OWN-PRICE ELASTICITY FOR A LINEAR DEMAND CURVE

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Let $Q^D(P)$ be a linear demand curve of the form

$$Q^D = a - bP.$$

Recall that the own-price elasticity of demand is given by

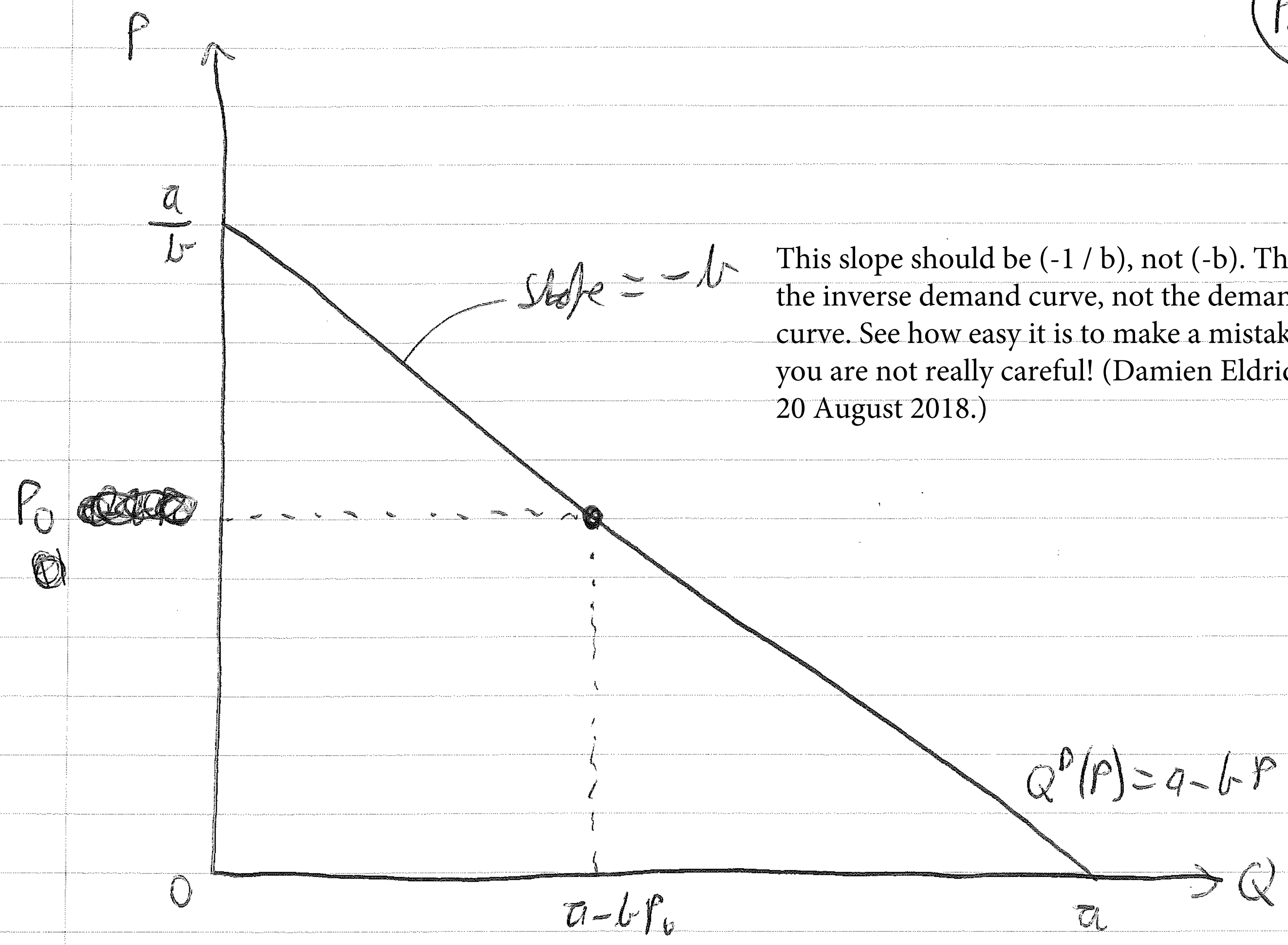
$$\varepsilon_P^D = \frac{P_{REF}}{Q_{REF}} \cdot \frac{\Delta Q^D(P)}{\Delta P}.$$

At every point on a linear demand curve, we have the same slope. This means that $\frac{\Delta Q^D}{\Delta P}$ does not change as P changes. In our example above, we have:

$$\frac{\Delta Q^D}{\Delta P} = -b = \text{slope of } Q^D(P).$$

Thus we can easily calculate the point own-price elasticity of demand in this example without the need for calculus. Specifically, the point own-price elasticity of demand at the point $(P_0, Q_0) = (P_0, a - bP_0)$ is:

$$\varepsilon_P^D(P_0) = \frac{P_0}{Q_0} \cdot (-b) = \frac{P_0}{a - bP_0} \cdot (-b) = \frac{-bP_0}{a - bP_0}.$$



This slope should be (-1 / b), not (-b). This is the inverse demand curve, not the demand curve. See how easy it is to make a mistake if you are not really careful! (Damien Eldridge, 20 August 2018.)

$$\varepsilon_P^D(P_0) = \frac{-bP_0}{a-bP_0}$$

When is $\varepsilon_P^D(P_0) = -1$?

$$\varepsilon_P^D(P_0) = -1$$

$$\Leftrightarrow \quad \frac{-bP_0}{a-bP_0} = -1 \qquad \text{Assuming } (a - bP_0 \neq 0)$$

$$\Leftrightarrow \quad -bP_0 = -(a-bP_0)$$

$$\Leftrightarrow \quad bP_0 = a - bP_0$$

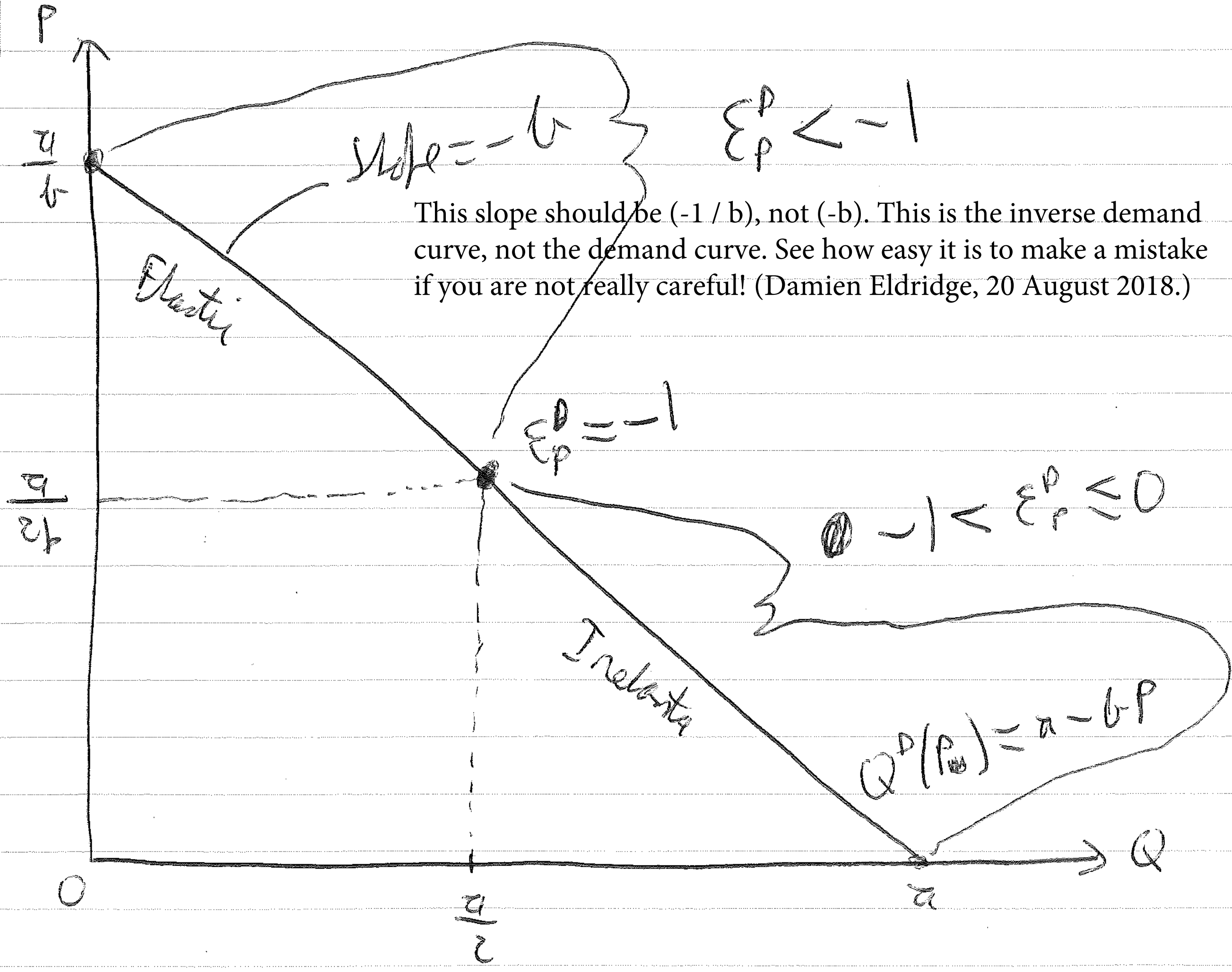
$$\Leftrightarrow \quad 2bP_0 = a$$

$$\Leftrightarrow \quad \boxed{P_0 = \frac{a}{2b}}$$

$$Q^D(P_0) = Q^D\left(\frac{a}{2b}\right) = a - b\left(\frac{a}{2b}\right) = a - \frac{a}{2} = \frac{a}{2}.$$

So we can conclude that $\varepsilon_P^D(P_0) = -1$ when

$P_0 = \frac{a}{2b}$ and hence $Q_0 = \frac{a}{2}$.



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