The relationship between marginal cost and average cost

Dr Damien S. Eldridge
Department of Economics and Finance
La Trobe University
Bundoora, Vic, 3086
Australia

7 May 2009

Recall that

$$AC(Q) = \frac{TC(Q)}{Q} = TC(Q)Q^{-1}.$$

As such, we have

$$\frac{dAC(Q)}{dQ} = \left(\frac{dTC(Q)}{dQ}\right)Q^{-1} + \left(\frac{dQ^{-1}}{dQ}\right)TC(Q),$$

whenever $Q > 0$. This can be simplified to

$$
\begin{aligned}
\frac{dAC(Q)}{dQ} &= \frac{MC(Q)}{Q} - \frac{TC(Q)}{Q^2} \\
&= \frac{1}{Q}\left(MC(Q) - \frac{TC(Q)}{Q}\right) \\
&= \frac{1}{Q}\left(MC(Q) - AC(Q)\right).
\end{aligned}
$$

As such, we know that following must be true whenever $Q > 0$:

$$
MC(Q) \begin{cases}
< & AC(Q) & \text{if } \frac{dAC(Q)}{dQ} < 0; \\[2mm]
= & AC(Q) & \text{if } \frac{dAC(Q)}{dQ} = 0; \\[2mm]
> & AC(Q) & \text{if } \frac{dAC(Q)}{dQ} > 0.
\end{cases}
$$