Exercise 9 : The pure deformation flow: stagnation flow –

A meteorologist and an astrophysicst are collaborating in a joint project of common interest. They encounter a flow field of the form $\mathbf{v} = v_x\,\mathbf{e}_x + v_y\,\mathbf{e}_y$ with

$$\begin{cases} v_x &= -\dfrac{x}{\tau}\,, \\ v_y &= \dfrac{y}{\tau}\,, \end{cases} \tag{1.25}$$

and τ a positive constant. They realize that this is what is called a *stagnation point flow*. On the other hand, they also realize that this is basically a particular case of the pure deformation flow studied in this section.

(a) In the section we said that the trajectory $\mathbf{x}_L(t)$ of an individual fluid element moving in any generic flow map $\mathbf{v}(\mathbf{x}, t)$ is given by

$$\frac{d\mathbf{x}_L}{dt} = \mathbf{v}[\mathbf{x}_L(t), t]\ . \tag{1.26}$$

Calculate, for the flow map (1.25), the trajectory of a generic fluid element that at time $t = 0$ is located at (x_0, y_0), with $x_0 \neq 0$ and $y_0 \neq 0$. Does the fluid element move towards the x-axis? And towards the y-axis? Which physical role does τ play in that motion?

(b) A streamline is defined as a curve that is everywhere tangent to the velocity vector. Calculate the generic expression for the streamlines $y(x)$ of the flow (1.25). Then, draw (approximately) a second diagram with various streamlines, checking if they match the velocity vectors you drew in (a). Check also if the streamlines you have calculated are the same curves as the trajectories calculated in (b) or not. Should they coincide?

(c) The dynamics of the system is such that the pressure field is given by $p(x, y) = p_0 + A\,y^2$, with p_0 and A two positive constants. Using the trajectories obtained in (b), calculate the time change of the pressure measured by an individual fluid element along its motion. Then calculate the Lagrange derivative of p. Are those values different or the same? Should they be? Give a physical explanation for all results.