ADAPTIVE IMAGE INPAINTING ALGORITHM BASED ON GENERALIZED PRINCIPAL COMPONENT ANALYSIS

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ABSTRACT

This paper proposes an image inpainting algorithm based on generalized principal component analysis. Several inpainting algorithms have been proposed based on the assumption that an image can be modeled by the autoregressive (AR) model. However, their performances are not good enough to apply to natural photographs because they assume that images are modeled by the position-invariant linear model. To improve the inpainting quality, this work introduces a multiple AR model based inpainting based on the generalized principle component analysis (GPCA) and proposes a new multiple matrix rank minimization approach. A practical algorithm is provided based on the iterative partial matrix shrinkage (IPMS) algorithm, and numerical examples show that the effectiveness of the proposed algorithm.

I. INTRODUCTION

This paper proposes an image inpainting algorithm based on generalized principal component analysis (GPCA). The image inpainting is the technique of restoration small damaged region of an image. Various methods have been proposed for image inpainting such as exemplar based approaches [1], [2] and partial differential equation (PDE) based approaches [3], [4]. The exemplar based approaches recover missing pixels of the image from known non local observed data, and the PDE based approaches estimate the missing pixels based on assuming that the pixels of the image is modeled by a partial differential equation.

Recently, matrix rank minimization based approach is focused on the signal restoration field. Several matrix rank minimization approaches are proposed to achieve image inpainting. Sznaier *et al.* have proposed Hankel operator approach to texture inpainting algorithm [5], and the authors proposed Hankel-like structured matrix rank minimization based image inpainting algorithm [6]. However restoration quality of these algorithms is not good enough to recover the general images because they assume that an image is modeled by the position-invariant linear systems.

This paper proposes an adaptive image inpainting method based on the AR model approach with GPCA. The GPCA is a subspace segmentation algorithm [7] which estimates L different linear subspaces groups given sample points into these subspaces. In the image inpainting problem, L different subspaces correspond to L different kinds of textures in the image, and GPCA leads to improve the inpainting algorithm by identifying L kinds of textures, that is, the algorithm achieves inpainting using L kinds of AR models. We have proposed GPCA algorithm [8] using the iterative partial matrix shrinkage (IPMS) algorithm [9]. Since the image inpainting problem requires simultaneously identifying multiple subspaces and restoring missing pixels, this work improves the GPCA algorithm to restore missing pixels and applies this algorithm to the image inpainting problem. Numerical examples show that the effectiveness of the proposed algorithm.

II. MAIN RESULT

II-A. Matrix rank minimization approach

Let $I_{i,j}$ denotes the value of (i,j) pixel of the image I which is $M \times N$ gray scale image or any layer of RGB of the color image. The image inpainting problem is formulated as follows,

$$\begin{array}{ll} \text{Find} & I_{i,j} \\ \text{subject to} & (i,j) \in \Omega \end{array} \tag{1}$$

where Ω denotes given index set of missing pixels. In the AR model based approach, we assume that the image is modeled by 2D-AR model as follows,

$$\sum_{l=-K}^{K} \sum_{m=-K}^{K} a_{l,m} I_{i+l,j+m} = 0, \tag{2}$$

where K and $a_{l,m}$ denote the model order and model coefficient at (l,m) lag, respectively. To find a solution of (2), the AR model based approach estimates the model coefficient and value of missing pixels simultaneously using a given model order K. However restoration quality of this approach depends heavily on the model order K in general. To overcome this limitations, structured matrix rank minimization based algorithm has been proposed [6], where the image inpainting problem is formulated as following,

$$\begin{array}{ll} \text{Minimize} & \textbf{rank} X \\ \text{subject to} & X \in \mathcal{H} \\ & x_{i,j} \in [0,1] \\ & x_{i,j} = I_{i,j}, \text{ for } (i,j) \in \Omega^c, \end{array} \tag{3}$$

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where $\mathcal{H}$ denotes set of the Hankel-like structured matrices X defined as follows,

$$X = \left[\boldsymbol{x}^T_{\hat{K}+1,\hat{K}+1}, \ldots, \boldsymbol{x}^T_{\hat{K}+1,N-\hat{K}}, \boldsymbol{x}^T_{\hat{K}+2,\hat{K}+1}, \ldots, \boldsymbol{x}^T_{M-\hat{K},N-\hat{K}}\right]^T \in \mathcal{H} \subset \boldsymbol{R}^{(M-2\hat{K})(N-2\hat{K})\times(2\hat{K}+1)^2}, \quad (4)$$

and

$$\begin{aligned}\boldsymbol{x}_{i,j} = [\ & x_{i-\hat{K},j-\hat{K}}, & x_{i-\hat{K},j-\hat{K}+1}, & \ \cdots & x_{i-\hat{K},j+\hat{K}}, \\ & x_{i-\hat{K}+1,j-\hat{K}}, & x_{i-\hat{K}+1,j-\hat{K}+1}, & \ \cdots & x_{i-\hat{K}+1,j+\hat{K}}, \\ & & \vdots & & \\ & x_{i+\hat{K},j-\hat{K}}, & x_{i+\hat{K},j-\hat{K}+1}, & \ \cdots & x_{i+\hat{K},j+\hat{K}}\ ] \\ \in \boldsymbol{R}^{(2\hat{K}+1)^2}, \end{aligned} \quad (5)$$

In the above equations, $\boldsymbol{x}_{i,j}$ is a vector generated as vectorize a part of $(2\hat{K}+1)\times(2\hat{K}+1)$ image centered on (i,j) and structured matrix X have the row vectors $\boldsymbol{x}_{i,j}$ for all index (i,j), where $\hat{K}$ is given upper limit of the model order. Because the rank of X is an increase function of the model order, we can estimate a valid model order by finding the missing elements of X such that its rank is minimal, that is, the problem (3) estimates missing pixels by minimizing $\textbf{rank}X$. Since this algorithm assumed a position invariant model (2), restoration quality of the inpainting is not good enough to inpaint real photographs. In order to improve the inpainting quality, this work assumes that the image is modeled by multiple AR models and introduces GPCA.

II-B. Generalized principal component analysis

GPCA is a problem to identify L different linear subspaces $\{S_i\}_{i=1}^L$ and relationships between each data and each subspace from given data set $\{\boldsymbol{y}_j\}_{j=1}^N$. If the data is mixture of some sub-data from each different linear space, GPCA is reasonable than PCA. Vidal *et al.* proposed a GPCA framework and a basic algorithm for GPCA in [7]. In [8], GPCA is formulated as the following feasibility problem subject to multiple matrix rank constraints,

$$\begin{array}{ll}\text{Find} & D_1, D_2, \ldots, D_L \\ \text{subject to} & \sum_{i=1}^{L} D_i = E, \\ & \textbf{rank}(D_iY) \le r_i, i = 1,2,\ldots,L \\ & \textbf{tr}(D_i^T D_j) = 0, \forall i \neq j \\ & D_i \in \mathcal{D}, i = 1,2,\ldots,L \end{array}, \quad (6)$$

where Y is a given data matrix defined as $Y = [\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_L]^T$, r_i is a known upper bound of dimension of each subspace. The matrices E and $\mathcal{D}$ denote an identity matrix and the set of diagonal matrices, respectively. The algorithm to provide an approximate solution of (6) has also been proposed in [8] based on the iterative partial shrinkage (IPMS) algorithm. The IPMS based GPCA algorithm gives a solution of (6) by relaxing it as the following problem,

$$\begin{array}{ll}\text{Minimize} & \sum_{i=1}^{\hat{L}} \|Z_i\|_{*,\hat{d}_i+1} \\ \text{subject to} & \sum_{i=1}^{\hat{L}} D_i = E \\ & Z_i = D_iY \end{array}, \quad (7)$$

where $\|Z\|_{*,r}$ denotes sum of non-dominant singular values defined as $\sum_{i=r+1}^{end} \sigma_i$ using ith singular value σ_i of Z, and D_i denotes diagonal matrices with elements $d_{j,j}^{(i)}$. While this problem has no the constraints of orthogonality between D_i and D_j, the following property is given in [8],

Property : Assume that each y_i belongs to only one subspace. Then $(D_1, D_2, \ldots, D_L)$ is the minimizer of (7) if and only if it is feasible solution of (6).

Note that the problem obtained by replacing $\|D_iY\|_{*,r_i}$ with the nuclear norm of D_iY never provides feasible solutions of (6) because it holds that $\sum_{i=1}^{L} \|D_iY\|_* \ge \|\sum_{i=1}^{L} D_iY\|_* = \|Y\|_*$, which implies that the nuclear norm minimization problem always gives $D_i = E$ for some i and $D_j = \mathbf{0}$ for all $i \neq j$, that is, we obtain non-switched linear model.

An approximate solution of problem (7) is obtained by the gradient projection method, which is described as the following update scheme,

Step 1 $\quad D_i \leftarrow \mathcal{T}_{\hat{r}+1,\lambda\sigma_{\hat{r}}}(D_i)$.
Step 2 $\quad$ Obtain D_i by solving the following problem,

$$\begin{array}{ll}\underset{\{D_i\}_{i=1}^{\hat{L}}}{\text{Minimize}} & \sum_{i=1}^{\hat{L}} \|D_iY - Z_i\|_F^2 \\ \text{subject to} & \sum_{i=1}^{\hat{L}} D_i = E \end{array}. \quad (8)$$

Step 3 $\quad Z_i \leftarrow D_iY$.

where $\mathcal{T}_{\hat{r}+1,\lambda\sigma_{\hat{r}}}(D)$ denotes soft thresholding operator which decreases $\hat{r}+1$th and more singular values of D by $\lambda\sigma_{\hat{r}}$. The optimal solution of (8) is obtained analytically as follows,

$$d_{j,j}^{(i)} = \frac{1}{\hat{L}}\left(1 - \sum_{k=1}^{\hat{L}} \frac{\boldsymbol{y}_j^T \boldsymbol{z}_j^{(k)}}{\boldsymbol{y}_j^T \boldsymbol{y}_j}\right) + \frac{\boldsymbol{y}_j^T \boldsymbol{z}_j^{(i)}}{\boldsymbol{y}_j^T \boldsymbol{y}_j}. \quad (9)$$

Algorithm 1 shows IPMS based GPCA algorithm proposed in [8]. In the next section, we apply the algorithm to image inpainting problem to implement multiple sub-model based inpainting algorithm.

II-C. GPCA based adaptive inpainting algorithm

This subsection gives an adaptive image inpainting algorithm based on GPCA. In GPCA approach to image inpainting, the missing pixels are restored by identifying multiple linear subspaces. To introduce this approach, we

Algorithm 1 IPMS based GPCA algorithm.

Input: Y, $\{D_i\}_{i=1}^{\hat{L}}$, λ, T_{max}.
  $t \leftarrow 0$.
  repeat
    for $i = 1$ to $\hat{L}$ **do**
      $D_i^{old} \leftarrow D_i$
      $[U, \Sigma, V] \leftarrow \mathrm{SVD}(D_i Y)$.
      $\hat{r} \leftarrow$ Rank estimation(Σ).
      $\acute{\Sigma} \leftarrow \mathcal{T}_{\hat{r}+1, \lambda\sigma_{\hat{r}}}(\Sigma)$.
      $Z_i \leftarrow U\acute{\Sigma}V^T$.
    end for
    for $i = 1$ to $\hat{L}$ **do**
      $d_{j,j}^{(i)} \leftarrow \frac{1}{\hat{L}}\left(1 - \sum_{k=1}^{\hat{L}} \frac{\boldsymbol{y}_j^T \boldsymbol{z}_j^{(k)}}{\boldsymbol{y}_j^T \boldsymbol{y}_j}\right) + \frac{\boldsymbol{y}_j^T \boldsymbol{z}_j^{(i)}}{\boldsymbol{y}_j^T \boldsymbol{y}_j}$.
    end for
    $t \leftarrow t + 1$
  until convergence or $t > T_{max}$
Output: Data set $D_i Y$ for each subspace i.

formulate an adaptive algorithm as following GPCA based algorithm,

$$\begin{aligned} \underset{Y, \{D_i\}_{i=1}^{\hat{L}}}{\text{Minimize}} \quad & \sum_{i=1}^{\hat{L}} \|D_i Y\|_{*, \hat{d}_i+1} + \gamma \sum_{i=1}^{\hat{L}} \|\mathrm{diag}(D_i)\|_1 \\ \text{subject to} \quad & \sum_{i=1}^{\hat{L}} D_i = I, Y \in \mathcal{H}, \\ & x_{i,j} = I_{i,j}, \text{ for } (i,j) \in \Omega^c. \end{aligned} \tag{10}$$

where diagonal matrices D_i denote model selector and the row vector of Y denotes a part of image. Thus row vector of $D_i Y$ is selected only modeled by ith liner model.

This problem is more difficult to solve than (7) because some missing elements of Y are unknown. In order to solve (10) this paper modifies Algorithm 1 and proposes Algorithm 2, where $\mathcal{S}$ and $\mathcal{P}$ denote the function of transforming an image into structured matrix defined by (4) and projection on the set of the structured matrices. Design matrices Y and D_i are estimated alternately. Solution of the proposed algorithm depend on the initial value of D_i.

In experimentally, it is a better scheme that inpainting the image using TV minimization, after that, make D_i using the result of mean shift clustering.

Algorithm 2 Proposed inpainting algorithm based on GPCA

Input: I, $\{D_i\}_{i=1}^{\hat{L}}$, λ, T_{max}.
  $Y \leftarrow \mathcal{S}(I)$.
  $t \leftarrow 0$.
  repeat
    for $i = 1$ to $\hat{L}$ **do**
      $Y^{old} \leftarrow Y$
      $[U, \Sigma, V] \leftarrow \mathrm{SVD}(D_i Y)$.
      $\hat{r} \leftarrow$ Rank estimation(Σ).
      $\acute{\Sigma} \leftarrow \mathcal{T}_{\hat{r}+1, \lambda\sigma_{\hat{r}}}(\Sigma)$.
      $Z_i \leftarrow U\acute{\Sigma}V^T$.
    end for
    for $i = 1$ to $\hat{L}$ **do**
      $d_{j,j}^{(i)} \leftarrow \frac{1}{\hat{L}}\left(1 - \sum_{k=1}^{\hat{L}} \frac{\boldsymbol{y}_j^T \boldsymbol{z}_j^{(k)}}{\boldsymbol{y}_j^T \boldsymbol{y}_j}\right) + \frac{\boldsymbol{y}_j^T \boldsymbol{z}_j^{(i)}}{\boldsymbol{y}_j^T \boldsymbol{y}_j}$.
    end for
    $Y \leftarrow \mathcal{P}(\sum_i Z_i)$.
    $t \leftarrow t + 1$
  until convergence or $t > T_{max}$
  $\hat{I} \leftarrow \mathcal{S}^{-1}(Y)$
Output: Inpainted image $\hat{I}$.

III. NUMERICAL EXAMPLES

This section presents numerical examples for the proposed algorithm. We utilize 4 kinds of 256×256 RGB images in the standard image database. The algorithms were applied to RGB layers of the image independently. We use $\hat{L} = 2$, $\lambda = 0.2$ and $T_{max} = 100$ in Algorithm 2.

Figure 1 and 2 show the results of inpainting by the proposed algorithm, our structured rank minimization method [6], and Wexler method [2], which is popular algorithm in exemplar based inpainting methods. Table I and II show the performance of the algorithms evaluated by PSNR and SSIM. We can see that the proposed algorithm achieves the best performance of all algorithms.

Table I. Performance of the algorithms (PSNR)

PSNR	Lenna	Mandrill	Parrots	Pepper
Rank min. single model [6]	34.60	33.41	37.99	36.04
Wexler's algorithm (exemplar)	33.68	33.35	36.04	34.55
Proposed based on GPCA	36.54	34.30	38.31	38.33

Table II. Performance of the algorithms (SSIM)

SSIM	Lenna	Mandrill	Parrots	Pepper
Rank min. single model [6]	0.9795	0.9716	0.9878	0.9855
Wexler's algorithm (exemplar)	0.9816	0.9717	0.9866	0.9843
Proposed based on GPCA	0.9858	0.9775	0.9906	0.9893

IV. CONCLUSION

This paper proposes a GPCA and low rank optimization based image inpainting algorithm. In order to model an image as a position variant linear system, we introduce the concept of GPCA and proposed a new inpatinting algorithm assuming that a image can be modeled by multiple AR models. The proposed inpainting algorithm is formulated as a multiple matrix rank minimization problem , which is difficult to solve. This paper modifies the IPMS algorithm to obtain approximate solution and proposes the GPCA based image inpainting algorithm. Numerical examples show that the effectiveness of the proposed algorithm.

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V. REFERENCES

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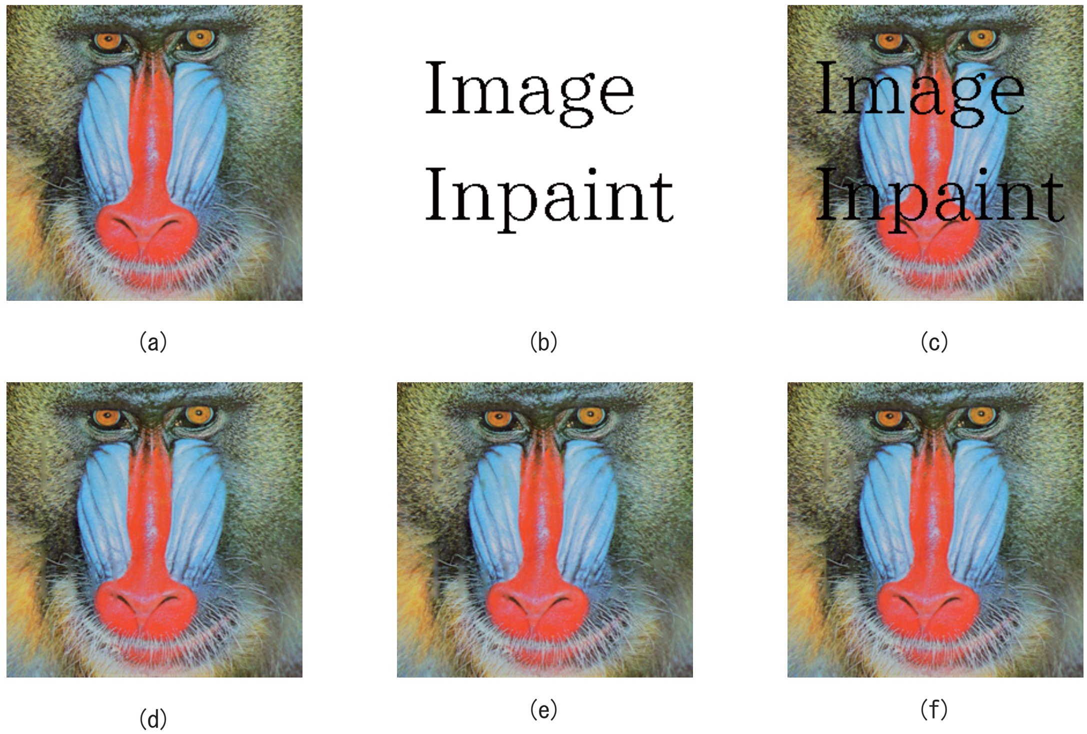

Fig. 1. Results of ‘Mandrill’ image. (a) original image, (b) missing mask, (c) observed image, (d) proposed algorithm, (e) rank minimization approach [6] and (f) Wexler’s exemplar algorithm.

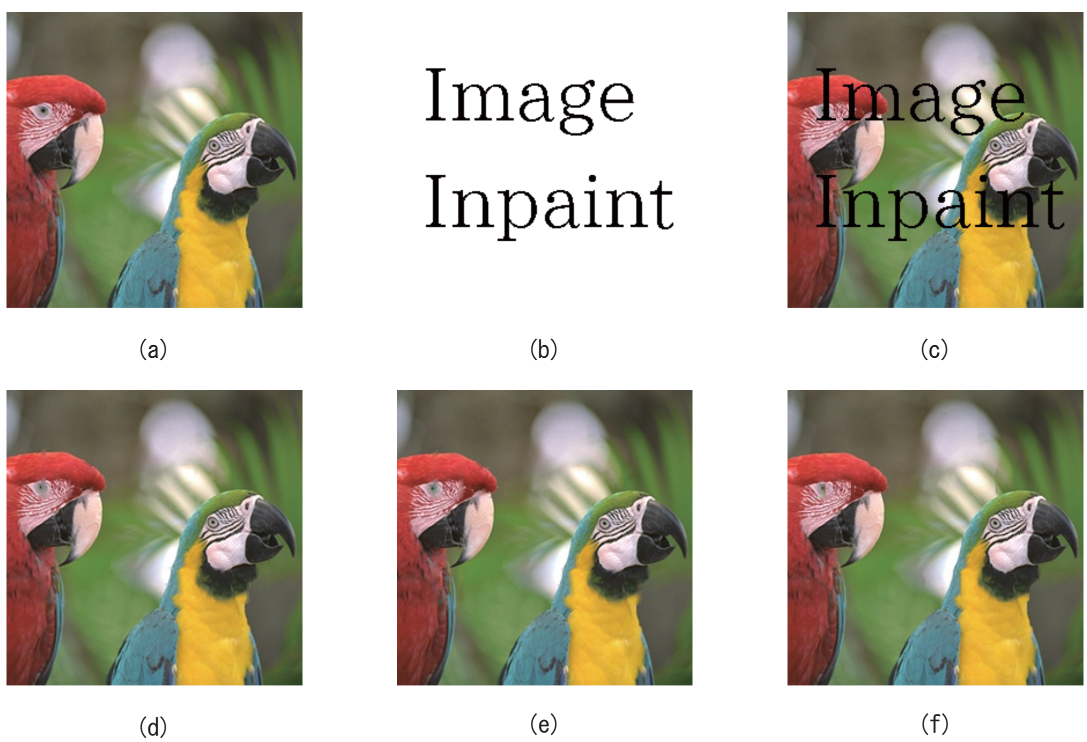

Fig. 2. Results of ‘Parrots’ image. (a) original image, (b) missing mask, (c) observed image, (d) proposed algorithm, (e) rank minimization approach [6] and (f) Wexler’s exemplar algorithm.

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