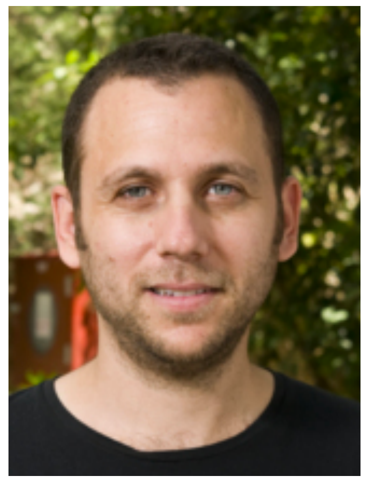

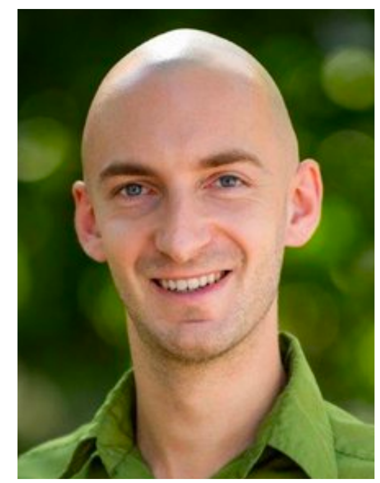

ITAI ARIELI (TECHNION)
YAKOV BABICHENKO (TECHNION)
FEDOR SANDOMIRSKIY (CALTECH)

PERSUASION AS TRANSPORTATION

N-AGENT PERSUASION

HOW TO SUPPLY INFORMATION OPTIMALLY TO MULTIPLE AGENTS?

today:
two agents, binary state

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information structure

$P(s_1, s_2 \mid \theta)$

2 receivers

state

$$\theta = \begin{cases} h & \text{prob. } p \\ \ell & \text{prob. } 1-p \end{cases}$$

$s_1 \in S_1$

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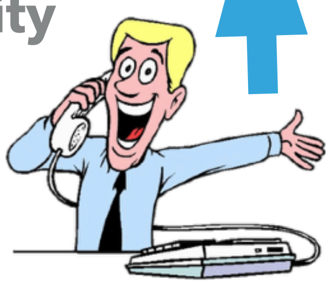

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- $N = 1$ **is easy:** sender's value $= \mathrm{cav}[u](p)$
  - Kamenica, Gentzkow (2011)
- $N \geq 2$ **is hard:** feasible distributions can be complex
  - Arieli, Babichenko, Sandomirskiy, Tamuz (2021), Brooks, Frankel, Kamenica (2022)

OUR CONTRIBUTION

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CONDITIONING ON THE STATE SIMPLIFIES THE PROBLEM

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DEFINITION

μ^ℓ and μ^h on $[0,1]^2$ is a feasible pair of conditional distributions

$\iff$ $\exists$ information structure s.t. $(p_1', p_2') \sim \mu^\theta$ conditional on θ

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MULTI-AGENT PERSUASION = OPTIMAL TRANSPORTATION PROBLEM!

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- **Archetypal coupling problem, many econ applications:**
  - Daskalakis et al. (2017), Kleiner, Manelli (2019), Boerma et al.(2021), Chiapporiet et al. (2010), Galichon (2021), Steinerberger, Tsyvinski (2019), Gensbittel (2015), Guo, Shmaya (2021), Cieslak, Malamud, Schrimpf (2011)

PERSUASION AS TRANSPORT

THEOREM

Value of a persuasion problem (p, u^ℓ, u^h) equals

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- **tractable dual extending 1-receiver results:**
  - $\mathrm{cav}[u]$-theorem by Kamenica, Gentzkow (2011) and duality by Dworczak, Kolotilin (2017)

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  - Gives a class of problems where full-information/partial-information signals are optimal

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THANK YOU!