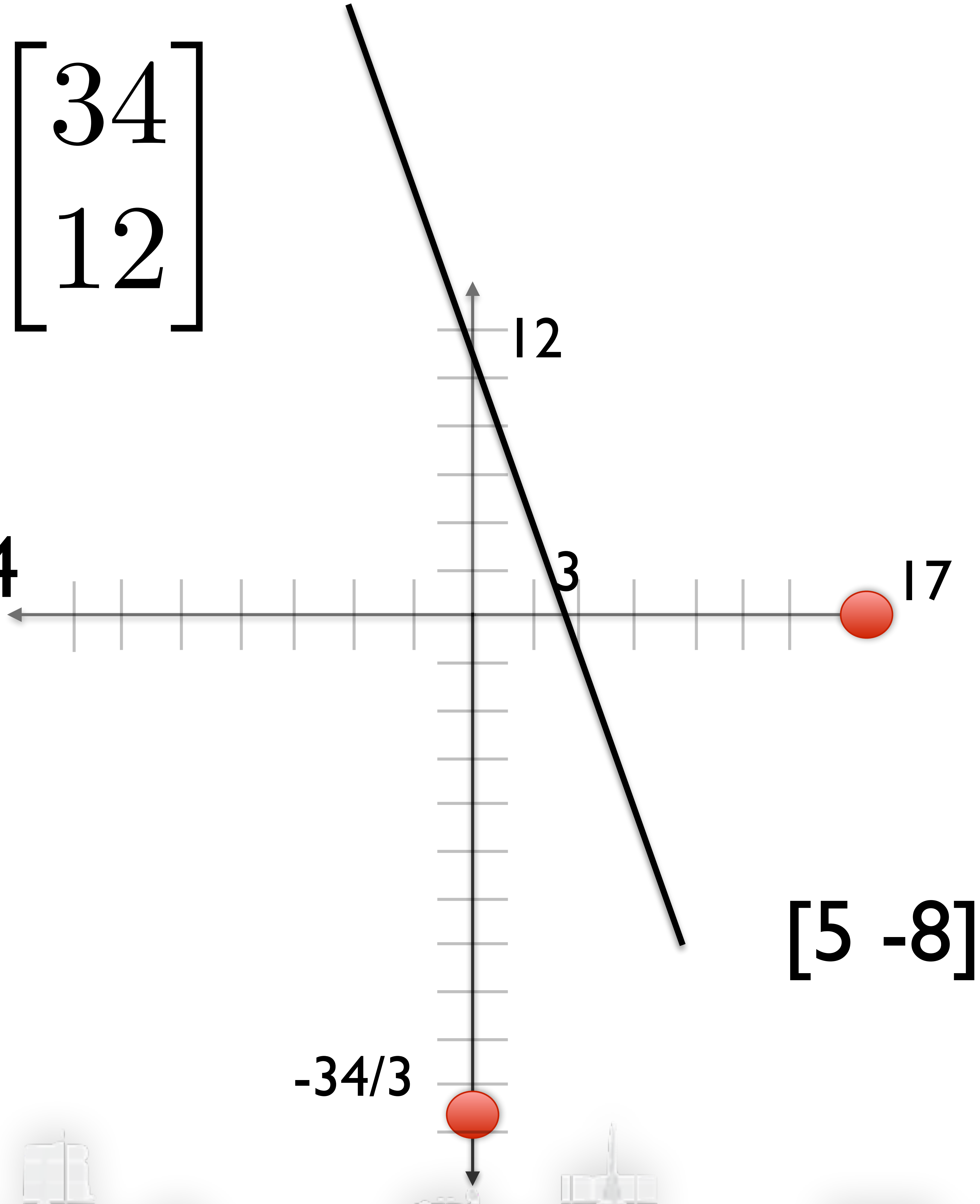
2x – 3y = 34
4x + y = 12



[5 -8]

$$\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 34 \\ 12 \end{bmatrix}$$

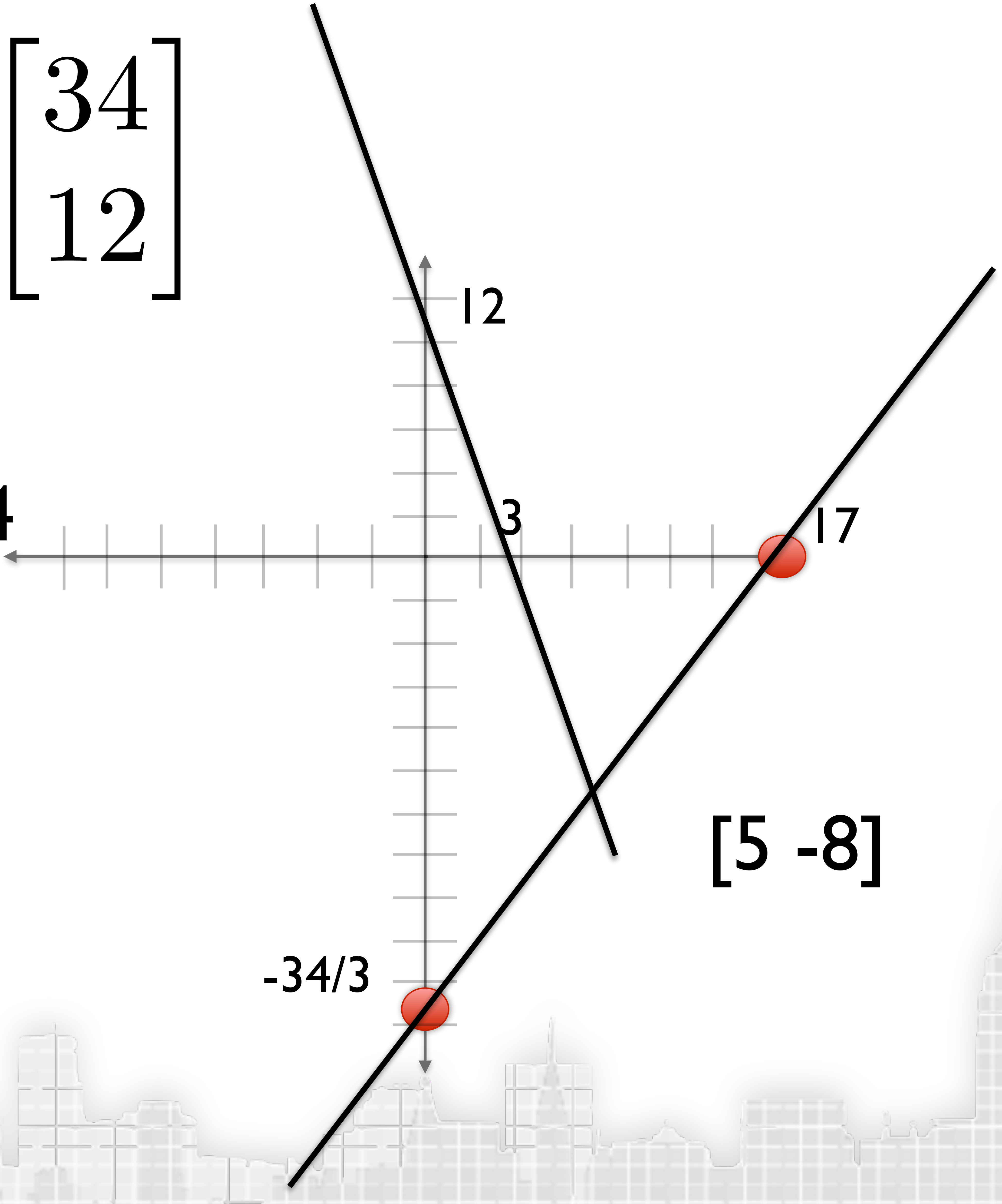
2x – 3y = 34
4x + y = 12



[5 -8]

$$\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 34 \\ 12 \end{bmatrix}$$

$2x - 3y = 34$
$4x + y = 12$

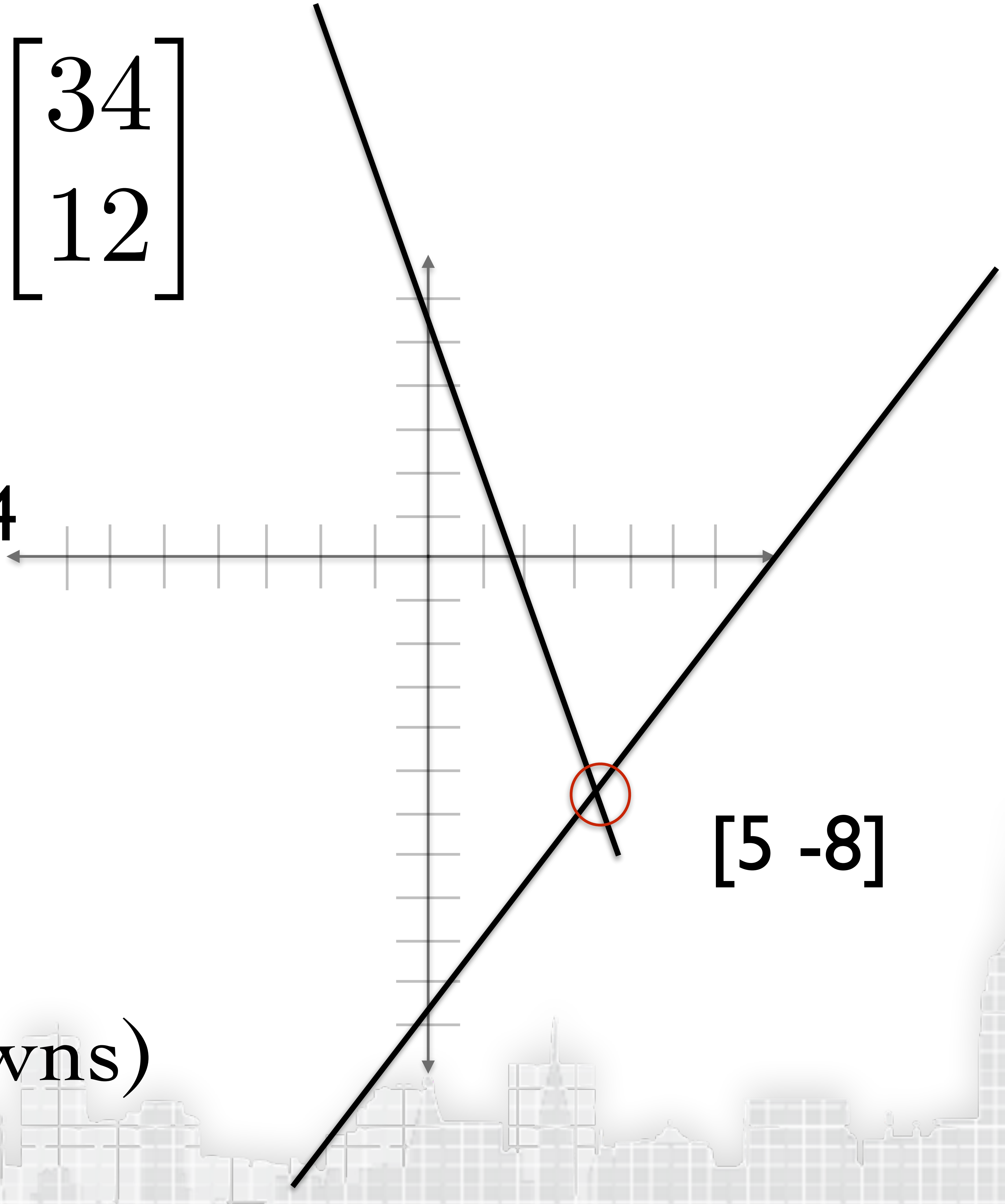


[5 -8]

$$\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 34 \\ 12 \end{bmatrix}$$

$2x - 3y = 34$

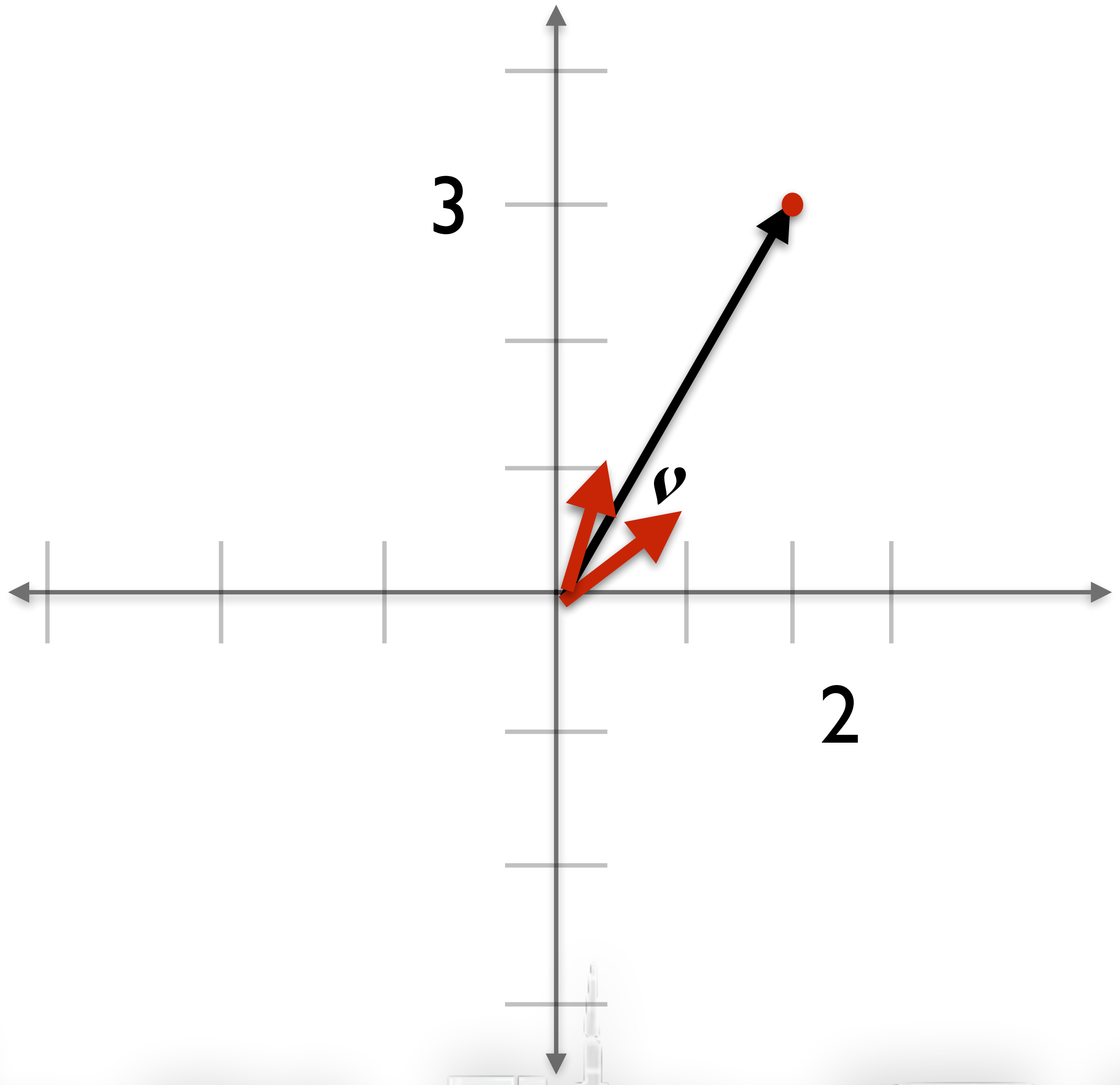
$4x + y = 12$



corresponds to finding the point of interction of vectors
(lines in 2D, planes in 3D ….
N equations, N dimensions, N unknowns)

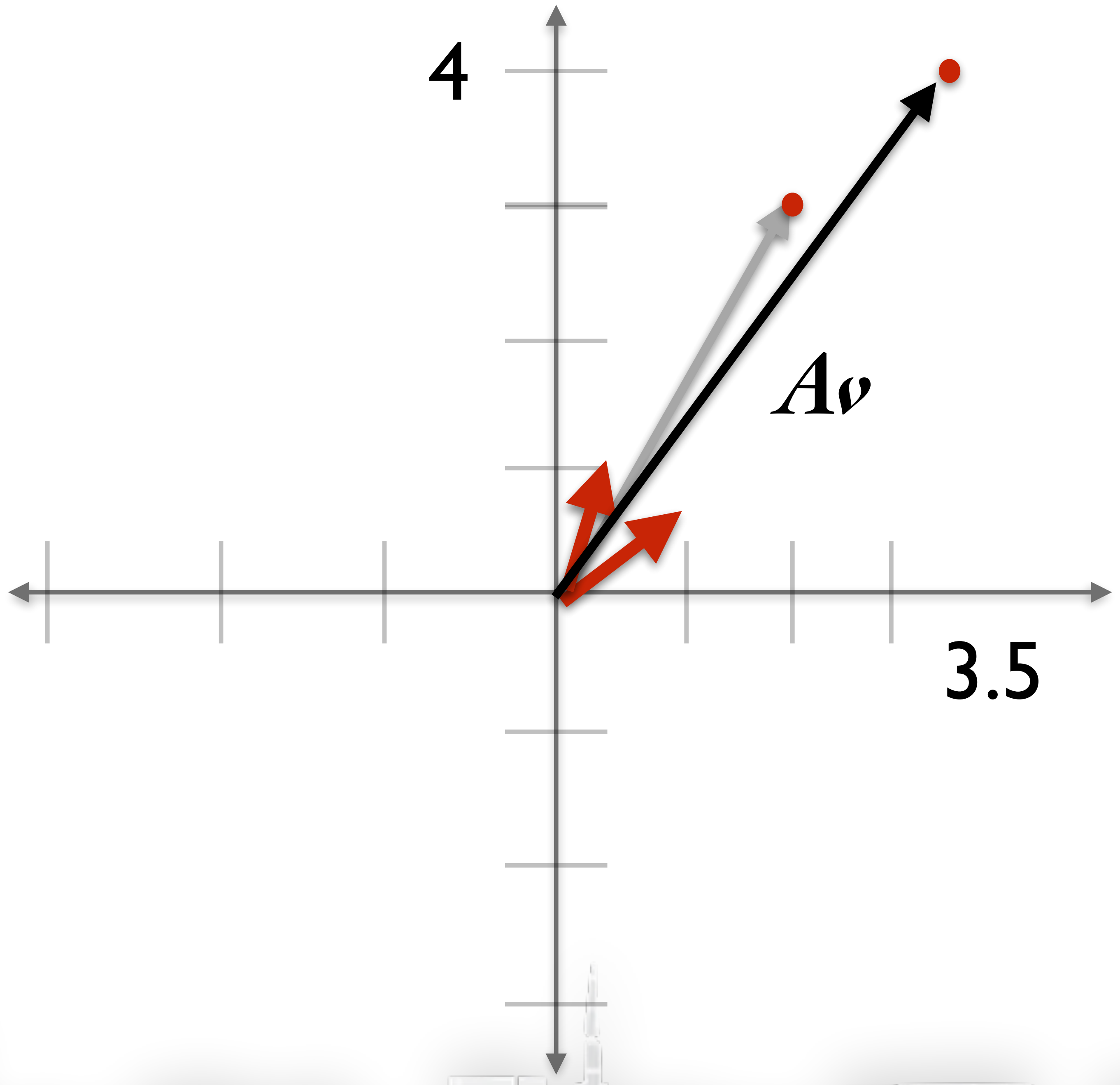
outline:

- What is a vector
- What is a matrix
- vector & matrix transformations
- Solving systems of linear equations
- Eigenvectors/Eigenvalues



$$v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

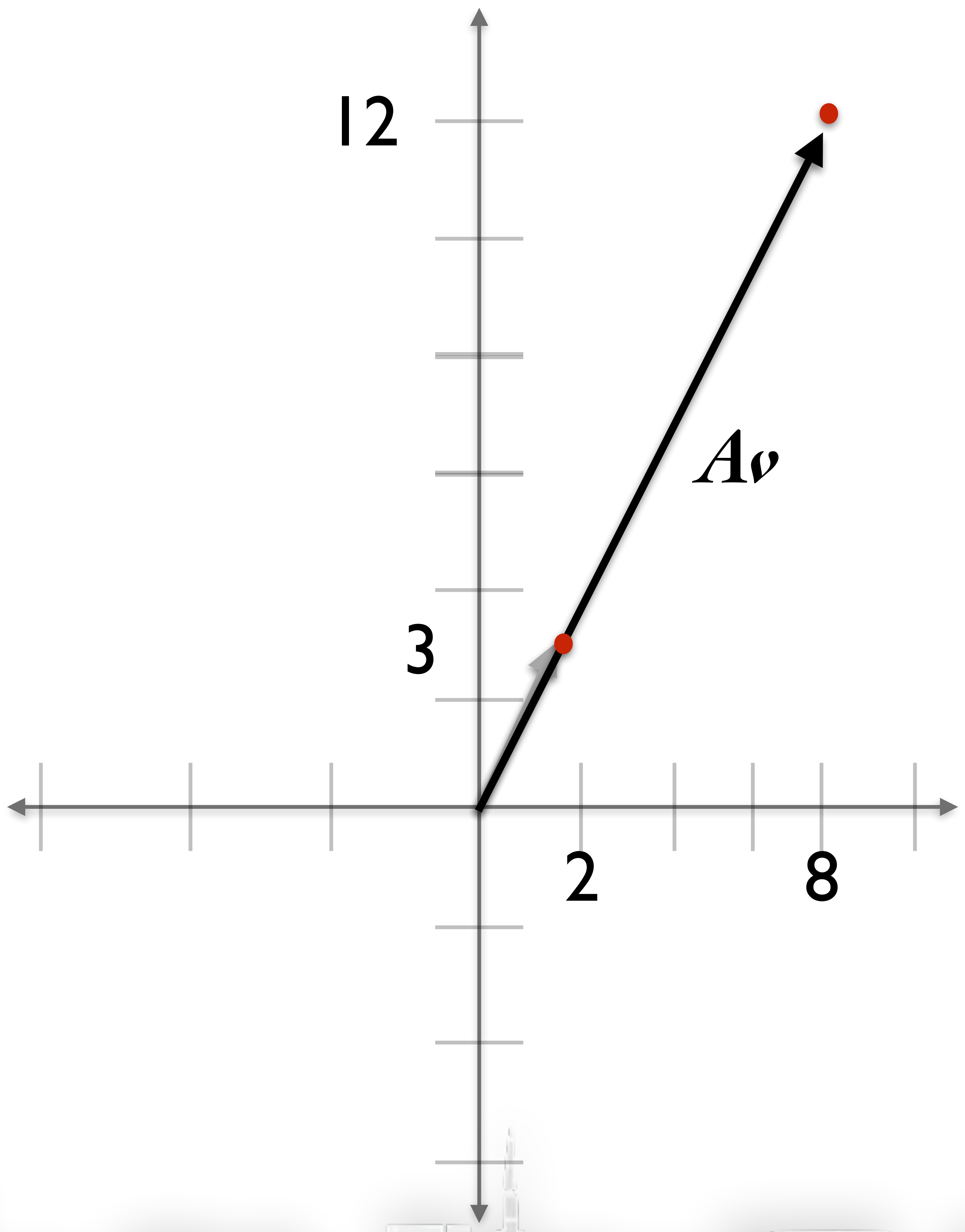
$$A = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$



$$v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

$$Av = \begin{bmatrix} 3.5 \\ 4 \end{bmatrix}$$



$$v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

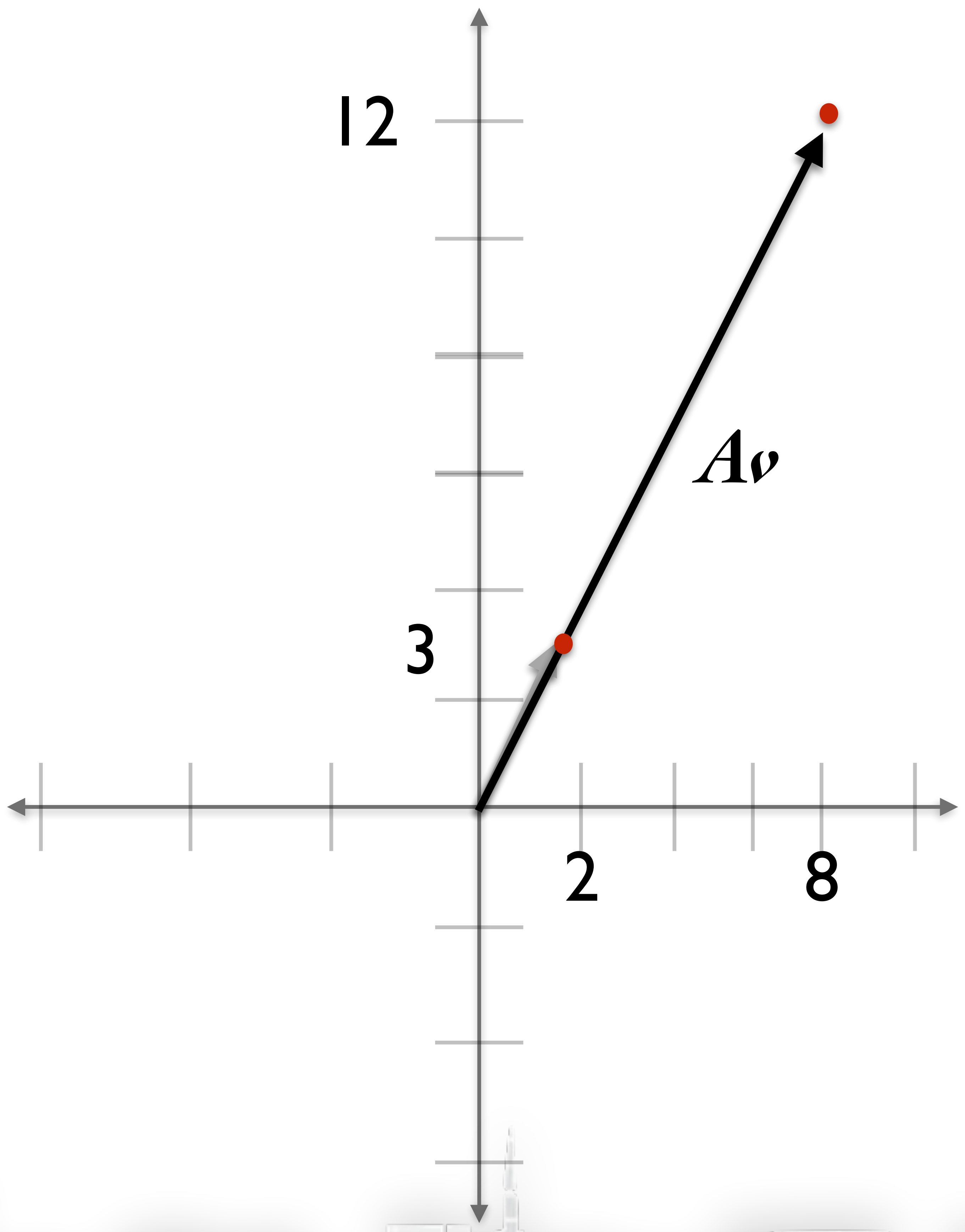
$$A = \begin{bmatrix} 1 & 2 \\ 6 & 0 \end{bmatrix}$$

$$Av = \begin{bmatrix} 8 \\ 12 \end{bmatrix} = 4v$$

A vector *v* is an ***eigenvector*** of the matrix *A* if there exist a λ such that

$$Av = \lambda v$$

and λ is then the eigenvalue of *A* associated to *v*



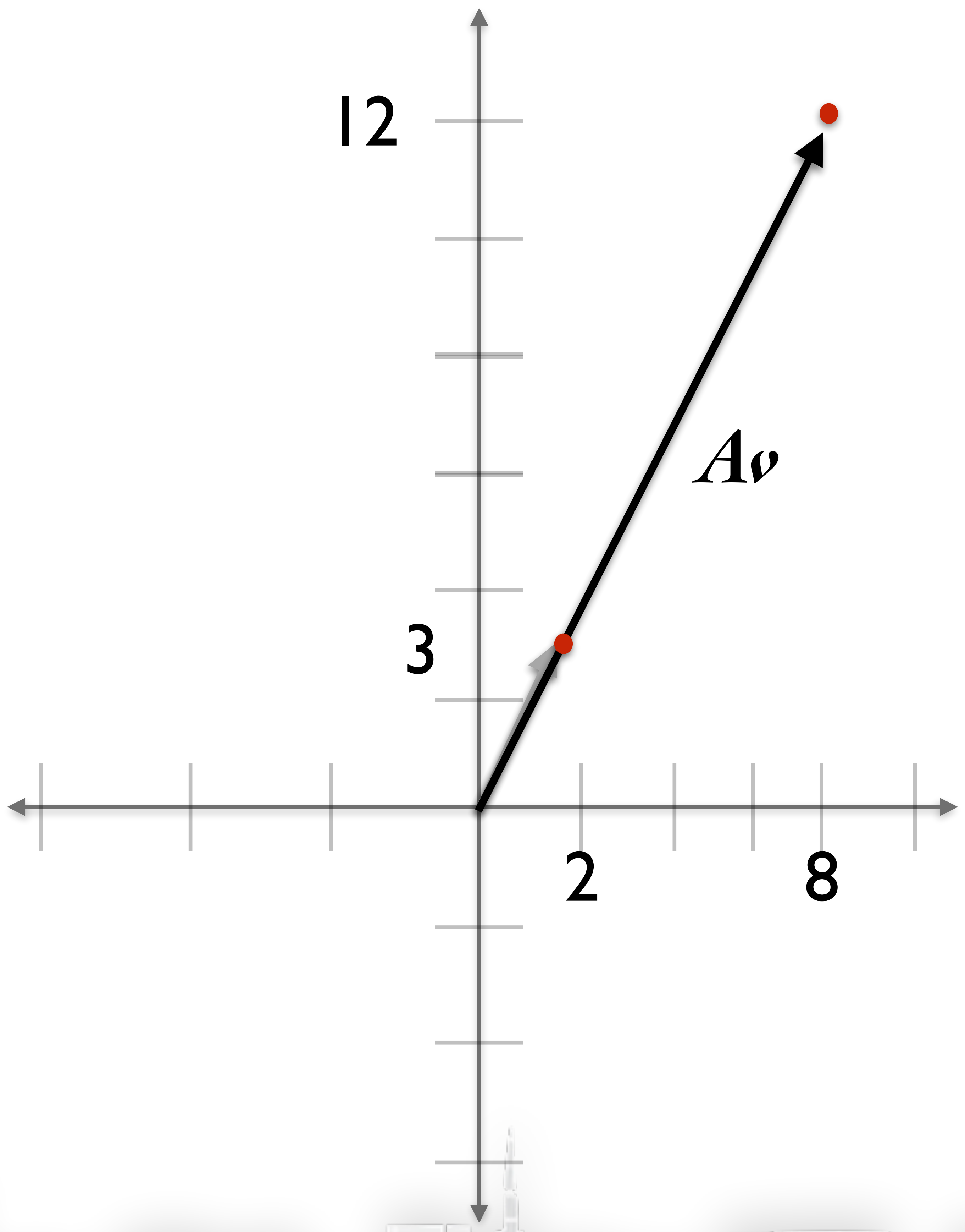
i.e. if you can draw a line through the origin (0,0) that passes through v and Av

i.e.: *v* is only scaled, and not rotated, by the transformation A

A vector *v* is an ***eigenvector*** of the matrix *A* if there exist a λ such that

$$Av = \lambda v$$

and λ is then the eigenvalue of *A* associated to *v*



every point on the same line as an eigenvector is an eigenvector. The line is called the *eigenspace*.

Facial recognition: Eigenfaces

https://github.com/fedhere/UInotebooks/tree/master/UCSL2016