SINGLE-PRICED

MARGINAL REVENUE FOR A MONOPOLIST

CASE ①°: LINER INVERSE DEMAND CURVE, DISCRETE UNITS

$$P^D(Q) = a - bQ \qquad \text{(Linear Inverse Demand)}$$

$$TR^m(Q) = P^D(Q) \cdot Q$$

$$= (a - bQ)Q$$

$$= aQ - bQ^2 \qquad \text{(Total Revenue)}$$

$$MR(Q) = \Delta TR(Q)/\Delta Q \qquad [\Delta Q = Q+1 - Q = 1 \text{ Unit}]$$

$$= \Delta TR(Q)$$

$$= TR(Q+1) - TR(Q)$$

$$= a(Q+1) - b(Q+1)^2 - [aQ - bQ^2]$$

$$= a(Q+1) - b(Q+1)^2 - aQ + bQ^2$$

$$= a(Q+1-Q) - b((Q+1)^2 - Q^2)$$

$$= a(1) - b((Q+1+Q)(Q+1-Q))$$

$$= a - b(2Q+1)(1)$$

$$= a - b(2Q+1)$$

$$= a - 2bQ - b$$

$$= (a-b) - 2bQ \qquad \text{(Marginal Revenue)}$$

Case (2) Linear Inverse Demand Curve, Continuous Units

$$P^D(Q) = a - bQ \qquad \text{(Linear Inverse Demand)}$$

$$TR^m(Q) = aQ - bQ^2 \qquad \text{(Total Revenue)}$$

$$MR(Q) = \Delta TR(Q) / \Delta Q$$

$$\Delta TR(Q) = TR(Q+\varepsilon) - TR(Q)$$

$$= a(Q+\varepsilon) - b(Q+\varepsilon)^2 - (aQ - bQ^2)$$

$$= a(Q+\varepsilon) - b(Q+\varepsilon)^2 - aQ + bQ^2$$

$$= a(Q+\varepsilon - Q) - b\left((Q+\varepsilon)^2 - Q^2\right)$$

$$= a(\varepsilon) - b\left((Q+\varepsilon+Q)(Q+\varepsilon-Q)\right)$$

$$= a\varepsilon - b(2Q+\varepsilon)(\varepsilon)$$

$$= a\varepsilon - 2bQ\varepsilon - b\varepsilon^2$$

$$= (a - b\varepsilon)\varepsilon - 2bQ\varepsilon$$

$$= \varepsilon\left((a - b\varepsilon) - 2bQ\right)$$

$$\Delta Q = (Q+\varepsilon) - Q$$

$$= Q + \varepsilon - Q$$

$$= \varepsilon$$

$$MR(Q) = \frac{\Delta TR(Q)}{\Delta Q}$$

$$= \frac{\xi\left((a-b\xi)-2bQ\right)}{\xi}$$

$$= a - b\xi - 2bQ .$$

Now, as ξ gets very small, we have:

$$MR(Q) = \lim_{\xi \to 0} (a - b\xi - 2bQ)$$

$$= a - b(0) - 2bQ$$

$$= a - 0 - 2bQ$$

$$= a - 2bQ \qquad \text{(Marginal Revenue)}.$$

Case (3): Linear Inverse Demand Curve, Using Calculus

$$P^D(Q) = a - bQ \qquad \text{(Linear Inverse Demand)}$$

$$TR^m(Q) = aQ - bQ^2 \qquad \text{(Total Revenue)}$$

$$MR^m(Q) = \frac{d\,TR(Q)}{dQ}$$

$$= \frac{d\left[aQ - bQ^2\right]}{dQ}$$

$$= a - 2bQ. \qquad \text{(Marginal Revenue)}$$

(A) Graph