13 Sept. 2021

Carry Trade with volatility proxy

Seminar in Quantitative trading with R

Federico Maggiore

M.Sc. in Data Science

Trier University

Supervisor:

Ph.D. Dennis Umlandt

Matr. nr: 1510137

Address: Bergheimer str. 59

Heidelberg, 69115

Email: s4femagg@uni-trier.de

Trier University, Germany

SoSe 2021

Contents

1 Introduction **1**

2 Literature Review **1**

2.1 An overview 1

2.2 Results 3

3 Methods & Data **3**

3.1 Portfolio Creation 4

3.1.1 Carry Trade Portfolio based on forward discounts 4

3.1.2 Carry Trade Portfolio based on Innovation 8

3.1.3 Low beta strategy 10

4 Conclusions **11**

A Menkhoff et al. **14**

B Maggiore **15**

1 Introduction

The purpose of this paper is to extend (in an admittedly simplified manner) the analysis performed by Menkhoff et al. [5]. The cited paper, indeed, analyses data going from 1983 to 2009. They investigate a (possible) relationship between FX volatility and excess returns. However, in this review, I will work with data ranging from 1983 to 2019, which allows me to have a better understanding of the overall approach and results.
The following section is going to be a (very) short review of the aforementioned paper, with a special focus on the parts that are particularly relevant for the paper I am presenting. From section 3 the focus is going to be shifted on my analysis while performing some comparisons with the aforementioned text.

2 Literature Review

2.1 An overview

The paper by Menkhoff et al. [5] focuses on the risk-return profile of a strategy that benefits from borrowing currencies with low-interest rates and investing in currencies that have a higher interest rate. Such approach to the market is called *carry trade* [5]. The choice of the Foreign Exchange markets is due to their peculiarity of being incredibly liquid, allowing traders to buy/sell among countries/currencies almost continuously. Menkfhoff et al. follow almost completely the approach proposed by Lusting et al. [3] [4], they create 5 portfolios ordering the currencies according to their forwards discount, ie. ”their relative interest rate differential versus U.S. money market interest rate at the end of each month” [5] as shown in eq. 1. In order to create the carry trade, they go long on portfolio 5 (highest interest rate) while being short on portfolio 1 (lowest relative interest rate). Note that f and s in eq. 1 represent forward and spot rate, respectively.

$$forwardDisc. = log(forwardRate) - log(spotRate) = f - s \tag{1}$$

When it comes to the computation of the excess returns, the authors rely on two different approaches:

- the monthly excess returns for holding a foreign currency k are computed as in eq. 2 [5][3],

$$rx_{t+1}^{k} \approx f_t^k - s_{t+1}^k \tag{2}$$

- the average log excess return of a portfolio i consists of the (equally weighted) average of the log currencies excess returns in that portfolio.

It is worth mentioning that they assume an investor has to establish a new position in every single currency in the first month and closes all positions in the last month. Moreover, they take into account transaction costs as they realized to have a portfolio turnover $\approx 30\%$ [5].

The return difference between portfolio 5 and 1 the is the *carry trade*, the H/L portfolio in their notation. A summary table of the descriptive statistics (taken from the original paper) is shown in fig. 4 in the appendix.

It is interesting to notice how the mean and median returns monotonically increase moving from portfolio 1 to portfolio 5 and H/L, while the standard deviation doesn't show any pattern.

As stated above, the main purpose of the paper is to analyze how volatility interferes with returns. In order to do so, the authors use a proxy to measure the volatility; they start from daily data to construct the proxy (recall that the portfolios are built on a monthly basis). More specifically, Menkhoff et al. compute the daily log return for each currency on each day. They then average over all the currencies available on a given day and finally average daily values up to the monthly frequency [5]. The final FX volatility proxy is obtained as follows:

$$\sigma_t^{FX} = \frac{1}{T_t} \sum_{\tau \in T_t} \left[\sum_{\kappa \in K_\tau} \left(\frac{|r_\tau^\kappa|}{K_\tau} \right) \right] \tag{3}$$

The last two sections from Menkhoff et al. that are relevant to this paper are the computation of the $betas$ and the construction of portfolios based on such βs. The βs are the results of eq. 4, where R_i and R_M represent the returns of currency i and returns of the overall market, respectively.

$$\beta_i = \frac{Cov(R_i, R_M)}{Var_M} \tag{4}$$

The intuition behind βs as a systemic risk measure is that low beta assets are not highly correlated with the market. Therefore, they represent an attractive opportunity for investors who dislike risk. This causes these assets to be highly demanded, which means higher prices. As result, assets with low beta are predicted to have lower returns than the market, section 3.1.3 deals with this statement.

2.2 Results

For a first analysis of the results, Menkhoff and al. divide the sample of 48 currencies into 4 subsamples depending on the value of global volatility innovations. These subsamples are sorted such that the first subsample contains the 25% currencies with the lowest $\Delta\sigma_t^{FX}$, whereas the last is composed by the ones with the highest volatility innovation. A graphical result is, again, available in fig. 5 (in the appendix) and clearly shows that high interest rate currencies yield higher excess returns when $\Delta\sigma_t^{FX}$ is low and vice versa. Intuitively, low interest rate currencies perform well compared to high interest rate currencies when the market is volatile [5].

3 Methods & Data

The data used is a collection of 10 tables divided as follows:

- 2 tables for the bid prices on a monthly basis,
- 2 tables for the ask prices on a monthly basis,
- 2 tables for the future bid prices on a monthly basis,
- 2 tables for the future ask prices on a monthly basis,
- 2 tables for the spot prices on a daily basis.

All the pairs are combined leaving 5 different tables. The 4 monthly basis tables have a shape of 427x48, whereas the daily spot table is a matrix of size 9329x48. The data-set is composed by 48 columns where each column represents a different currency, while rows represent rates.

The data-set is then partially changed in many regards. First of all, some of the currencies were in ”reverse” order with respect to the others. While the general structure of the data is $currency_i$ to \$, for some of the columns it was the reverse, i.e. \$ to $currency_i$. In order to revert the relationship the inverse is used. Secondly, some of the currencies enter and exit the data-set during the entire period. An example of this phenomenon is the euro, between 1983 and early 2000 the euro was not adopted by any country, whereas after the start of the new century some countries embraced it as their official currency. For some reason, in the provided data-set the old currencies (eg. Italian Lira) had entries until 2019. This has been changed in the final data-set for all currencies that joined the euro (according to the joining date). The same actions are performed on both monthly and daily tables.

Simplifying the approach taken from Menkhoff et al., rather than creating 5 different portfolios plus the ”combination” of them, the number is reduced to 2 plus their ”combination” (represented by the *carry trade*, ”H/L” in Menkhoff’s notation). Nonetheless, the underlying concepts remain a core for this paper.

3.1 Portfolio Creation

In this subsection, 4 different portfolios are created and analyzed. All the assets allocations described are built using different portfolio sizes. Each portfolio varies according to the number of currencies available at any given month, ie. if 20% is used, then in a month with 10 disposable currencies the portfolio will comprehend 2 currencies, whereas a later point in time might be composed by 3 or more.

3.1.1 Carry Trade Portfolio based on forward discounts

The starting point for this portfolio are eq. 1 and eq. 2 introduced in section 2.1.

Since the portfolios are adjusted at the end of every month there is the need of a way to compute returns for currencies entering / exiting the portfolio. A similar (simplified) approach as in Menkhoff et al. [5] is used:

- monthly excess returns for a currency that enters and stays in the portfolio at the end of the month

$$rx_{t+1}^{l} = f_t^b - s_{t+1} \tag{5}$$

$$rx_{t+1}^{s} = -f_t^a + s_{t+1} \tag{6}$$

- monthly excess returns for a currency that was in the portfolio but exits at the end of the month

$$rx_{t+1}^{l} = f_t - s_{t+1}^a \tag{7}$$

$$rx_{t+1}^{s} = -f_t + s_{t+1}^b \tag{8}$$

Note that, in the above equations, the superscripts l and s refer to long and short positions, whereas the superscripts b and a are relate to bid and ask prices.

The matrix output of eq. 2 plays a vital role as, for each month, all the currencies are sorted according to the value of rx_{t+1}^{k}. The ordered currencies are then used to build the two portfolios for the following month. The lowest interest rate currencies are shorted while going long on the ones with the highest interest rate (or forward discount). Fig. 1 shows the result of running the above-mentioned strategy on the different portfolios.

The percentages in the legend are interpreted as follows: each month all the available currencies are taken into consideration, the number of such available currencies is multiplied by the percentage and rounded. In particular, the 10% portfolio contains (on average) 3 currencies in each the long and the short; this number goes up to 7, 10 and 13 for 20%, 30% and 40%, respectively. For the most balanced configuration (20%) a descriptive statistics table is shown in tab. 1. Statistics for all the other sizes are available in the appendix, tab. 3.

As fig. 1 clearly depicts, after almost 40 years, an investor would have a higher cumulative return investing only in the 10% currencies having the highest (and lowest) rx_{t+1}^{k}. Going deeper in the analysis, though, it is possible to appreciate that the lower the percentage of the market the investor is involved in, the higher the standard deviation (and therefore the fluctuations) he/she must accept. In order to get an increment of the log excess monthly return of 0.001, for example when moving from the 20% portfolio to the 10% portfolio, the standard deviation increases from 0.029 for the 0.039.

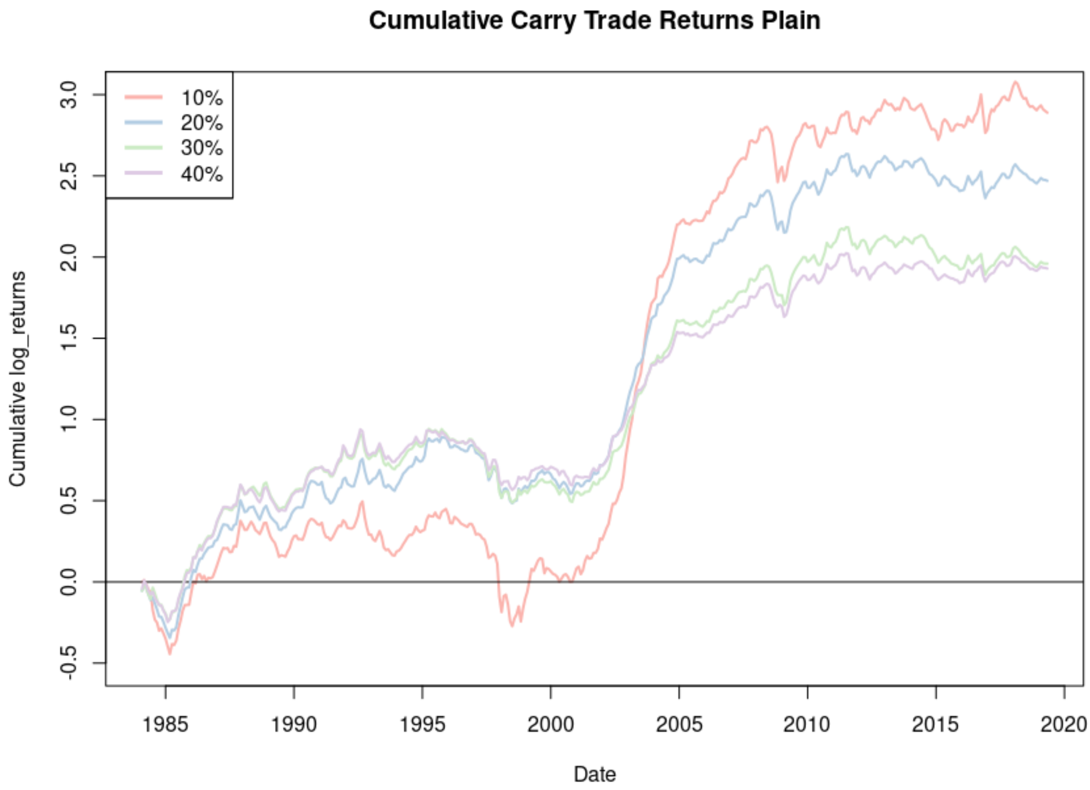

Figure 1: Cumulative excess log return over time for different Carry Trade portfolio sizes.

Looking at tab. 3 it is possible to notice that the standard deviation decreases monotonically when increasing the number of currencies in the portfolio. Intuitively, the more currencies are in a portfolio the more the gains and the losses are flattened towards the overall market average. This also explains why, in a time of good overall performance (as after the year 2000 in fig. 1), the portfolios containing 30 and 40% are those that performed the worst.

Moving to the analysis of the skewness it is possible to see that all the H/L portfolios show a value that is in line with both the first stylized fact, according to which the distribution of returns is negatively skewed, and the paper by Menkhoff et al [5]. The same doesn't apply for the kurtosis which does not decrease (for every portfolio size) when moving from short to long and from long to H/L.

With regards to risk measures, I used Value at Risk, Expected Shortfall and Sharpe ratio. In tab. 1 and tab. 3 VaR, ES and Sharpe, respectively.

A brief explanation of their meaning is given below:

Portfolio 20%	short	long	H/L
Mean	-0.004	0.001	0.006
Median	-0.004	0.000	0.006
Std.Dev.	0.016	0.019	0.029
Skewness	0.477	0.289	-0.275
Kurtosis	1.637	1.426	0.531
VaR 5%	-0.030	-0.027	-0.040
ES 5%	-0.033	-0.037	-0.062
Sharpe	-0.264	0.076	0.196

Table 1: Descriptive statistics for the 20% portfolio. All the values are expressed in monthly values. Mean and median refer to monthly log excess returns.

- **Value at Risk.** The VaR deals with the potential losses that a portfolio can suffer. Its value is derived as the maximum percentage that a portfolio can, with 95% confidence, expect to lose over any given year. Although it's a useful indication of risk in a portfolio, it's not a perfect measure as it assumes a normal distribution for returns. Moreover, it doesn't say anything about the potential losses outside of the 95% confidence level, ie. VaR does not tell the amount of the loss when this loss is higher than the VaR.

- **Expected shortfall.** The ES aims to improve on some of the pitfalls of the VaR. Its focus is on the tail of the loss distribution (ignored by VaR). For example, the ES for the H/L portfolio in tab. 1 is measured to be -0.062. This means that in case of extreme losses, the amount of such loss would be 0.0662, on average.

- **Sharpe ratio.** The Sharpe ratio measures the performance of an investment comparing it to a risk-free return, divided by the standard deviation of the risky investment. In other words, if two portfolios have the same return but one of them has a lower risk, its Sharpe ratio will be higher. For simplicity, a risk-free return $R_f = 0$ is considered.

Breaking down Sharpe ratios, it is possible to notice how the H/L always outperforms the individual long and short strategies. Also, comparing all the different portfolio sizes, the 20% portfolio obtains the best results with a value of 0.196. Interestingly, the 40% portfolio scores a Sharpe ratio of 0.187 which is the second best over all sizes, even though such portfolio has the lowest returns, the smaller variance makes it a solid investment in terms of return/risk. Note, however, that should be kept in mind that investments with a higher Sharpe ratio can be more volatile than those with a lower ratio. The higher Sharpe ratio only indicates that risk/reward profile of an investment is more optimal than another, significant risk or volatility might still be involved.

3.1.2 Carry Trade Portfolio based on Innovation

As stated in Menkhoff [5], the finance theory suggests that there must be a volatility risk premium as unexpected high volatility worsens the investor's risk-return profile. Citing Ang et al. [1], "Risk-averse agents demand stocks that hedge against this risk. Periods of high volatility also tend to coincide with downward market movements assets with high sensitivities to market volatility risk provide hedges against market downside risk. The higher demand for assets with high systematic volatility loadings increases their price and lowers their average return".

Following the approach from Menkhoof et al. [5] a simple measure of a proxy for global FX volatility is constructed, according to eq. 3. The core of the empirical analysis is the volatility *innovations* ($\Delta\sigma_t^{FX}$) which is firstly computed as the first difference of the volatility found in eq. 3, as suggested by Ang et al. [1]. The authors [5], however, find that there is significant autocorrelation in the results of such a method and, therefore, use a simple AR(1) and take the (uncorrelated) residuals as a proxy for volatility innovation. In the case of this paper, I had better results using an ARIMA(1,0,1)(2,0,0) leading to the residuals depicted in fig. 6 in the appendix. In this data-set a simple AR(1) led to autocorrelated residuals that failed the Ljung-Box test. The residuals obtained are then used as a proxy for volatility, leading to a new collection of portfolios in which the currencies are sorted depending on the value of $\Delta\sigma_t^{FX}$.

The descriptive statistics for all the portfolio sizes are structured as in the previous section, tab. 2 refers to the 20% portfolio, whereas for all the other sizes tab. 4 is available in the appendix.

Reading through tab. 2 it seems clear that the depicted scenario does not represent a

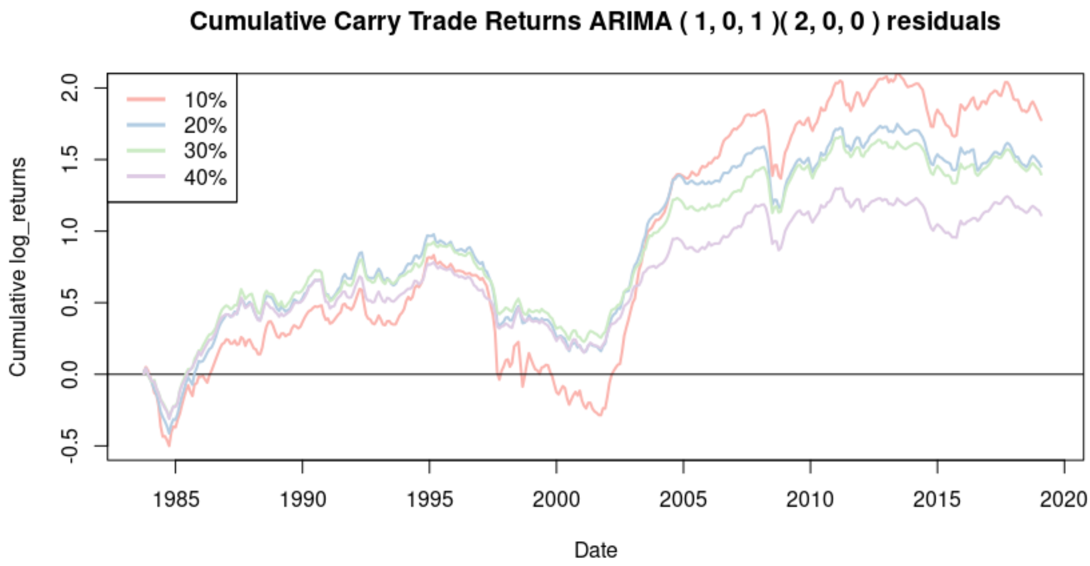

Figure 2: Cumulative excess log return over time for different Carry Trade portfolio sizes based on volatility innovation measured by the residuals of an ARIMA(1,0,1)(2,0,0).

huge deviation from the one shown in tab. 1. The rx^k_{t+1} is still increasing steadily when moving from short to long and from long to H/L. The same happens, once again, for the standard deviation: increases intra-portfolio (from short to H/L) and decreases as the portfolio involves a higher share of the available currencies. Also the first stylized is still corroborated by all H/L portfolios.

When it comes to risk measures something changes. Among all possible combinations of portfolio sizes, the best investment in terms of Sharpe ratio is the 30%, while the second best is represented by the portfolio containing 20% of the currencies. While the difference is small, 0.113 vs 0.103, it might be a suggestion that, with respect to volatility, a more diverse portfolio might represent a safer bet.

Considering the structure of the results and their tendency to behave almost identically when sorting on forward discounts and on volatility proxy, it seems clear that a relation between returns and volatility exists. The two portfolio groups are, indeed, pretty similar even though the magnitude is somehow different, this is a hint that returns probably incorporate volatility information.

Portfolio 20%	short	long	H/L
Mean	-0.003	0.000	0.003
Median	-0.002	0.002	0.005
Std.Dev.	0.017	0.021	0.033
Skewness	0.663	-0.547	-0.673
Kurtosis	2.604	1.745	2.149
VaR 5%	-0.030	-0.032	-0.049
ES 5%	-0.031	-0.054	-0.088
Sharpe	-0.151	0.037	0.103

Table 2: Descriptive statistics for the 20% portfolio. All the values are expressed in monthly values. Mean and median refer to monthly log excess returns.

3.1.3 Low beta strategy

In the last part of the paper, I want to analyze a different scenario. The focus is the volatility beta (eq. 4) on different time frames. Note that another step in terms of simplification is taken: there is no carry trade, the adjustment of currencies enter or leaving the portfolios is not taken into account and only 2 portfolios are created (20% and 30%).

One of the most important aspects of this analysis is that the portfolios are going to be adjusted every 6 and 18 months. The idea is that, as stated in 3.1.2, the theory suggests that low beta assets should have lower returns as their price is inflated by higher demand since they represent a safer asset, at least with respect to volatility.

Fig. 3 shows that, interestingly, in all 4 scenarios the low beta portfolio outperforms the high beta counterpart. Tab. 5 and tab. 6, in the appendix, confirm what is visually depicted. In all scenarios, low beta currencies have a higher mean, while keeping the standard deviation lower. This is reflected also in the risk measures, where the low beta assets greatly outperform the high beta currencies.

Whether this result stems from the oversimplified framework or not is left as future work. However, it must be noted that in the literature the *low beta anomaly* received interest in recent years as in Frazzini and Pedersen (2011) [6] or Baker, Bradley, and Wurgler (2010)[2]. The latter, in particular, offer an interesting behavioral explanation of why such anomaly *might* exist in the stock market: ”In the context of the low-risk anomaly, we believe that an important subset of investors has a preference for risky stocks. This preference derives from the biases that afflict the individual investor. We believe individuals’ preferences for lotteries and well-established biases of representativeness and overconfidence lead to demand for risk that is not warranted by fundamentals. This irrational demand causes such high-risk stocks to be overpriced, which, all else equal, leads to lower future returns”.

It must be noted, however, that low beta doesn’t necessarily mean low risk. From eq. 4 is clear that an asset beta can be low if it has low volatility of returns or it has a low correlation with the market’s returns. This means that even a low beta asset can still be extremely risky.

4 Conclusions

Throughout this paper I used currency exchange rates to build different portfolios.

Starting from the typical carry trade, created according to forward discounts, in which the investor goes short on low interest currency to fund the purchase of high interest rate currencies. The second part of the paper had a focus on volatility measures and volatility proxy. From the realized volatility I moved to a volatility proxy created using an ARIMA model. Such proxy was then used to generate a new set of portfolios that, as seen, show a pattern that is very similar to the one obtained in the carry trade portfolio. This led to a hint on how volatility and returns are indeed linked.

Finally, a brief study was conducted on low beta portfolios. It showed that, at least in a very simplified environment, the low beta anomaly exists and might be exploited to generate high returns while reducing, in some cases, the risk.

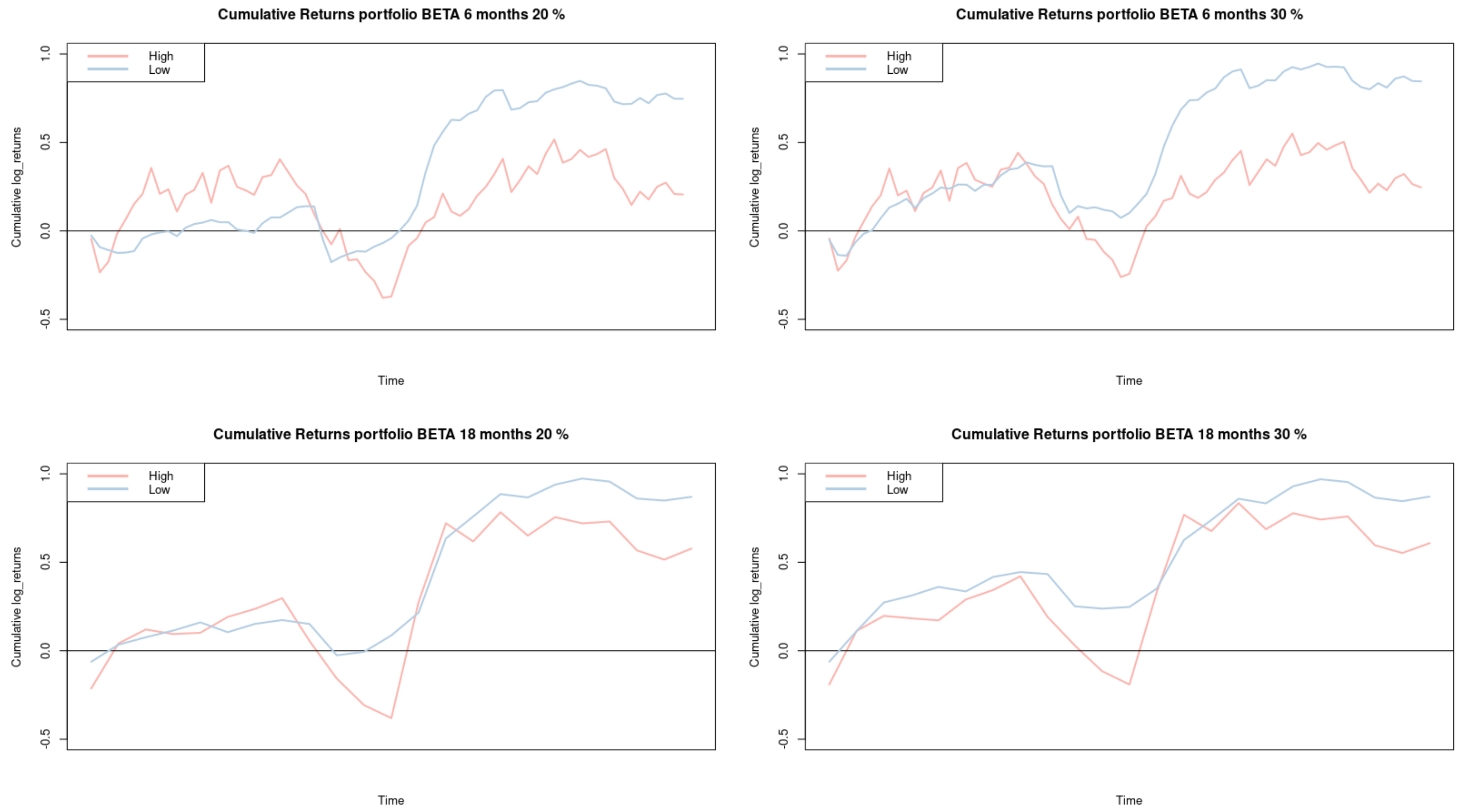

Figure 3: Cumulative excess log return over time for different beta portfolios.

References

[1] Andrew Ang, Robert J Hodrick, Yuhang Xing, and Xiaoyan Zhang. The cross-section of volatility and expected returns. *The Journal of Finance*, 61(1):259–299, 2006.

[2] Malcolm P Baker, Jeffrey Wurgler, and Brendan Bradley. A behavioral finance explanation for the success of low volatility portfolios. 2010.

[3] Hanno Lustig, Nikolai Roussanov, and Adrien Verdelhan. Common risk factors in currency markets. *The Review of Financial Studies*, 24(11):3731–3777, 2011.

[4] Hanno Lustig and Adrien Verdelhan. The cross-section of foreign currency risk premia and consumption growth risk: Reply. *American Economic Review*, 101(7):3477–3500, 2011.

[5] Lukas Menkhoff, Lucio Sarno, Maik Schmeling, and Andreas Schrimpf. Carry trades and global foreign exchange volatility. *The Journal of Finance*, 67(2):681–718, 2012.

[6] Andrea Frazzini Pedersen and H Lasse. Betting against beta. 2011.

A Menkhoff et al.

All Countries (with b-a)							
Portfolio	1	2	3	4	5	Avg.	H/L
Mean	−1.46	−0.10	2.65	3.18	5.76	2.01	7.23
	[−0.80]	[−0.06]	[1.43]	[1.72]	[2.16]	[1.18]	[3.13]
Median	−2.25	0.77	1.96	4.09	10.17	2.87	11.55
Std. dev.	8.50	7.20	8.11	8.39	10.77	7.39	9.81
Skewness	0.18	−0.23	−0.28	−0.55	−0.66	−0.40	−1.03
Kurtosis	3.77	4.11	4.34	4.78	5.08	3.98	4.79
Sharpe ratio	−0.17	−0.01	0.33	0.38	0.54	0.27	0.74
AC(1)	0.04	0.09	0.14	0.11	0.23	0.14	0.18
	(0.74)	(0.27)	(0.04)	(0.14)	(0.00)	(0.04)	(0.01)
Coskew (DOL)	0.38	−0.07	−0.14	−0.15	−0.06	0.38	−0.21
Coskew (MKT)	0.18	0.03	0.11	0.10	0.04	0.10	−0.12

Figure 4: Descriptive statistics for portfolios 1-5 and H/L as in [5] p.690

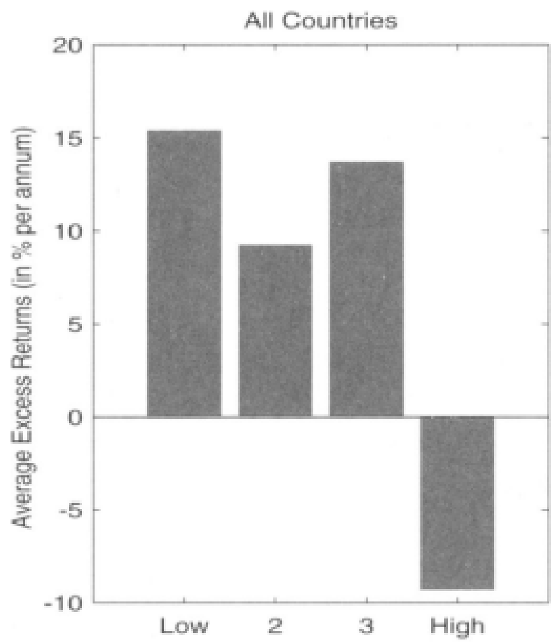

Figure 5: Excess returns and volatility [5] p.694

B Maggiore

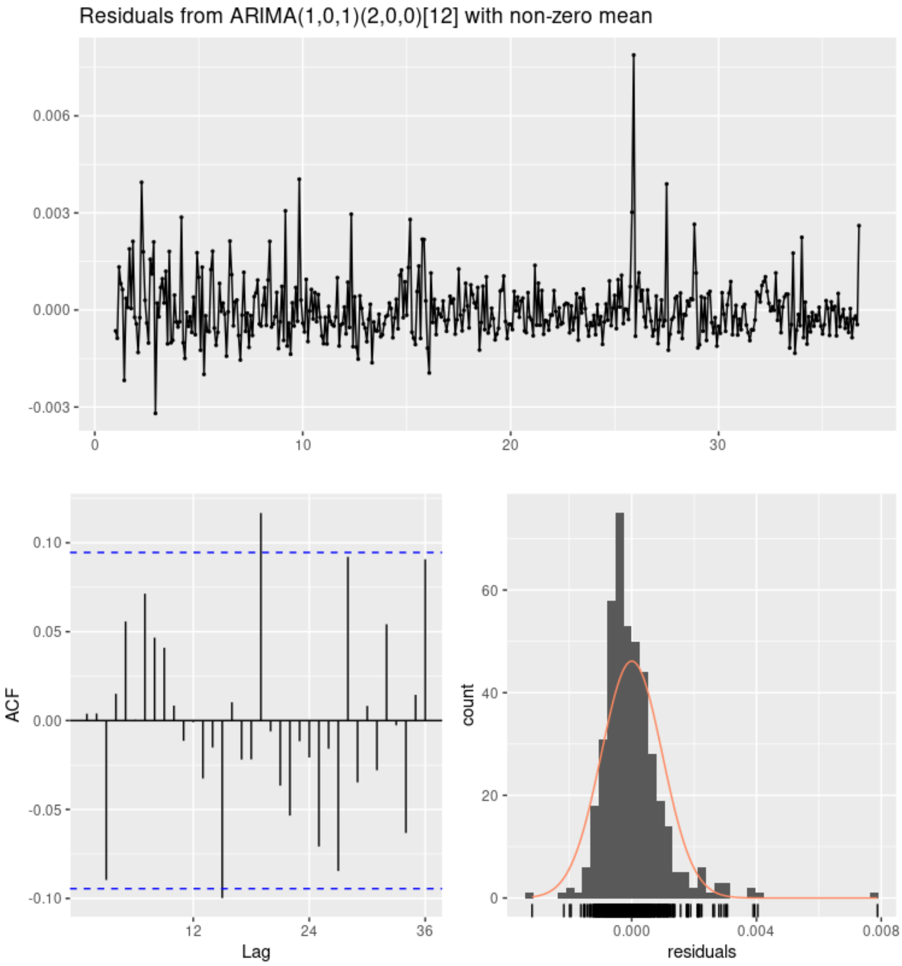

Figure 6: Analysis of the residual of an ARIMA(1,0,1)(2,0,0) on realized volatility. Ljung-Box test p-value = 0.2445

Portfolio	short 10%	long 10%	H/L 10%	short 30%	long 30%	H/L 30%	short 40%	long 40%	H/L 40%
Mean	-0.003	0.004	0.007	-0.004	0.000	0.005	-0.005	0.001	0.004
Median	-0.003	0.004	0.005	-0.004	-0.001	0.005	-0.004	-0.002	0.004
Std.Dev.	0.024	0.025	0.039	0.014	0.018	0.026	0.012	0.018	0.024
Skewness	0.838	0.239	-0.532	-0.252	0.149	-0.319	0.348	0.183	-0.108
Kurtosis	5.239	4.175	2.456	4.914	0.780	0.908	3.366	0.529	0.741
VaR 5%	-0.037	-0.029	-0.052	-0.026	-0.026	-0.038	-0.024	-0.028	-0.036
ES 5%	-0.033	-0.046	-0.103	-0.044	-0.037	-0.058	-0.028	-0.036	-0.049
Sharpe 5%	-0.110	0.165	0.172	-0.339	-0.013	0.173	-0.455	-0.055	0.187

Table 3: All the values are expressed in monthly values. Mean and Median refer to monthly log excess returns.

Portfolio	short 10%	long 10%	H/L 10%	short 30%	long 30%	H/L 30%	short 40%	long 40%	H/L 40%
Mean	-0.003	0.000	0.004	-0.002	0.000	0.003	-0.002	0.000	0.002
Median	-0.003	0.002	0.006	-0.002	0.001	0.004	-0.001	0.002	0.004
Std.Dev.	0.025	0.027	0.043	0.014	0.019	0.029	0.011	0.019	0.026
Skewness	1.285	-0.494	-1.107	0.319	-0.448	-0.484	0.286	-0.451	-0.389
Kurtosis	9.183	3.471	5.252	1.836	1.429	1.476	1.935	0.959	0.896
VaR 5%	-0.040	-0.0417	-0.064	-0.026	-0.030	-0.045	-0.022	-0.032	-0.040
ES 5%	-0.038	-0.079	-0.152	-0.030	-0.047	-0.071	-0.025	-0.047	-0.061
Sharpe	-0.142	0.020	0.095	-0.177	0.039	0.112	-0.155	0.042	0.098

Table 4: All the values are expressed in monthly values. Mean and Median refer to monthly log excess returns.

Portfolio 6 months	low 20%	high 20%	low 30%	high 30%
Mean	0.010	0.0029	0.012	0.003
Median	0.009	0.021	0.0139	0.019
Std.Dev.	0.052	0.093	0.051	0.088
Skewness	-0.342	-0.301	-0.476	-0.297
Kurtosis	4.346	-0.798	2.039	-0.674
VaR 5%	-0.071	-0.167	-0.081	-0.150
ES 5%	-0.138	-0.185	-0.123	-0.176
Sharpe	0.204	0.031	0.237	0.039

Table 5: All the values are expressed in monthly values. Mean and Median refer to monthly log excess returns.

Portfolio 18 months	low 20%	high 20%	low 30%	high 30%
Mean	0.037	0.025	0.037	0.026
Median	0.034	0.007	0.027	-0.011
Std.Dev.	0.111	0.210	0.096	0.191
Skewness	1.456	1.369	0.229	1.066
Kurtosis	4.442	2.006	0.551	0.740
VaR 5%	-0.092	-0.212	-0.085	-0.187
ES 5%	-0.145	-0.377	-0.149	-0.300
Sharpe	0.339	0.119	0.390	0.138

Table 6: All the values are expressed in monthly values. Mean and Median refer to monthly log excess returns.