Building a Robot Judge: Data Science for Decision-Making

8. Instrumental Variables

Q&A Padlet

https://padlet.com/eash44/58d15s2wnv1rp7re

Recap: Machine Learning Pitfalls

- Not even looking at the test set data.
- Are these rules too strict to be practical?
- Data / interpretability issues with deep learning
- How to deal with out-of-distribution data points, or adversarial attacks.

Learning Objectives

1. Implement and evaluate machine learning pipelines.
2. **Implement and evaluate causal inference designs.**
   - Today: Instrumental Variables
3. Understand how (not) to use data science tools (ML and CI) to support expert decision-making.

Objectives in an Empirical Project

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   - Produce descriptive visuals and statistics on the text and metadata
3. Econometrics:
   - Articulate a research design and the identification assumptions for procuring causal estimates.
   - Run regressions to produce the estimates.
   - Run identification checks and specification checks to enhance confidence in results.

Outline

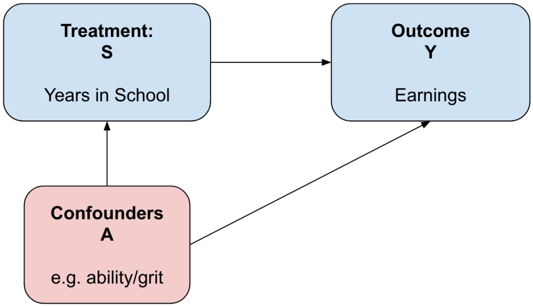
Instrumental Variables

IV with Machine Learning

Deep IV

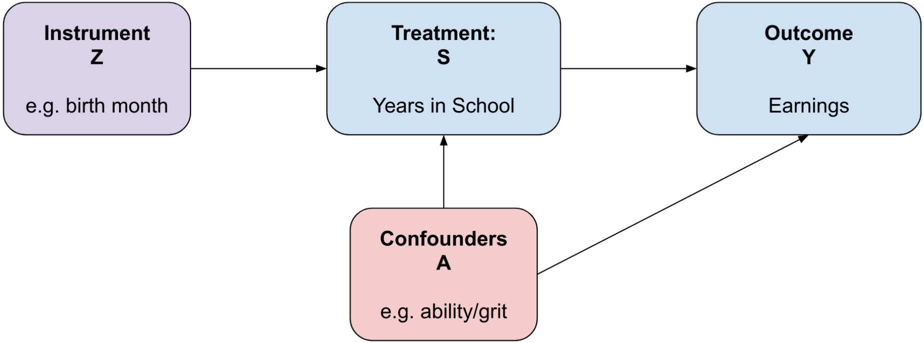
- Example from Week 2: Causal effect of schooling S_i on earnings Y_i.
- There is an unobserved confounder (say ability A_i) correlated with schooling and earnings

$$Y_i = \alpha + \rho S_i \underbrace{+\phi A_i}_{\text{unobserved}} + \eta_i$$



- OLS estimates for $\hat{\rho}$ will be biased.

Instrumental Variable (IV): a variable Z_i, that is correlated with S_i, but not correlated with anything else affecting Y_i.



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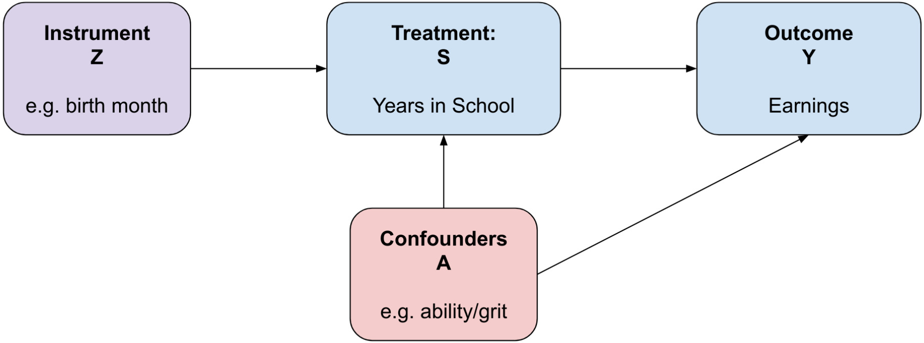
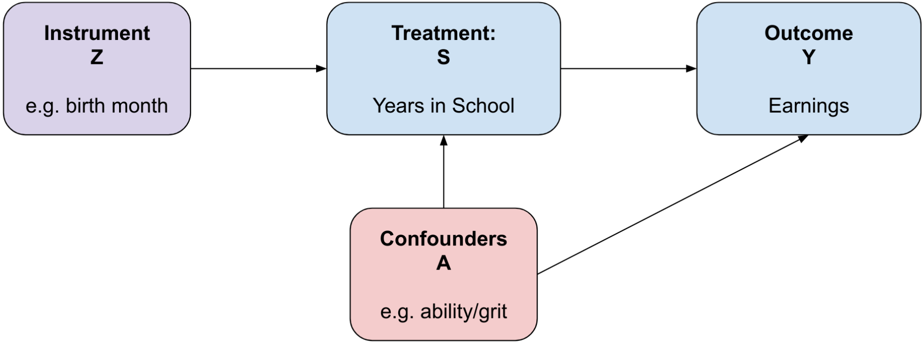


$$Y_i = \alpha + \rho S_i + \underbrace{(+\phi A_i)}_{\text{unobserved}} + \epsilon_i$$

$$\text{Cov}[Z_i, S_i] \neq 0, \text{Cov}[Z_i, A_i] = 0$$

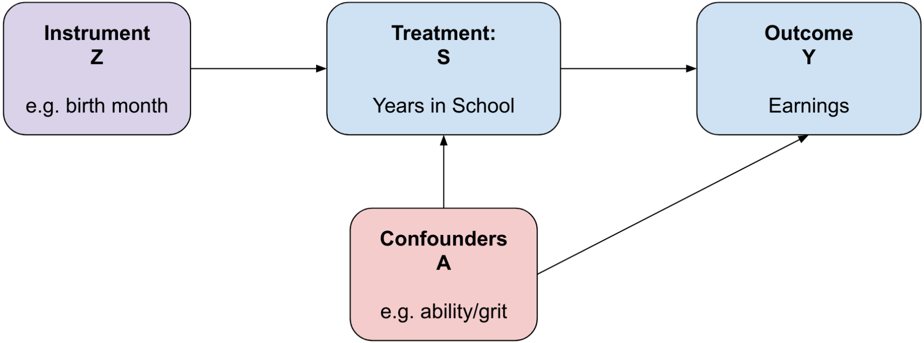
- With a valid instrument, can procure causal estimates for $\hat{\rho}$

Instrumental Variables: Main Intuition



- We identify a source of variation in treatment assignment that is as good as random – orthogonal to any relevant unobserved confounder.
- We compare individuals that, due to the instrument, are shifted between the control group and treatment group.

What is a valid instrumental variable?



1. Correlated with the causal variable, e.g. S_i:

$$\text{Cov}[Z_i, S_i] \neq 0$$

2. Uncorrelated with any other determinants of outcome Y:

$$\text{Cov}[Z_i, \epsilon_i] = 0$$

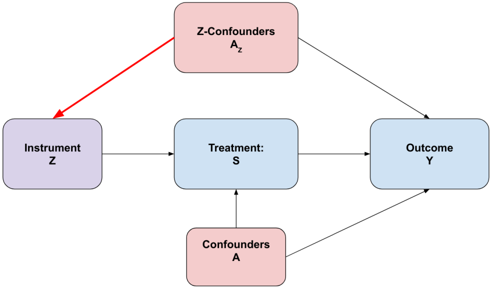
Identification requirement has two dimensions:

Exogeneity: None of the unobserved factors affects the instrument:

$$\epsilon_i \nrightarrow Z_i$$

- No "Z-confounders"

Violation of exogeneity:



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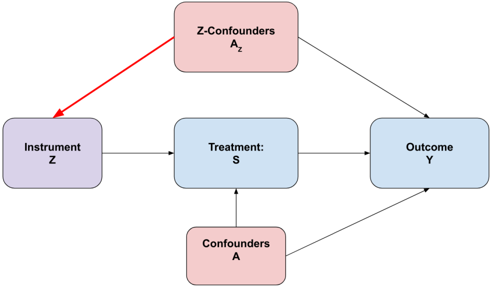
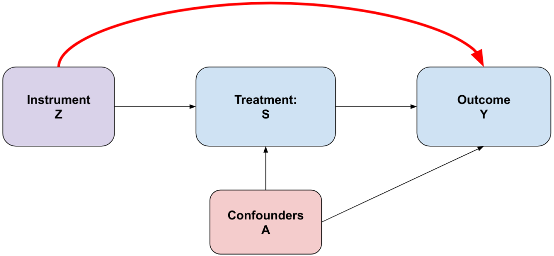


Exclusion: Instrument only affects outcome through treatment variable:

$$Z_i \nrightarrow \epsilon_i$$

Violation of exclusion:



Good instruments are hard to find

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- Some usual sources of instruments:
  - Nature (e.g. genes, weather)
  - Assignment rules (e.g. random assignment of judges to cases)
  - 'Natural' experiments (e.g. the quarter of birth, conscription lottery, electoral timing...)

Zoom Poll 8.1: Good instruments for schooling

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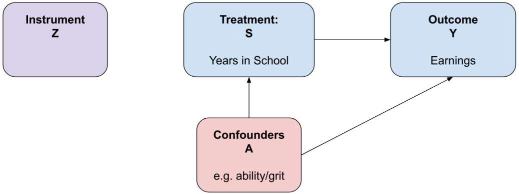
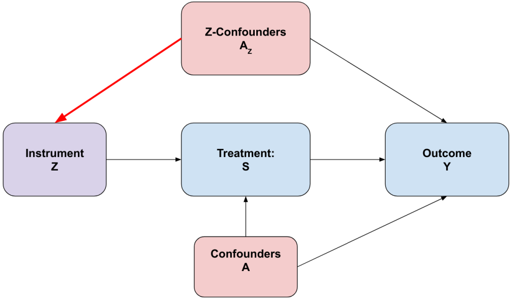
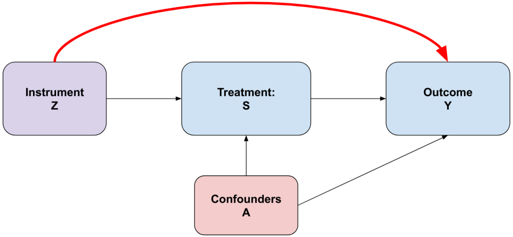
Violation of relevance:



Violation of exogeneity:



Violation of exclusion:



IV estimator

We have

$$Y_i = \alpha + \rho S_i + \epsilon_i$$

and an instrument Z_i where $\text{Cov}[Z_i, S_i] \neq 0$ and $\text{Cov}[Z_i, \epsilon_i] = 0$.

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- Thus:

$$\rho = \frac{\text{Cov}[Z_i, Y_i]}{\text{Cov}[Z_i, S_i]}$$

  with sample estimate

$$\hat{\rho}_{\text{IV}} = \frac{\sum_{i=1}^{n} Z_i Y_i}{\sum_{i=1}^{n} Z_i S_i}$$

```
from linearmodels.iv import IV2SLS
eq = "wages ~ 1 + [schooling ~ instrument] + C(fixed_effect)"
iv = IV2SLS.from_formula(eq,data=df).fit()
```

Examples

Look at papers if curious

- Immigration
  - Networks of immigrants (Card 1991)
- Does police decrease crime?
  - Electoral cycles (Levitt 1997)
- The impact of violent movies on crime
  - Blockbuster movies (Dahl and DellaVigna 2009)
- The effect of preschool television exposure on standardized test scores during adolescence:
  - Gentzkow and Shapiro 2008
- The Potato's Contribution to Population and Urbanization:
  - Nunn and Nancy Qian 2011
- Influence of mass media on U.S. government response to natural disasters
  - Eisensee and Strömberg 2007

Two-Stage Least Squares (2SLS)

IV estimates are equivalent to running two separate OLS regressions:

1. Estimate"first stage", regressing treatment on instrument:

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1. Estimate"first stage", regressing treatment on instrument:

$$S_i = \gamma Z_i + \nu_i$$

2. Form prediction $\hat{S}_i = \hat{\gamma} Z_i$ and estimate the"second stage", regressing outcome on first-stage-predicted treatment:

$$Y_i = \rho \hat{S}_i + \epsilon_i$$

2SLS Matrix Notation compared to OLS

- With model $Y = X'\beta + U$ and instrument Z, we have

$$\beta_{OLS} = (X'X)^{-1}(X'Y)$$
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- which estimate is more biased?

$$\mathbb{E}[(X'X)^{-1}(X'U)] \gtrless \mathbb{E}[(Z'X)^{-1}(Z'U)]?$$

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- Is Z_i correlated with causal variable of interest, S_i?
  - YES: check for significance of first stage (first-stage F-statistic)
- Is Z_i uncorrelated with any other determinants of Y_i?
  - Not directly testable – relies on institutional knowledge
  - but often indirect ways to probe exogeneity and exclusion

Weak Instruments

The bias of 2SLS can be written as:

$$\text{plim}\hat{\rho} = \rho + \frac{\text{Corr}[Z, \epsilon]}{\text{Cov}[S, Z]} \cdot \frac{\sigma_\epsilon}{\sigma_S}$$

- When the instrument is weakly correlated with the endogenous regressor, the bias increases.
- Kleibergen-Paap First-stage F-statistic should be higher than 10.

Reduced Form

"Reduced Form" (RF) means regressing the outcome directly on the instrument:

$$Y_i = \alpha + \phi Z_i + \epsilon_i$$

- papers will normally report this along with 2SLS estimates.
- for causal interpretation, RF requires exogeneity but not exclusion.

Instruments with Observed Confounders

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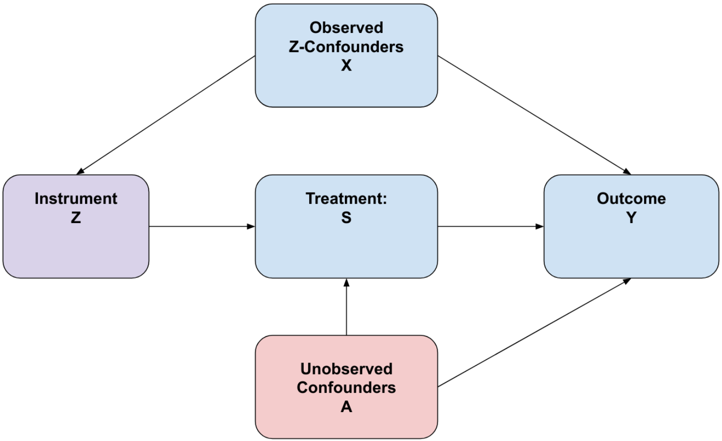
- With Z-confounders, we have the same property.



- IV independence assumption can be written as $\text{Cov}[Z_i, \epsilon_i | X] = 0$.

Practice: Effect of Fox News on COVID-19 Social Distancing

http://bit.ly/BRJ-W7-FNC-doc

Fuzzy RD $=$ IV

- **Sharp RD (regression discontinuity):** treatment status is **deterministic/discontinuous** function of running variable (x_i), with cutoff c:

$$Y_i = \alpha + \rho \mathbb{I}[x_i > c] + f(x_i)'\beta + \epsilon_i$$

```
eq = "death_rate ~ above_21 + age + age_squared"
rdd = smf.ols(formula=eq, data=df).fit()
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- **Fuzzy RD:** being above threshold increases **probability** of receiving treatment, rather than deterministically changing treatment. Use RD as first stage in 2SLS:

$$D_i = \alpha + \gamma \mathbb{I}[x_i > c] + \eta_i$$
$$Y_i = \alpha + \rho D_i + \epsilon_i$$

  - instrument is a dummy variable for being above cutoff
  - endogenous variable is whether treatment is actually assigned.
  - include polynomials in running variable as covarates.

```
eq = "death_rate ~ age + age_squared + [drinker ~ above_21]"
iv = IV2SLS.from_formula(eq,data=df).fit()
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Outline

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IV with Machine Learning

Deep IV

Lasso IV with Weak Instruments

Consider the problem of a sparse first stage:

$$S_i = \alpha + \boldsymbol{Z}_i'\boldsymbol{\phi} + \nu_i$$

- $\boldsymbol{Z}_i$ is a high-dimensional vector
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Solution:

- Train lasso (or elastic net), $S \sim \text{Lasso}(\boldsymbol{Z})$
  - use CV grid search across the whole dataset to select L1 penalty
  - get subset of instruments with non-zero coefficients, Z_{Lasso}.
- Run 2SLS with Z_{Lasso} as instrument(s).
- This is the "optimal" set of instruments under sparsity (Belloni et al 2014).

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  - e.g., some people won't go to school even if they win a scholarship.
  - first stage is driven by"compliers" (responders to instrument).
- Standard 2SLS estimates give a "local average treatment effect" on the complier population.

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  - Learn $\eta_0(X) = \mathbb{E}(S|X, Z = 0)$
  - Learn $\eta_1(X) = \mathbb{E}(S|X, Z = 1)$
- Conditional first stage effect estimate is $\hat{\gamma}(X) = \eta_1(X) - \eta_0(X)$.

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- Conditional first stage effect estimate is $\hat{\gamma}(X) = \eta_1(X) - \eta_0(X)$.
- Can be used to analyze complier population, or to re-weight regressions to get closer to an average treatment effect (Coussens and Spiess 2021).

Practice: Adding Instruments to Custom Causal Graphs

http://bit.ly/BRJ-W7-graphs-doc

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Deep Instrumental Variables

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- *Deep IV: A Flexible Approach for Counterfactual Prediction*
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- *Deep IV: A Flexible Approach for Counterfactual Prediction*
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  - use deep learning to extend 2SLS to high-dimensional settings
- Causal effect of interest:

$$f(S;\theta) = \mathbb{E}\{Y|S\}$$

where w could be high-dimensional and $f(\cdot)$ could be highly non-linear.

First stage

In first stage, approximate $g(S|\gamma(Z))$, the distribution of S:

- assume that $g(\cdot)$ is a mixture density network (a mixture of gaussian distributions) where the parameter vector $\gamma(\cdot)$ includes the weights, means, and variances (Bishop 2006).
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  - $\gamma(Z)$ is a modeled as a feed-forward neural network.
- $g(\cdot)$ has to be a parametrized distribution because Deep IV requires that the distribution be integrated in the second stage.
- validate first-stage relevance in in held-out test set.

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  - this is the true Y minus predicted $\hat{Y}$, but $\hat{Y}$ is conditioned on the instrument-predicted treatment distribution $\hat{g}$.
- The integral in $\mathcal{L}(\theta)$ is approximated by

$$\int \hat{Y}(S;\theta) d\hat{g}(S|\gamma(Z_i)) \approx \frac{1}{m} \sum_j^m \hat{Y}(\tilde{S}(Z_i);\theta)$$

where you make m draws from the estimated treatment distribution given Z_i (the instruments for observation i).

  - Like 2SLS, a prediction for the endogenous regressor with the instruments is used during second-stage estimation.