Ayudantía #1

Lógica Digital

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Convierte de base decimal a base 2

1) $(10)_{10}$

1) $(457)_{10}$

1) $(2)_{10}$

1) $(10)_{10}$

1) $(10)_{10}$

Resto

$10 : 2 = 5$ | 0

1) $(10)_{10}$

	Resto
10 : 2 = 5	0
5 : 2 = 2	10

1) $(10)_{10}$

	Resto
10 : 2 = 5	0
5 : 2 = 2	10
2 : 2 = 1	010

1) $(10)_{10}$

	Resto
10 : 2 = 5	0
5 : 2 = 2	10
2 : 2 = 1	010
1 : 2 = 0	1010

1) $(10)_{10}$

	Resto
10 : 2 = 5	0
5 : 2 = 2	10
2 : 2 = 1	010
1 : 2 = 0	1010

R: $(1010)_2$

1) $(10)_{10}$

En 8 bits?

1) $(10)_{10}$

En 8 bits?

R: $(00001010)_2$

2) $(457)_{10}$

2) $(457)_{10}$

Resto

457 : 2 = 228 | 1

2) $(457)_{10}$

	Resto
457 : 2 = 228	1
228 : 2 = 114	01

2) $(457)_{10}$

	Resto
457 : 2 = 228	1
228 : 2 = 114	01
114 : 2 = 57	001

2) $(457)_{10}$

	Resto
457 : 2 = 228	1
228 : 2 = 114	01
114 : 2 = 57	001
57 : 2 = 28	1001

2) $(457)_{10}$

	Resto
457 : 2 = 228	1
228 : 2 = 114	01
114 : 2 = 57	001
57 : 2 = 28	1001
28 : 2 = 14	01001

2) $(457)_{10}$

	Resto
457 : 2 = 228	1
228 : 2 = 114	01
114 : 2 = 57	001
57 : 2 = 28	1001
28 : 2 = 14	01001
...	...
1 : 2 = 0	R: $(111001001)_2$

2) 457_{10}

Tiene más de 8 bits???

2) $(457)_{10}$

Tiene más de 8 bits???

Desde el 256 ya supera los 8 bits

3) $(2)_{10}$

3) $(2)_{10}$

Resto

$2 : 2 = 1$ 0

3) $(2)_{10}$

Resto

$2 : 2 = 1$ 0

$1 : 2 = 0$ 10

3) $(2)_{10}$

Resto

$2 : 2 = 1$ 0

$1 : 2 = 0$ 10

R: $(10)_2$

3) $(2)_{10}$

En 8 bits?

3) $(2)_{10}$

En 8 bits?

R: $(00000010)_2$

Hex
0
1
2
3
4
5
6
7
8
9
A
B
C
D
E
F

Convierte de base 2 a base 16

1) $(1000)_2$

1) $(00101001)_2$

1) $(0101100)_2$

1) $(1000)_2$

1) $(1000)_2$

en base 10!

$1 + 0 + 0 + 0$

$\times 2^3$ $\quad \times 2^1$ $\quad \times 2^2$ $\quad \times 2^0$

1) $(1000)_2$

en base 16!

$$1 \times 2^3 + 0 \times 2^1 + 0 \times 2^2 + 0 \times 2^0$$

1) $(1000)_2$

en base 16!

$$1 \times 2^3 + 0 \times 2^1 + 0 \times 2^2 + 0 \times 2^0$$

R: $(8)_{16}$

2) $(00101001)_2$

2) $(00101001)_2$

en base 10!

0	0	1	0	1	0	0	1
$\times 2^7$	$\times 2^6$	$\times 2^5$	$\times 2^4$	$\times 2^3$	$\times 2^2$	$\times 2^1$	$\times 2^0$

2) $(00101001)_2$

0 0 1 0

$x2^7$ 1 $x2^6$ 0 $x2^5$ 0 $x2^4$ 1

$x2^3$ $x2^2$ $x2^1$ $x2^0$

2) $(00101001)_2$

en base 16!

0	1	0	0
$\times 2^3$	$\times 2^3$	$\times 2^2$	$\times 2^2$

1	0	0	1
$\times 2^1$	$\times 2^1$	$\times 2^0$	$\times 2^0$

2) $(00101001)_2$

0	1	0	0	1	0	0	1
$\times 2^3$	$\times 2^3$	$\times 2^2$	$\times 2^2$	$\times 2^1$	$\times 2^1$	$\times 2^0$	$\times 2^0$

R1: 2

R2: 9

2) $(00101001)_2$

0 $x2^3$ 1 $x2^3$ 0 $x2^2$ 0 $x2^2$

1 $x2^1$ 0 $x2^1$ 0 $x2^0$ 1 $x2^0$

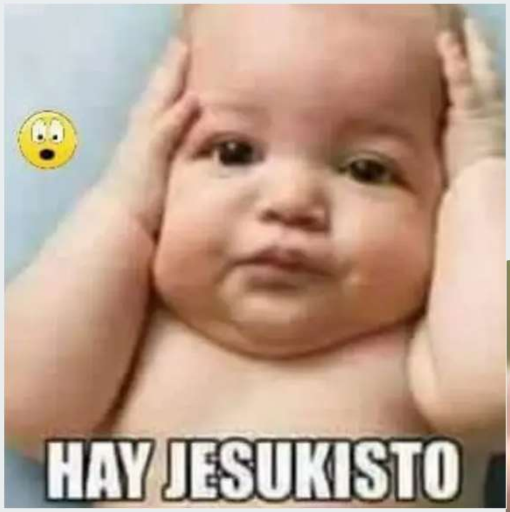
R1: 2

R2: 9

R: $(29)_{16}$

3) $(0101100)_2$

3) $(0101100)_2$



!!!?!?Solo tengo 7 bits???

3) $(0101100)_2$

0		0		1		0	
$x2^7$	1	$x2^6$	1	$x2^5$	0	$x2^4$	0
	$x2^3$		$x2^2$		$x2^1$		$x2^0$

3) $(0101100)_2$

0 0 1 0

$\times 2^7$ 1 $\times 2^6$ 1 $\times 2^5$ 0 $\times 2^4$ 0

$\times 2^3$ $\times 2^2$ $\times 2^1$ $\times 2^0$

3) $(0101100)_2$

0 $\times 2^3$ $\quad 1$ $\times 2^3$ $\quad 0$ $\times 2^2$ $\quad 1$ $\times 2^2$

1 $\times 2^1$ $\quad 0$ $\times 2^1$ $\quad 0$ $\times 2^0$ $\quad 0$ $\times 2^0$

3) $(0101100)_2$

0 $x2^3$ 1 $x2^3$ 0 $x2^2$ 1 $x2^2$

1 $x2^1$ 0 $x2^1$ 0 $x2^0$ 0 $x2^0$

R1: 2

R2: 12 = C

3) $(0101100)_2$

0	1	0	1	1	0	0	0
$x2^3$	$x2^3$	$x2^2$	$x2^2$	$x2^1$	$x2^1$	$x2^0$	$x2^0$

R1: 2

R2: 12 = C

$$R: (2C)_{16}$$

Cómo convertir de hexadecimal a binario?

Cómo convertir de hexadecimal a binario?

proceso inverso

Convierte de base 16 a base 2

1) $(8)_{16}$

1) $(2C)_{16}$

1) $(98BA)_{16}$

1) $(8)_{16}$

1) $(8)_{16}$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7

Binary	Hex
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

1) $(8)_{16}$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7

Binary	Hex
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

1) $(8)_{16}$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7

Binary	Hex
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

R: $(1000)_2$

2) $(2C)_{16}$

2) $(2C)_{16}$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7

Binary	Hex
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

2) $(2C)_{16}$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7

Binary	Hex
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

2) $(2C)_{16}$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4

Binary	Hex
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D

R: $(0010\ 1100)_2$

3) $(98BA)_{16}$

3) $(98BA)_{16}$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7

Binary	Hex
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

3) $(98BA)_{16}$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7

Binary	Hex	
1000	8	segundo
1001	9	primero
1010	A	cuarto
1011	B	tercero
1100	C	
1101	D	
1110	E	
1111	F	

3) $(98BA)_{16}$

Binary	Hex	
1000	8	segundo
1001	9	primero
1010	A	cuarto
1011	B	tercero
1100	C	

R: $(1001\ 1000\ 1011\ 1010)_2$

Algebra Booleana

Demostrar la propiedad de Idempotencia 1:

$$x + x = x$$

$$x + x = x$$

$$x + x = x$$
$$(x + x) \cdot 1$$

$$x + x = x$$

$$(x + x) \cdot 1$$

$$(x + x) \cdot (x + \overline{x})$$

$$x + x = x$$

$$(x + x) \cdot 1$$

$$(x + x) \cdot (x + \overline{x})$$

$$xx + x\overline{x}$$

$$x + x = x$$

$$(x + x) \cdot 1$$

$$(x + x) \cdot (x + \overline{x})$$

$$xx + x\overline{x}$$

$$x + \underbrace{x\overline{x}}_{0}$$

$$x + x = x$$

$$(x + x) \cdot 1$$

$$(x + x) \cdot (x + \overline{x})$$

$$xx + x\overline{x}$$

$$x + \underbrace{x\overline{x}}_{0}$$

$$\boxed{x = x}$$

Demostrar la propiedad de Idempotencia 2:

$$x \cdot x = x$$

$$x \cdot x = x$$

$$\boldsymbol{x \cdot x = x}$$

$$x \cdot x + 0$$

$$x \cdot x = x$$

$$x \cdot x + \underbrace{0}_{x\bar{x}}$$

$$x \cdot x = x$$

$$x \cdot x + 0$$

$$xx + x\overline{x}$$

$$\boldsymbol{x \cdot x = x}$$

$$x \cdot x + 0$$

$$xx + x\overline{x}$$

$$x \cdot (x + \overline{x})$$

$$x \cdot x = x$$

$$x \cdot x + 0$$

$$xx + x\overline{x}$$

$$x \cdot \underbrace{(x + \overline{x})}_{1}$$

$$x \cdot x = x$$

$$x \cdot x + 0$$

$$xx + x\overline{x}$$

$$x \cdot (x + \overline{x})$$

$$\boxed{x \cdot 1 = x}$$

Demostrar la propiedad de Ley del consenso:

$$x + \overline{x}y = x + y$$

$$x + \overline{x}y = x + y$$

$$x + \overline{x}y = x + y$$

Por propiedad distributiva

$$x + y \cdot z = (x + y) \cdot (x + z)$$

$$x + \overline{x}y = x + y$$

Por propiedad distributiva

$$x + y \cdot z = (x + y) \cdot (x + z)$$

$$\boxed{x + \overline{x} \cdot y = (x + \overline{x}) \cdot (x + y)}$$

$$x + \overline{x}y = x + y$$

$$(x + \overline{x}) \cdot (x + y)$$

$$x + \overline{x}y = x + y$$

$$\underbrace{(x + \overline{x})}_{1} \cdot (x + y)$$

$$x + \overline{x}y = x + y$$

$$(x + \overline{x}) \cdot (x + y)$$

$$1 \cdot (x + y)$$

$$x + \overline{x}y = x + y$$

$$(x + \overline{x}) \cdot (x + y)$$

$$1 \cdot (x + y)$$

$$\boxed{x + y = x + y}$$

Demostrar la propiedad de Ley De-Morgan:

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

$$\Rightarrow$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$\begin{aligned} A &= x \cdot y \\ B &= \overline{x} + \overline{y} \end{aligned} \quad \Rightarrow \quad \overline{A} = B$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Se debe cumplir que:

$$A + \overline{A} = 1$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Se debe cumplir que:

$$A + \overline{A} = 1$$

$$A + B = 1$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Se debe cumplir que:

$$A + B = 1$$

$$x \cdot y + \overline{x} + \overline{y}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Se debe cumplir que:

$$A + B = 1$$

$$x \cdot y + \overline{x} + \overline{y}$$

$$(x + \overline{x}) \cdot (y + \overline{x}) + \overline{y}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Se debe cumplir que:

$$A + B = 1$$

$$x \cdot y + \overline{x} + \overline{y}$$

$$\underbrace{(x + \overline{x})}_{1} \cdot (y + \overline{x}) + \overline{y}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Se debe cumplir que:

$$A + B = 1$$

$$x \cdot y + \overline{x} + \overline{y}$$

$$y + \overline{x} + \overline{y}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Se debe cumplir que:

$$A + B = 1$$

$$x \cdot y + \overline{x} + \overline{y}$$

$$\overline{x} + \underbrace{y + \overline{y}}_{1}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Se debe cumplir que:

$$A + B = 1$$

$$x \cdot y + \overline{x} + \overline{y}$$

$$\overline{x} + 1 = 1$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Se debe cumplir que:

$$A + B = 1$$

$$x \cdot y + \overline{x} + \overline{y}$$

$$\overline{x} + 1 = 1$$

$$\boxed{A + \overline{A} = 1}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Simultáneamente se debe cumplir que:

$$A \cdot \overline{A} = 0$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Simultáneamente se debe cumplir que:

$$A \cdot \overline{A} = 0$$

$$A \cdot B = 0$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Simultáneamente se debe cumplir que:

$$A \cdot B = 0$$

$$(xy) \cdot (\overline{x} + \overline{y})$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Simultáneamente se debe cumplir que:

$$A \cdot B = 0$$

$$(xy) \cdot (\overline{x} + \overline{y})$$

$$xy\overline{x} + xy\overline{y}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Simultáneamente se debe cumplir que:

$$A \cdot B = 0$$

$$(xy) \cdot (\overline{x} + \overline{y})$$

$$\underbrace{xy\overline{x}}_{0} + \underbrace{xy\overline{y}}_{0}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Simultáneamente se debe cumplir que:

$$A \cdot B = 0$$

$$(xy) \cdot (\overline{x} + \overline{y})$$

$$0 + 0$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

Simultáneamente se debe cumplir que:

$$A \cdot B = 0$$

$$(xy) \cdot (\overline{x} + \overline{y})$$

$$0 + 0$$

$$\boxed{A \cdot \overline{A} = 0}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$A = x \cdot y$$

$$B = \overline{x} + \overline{y}$$

(1) $A \cdot \overline{A} = 0$

(2) $A + \overline{A} = 1$

Como (1) y (2) se cumplen, se comprueba la igualdad.

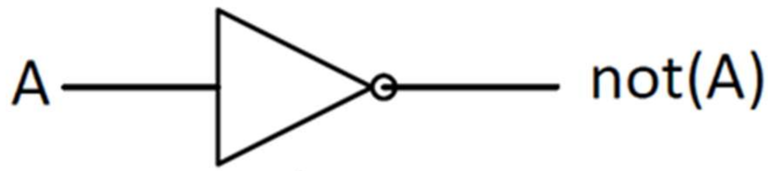
$$\overline{A} = B$$

$$\boxed{\overline{x \cdot y} = \overline{x} + \overline{y}}$$

Repaso de las compuertas lógicas básicas

Compuerta NOT, de un input:

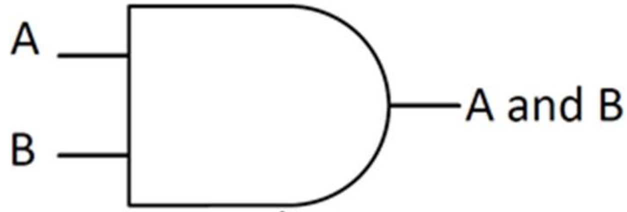
output = $\tilde{A} = \neg A = \mathrm{not}(A)$



A	not(A)
0	1
1	0

Compuerta AND, de dos inputs:

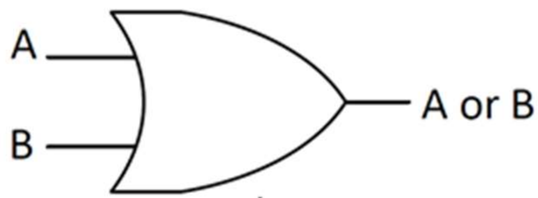
output = $A \bullet B = A \land B = A$ and B



A	B	A and B
0	0	0
0	1	0
1	0	0
1	1	1

Compuerta OR, de dos inputs:

output = $A + B = A \vee B = A \text{ or } B$



A	B	A or B
0	0	0
0	1	1
1	0	1
1	1	1

Construir la tabla de verdad de la siguiente expresión:

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$$(x + y + z)$$

$$(x + \overline{y} + \overline{z})$$

$$(\overline{x} + y + \overline{z})$$

$$(\overline{x} + \overline{y} + \overline{z})$$

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
$\boxed{0}$	$\boxed{0}$	$\boxed{0}$	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x + y + z)$ **0**

$(x + \overline{y} + \overline{z})$

$(\overline{x} + y + \overline{z})$

$(\overline{x} + \overline{y} + \overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **0**

$(x+\overline{y}+\overline{z})$ **1**

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **0**

$(x+\overline{y}+\overline{z})$ **1**

$(\overline{x}+y+\overline{z})$ **1**

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x + y + z)$ **0**

$(x + \overline{y} + \overline{z})$ **1**

$(\overline{x} + y + \overline{z})$ **1**

$(\overline{x} + \overline{y} + \overline{z})$ **1**

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$$(x+y+z)$$

$$(x+\overline{y}+\overline{z})$$

$$(\overline{x}+y+\overline{z})$$

$$(\overline{x}+\overline{y}+\overline{z})$$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$ **1**

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x + y + z)$ **1**

$(x + \overline{y} + \overline{z})$ **1**

$(\overline{x} + y + \overline{z})$ **1**

$(\overline{x} + \overline{y} + \overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$ **1**

$(\overline{x}+y+\overline{z})$ **1**

$(\overline{x}+\overline{y}+\overline{z})$ **1**

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$$(x+y+z)$$

$$(x+\overline{y}+\overline{z})$$

$$(\overline{x}+y+\overline{z})$$

$$(\overline{x}+\overline{y}+\overline{z})$$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$ **1**

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$ **1**

$(\overline{x}+y+\overline{z})$ **1**

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x + y + z)$ **1**

$(x + \overline{y} + \overline{z})$ **1**

$(\overline{x} + y + \overline{z})$ **1**

$(\overline{x} + \overline{y} + \overline{z})$ **1**

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$

$(x+\overline{y}+\overline{z})$

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$ **0**

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$ **0**

$(\overline{x}+y+\overline{z})$ **1**

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x + y + z)$ **1**

$(x + \overline{y} + \overline{z})$ **0**

$(\overline{x} + y + \overline{z})$ **1**

$(\overline{x} + \overline{y} + \overline{z})$ **1**

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$$(x+y+z)$$

$$(x+\overline{y}+\overline{z})$$

$$(\overline{x}+y+\overline{z})$$

$$(\overline{x}+\overline{y}+\overline{z})$$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$ **1**

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$ **1**

$(\overline{x}+y+\overline{z})$ **1**

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$(x + y + z)$ **1**

$(x + \overline{y} + \overline{z})$ **1**

$(\overline{x} + y + \overline{z})$ **1**

$(\overline{x} + \overline{y} + \overline{z})$ **1**

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	
1	1	0	
1	1	1	

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	
1	1	0	
1	1	1	

$$(x+y+z)$$

$$(x+\overline{y}+\overline{z})$$

$$(\overline{x}+y+\overline{z})$$

$$(\overline{x}+\overline{y}+\overline{z})$$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$ **1**

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$ **1**

$(\overline{x}+y+\overline{z})$ **0**

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	
1	1	0	
1	1	1	

$(x + y + z)$ **1**

$(x + \overline{y} + \overline{z})$ **1**

$(\overline{x} + y + \overline{z})$ **0**

$(\overline{x} + \overline{y} + \overline{z})$ **1**

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	
1	1	1	

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	
1	1	1	

$$(x+y+z)$$

$$(x+\overline{y}+\overline{z})$$

$$(\overline{x}+y+\overline{z})$$

$$(\overline{x}+\overline{y}+\overline{z})$$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	
1	1	1	

$(x + y + z)$ **1**

$(x + \overline{y} + \overline{z})$ **1**

$(\overline{x} + y + \overline{z})$

$(\overline{x} + \overline{y} + \overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$ **1**

$(\overline{x}+y+\overline{z})$ **1**

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	
1	1	1	

$(x + y + z)$ **1**

$(x + \overline{y} + \overline{z})$ **1**

$(\overline{x} + y + \overline{z})$ **1**

$(\overline{x} + \overline{y} + \overline{z})$ **1**

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	**1**
1	1	1	

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	**1**
1	1	1	

$(x+y+z)$

$(x+\overline{y}+\overline{z})$

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	**1**
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$

$(\overline{x}+y+\overline{z})$

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	**1**
1	1	1	

$(x + y + z)$ **1**

$(x + \overline{y} + \overline{z})$ **1**

$(\overline{x} + y + \overline{z})$

$(\overline{x} + \overline{y} + \overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	**1**
1	1	1	

$(x+y+z)$ **1**

$(x+\overline{y}+\overline{z})$ **1**

$(\overline{x}+y+\overline{z})$ **1**

$(\overline{x}+\overline{y}+\overline{z})$

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	**1**
1	1	1	

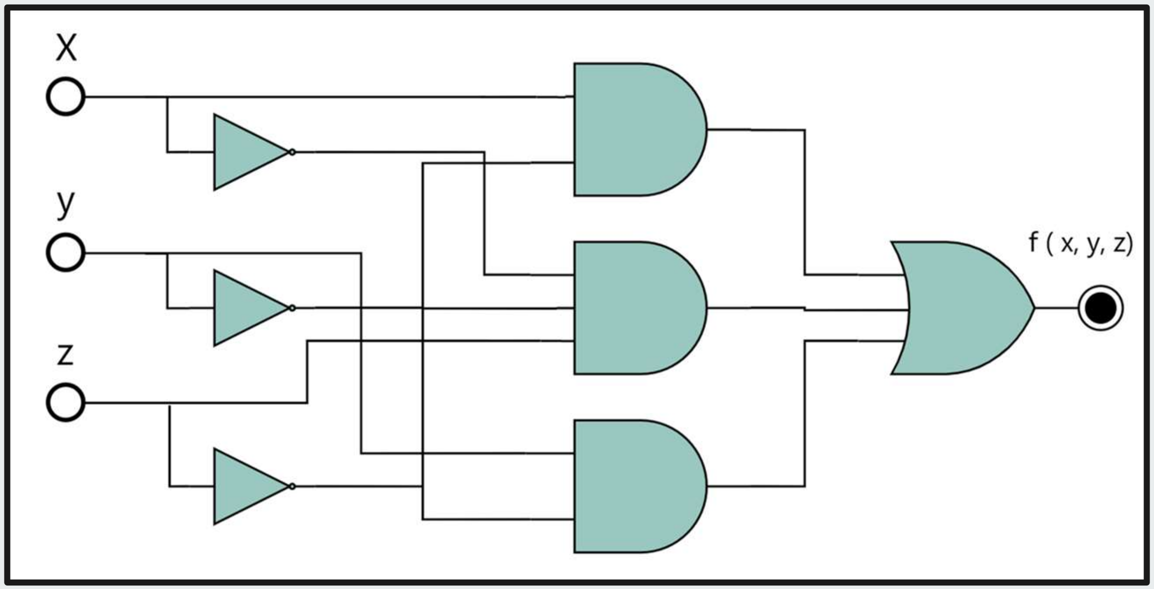
$(x+y+z)$	**1**
$(x+\overline{y}+\overline{z})$	**1**
$(\overline{x}+y+\overline{z})$	**1**
$(\overline{x}+\overline{y}+\overline{z})$	**0**

$$f(x, y, z) = (x + y + z) \cdot (x + \overline{y} + \overline{z}) \cdot (\overline{x} + y + \overline{z}) \cdot (\overline{x} + \overline{y} + \overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	**1**
1	1	1	**0**

$$f(x,y,z) = (x+y+z)\cdot(x+\overline{y}+\overline{z})\cdot(\overline{x}+y+\overline{z})\cdot(\overline{x}+\overline{y}+\overline{z})$$

x	y	z	f (x, y, z)
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	**1**
1	1	1	**0**



$$f(x,y,z) = (x \cdot \overline{z}) + (\overline{x} \cdot \overline{y} \cdot z) + (y \cdot \overline{z})$$