Constrained optimization: practical session

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Plan for today

1. (very) brief introduction to numerical methods for optimization
2. how to practical implement these methods

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Useful online resources if you want to know more

- Convex optimization, Stephen Boyd and Lieven Vandenberghe
- Youtube channel, Michel Bielaire
- Foundations of Computational Economics, Fedor Iskhakov
- QuantEcon, John Stachurski and Thomas Sargent
- NumEconCPH, Jeppe Druedahl, Asker Christensen, and Christian Carstensen
- Note on optimization, Anders Munk-Nielsen

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Unconstrained optimization

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Recall, that we can transform any maximization problem into a minimization problem.

Examples I

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As the FOC is linear in x, this optimization problem has a closed form solution

Examples II

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- $FOC : f'(x) = e^{x} + 4e^{-2x} - 3e^{-3x} = 0$
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As the FOC is none-linear in x, this optimization problem has no closed form solution

Aim for the first half of the lecture

Introduce you to numerical methods used to solve optimization problems

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Two classes of optimizers:

1. Gradient based (our focus)
2. None-gradient based

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Two classes of optimizers:

1. Gradient based (our focus)
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Gradient based optimizers include (not conclusive):

- Newton's method
- BFGS
- BHHH
- Gradient descent

Newton's method

- Case: we cannot solve the optimization problem analytically.

$$\min_{x \in \mathbb{R}^k} \quad f(x)$$

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- Idea: A second order polynomial has a closed form solution. So, let's approximate $f(x)$ by a 2nd order Taylor polynomial in the point x_0

$$\min_{x \in \mathbb{R}^k} \quad f(x_0) + (x - x_0)^T \nabla f(x_0) + \tfrac{1}{2}(x - x_0)^T \nabla^2 f(x_0)(x - x_0)$$

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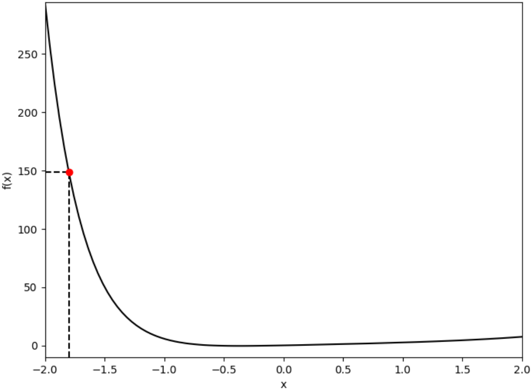
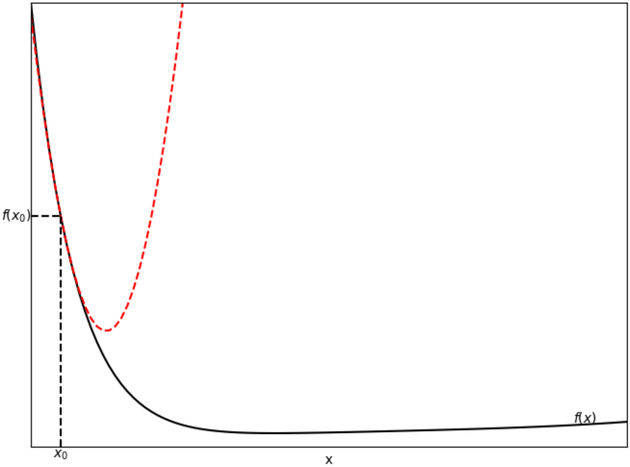
- SOC: $[\nabla^2 f(x_0)]^{-1} \geq 0$

Example I: Consider the minimization problem without closed-form solution

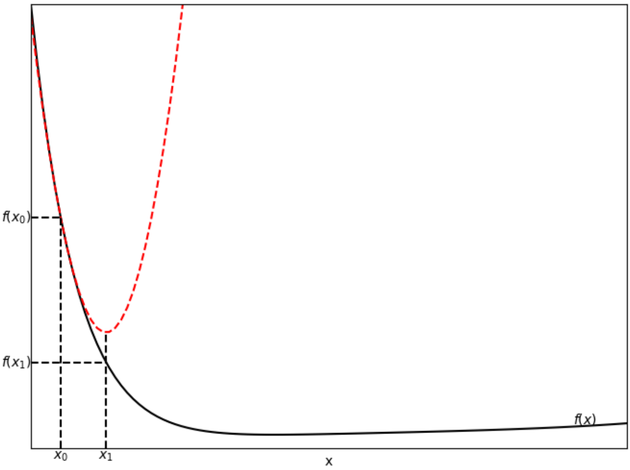
- $f(x) = e^{x} - 2e^{-2x} + e^{-3x}$
- $f'(x) = e^{x} + 4e^{-2x} - 3e^{-3x}$
- $f''(x) = e^{x} - 8e^{-2x} + 9e^{-3x}$



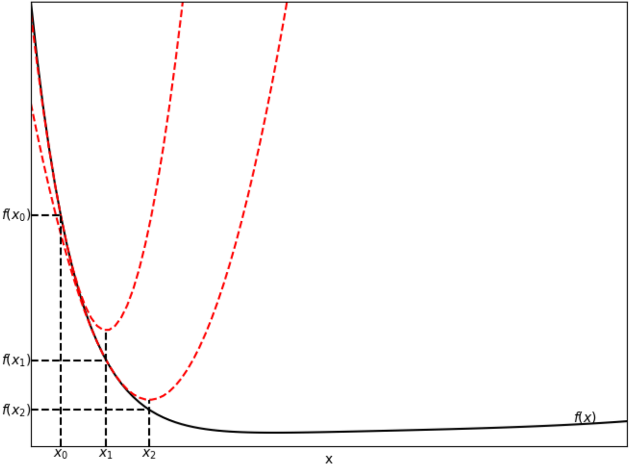
Example I: Approximate the function by the 2nd order Taylor approximation



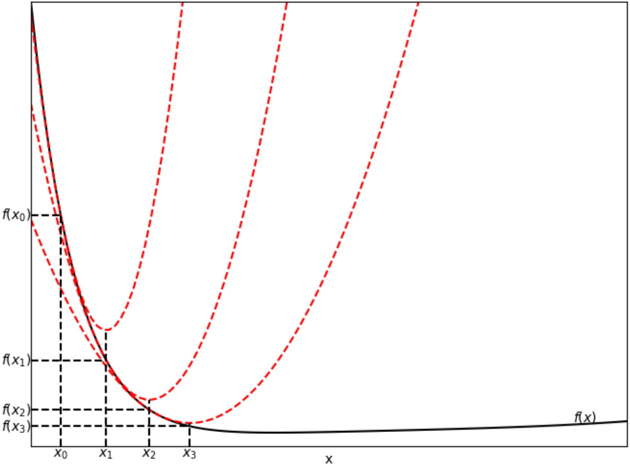
Example I: Find the minimum of the 2nd order Taylor approximation



Example I: Repeat



Example I: Repeat, repeat



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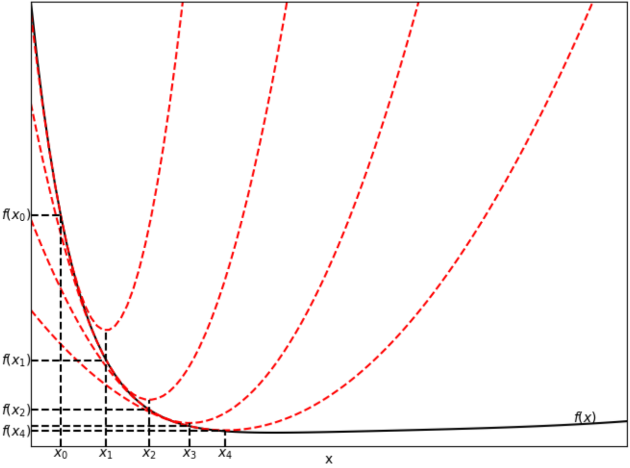
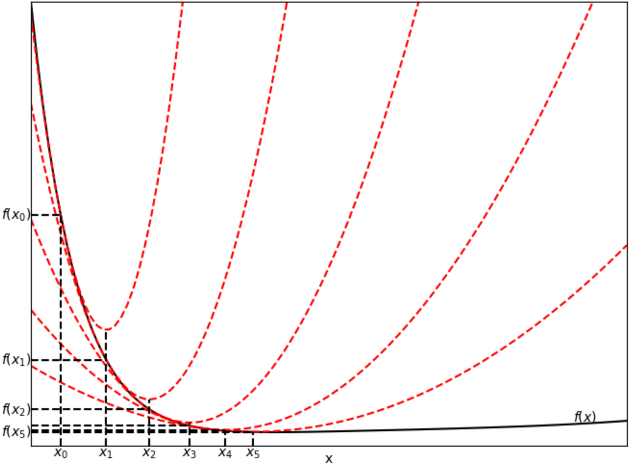


Example I: Repeat, repeat, repeat, ...



Newton's method

The simplest implementation of the Newton's method starts from an initial guess, x_0, and then iterative update the solution of the FOC

$$x_{k+1} = x_k - [\nabla^2 f(x_k)]^{-1} \nabla f(x_k)$$

until the norm of the gradient is sufficiently close to zero, $\|\nabla f(x_k)\| < \varepsilon$.

Simple implementation of the Newton's method

```
def NewtonsMethod(x,grad,hess):
  convergence = 'failed'
  for k in range(1000):
    gradx = grad(x) #evaluate the gradient in x_{k}

    norm_grad = np.sum(np.abs(gradx), axis=None) #calculate the norm of the gradient
    if norm_grad < 1e-10: #stop if gradient close to zero
      convergence = 'converged'
      break

    dx =-np.linalg.solve(hess(x), gradx) #calculate the newton step
    x = x + dx #calculate x_{k+1}

    norm_step = np.sum(np.abs(dx), axis=None) #calculate the norm of the gradient
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Let's take a closer look at how this works

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- For function that are not well approximated by the 2nd order Taylor approximation we can improve the performance of Newton's method by using line search

Line search

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$$\Delta x \equiv -[\nabla^2 f(x_0)]^{-1} \nabla f(x_0)$$

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- Exact line search for the optimal t

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- Inexact line search just tries to find an adequately t

Backtracking line search

Backtracking line search is a very simple inexact line search algorithm based on the Armijo–Goldstein condition

$$f(x_k + t\Delta x) < f(x_k) + \alpha t \nabla f(x_k)^T \Delta x$$

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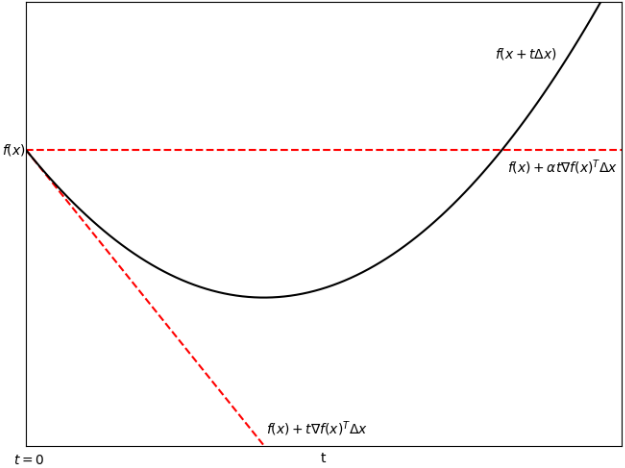
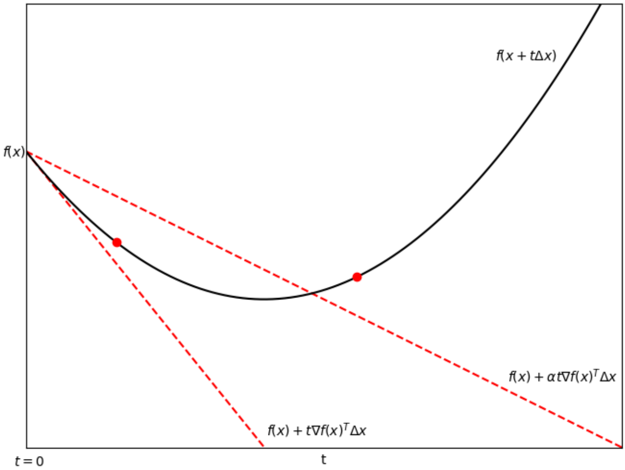
- Best practice is to set β between 0.10 and 0.80

Backtracking line search with $\alpha = 0$



Backtracking line search with $\alpha < 1$



Implementation of Newton's method with backtracking

```python
def NewtonsMethodBacktracking(fun,x0,grad,hess):
  convergence = 'failed'
  a, b = 0.2, 0.6 #backtracking parameters
  for k in range(1000):
    fun0 = fun(x0) #evaluate the function value in x_{k}
    grad0 = grad(x0) #evaluate the gradient in x_{k}

    norm_grad = np.sum(np.abs(grad0), axis=None) #calculate the norm of the gradient
    if norm_grad < 1e-10: #stop if gradient close to zero
      convergence = 'converged'
      break

    dx =-np.linalg.solve(hess(x0), grad0) #calculate the newton step

    t = 1 #initiate t step length
    x1 = x0 + dx #calculate initial x_{k+1}
    while (fun(x1) > fun0 + a * t * grad0 * dx): # Armijo-Goldstein condition
      t = b * t #update t if predicted improvement in f(x) is not adequately large
      x1 = x0 + t * dx #update x_{k+1}

    norm_step = np.sum(np.abs(t * dx), axis=None) #calculate the norm of the step size
    if norm_step < 1e-16: #stop if step is close to zero
      convergence = 'stopped early'
      break
  return x1, convergence
```

Small exercise

Follow the link to Google Colab and do this small exercise:

1. fill out the missing lines in order to calculate the quadratic function, and its first and second derivative
2. choose an initial guess and use Newton's method to minimize the quadratic function
3. what do you find?
4. does your result change if you change the initial guess or the quadratic function?

Calculating the gradient and hessian

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As the hessian, $\nabla^2 f(x)$, is the second derivative we can also use numerical and automatic differentiation to calculate the hessian by simply applying the method twice.

BFGS

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\begin{aligned}
H_{k+1} &= H_k + \frac{yy^T}{y^T s} - \frac{H_k s s^T H_k^T}{s^T H_k s}, \\
y &\equiv \nabla f(x_{k+1}) - \nabla f(x_k), \\
s &\equiv x_{k+1} - x_k
\end{aligned}
$$

where H_0 typically is set to the identity matrix, $H_0 = I$

Example II: Random utility model

Let's consider the random utility model, where the agent i has to choose between two alternatives, $d_i \in (0,1)$

$$\max_{d_i \in (0,1)} v_i(d_i) + \varepsilon_i(d_i)$$

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the payoffs, $v_i(d_i)$, are given as

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the payoffs, $v_i(d_i)$, are given as

$$v_i(d_i = 0) = 0,$$
$$v_i(d_i = 1) = x_i\beta,$$

If the taste-shocks are extreme value type-I distributed the choice probability of choosing alternative 1 is given by a closed form solution

$$Pr(d_i = 1|x_i) = \frac{e^{v_i(d_i=1)}}{1 + e^{v_i(d_i=1)}}$$

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We can then estimate β by maximum likelihood estimation (MLE)

$$\hat{\beta} = \arg\max_{\beta \in \mathbf{R}^k} \prod_{i=1}^{N} Pr(d_i = 1|x_i)^{d_i}(1 - Pr(d_i = 1|x_i)^{1-d_i})$$

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Taking the logarithm (monotone transformation) of the likelihood function preserves the solution of the maximization problem

$$\hat{\beta} = \arg\max_{\beta \in \mathbf{R}^k} \sum_{i=1}^{N} d_i \log Pr(d_i = 1|x_i) + (1 - d_i) \log(1 - Pr(d_i = 1|x_i))$$

Example II: Implementation in JAX

We will use the python package JAX to solve this optimization problem.

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Let's look at how we can estimate the parameters of this model using JAX

Example III: Two-sided matching model

Consider a matching market that consist of X worker types and Y firm types. It is assumed that there exists a continum of each type, and the marginal distribution of worker and firm types are denoted by n_x and n_y, respectively.

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Each worker of type x face the discrete choice of working for one of the Y types of firms or become unemployed

$$\max_{y} \tilde{u}_{xy} + \epsilon_{xy},$$

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the deterministic utility term, $\tilde{u}_{xy}$, is defined as

$$\tilde{u}_{xy} = u_{xy} + w_{xy}, \quad for \quad y = 1, ..., Y,$$
$$\tilde{u}_{x0} = 0.$$

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the deterministic productivity term, $\tilde{v}_{xy}$, is defined as

$$\begin{aligned}
&\tilde{v}_{xy} = v_{xy} - w_{xy}, \quad for \quad x = 1, ..., X, \\
&\tilde{v}_{0y} = 0.
\end{aligned}$$

Example III: Market clearing

If the taste-shocks $(\epsilon_{xy}, \eta_{xy})$ are assumed iid type-I extreme value distributed the choice probabilities of the workers and firms (p_{xy}, q_{xy}) are given by the logit choice probabilities

$$
\begin{aligned}
p_{xy} &= \frac{\exp(u_{xy} + w_{xy})}{1 + \sum_{y=1}^{Y} \exp(u_{xy} + w_{xy})}, \quad \forall (x, y), \\
q_{xy} &= \frac{\exp(v_{xy} - w_{xy})}{1 + \sum_{x=1}^{X} \exp(v_{xy} - w_{xy})}, \quad \forall (x, y).
\end{aligned}
$$

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$$q_{xy} = \frac{\exp(v_{xy} - w_{xy})}{1 + \sum_{x=1}^{X} \exp(v_{xy} - w_{xy})}, \quad \forall (x, y).$$

The wages, w_{xy}, are determined by a set of market clearing conditions, such that excess demand is zero, $z_{xy} = 0$

$$z_{xy}(W) \equiv q_{xy} \cdot n_y - p_{xy} \cdot n_x = 0, \quad \forall (x, y).$$

We can use Newton's method to set excess demand to zero. The idea is now to approximate $Z(W) \equiv (z_{11}, ..., z_{1Y}, ..., z_{X1}, ..., z_{XY})^T$ by a 1st order Taylor approximation in the point W_0

$$Z(W) \approx Z(W_0) + \nabla Z(W_0)(W - W_0)$$

Example III: Newton's method for solving systems of equations

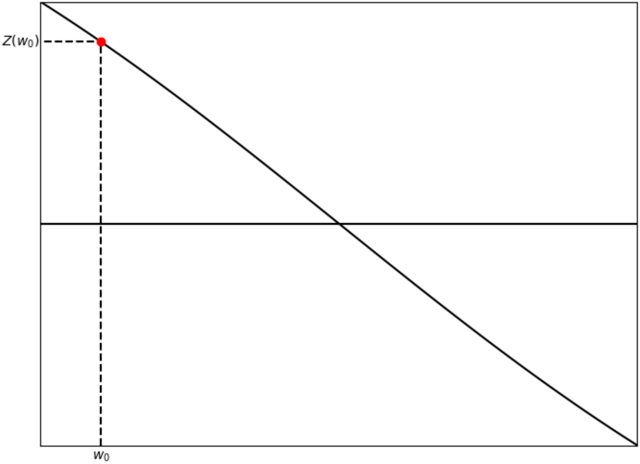
We can use Newton's method to set excess demand to zero. The idea is now to approximate $Z(W) \equiv (z_{11}, ..., z_{1Y}, ..., z_{X1}, ..., z_{XY})^T$ by a 1st order Taylor approximation in the point W_0

$$Z(W) \approx Z(W_0) + \nabla Z(W_0)(W - W_0)$$

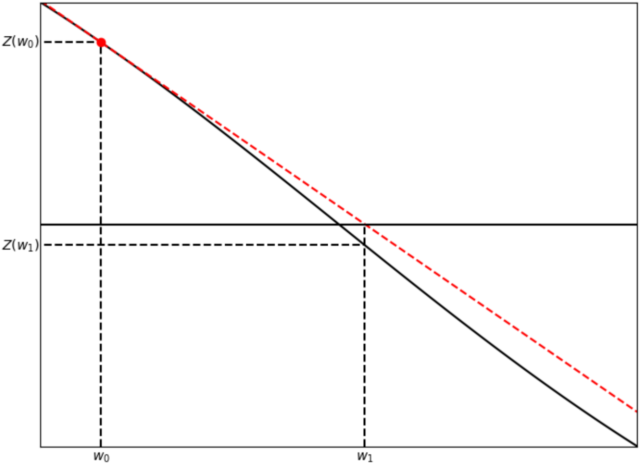
As this a system of linear equation it has a closed form solution

$$Z(W_0) + \nabla Z(W_0)(W - W_0) = 0 \Leftrightarrow W^* = W_0 - [\nabla Z(W_0)]^{-1} Z(W_0)$$

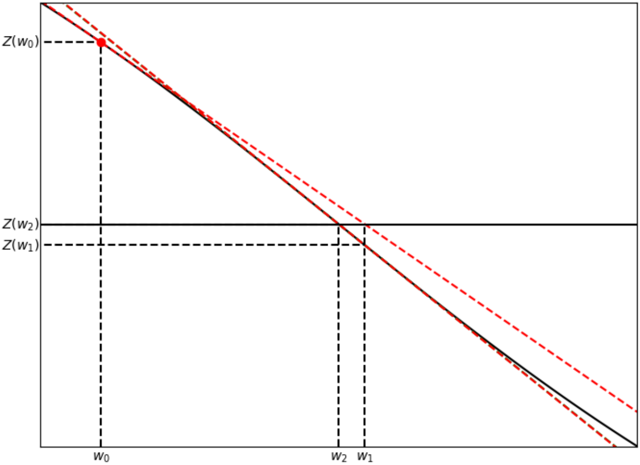
Example III: Excess demand for labor in initial guess, $Z(W_0)$



Example III: Excess demand for labor after first Newton step, $Z(W_1)$



Example III: Excess demand for labor after second Newton step, $Z(W_2)$



Example III: Implementation in JAX

JAX has not implemented Netwon's method for solving systems of none-linear equations. Hence, we will use the package Scipy to solve this matching model.

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1. evaluating excess demand, $Z(W)$
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Let's look at how we can solve this model using JAX and Scipy

Thank you for today :)