

Building a Robot Judge:  
Data Science for Decision-Making  
4. Regression Discontinuity and Diff-in-Diff

17th October 2021

<https://padlet.com/eash44/bkvg9vmiix3dtpoy>

# Machine Learning vs Causal Inference

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- ▶ estimate a low-dimensional **causal parameter**  $\rho$  using

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where  $i$  indexes over documents,  $\alpha_i$  includes control variables (and fixed effects),  $\cdot$  is dot product, and  $\epsilon_i$  is the error residual.

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- ▶ **Glossary for machine learning vs causal inference terms:**

<https://bit.ly/ML-Econ-Glossary>.

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- ▶  $\rho$  gives a prediction how outcome  $y$  would change if treatment variable  $x$  were **exogenously shifted**.
- ▶ useful for policy evaluation.



# Outline

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Fixed Effects

In-Group Bias in the Indian Judiciary

Panel Data / Differences-in-Differences

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- ▶ If there is some randomness in the running variable, being just above or just below the threshold is randomly assigned.

# Example: Effect of Minimum Legal Drinking Age on Death Rates

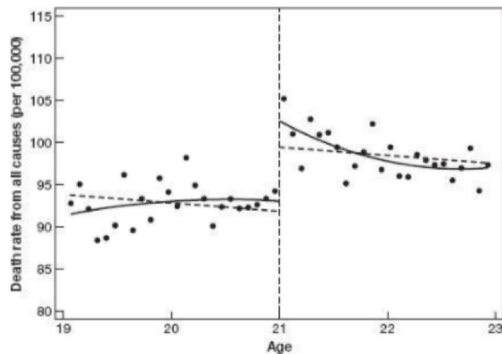
Carpenter and Dobkin (2009)

- ▶ outcome variable  $Y_i$  : death rate
- ▶ running variable  $x_i$  : age
- ▶ cutoff:  $c = 21$ , age where minors can suddenly drink legally
- ▶ treatment  $D = \mathbb{I}[x_i > c]$  : legal drinking status

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# RDD Estimation

- ▶ OLS regression:

$$Y_i = \alpha + \rho \mathbb{I}[x_i > c] + f(x_i)' \beta + \epsilon_i$$

- ▶  $f(x_i)$  includes polynomials in the forcing variable
  - ▶ generally linear or quadratic
  - ▶ can also interact with being above or below the cutoff

```
rdd = smf.ols(formula="death_rate ~ above_21 + age + age_squared", data=df).fit()
```



## Localizing around cutoff

- ▶ Standard practice is to limit sample to a small bandwidth around the cutoff point
  - ▶ treatment more likely to be exogenous.

```
df_rdd = df[(df.age >= 19) & (df.age <= 22)]  
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- ▶ How to choose the bandwidth?
  - ▶ Trade-off: the closer you get the better it is for identification, but the less data you have.
  - ▶ there are formulas for "optimal bandwidth" (e.g.: Imbens-Kalyanaraman 2011, Calonico, Cattaneo and Titiunik 2014).
  - ▶ can use the rdrobust package.
  - ▶ should also explore robustness to different bandwidths

# Testing the validity of RDD

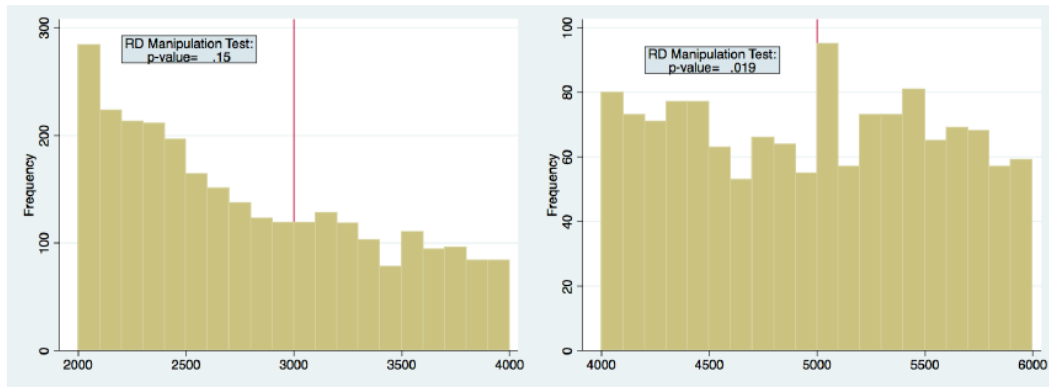
- ▶ RD Design can be invalid if individuals can precisely manipulate the assignment variable  $x_i$  in order to get (or to avoid) treatment.
- ▶ Testing for validity:
  1. Density of the running variable should be continuous (McCrary test)
  2. Predetermined characteristics should have the same distribution just above and just below the cut off

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- ▶ Testing for validity:
  1. Density of the running variable should be continuous (McCrary test)
  2. Predetermined characteristics should have the same distribution just above and just below the cut off
- ▶ Another problem: other important variables are changing at the cutoff besides the treatment you had in mind.
  - ▶ have to think carefully / check if observable / run placebos.

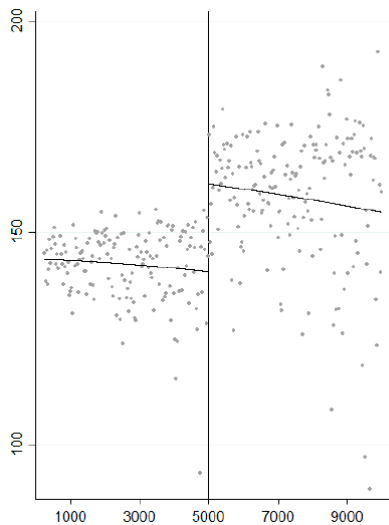
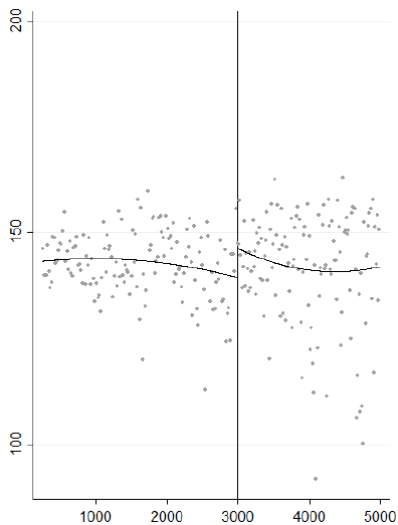
# Manipulation Test: Density Around Cutoff

Bagues and Campa (2017): Histograms of Population Around Population Thresholds



# Manipulation Test: Effect on Past Covariates

Bagues and Campa (2017): Federal Transfers Per Capita



## RDD: Recap

- ▶ Useful method to analyze the impact of treatment when the assignment varies discontinuously due to some rules!
  - ▶ (test score, electoral results, income threshold, etc.)
- ▶ Graphical analysis is key, and can be very convincing
- ▶ Need a large sample around the threshold
- ▶ Have to check for manipulation at the threshold

## Activity: Think of an RD Design

- ▶ Think of an idea for a regression discontinuity design
  - ▶ something from your field/hobby/etc
- ▶ write down the associated variables:
  - ▶ outcome, running variable, threshold
- ▶ how would manipulation around the cutoff happen in your example? could other relevant variables be changing at the cutoff besides the treatment you had in mind? how would you test for that?
- ▶ type out your answer privately. in ~4 minutes, we will paste them into this padlet:  
<https://padlet.com/eash44/u9o5ktslocc203di>



# Outline

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In-Group Bias in the Indian Judiciary

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## Week 2 Recap: Adjusting for Confounders

- ▶ Want to estimate effect of an explanatory variable  $D$  on an outcome  $Y$ .
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  - ▶ e.g. effect of drinking coffee on study productivity; confounders could be the time of day.
- ▶ If confounders are observed, can identify effect of  $D$  on  $Y$  by “adjusting for” or “controlling for”  $A$ .
- ▶ two ways to do that:
  1. residualize  $D$  and  $Y$  on  $A$  and estimate relationship between  $\tilde{D}$  and  $\tilde{Y}$ .
  2. include  $A$  in a linear regression with outcome  $Y$  and predictor  $D$ .

## Fixed Effects: Intuition

- ▶ Most of the time, there are many potential confounders that cannot be observed.
- ▶ in the coffee-productivity example, for each person  $i$ :
  - ▶ whether  $i$ 's parents drink coffee
  - ▶ how close  $i$  live to a coffee shop
  - ▶ etc.

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  - ▶ whether  $i$ 's parents drink coffee
  - ▶ how close  $i$  live to a coffee shop
  - ▶ etc.
- ▶ What if we can observe  $i$ 's productivity multiple times?
  - ▶ sometimes  $i$  had coffee, and sometimes not.
  - ▶ then could “control” *for the person themselves*, rather than *their individual characteristics*.
  - ▶ this adjusts for everything unique to the individual  $i$ , whether it is observed or not.

## Fixed Effects: Residualization Approach

In Week 2 we had outcome  $Y$ , treatment  $D$ , confounder  $A$ . We adjusted for  $A$  by:

1. learn the function  $\hat{D}(A)$ , compute residual  $\tilde{D} = D - \hat{D}$
2. learn the function  $\hat{Y}(A)$ , compute residual  $\tilde{Y} = Y - \hat{Y}$
3. if  $A$  is the only confounder, the relationship between  $\tilde{D}$  and  $\tilde{Y}$  is causal.

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With fixed effects, we have  $N$  individuals, indexed by  $i$ , and  $T$  periods, indexed by  $t$ :

1. de-mean (center)  $D_{it}$  for each  $i$  – i.e., form  $\bar{D}_i = \frac{1}{T} \sum_t D_{it}$ , then compute residual  $\tilde{D}_{it} = D_{it} - \bar{D}_i, \forall i$ .
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2. de-mean  $Y_{it}$  the same way  $\rightarrow \tilde{Y}_{it}$
3. if all confounders are at the level of  $i$  (there are no confounders that vary over time within  $i$ ), the relationship between  $\tilde{D}_{it}$  and  $\tilde{Y}_{it}$  is causal.

## Fixed Effects: Regression Approach

- ▶ In Week 2 we had the linear model

$$Y_i = \alpha + \beta D_i + \gamma a_i + \eta_i$$

- ▶ could adjust for observed confounder  $a_i$  by including it as a linear predictor in the regression.

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- ▶ Now we have

$$Y_{it} = \alpha_i + \beta D_{it} + \epsilon_{it}$$

where  $t$  indexes time, and  $\alpha_i$  is a “fixed effect” for person/group  $i$ .

- ▶  $\alpha_i$  includes a set of binary variables that equal one for observations in  $i$ .
- ▶ in machine learning this is called a one-hot-encoded categorical variable.

```
fe = smf.ols(formula="product ~ coffee + C(person_id)", data=df).fit()
```

## Notes on fixed effects

- ▶ Can be used in many contexts:
  - ▶ the “entity”  $i$  could be people or firms or cities or countries, etc
- ▶ Usually, there are many confounders in a regression, many of which we can't measure.
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  - ▶ we are comparing  $i$  to itself at a different time –  $i$  is its own control group!
- ▶ With the regression approach, we can add multiple sets of fixed effects, e.g.:

$$Y_{it} = \alpha_i + \alpha_t + \beta D_{it} + \epsilon_{ict}$$

where now we have  $\alpha_t$ , a “time fixed effect” which for example could represent time of day or day of the week – a set of dummies for observations at period  $t$ .

```
fe2 = smf.ols(formula="product ~ coffee + C(person_id) + C(time)", data=df).fit()
```

- ▶ this is a “two-way fixed-effects” model, which we will come back to shortly.

## Randomization Blocks

- ▶ Consider an outcome  $Y_{ijc}$  in case  $i$  for judge  $j$  on court  $c$ , e.g. guilty/innocent.
- ▶ We want to estimate the effect of judge characteristic  $D_j$ , e.g. political party.
- ▶ If judges get different types of cases, estimating  $\hat{\beta}$  from

$$Y_{ijc} = \alpha + \beta D_j + \epsilon_{ijc}$$

would be biased ( $\text{cov}(D_j, \epsilon_{ijc}) \neq 0$ ) because the unobserved case characteristics (in  $\epsilon_{ijc}$ ) are affecting both  $D_j$  and  $Y_{ijc}$ .

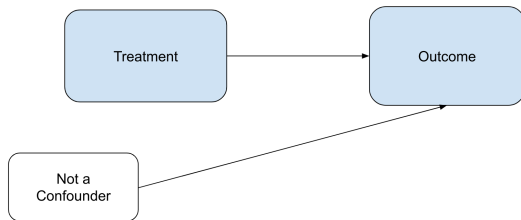
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- ▶ But say judges are randomly assigned within court.
  - ▶ Then, after conditioning on a court fixed effect  $\alpha_c$ , there is no influence of the case-type confounders on the assigned judge characteristic (the treatment):



- ▶ Hence, we get causal estimates of  $\hat{\beta}$  from

$$Y_{ijc} = \alpha_c + \beta D_j + \eta_{ijc}$$

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## Class Activity on India Courts In-Group Bias

[https://docs.google.com/document/d/1LVnl15wSCGD3rinuShjPzmndt\\_ALEk0XV39ZY0ktds/edit?usp=sharing](https://docs.google.com/document/d/1LVnl15wSCGD3rinuShjPzmndt_ALEk0XV39ZY0ktds/edit?usp=sharing)

## Reading Regression Tables

Table 6: Impact of assignment to a judge with the same last name on defendant outcomes

	(1)	(2)	(3)	(4)
	Acquitted	Acquitted	Acquitted	Acquitted
Same last name	0.013** (0.006)	0.014** (0.006)	0.019* (0.011)	0.025*** (0.009)
Observations	2239516	2237502	2258437	2256242
Fixed Effect	Court-month	Court-month	Court-year	Court-year
Judge Fixed Effect	No	Yes	No	Yes

Standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

*Notes:* This table reports results from a test of the impact of random assignment to a judge with the same last name as the defendant on likelihood of acquittal, see Equation 5. Charge section and last name fixed effects have been used across all columns reported.

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## Panel Data (Longitudinal Data) is Data Over Time

- ▶ we have outcomes  $y_{it}$  for “unit” (individual/group)  $i$  at time  $t$
- ▶  $N$  units and  $T$  time periods
  - ▶ a “balanced” dataset will have  $NT$  observations.
  - ▶ “unbalanced” panel data means that some unit-period pairs are missing – e.g. due to entering or leaving the sample. this is not a problem in practice.
- ▶ The goal of panel data methods is to construct counterfactuals using the longitudinal structure of the data.

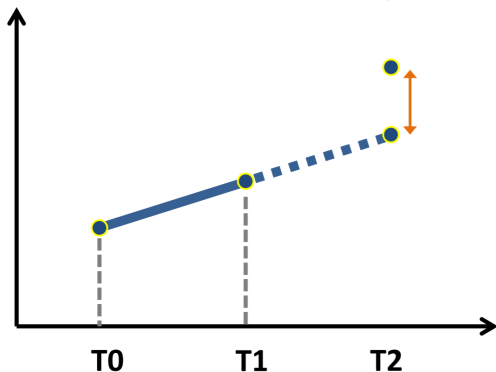
## What if there is only one unit? Time Series Analysis

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- ▶ In macroeconomics (analysis of the whole economy), you only observe one unit (the economy).
  - ▶ How to estimate causal effect of a macroeconomic policy like changing interest rates?

Solution 1: Time Series Analysis



- ▶ “time series analysis” assumes economy continues on current trend in absence of intervention.

Source: Yixing Zu slides.

## Example where previous methods fail

- ▶ Example: taxes raised in canton A, but **not** in canton B
  - ▶ we observe employment  $Y_{jt}$  in time periods  $t$  before and after the reform in both cantons  $j$
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$$Y_{jt} = \alpha + \gamma D_{jt} + \varepsilon_{jt}$$

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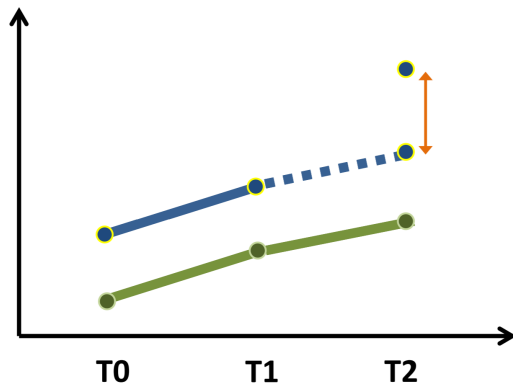
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- ▶ there are canton-level confounders biasing the estimate.
- ▶ fixed effects approach:

$$Y_{jt} = \alpha_j + \gamma D_{jt} + \varepsilon_{jt}$$

- ▶  $\hat{\gamma}$  estimates the pre/post change in employment for canton A
  - ▶ **but:**
    - ▶ what if employment was already going up over time in all of switzerland?
    - ▶ the post-treatment estimate  $\hat{\gamma}$  is biased upward by the time confounder.

# Differences-in-Differences

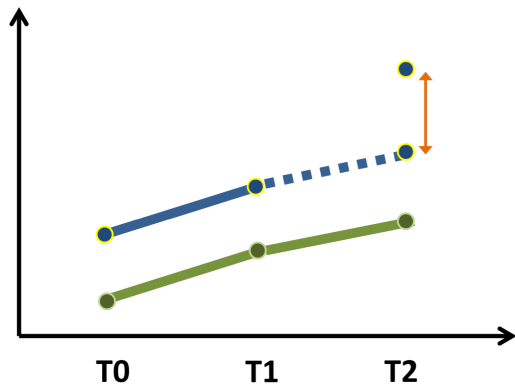


- ▶ use canton B as a counterfactual to adjust for the time trend.
- ▶ In this example, the DD estimator is

$$[Y_{A1} - Y_{A0}] - [Y_{B1} - Y_{B0}]$$

= employment change in treated canton, relative to employment change in comparison canton.

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= **employment change in treated canton, relative to employment change in comparison canton.**

- ▶ in regression form, we estimate

$$Y_{jt} = \alpha_j + \alpha_t + \gamma D_{jt} + \varepsilon_{jt}$$

where  $\alpha_t$  is a **time fixed effect** – an indicator variable for each time period  $t$ .

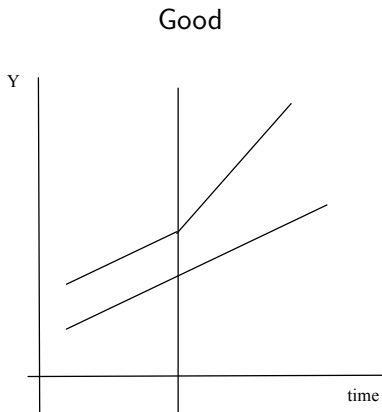
```
dd = smf.ols(formula="emp ~ tax + C(canton) + C(time)", data=df).fit()
```

## Diff-in-diff: Checking for Parallel trends

- ▶ The identification assumption for diff-in-diff is “**parallel trends**” :
  - ▶ e.g., absent tax change, trend in employment would have been the same in cantons A and B.

## Diff-in-diff: Checking for Parallel trends

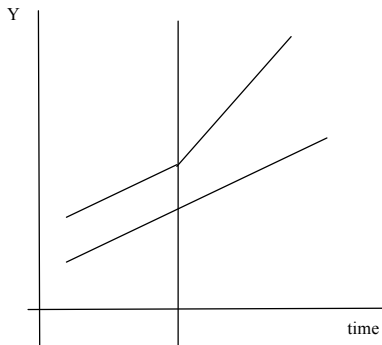
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  - ▶ e.g., absent tax change, trend in employment would have been the same in cantons A and B.



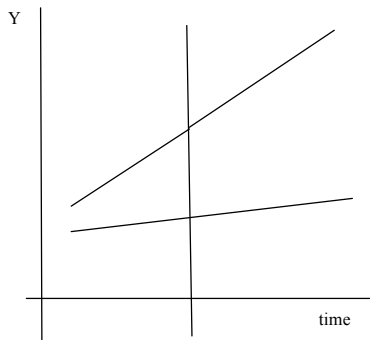
## Diff-in-diff: Checking for Parallel trends

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Good



Not Good



# Two-Way Fixed-Effects Regression

- ▶ The regression form

$$Y_{jt} = \alpha_j + \alpha_t + \gamma D_{jt} + \varepsilon_{jt}$$

generalizes to  $> 2$  groups and  $> 2$  periods

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- ▶ TWFE is an empirical workhorse.
  - ▶ e.g., in our example, taxes and employment across cantons could be correlated for many confounding reasons.
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    - ▶ time-invariant canton-level factors
    - ▶ nationwide time-varying factors
- ▶ Potential confounders must
  - ▶ vary over time by canton
  - ▶ be correlated with outcome variable
  - ▶ be correlated with the timing of treatment/reforms

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- ▶ Skeptical questions to ask:
  - ▶ Why did the treatment group adopt the policy, and not the control group?
  - ▶ Were other policies adopted at the same time that might also affect the outcome?
  - ▶ Could the treatment spill over into the comparison cantons?

## A note on standard errors

- ▶ Consider the regression for cantonal tax cuts and employment. We have 26 cantons.
  - ▶ the default standard errors formula for OLS assume that all observations are independent realizations.
- ▶ Compare the following analyses:
  - ▶ including the 10 years before and after the reform ( $N = 260$ )
  - ▶ including the 20 years before and after ( $N = 520$ )

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- ▶ Using the default SE's, the second analysis would give much more precise estimate, even though the data contain nearly equivalent information.

# Clustering Standard Errors

Cluster standard errors:

- ▶ statistically acknowledges how many independent sources of information there are in the data.
- ▶ the standard approach is to cluster at the unit where treatment is assigned.
  - ▶ in this example, by canton.

```
dd = smf.ols(formula="emp ~ tax + C(canton) + C(time)", data=df)
result = dd.fit(cov_type="cluster", cov_kwds={"groups":df["canton"]})
```

- ▶ for city-level reforms cluster by city, etc.

## Event Study: Dynamic Treatment Effects

- ▶ So far we have estimated regressions like

$$Y_{jt} = \alpha_j + \alpha_t + \beta D_{jt} + \varepsilon_{jt}$$

- ▶  $\hat{\beta}$  will give us the average effect in the post-treatment period, relative to pre-treatment and to the control group.
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- ▶ What if we care about the dynamics of the effect? How it changes over time?
- ▶ The proper way to do this is with a “panel event study”, where we estimate

$$Y_{jt} = \alpha_j + \alpha_t + \sum_{\tau=-W, \tau \neq -1}^W \beta_{\tau} D_{jt}^{\tau} + \varepsilon_{jt}$$

- ▶ here, each item  $D_{jt}^{\tau}$  represents a “lead” or a “lag” of treatment time. so, e.g.,  $\tau = 0$  for the period of treatment,  $\tau = 1$  is the year after,  $\tau = -2$  is two years before, etc.
- ▶  $\tau = -1$ , the year before treatment, is dropped  $\rightarrow$  it is the reference year, and  $\hat{\beta}_{\tau}$  measures the difference relative to  $\tau = -1$ .
- ▶ see “The Effect”, Section 18.2 and 18.3 for more detail.



## Practice with TWFE

- ▶ Formulate the “Effect of Coffee on Productivity” question as a two-way fixed-effects (TWFE) analysis.
  - ▶ write down the estimating equation
  - ▶ label the outcome, the treatment, the error term, and the fixed effect terms.
  - ▶ specify the “treated” group(s) and the “control” group(s)
  - ▶ what fixed effects are included? what do they control for?
  - ▶ what are some threats to the parallel trends assumption?
  - ▶ how do you cluster standard errors?
- ▶ write down your answer on paper.
  - ▶ in 4 mins, we will show to the webcam.
  - ▶ also take a picture of it and email to claudia.