#### Consumer Demand and Consumer Welfare

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#### Sources

- None of this material originates with me.
- It has been drawn from, or influenced by, a number of sources.
- Some of these might include some or all of the sources listed in the references below.

#### Introductory Level References

These are references that are suitable for a first-year undergraduate economics course.

- Alchian, AA, and WR Allen (1983), Exchange and production: Competition, coordination and control (third edition), Wadsworth Publishing Company, USA: Chapter 2 (pp. 13–44).
- Case, KE, and RC Fair (1989), *Principles of Economics*, Prentice-Hall, USA: Chapters 4–6 (pp. 77–162).
- Gans, J, S King and NG Mankiw (2009), *Principles of microeconomics (fourth edition)*, Cengage Learning Australia, China: Chapters 4, 5, 7 and 22 (pp. 62–112, 134–154 and 486–515).
- Hamermesh, DS (2006), Economics is everywhere (second edition),
   McGraw-Hill-Irwin, USA: Chapters 2, 5 and 6 (pp. 15–24 and 49–77).
- Heyne, P (1991), The economic way of thinking (sixth edition),
   Macmillan Publishing Company, USA: Chapter 2 (pp. 15–45).

#### Intermediate Level References

These are references that are suitable for a second-year undergraduate economics course.

- Hirshleifer, J (1988), *Price theory and applications (fourth edition)*, With the assistance of M Sproul, Prentice-Hall, USA: Chapters 2 and 7E (pp. 23–54 and 204–212).
- Hirshleifer, J, A Glazer, and D Hirshleifer (2005), Price theory and applications: Decisions, markets, and information (seventh edition), Cambridge University Press, New York: Chapters 2 and 7.3.
- Nicholson, W (1987), Intermediate microeconomics and its applications (fourth edition), The Dryden Press, USA: Chapters 1 and 12 (Consumers Surplus) (pp. 5–21 and 334).
- Varian, HR (1987), Intermediate microeconomics: a modern approach, WW Norton and Company, USA: Chapters 1 and 15 (pp. 1–19 and 242–266).

#### Advanced Level References

These are references that are suitable for a third-year undergraduate economics course.

- Gravelle, H, and R Rees (1981), *Microeconomics*, Longman Group, USA: Section C of Chapter 4 (pp. 103–111).
- Nicholson, W (1998), Microeconomic theory: Basic principles and extensions (seventh edition), The Dryden Press, USA: Chapters 1, 5 (Consumer Surplus), and 15 (pp. 3–22, 152–157, and 438–458).
- Takayama, A (1993), Analytical methods in economics, The University of Michigan Press, USA: Appendix C (pp. 621–647).
- Varian, HR (1992), Microeconomic analysis (third edition), WW Norton and Company, USA: Chapter 10 (pp. 160–171).

# Marginal Private Benefit and Demand

- The marginal benefit of an additional unit of a good to you is the maximum amount that you would pay for that additional unit, given your consumption level of the good without the additional unit.
- The marginal private benefit schedule for a good is a function of the form MB (Q; other), where Q is the quantity of the good that is consumed, other is the values that are taken by a set of other factors that affect the marginal benefit of the good in question and MB (Q; other) is the benefit that the consumer receives from the last of the Q units of the good that he consumed.
- As a matter of convenience, we will often write  $MB\left(Q;other\right)$  as simply  $MB\left(Q\right)$ .

# **Diminishing Marginal Benefits**

- We will assume that the marginal private benefit schedule displays diminishing marginal returns.
- In other words, holding the values of the things that are in *other* fixed, the benefit from consuming a further unit of the good (that is, the marginal benefit) will decrease as *Q* increases.
- This means that the marginal private benefit schedule for a good will be downward sloping in (Q, P)-space.

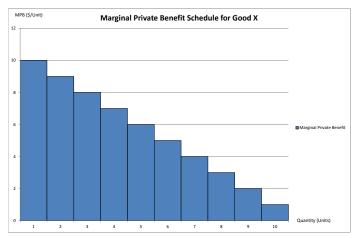
# Discrete MB Schedule Example Part 1

- This example comes from Alchian and Allen (1983, p. 43, Question 43).
- Suppose that only integer (whole number) amounts of a particular good (call it X) may be consumed.
- The marginal benefit schedule for good *X* is given by the following table.

| Q  | 1    | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|----|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| MB | \$10 | \$9 | \$8 | \$7 | \$6 | \$5 | \$4 | \$3 | \$2 | \$1 |

# Discrete MB Schedule Example Part 2

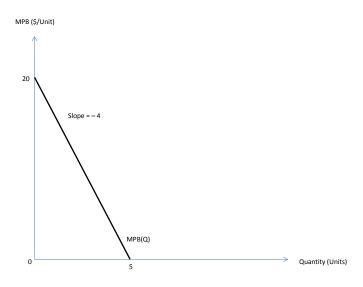
 The following diagram is the graph of this marginal private benefit schedule.



# Linear MB Schedule Example Part 1

- Suppose that the marginal private benefit schedule is given by the equation MPB = 20 4Q.
- This is the equation of a straight line in (Q, MPB)-space with a MPB intercept equal to \$20 and a slope equal to (-4) (\$/Unit).
- The diagram on the next page is a graph of this marginal private benefit schedule.

# Linear MB Schedule Example Part 2



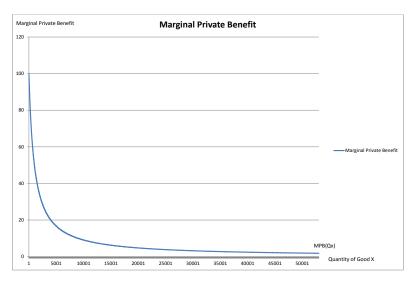
### Non-Linear MB Schedule Example Part 1

 Suppose that the marginal private benefit schedule is given by the equation

$$MPB = \frac{100}{(Q+1)}.$$

- This is the equation of a rectangular hyperbola in (Q, MPB)-space (or, more accurately, in (Q+1, MPB)-space).
- The diagram on the next page is a graph of this marginal private benefit schedule.

# Non-Linear MB Schedule Example Part 2



# Marginal Benefit and Demand Part 1

- A MPB schedule tells us how much an individual benefits from an additional unit of a good or service.
- This is, effectively, the maximum amount that the individual would be willing to pay for that additional unit.
- A demand schedule tells us how many units of an item a consumer would be willing to purchase at a particular price.
- Clearly, a consumer will only choose to purchase an additional unit of a good if the MPB from that unit either matches or exceeds the price that must be paid for that unit.
- Since the MPB schedule is downward sloping in (Q, MPB)-space, an individual will purchase all units up to the point where MPB is equal to the price.

# Marginal Benefit and Demand Part 2

- Let *P* denote the price of the good or service.
- Since the last unit that is purchased will be the one for which MPB = P, we know that for any given price P, the quantity demanded will be given by the point on the MPB schedule where MPB = P.
- Thus the inverse demand schedule can be found by replacing MPB (the LHS of the equation for the MPB schedule) with the price of the commodity, P.
- This yields an equation of the form P = MB(Q). Since this is the inverse demand equation, we will sometimes write this as  $P = D^{-1}(Q)$ .
- If we invert this inverse demand equation (that is, rearrange it to make Q the subject), then we obtain the demand equation, Q = D(P).



- In economics, we employ an unusual convention for graphing demand (and supply) functions. This convention is known as the Marshallian graphing convention.
  - You might recall from high school mathematics that the usual convention for graphing functions of the form y = f(x) is that the independent variable (x) is depicted on the horizontal axis and the dependent variable (y) is depicted on the vertical axis.
- We typically view demand functions (and supply functions) as expressing quantity demanded (or supplied) as a function of price.
  - As such, under the standard graphing convention, we would place price on the horizontal axis and quantity on the vertical axis.
  - Instead of this standard convention, economists typically place quantity on the horizontal axis and price on the vertical axis.
  - What might be referred to as a demand curve (or a supply curve) under the Marshallian graphing convention would really be an inverse demand curve (or inverse supply curve) under the standard graphing convention.

- The Marshallian convention means that, while we talk about demand curves (or supply curves), we actually draw inverse demand curves (or inverse supply curves).
- This means that you need to be careful when moving from the equation for a demand curve (or a supply curve) to the graph of that demand curve (or supply curve) if you are going to follow the Marshallian convention when graphing demand curves (or supply curves).

Suppose that we have a linear demand curve of the form

$$Q = a - bP$$
,

where a > 0 and b > 0.

• The corresponding inverse-demand curve can be found as follows:

$$Q = a - bP$$

$$\iff Q + bP = a$$

$$\iff bP = a - Q$$

$$\iff P = \frac{a}{b} - \left(\frac{1}{b}\right)Q.$$

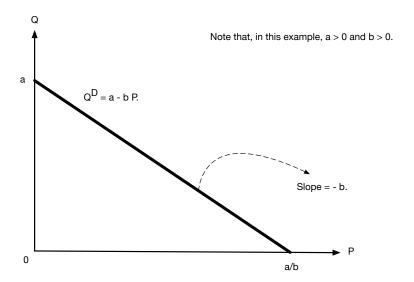
• Note the inverse demand curve is also linear when the demand curve itself is linear.

Recall that we have a linear demand curve: of the form

$$Q = a - bP$$
,

where a > 0 and b > 0.

- The slope of this demand curve is equal to (-b) and the Q-intercept is the point (0, a) in (P, Q)-space.
- It can be shown that the P-intercept is the point  $(\frac{a}{b},0)$  in (P,Q)-space.
- This is illustrated on the next page using the standard mathematical convention for choice of axes.

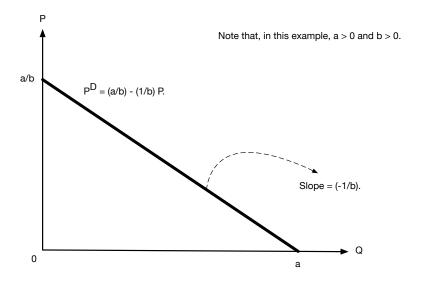


Recall that the corresponding inverse demand curve is

$$P = \frac{a}{b} - \left(\frac{1}{b}\right)Q,$$

where a > 0 and b > 0.

- The slope of this demand curve is equal to  $\left\{-\left(\frac{1}{b}\right)\right\}$  and the P-intercept is the point  $\left(0,\frac{a}{b}\right)$  in (Q,P)-space.
- It can be shown that the P-intercept is the point (a,0) in (Q,P)-space.
- This is illustrated on the next page using the standard mathematical convention for choice of axes (which is equivalent to graphing the original demand curve using the Marshallian graphing convention).



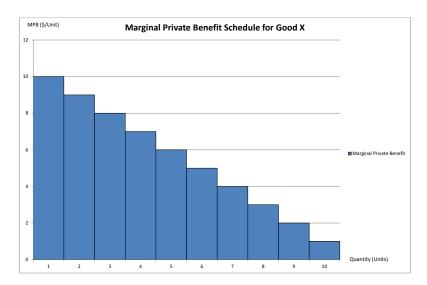
- Suppose that an individual consumes Q units of a good or service.
   What is the total benefit to the individual of this level of consumption?
  - We will assume for the moment that the good or service in question can only be consumed in integer (that is, whole number) amounts.
  - This is the type of situation that was depicted in the discrete MPB schedule example provided earlier in these notes.
- We can find the marginal benefits for each of the units that are consumed from the individual's MPB schedule for this good or service.
  - The marginal benefit of the first unit is given by MPB(1), the marginal benefit of the second unit is given by MPB(2),  $\cdots$ , and the marginal benefit of the Qth unit is given by MPB(Q).

- The total benefit that the individual obtains from consuming *Q* units of this good or service is simply the sum over all units that are consumed of the MPB from each of those units.
- In other words,

$$TPB(Q) = \sum_{q=1}^{Q} MPB(q)$$
  
=  $MPB(1) + MPB(2) + \cdots + MPB(Q)$ .

- Note that this is simply the area below the MPB schedule.
- Since the MPB schedule coincides with the inverse demand schedule, this is also the area under the inverse demand schedule.

 Graphically, the total benefit associated with consuming 10 units of a good or service for the case considered in the discrete MPB schedule example provided earlier is the shaded area under the MPB schedule in the diagram on the following page.



 Numerically, the total benefit for this example can be calculated as follows.

$$TPB(10) = \sum_{q=1}^{10} MPB(q)$$

$$= MPB(1) + MPB(2) + \dots + MPB(10)$$

$$= \$10 + \$9 + \$8 + \$7 + \$6 + \$5 + \$4 + \$3 + \$2 + \$1$$

$$= \$55.$$

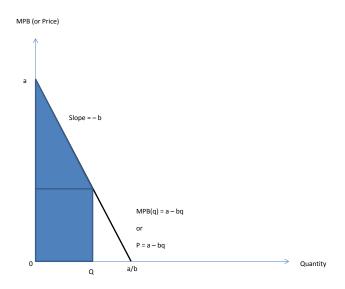
 Recall that when consumption must involve integer (that is, whole number) units, the relationship between total benefit and the relevant marginal private benefits is given by

$$\mathit{TPB}(\mathit{Q}) = \sum_{q=1}^{\mathit{Q}} \mathit{MPB}(q).$$

- This relationship can be extended to the case where consumption does not have to be measured in integer (that is, whole number) units.
- In the case where consumption of infinitesimal amounts is possible, the relevant equation becomes

$$TPB(Q) = \int\limits_{q=0}^{Q} MPB(q)dq.$$

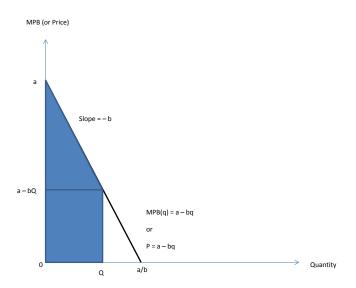
- As before, this is simply the area under the relevant MPB (or inverse demand) schedule between the points q = 0 and q = Q.
- ullet Graphically, the total benefits associated with consuming Q units of a good or service is the shaded area under the MPB schedule in the following diagram.
  - While the diagram employs a linear inverse demand curve, it did not have to do so.
  - The same basic idea applies to non-linear inverse demand curves.
  - However, in non-linear cases, the area under the curve will not be equal to sum of the area of a rectangle and the area of a triangle.
  - Hence the need for the definite integral in the TPB formula provided earlier.



- In the case where the MPB (or inverse demand) schedule is linear, the total private benefit can be calculated by using some simple geometry results from high school mathematics.
- Suppose that the inverse demand curve for an individual is given by

$$P = a - bq$$

- and that the individual chooses to consume Q units of the good or service, where  $0 < Q < \frac{a}{h}$ .
- Note that the marginal benefit from the Qth unit of this good or service is simply (a bQ).
- The total benefit for this case is illustrated in the diagram on the following slide.



- In this particular example, the total private benefit from consumng Q units of the good or service is equal to the sum of the area of a rectangle and the area of a triangle.
- The base of the rectangle is Q-0=Q units and the height of the rectangle is (a-bQ)-0=(a-bQ) \$/unit.
  - As such, the area of the rectangle is  $Q(a bQ) = (aQ bQ^2)$  (units)(\$/unit), which is simply \$ $(aQ bQ^2)$ .
- The base of the triangle is Q-0=Q units and the perpendicular height of the triangle is a-(a-bQ)=bQ \$/unit.
  - As such, the area of the triangle is  $\left(\frac{1}{2}\right)Q\left(bQ\right)=\frac{bQ^2}{2}$  (units)(\$/unit), which is simply \$ $\left(\frac{bQ^2}{2}\right)$ .

 This means that, in this particular example, the total private benefit from consuming Q units of the good or service is

$$TPB(Q) = \$(aQ - bQ^2) + \$\left(\frac{bQ^2}{2}\right)$$
$$= \$\left(aQ - bQ^2 + \frac{bQ^2}{2}\right)$$
$$= \$\left(aQ - \frac{bQ^2}{2}\right).$$

# The paradox of value

- The "Paradox of Value" is a term used to describe a potential confusion between marginal benefit and total benefit.
- It is sometimes known as the "Water-Diamond Paradox" because an example of this potential confusion that is commonly proposed concerns the value of water and the value of diamonds.
  - Water is needed to sustain life, while diamonds are not (or at least do not appear to be) essential for sustaining life. As such, some people might argue that water is much more valuable than diamonds.
  - However, the price of diamonds is very high compared to the price of water in at least some parts of the world (although the price of water might well be higher than it used to be).
  - A situation in which water is much more valuable than diamonds but the price of diamonds is much higher than the price of water might appear to be somewhat paradoxical.
  - But this is only the case if you confuse marginal benefit with total benefit.

## Consumer Surplus Part 1

- We have seen how to calculate the total benefit to a consumer of Q units of some commodity.
- But in general, the consumer will need to purchase those Q units.
- This involves giving up resources that could have been used to purchase other commodities.
- As such, the "net gain" to the consumer from Q units of a commodity is usually smaller than the total benefit of those units to the consumer.
- How might we measure the "net gain" to the consumer from purchasing Q units of a commodity?

- Suppose that the consumer would optimally choose to purchase Q units of a commodity given its current price.
- We want to measure how much the consumer values being allowed to purchase those *Q* units of the commodity (given its current price).
- There are two common ways of answering this question. These are the "willingness to pay" approach and the "willingness to accept" approach.
  - The "willingness to pay" approach attempts to identify the maximum amount that a consumer would be willing to pay in order to avoid having the right to purchase any amount of the commodity (given its current price) taken away.
  - The "willingness to accept" approach attempts to identify the minimum amount that the consumer would need to be paid in order to agree to the removal of the right to purchase any amount of the commodity (given its current price).

- In general, the "willingness to pay" and "willingness to accept" measures of the net gain to the consumer will be different.
  - However, under some circumstances they will be the same.
  - In order to keep things relatively simple, we will assume that are the same for the moment.
  - We will revisit this issue later on when we discuss the compensating variation and equivalent variation concepts.
- The net gain to the consumer is basically equal to the difference between the total benefit that the consumer receives from purchasing Q units of the commodity and the total cost to the consumer of purchasing those units.
  - This difference is known as the "consumer surplus" from purchasing Q units of the commodity.
  - We have already discussed how to find the total benefit of Q units of the commodity to the consumer.
  - But what is the total cost of these units to the consumer?

- Clearly we want to find a measure of the opportunity cost of purchasing Q units of the commodity.
- Under some circumstances, this opportunity cost will simply be equal
  to the expenditure that the consumer must make to purchase those Q
  units at the current price of the commodity.
  - Let's assume that these circumstances apply for the remainder of this discussion.
- Suppose that the consumer is a price taker in the market for this commodity and that the price of the commodity is P per unit.
- Clearly the consumer's expenditure on this commodity will be PQ under these circumstances.
- This means that the consumer surplus from purchasing Q units of this commodity is given by

$$CS(Q) = TPB(Q) - PQ = \int\limits_{q=0}^{Q} MPB(q)dq - PQ.$$

 We have shown that the consumer surplus from purchasing Q units of this commodity is given by

$$CS(Q) = TPB(Q) - PQ = \int_{q=0}^{Q} MPB(q)dq - PQ.$$

Note that

$$\int_{q=0}^{Q} Pdq = [Pq]_{q=0}^{Q} = PQ - 0 = PQ.$$

Thus we have

$$CS(Q) = \int\limits_{q=0}^{Q} MPB(q)dq - \int\limits_{q=0}^{Q} Pdq.$$

This can be rewritten as

$$CS(Q) = \int_{q=0}^{Q} (MPB(q) - P) dq.$$

• In other words, if  $0 \leqslant Q \leqslant Q^D(P)$ , then under some circumstances, the consumer surplus from purchasing Q units of a commodity is equal to the area below the inverse demand (or marginal benefit) schedule and above the price line from q=0 to q=Q.

 Recall our linear marginal benefit schedule example in which the marginal benefit schedule is given by

$$MB(q) = a - bq.$$

 The demand curve associated with this linear marginal benefit schedule is also linear. It is given by

$$Q^D(P) = \frac{a}{b} - \frac{1}{b}P.$$

- If the prevailing price is equal to  $P_0$ , then the consumer will choose to purchase  $Q_0 = Q^D(P_0) = \frac{a}{b} \frac{1}{b}P_0$  units of the commodity.
- The consumer surplus associated with this purchase is equal to the area of a triangle with a base length of  $(Q_0 0) = Q_0$  units and a perpendicular height length of  $(a P_0) = P_0$  dollars-per-unit (assuming that prices are expressed in dollars).

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Thus we have

$$CS(Q_0) = \frac{1}{2} \{ (a - P_0) Q_0 \}$$
 dollars.

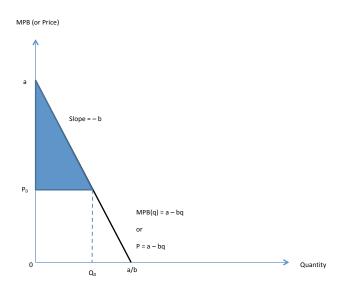
• If we substitute in the value for  $Q_0$ , we obtain:

$$CS(Q_0) = \frac{1}{2} \left\{ (a - P_0) \left( \frac{a}{b} - \frac{1}{b} P_0 \right) \right\}$$
 dollars.

This can be rewritten as

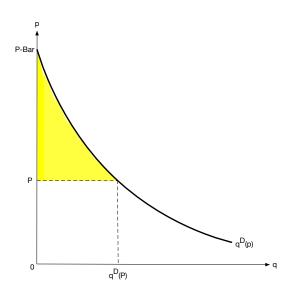
$$CS(Q_0) = rac{1}{2} \left( rac{a^2}{b} - 2rac{a}{b} P_0 + rac{1}{b} P_0^2 
ight) \; \; ext{dollars}.$$

• This example is illustrated in the diagram on the next slide. The consumer surplus is equal to shaded region in that diagram.



- Up until now, we have been calculating consumer surplus from the marginal benefit (or inverse demand) schedule by summing or integrating with respect to quantity.
- We can also calculate consumer surplus from the (direct) demand schedule by summing or integrating with respect to price.
- Suppose that  $\overline{P}$  is the lowest price at which the quantity demanded is zero. We will call this the consumer's "reservation price" for the commodity in question. In some cases, this reservation price will be finite. In other cases, it will be infinite.
- Note that  $q^D(p)=0$  for all  $p\geqslant \overline{P}$ , while  $q^D(p)>0$  for all  $p<\overline{P}$ .
- The consumer surplus that results when the price of the commodity is
   P is

$$CS(P) = \int\limits_{p=P}^{\overline{P}} q^D(p) dp.$$



- We are sometimes interested in the impact of a change in the price of a commodity on the welfare of a consumer.
- This can be measured by the change in consumer surplus that results from the change in the price of the commodity.
- Suppose that the price of the commodity is initially  $P_0$ . At this price, the consumer demands  $Q_0 = q^D(P_0)$  units of the commodity.
- The initial consumer surplus in terms of quantity is

$$CS(Q_0) = \int_{q=0}^{Q_0} (MPB(q) - P_0) dq.$$

• The initial consumer surplus in terms of price is

$$CS(P_0) = \int\limits_{p=P_0}^{\overline{P}} q^D(p) dp.$$

- Suppose that the price of the commodity increases from  $P_0$  to  $P_1 > P_0$ . At the new price, the consumer demands  $Q_1 = q^D(P_1)$  units of the commodity.
- The new consumer surplus in terms of quantity is

$$CS(Q_1) = \int_{q=0}^{Q_1} (MPB(q) - P_1) dq.$$

• The new consumer surplus in terms of price is

$$CS(P_1) = \int\limits_{p=P_1}^{\overline{P}} q^D(p) dp.$$

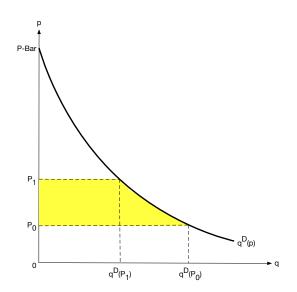
Using the price approach, the change in consumer surplus that results from the increase in price from  $P_0$  to  $P_1 > P_0$  is

$$\Delta CS(P_{0}, P_{1}) = CS(P_{1}) - CS(P_{0}) = \int_{p=P_{1}}^{\overline{P}} q^{D}(p)dp - \int_{p=P_{0}}^{\overline{P}} q^{D}(p)dp$$

$$= \int_{p=P_{1}}^{\overline{P}} q^{D}(p)dp - \left\{ \int_{p=P_{0}}^{P_{1}} q^{D}(p)dp + \int_{p=P_{1}}^{\overline{P}} q^{D}(p)dp \right\}$$

$$= \int_{p=P_{1}}^{\overline{P}} q^{D}(p)dp - \int_{p=P_{0}}^{P_{1}} q^{D}(p)dp - \int_{p=P_{1}}^{\overline{P}} q^{D}(p)dp$$

$$= - \int_{p}^{P_{1}} q^{D}(p)dp$$



Using the quantity approach, the change in consumer surplus that results from the increase in price from  $P_0$  to  $P_1 > P_0$  is

$$\Delta CS(Q_0, Q_1) = CS(Q_1) - CS(Q_0)$$

$$= \int_{q=0}^{Q_1} (MPB(q) - P_1) dq - \int_{q=0}^{Q_0} (MPB(q) - P_0) dq$$

$$= \int_{q=0}^{Q_1} (MPB(q) - P_1) dq$$

$$- \left\{ \int_{q=0}^{Q_1} (MPB(q) - P_0) dq + \int_{q=Q_1}^{Q_0} (MPB(q) - P_0) dq \right\}$$

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$$= \int_{q=0}^{Q_1} (MPB(q) - P_1) dq$$

$$- \int_{q=0}^{Q_1} (MPB(q) - P_0) dq - \int_{q=Q_1}^{Q_0} (MPB(q) - P_0) dq$$

$$= \int_{q=0}^{Q_1} [(MPB(q) - P_1) - (MPB(q) - P_0)] dq$$

$$- \int_{q=Q_1}^{Q_0} (MPB(q) - P_0) dq$$

(Continued on next slide.)



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$$\begin{split} &= \int\limits_{q=0}^{Q_1} \left[ \mathit{MPB}(q) - \mathit{P}_1 - \mathit{MPB}(q) + \mathit{P}_0 \right] \mathit{dq} - \int\limits_{q=Q_1}^{Q_0} \left( \mathit{MPB}(q) - \mathit{P}_0 \right) \mathit{dq} \\ &= \int\limits_{q=0}^{Q_1} \left( \mathit{P}_0 - \mathit{P}_1 \right) \mathit{dq} - \int\limits_{q=Q_1}^{Q_0} \left( \mathit{MPB}(q) - \mathit{P}_0 \right) \mathit{dq} \\ &= - \int\limits_{q=0}^{Q_1} \left( \mathit{P}_1 - \mathit{P}_0 \right) \mathit{dq} - \int\limits_{q=Q_1}^{Q_0} \left( \mathit{MPB}(q) - \mathit{P}_0 \right) \mathit{dq} \\ &= - \left\{ \int\limits_{q=0}^{Q_1} \left( \mathit{P}_1 - \mathit{P}_0 \right) \mathit{dq} + \int\limits_{q=Q_1}^{Q_0} \left( \mathit{MPB}(q) - \mathit{P}_0 \right) \mathit{dq} \right\}. \end{split}$$

(Continued on next slide.)



Thus we have

$$\begin{split} &\Delta \textit{CS}\left(\textit{Q}_{0},\textit{Q}_{1}\right) = \textit{CS}\left(\textit{Q}_{1}\right) - \textit{CS}\left(\textit{Q}_{0}\right) \\ &= -\left\{ \int\limits_{q=0}^{Q_{1}} \left(\textit{P}_{1} - \textit{P}_{0}\right) \textit{d}q + \int\limits_{q=Q_{1}}^{Q_{0}} \left(\textit{MPB}(q) - \textit{P}_{0}\right) \textit{d}q \right\} \\ &= -\left\{ \text{Area A} + \text{Area B} \right\}. \end{split}$$

