Power Analysis for Gift Exchange Experiment

```r
suppressMessages({
  library(tidyverse)
  library(fixest)
  library(pwr)
})

set.seed(42)

cost <- function(e) {
  case_when(
    e == 0.1 ~ 0,
    e == 0.2 ~ 1,
    e == 0.3 ~ 2,
    e == 0.4 ~ 4,
    e == 0.5 ~ 6,
    e == 0.6 ~ 8,
    e == 0.7 ~ 10,
    e == 0.8 ~ 12,
    e == 0.9 ~ 15,
    e == 1.0 ~ 28,
    TRUE ~ NA
  )
}


grounds <- read_csv("../data/generated/gift_rounds.csv", show_col_types = FALSE) %>%
  mutate(
    experiment = factor(ifelse(
      experiment == "fgift",
      "Business Framing", "Neutral Framing"
    ), c("Neutral Framing", "Business Framing"))
```

```
  )

part <- grounds %>%
  group_by(experiment, session_code, group_id) %>%
  summarise(
    payoff_1 = sum((100 - wage)*effort),
    payoff_2 = sum(wage - cost(effort)),
    .groups = "drop"
  ) %>%
  pivot_longer(
    c(payoff_1, payoff_2), values_to = "payoff", names_to = "player_id",
    names_prefix = "payoff_", names_transform = as.integer
  )
```

Descriptive statistics of pretest data to standardize the power tests

```
mn_wage_start <- mean(grounds$wage[grounds$round == 1])
sd_wage <- sd(grounds$wage)
mn_effort_start <- mean(grounds$effort[grounds$round == 1])
sd_effort <- sd(grounds$effort)

sprintf(
  "Mean wage start (SD all): %.2f (%.2f)", mn_wage_start, sd_wage
)
```

```
[1] "Mean wage start (SD all): 50.00 (17.68)"
```

```
sprintf(
  "Mean effort start (SD all) sent: %.2f (%.2f)", mn_effort_start, sd_effort
)
```

```
[1] "Mean effort start (SD all) sent: 0.47 (0.04)"
```

```
# Some pretest regressions to see how rounds affect our DVs:

ols_wage <- lm(wage ~ round, data = grounds)
summary(ols_wage)
```

```
Call:
lm(formula = wage ~ round, data = grounds)

Residuals:
    Min      1Q  Median      3Q     Max
-30.861 -12.397   3.245  11.961  30.676

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  43.9333     3.4072  12.894  < 2e-16 ***
round         2.8212     0.5491   5.138 1.42e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.77 on 98 degrees of freedom
Multiple R-squared:  0.2122,    Adjusted R-squared:  0.2042
F-statistic:  26.4 on 1 and 98 DF,  p-value: 1.417e-06
```

```r
ols_effort <- lm(effort ~ round, data = grounds)
summary(ols_effort)
```

```
Call:
lm(formula = effort ~ round, data = grounds)

Residuals:
      Min        1Q    Median        3Q       Max
-0.192727 -0.011818 -0.003636  0.002500  0.104545

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.490000   0.008766   55.90   <2e-16 ***
round       0.002727   0.001413    1.93   0.0564 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.04058 on 98 degrees of freedom
Multiple R-squared:  0.03664,   Adjusted R-squared:  0.0268
F-statistic: 3.727 on 1 and 98 DF,  p-value: 0.05644
```

```r
ols_effort_wage <- lm(effort ~ wage, data = grounds)
summary(ols_effort_wage)
```

```
Call:
lm(formula = effort ~ wage, data = grounds)

Residuals:
      Min        1Q    Median        3Q       Max
-0.198503 -0.012253  0.001497  0.008372  0.098059

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.4641272  0.0139199  33.343  < 2e-16 ***
wage        0.0006875  0.0002245   3.062  0.00284 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0395 on 98 degrees of freedom
Multiple R-squared:  0.08732,   Adjusted R-squared:  0.07801
F-statistic: 9.377 on 1 and 98 DF,  p-value: 0.002837
```

```r
summary(lm(effort ~ wage*experiment, data = grounds))
```

```
Call:
lm(formula = effort ~ wage * experiment, data = grounds)

Residuals:
      Min        1Q    Median        3Q       Max
-0.193794 -0.014617  0.005495  0.006206  0.096911

Coefficients:
                                 Estimate Std. Error t value Pr(>|t|)
(Intercept)                     4.973e-01  2.633e-02  18.887   <2e-16 ***
wage                           -7.109e-05  5.464e-04  -0.130    0.897
experimentBusiness Framing     -3.653e-02  3.858e-02  -0.947    0.346
wage:experimentBusiness Framing  8.396e-04  6.684e-04   1.256    0.212
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 0.03942 on 96 degrees of freedom
Multiple R-squared:  0.1094,    Adjusted R-squared:  0.08154
F-statistic:  3.93 on 3 and 96 DF,  p-value: 0.01081
```

```r
summary(feols(
  effort ~ wage*experiment | round,
  cluster = c("round", "group_id"),
  data = grounds
))
```

```
OLS estimation, Dep. Var.: effort
Observations: 100
Fixed-effects: round: 10
Standard-errors: Clustered (round & group_id)
                                Estimate Std. Error   t value   Pr(>|t|)
wage                            0.000102   0.000083   1.22499 2.8778e-01
experimentBusiness Framing     -0.000545   0.000010 -52.75127 7.7300e-07 ***
wage:experimentBusiness Framing 0.000279   0.000197   1.41511 2.2996e-01
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 0.036878     Adj. R2: 0.076097
                 Within R2: 0.087277
```

```r
fe_wage <- feols(
  wage ~ experiment | round, cluster = c("round", "group_id"),
  data = grounds
)
summary(fe_wage)
```

```
OLS estimation, Dep. Var.: wage
Observations: 100
Fixed-effects: round: 10
Standard-errors: Clustered (round & group_id)
                           Estimate Std. Error t value  Pr(>|t|)
experimentBusiness Framing     24.7    4.72028 5.23274 0.0063718 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 9.36763     Adj. R2: 0.684559
                Within R2: 0.634783
```

```r
fixef(fe_wage)$round
```

```
    1     2     3     4     5     6     7     8     9    10
37.65 36.15 40.15 42.65 44.15 45.15 50.15 55.15 59.65 60.15
```

```r
# Clearly not exponential but decreasing positive trend for rounds.
# Sticking to a linear trend for the estimation.

summary(lm(wage ~ round*experiment, data = grounds))
```

```
Call:
lm(formula = wage ~ round * experiment, data = grounds)

Residuals:
     Min       1Q   Median       3Q      Max
-21.4909  -3.9470   0.0758   4.7727  26.0667

Coefficients:
                                 Estimate Std. Error t value Pr(>|t|)
(Intercept)                       40.1333     2.6333  15.241  < 2e-16 ***
round                              1.2667     0.4244   2.985   0.0036 **
experimentBusiness Framing         7.6000     3.7241   2.041   0.0440 *
round:experimentBusiness Framing   3.1091     0.6002   5.180 1.22e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.62 on 96 degrees of freedom
Multiple R-squared:  0.7695,    Adjusted R-squared:  0.7623
F-statistic: 106.8 on 3 and 96 DF,  p-value: < 2.2e-16
```

```r
fe_effort <- feols(
  effort ~ experiment | round, cluster = c("round", "group_id"),
  data = grounds
)
summary(fe_effort)
```

```
OLS estimation, Dep. Var.: effort
Observations: 100
```

```
Fixed-effects: round: 10
Standard-errors: Clustered (round & group_id)
                           Estimate Std. Error t value Pr(>|t|)
experimentBusiness Framing    0.022   0.016069 1.36911  0.24281
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 0.037     Adj. R2: 0.090854
              Within R2: 0.081208
```

```
fixef(fe_effort)$round
```

```
    1     2     3     4     5     6     7     8     9    10
0.459 0.479 0.509 0.499 0.499 0.499 0.499 0.499 0.499 0.499
```

```
# No round trend in pct_returned based on pretest data
```

Equation-based Power Analysis based on Pretest Data

Round Based Analysis

```
pwr.t.test(500, 0.1*mn_wage_start/sd_wage)
```

```

     Two-sample t test power calculation

              n = 500
              d = 0.2828096
      sig.level = 0.05
          power = 0.9939178
    alternative = two.sided

NOTE: n is number in *each* group
```

```
pwr.t.test(d = 0.1*mn_wage_start/sd_wage, power = 0.8)
```

```
     Two-sample t test power calculation

              n = 197.2325
              d = 0.2828096
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number in *each* group
```

```
pwr.t.test(500, 0.1*mn_effort_start/sd_effort)
```

```
     Two-sample t test power calculation

              n = 500
              d = 1.142636
      sig.level = 0.05
          power = 1
    alternative = two.sided

NOTE: n is number in *each* group
```

```
pwr.t.test(d = 0.1*mn_effort_start/sd_effort, power = 0.8)
```

```
     Two-sample t test power calculation

              n = 13.05699
              d = 1.142636
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number in *each* group
```

```
pr <- pwr.t.test(n = 500, power = 0.8)
sprintf("MDE Wage: %.2f", pr$d * sd_wage)
```

```
[1] "MDE Wage: 3.14"
```

```
sprintf("MDE Effort: %.2f", pr$d * sd_effort)
```

```
[1] "MDE Effort: 0.01"
```

Participant and Dyad Based Analysis

```
mn_payoff_part <- mean(part$payoff)
sd_payoff_part <- sd(part$payoff)
sprintf("Mean (SD) of part payoff: %.2f (%.2f)", mn_payoff_part, sd_payoff_part)
```

```
[1] "Mean (SD) of part payoff: 368.07 (198.99)"
```

```
pwr.t.test(100, (0.1*mn_payoff_part)/sd_payoff_part)
```

```

     Two-sample t test power calculation

              n = 100
              d = 0.1849716
      sig.level = 0.05
          power = 0.2557083
    alternative = two.sided

NOTE: n is number in *each* group
```

```
pr <- pwr.t.test(n = 100, power = 0.8)
sprintf(
  "MDE Payoff part: %.2f (%.1f %% of mean)", pr$d * sd_payoff_part,
  100*(pr$d * sd_payoff_part)/mn_payoff_part
)
```

```
[1] "MDE Payoff part: 79.23 (21.5 % of mean)"
```

```r
dyads <- part %>%
  group_by(experiment, session_code, group_id) %>%
  summarise(sum_payoff = sum(payoff),.groups = "drop")

mn_payoff_dyads <- mean(dyads$sum_payoff)
sd_payoff_dyads <- sd(dyads$sum_payoff)
sprintf("Mean (SD) of dyad payoff: %.2f (%.2f)", mn_payoff_dyads, sd_payoff_dyads)
```

```
[1] "Mean (SD) of dyad payoff: 736.15 (70.22)"
```

```r
pwr.t.test(50, (0.1*mn_payoff_dyads)/sd_payoff_dyads)
```

```

     Two-sample t test power calculation

              n = 50
              d = 1.048414
      sig.level = 0.05
          power = 0.9993813
    alternative = two.sided

NOTE: n is number in *each* group
```

```r
pr <- pwr.t.test(n = 50, power = 0.8)
sprintf(
  "MDE Payoff dyads: %.2f (%.1f %% of mean)", pr$d * sd_payoff_dyads,
  100*(pr$d * sd_payoff_dyads)/mn_payoff_dyads
)
```

```
[1] "MDE Payoff dyads: 39.73 (5.4 % of mean)"
```

Simulation for regression based tests

```r
if (file.exists("../data/generated/giftex_sim_results.csv")) {
  giftex_sim_results <- read_csv("../data/generated/giftex_sim_results.csv", show_col_type
} else {
```

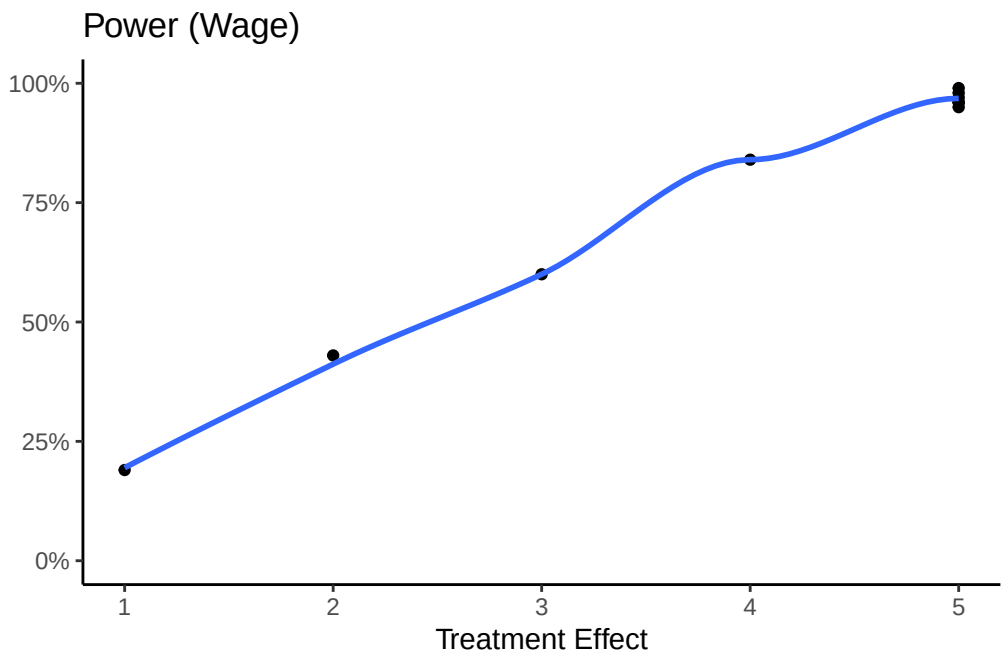
```r
sim_data <- function(parms, runs = 50, rounds = 10) {
  cl <- function(val, vmin = 0, vmax = 100) {
    if (val > vmax) return(as.integer(vmax))
    if (val < vmin) return(as.integer(vmin))
    as.integer(round(val))
  }
  el <- function(x) {
    steps <- seq(0.1, 1, by = 0.1)
    steps[which.min(abs(steps - x))][1]
  }
  gr <- function(rd, exp, g, parms) {
    wage_start = ifelse(
      exp == "fgift",
      parms$wage_start + parms$wage_start_teffect,
      parms$wage_start
    )
    wage_grate = ifelse(
      exp == "fgift",
      parms$wage_grate + parms$wage_grate_teffect,
      parms$wage_grate
    )
    effort_start = ifelse(
      exp == "fgift",
      parms$effort_start + parms$effort_start_teffect,
      parms$effort_start
    )
    effort_grate = ifelse(
      exp == "fgift",
      parms$effort_grate + parms$effort_grate_teffect,
      parms$effort_grate
    )
    effort_wage = ifelse(
      exp == "fgift",
      parms$effort_wage + parms$effort_wage_teffect,
      parms$effort_wage
    )

    tibble(
      experiment = factor(ifelse(
        exp == "fgift",
        "Business Framing", "Neutral Framing"
```

```
      ), c("Neutral Framing", "Business Framing")),
      group_id = g,
      round = rd,
      wage = cl(
        wage_start + (rd-1)*wage_grate + rnorm(1, 0, parms$wage_evar),
        0, 100
      ),
      effort = el(
        effort_start + effort_wage*wage + (rd-1)*effort_grate +
            rnorm(1, 0, parms$effort_evar)
      )
    )
  }
  bind_rows(
    lapply(
      c("gift", "fgift"),
      function(e) bind_rows(
        lapply(
          1:runs,
          function(g) bind_rows(lapply(1:rounds, gr, e, g, parms))
        )
      )
    )
  )
}

run_giftex_sim <- function(te) {
  parms <- tibble(
    wage_start = mn_wage_start,
    wage_start_teffect = te$teffect_wage,
    wage_grate = coef(ols_wage)[2],
    wage_grate_teffect = te$teffect_wage_grate,
    wage_evar = sd_wage,
    effort_start = mn_effort_start,
    effort_start_teffect = te$teffect_effort,
    effort_grate = coef(ols_effort)[2],
    effort_grate_teffect = te$teffect_effort_grate,
    effort_wage = coef(ols_effort_wage)[2],
    effort_wage_teffect = te$teffect_effort_wage,
    effort_evar = sd_effort
  )
```

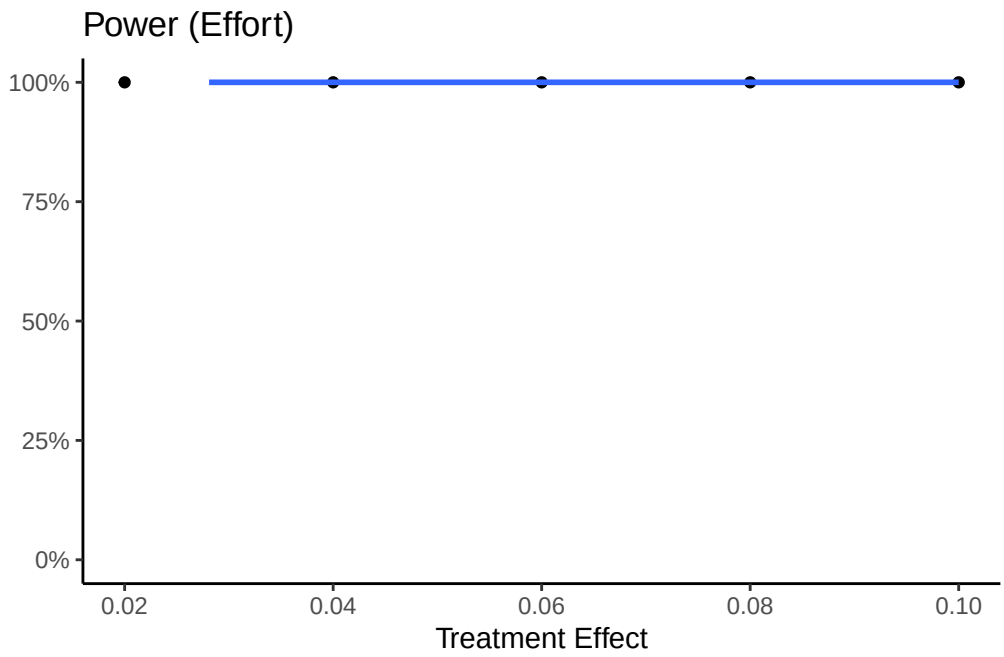
```
  smp <- sim_data(parms)
  ci_wage_fe <- confint(
    feols(wage ~ experiment | round, cluster = c("round", "group_id"), data = smp)
  )
  ci_wage_round_fe <- confint(
    feols(wage ~ experiment*round, cluster = c("round", "group_id"), data = smp)
  )
  ci_effort_fe <- confint(
    feols(effort ~ experiment | round, cluster = c("round", "group_id"), data = smp)
  )
  ci_effort_round_fe <- confint(
    feols(effort ~ experiment*round, cluster = c("round", "group_id"), data = smp)
  )
  ci_effort_wage_fe <- confint(
    feols(effort ~ experiment*wage | round, cluster = c("round", "group_id"), data = smp
  )
  tibble(
    wage_teffect_lb = pull(ci_wage_fe[1]),
    wage_teffect_ub = pull(ci_wage_fe[2]),
    wage_round_teffect_lb = ci_wage_round_fe[4, 1],
    wage_round_teffect_ub = ci_wage_round_fe[4, 2],
    effort_teffect_lb = pull(ci_effort_fe[1]),
    effort_teffect_ub = pull(ci_effort_fe[2]),
    effort_round_teffect_lb = ci_effort_round_fe[4, 1],
    effort_round_teffect_ub = ci_effort_round_fe[4, 2],
    effort_wage_teffect_lb = ci_effort_wage_fe[4, 1],
    effort_wage_teffect_ub = ci_effort_wage_fe[4, 2],
  )
}

sim_power_giftex <- function(plan) {
  sim_results <- bind_rows(
    lapply(
      1:nrow(plan),
      function(x) {
        message(
          sprintf("Running giftex sim, plan row %d of %d...", x, nrow(plan)),
          appendLF = F
        )
        rv <- bind_cols(plan[x,], run_giftex_sim(plan[x,]))
        message("")
```

```
          rv
        }
      )
    )
  }
  plan <- bind_rows(
    expand_grid(
      n = 1:100,
      tibble(
        teffect_wage = 1:5,
        teffect_effort = c(0.02, 0.04, 0.06, 0.08, 0.1)
      ),
      teffect_wage_grate = 0,
      teffect_effort_grate = 0,
      teffect_effort_wage = 0,
    ),
    expand_grid(
      n = 1:100,
      tibble(
        teffect_wage = 0,
        teffect_wage_grate = c(0.5, 1, 1.5, 2, 2.5),
        teffect_effort = 0,
        teffect_effort_grate = c(0.002, 0.004, 0.006, 0.008, 0.01),
        teffect_effort_wage = 0
      )
    ),
    expand_grid(
      n = 1:100,
      teffect_wage = c(0, 5),
      teffect_wage_grate = 0,
      teffect_effort = 0,
      teffect_effort_grate = 0,
      teffect_effort_wage = c(0.0002, 0.0004, 0.0006, 0.0008, 0.001)
    )
  )

  message(sprintf("Starting giftex power simulations (%d runs): %s", nrow(plan), Sys.time(
  giftex_sim_results <- sim_power_giftex(plan)
  write_csv(giftex_sim_results, "../data/generated/giftex_sim_results.csv")
  message(sprintf("Done: %s", Sys.time()))
}
```

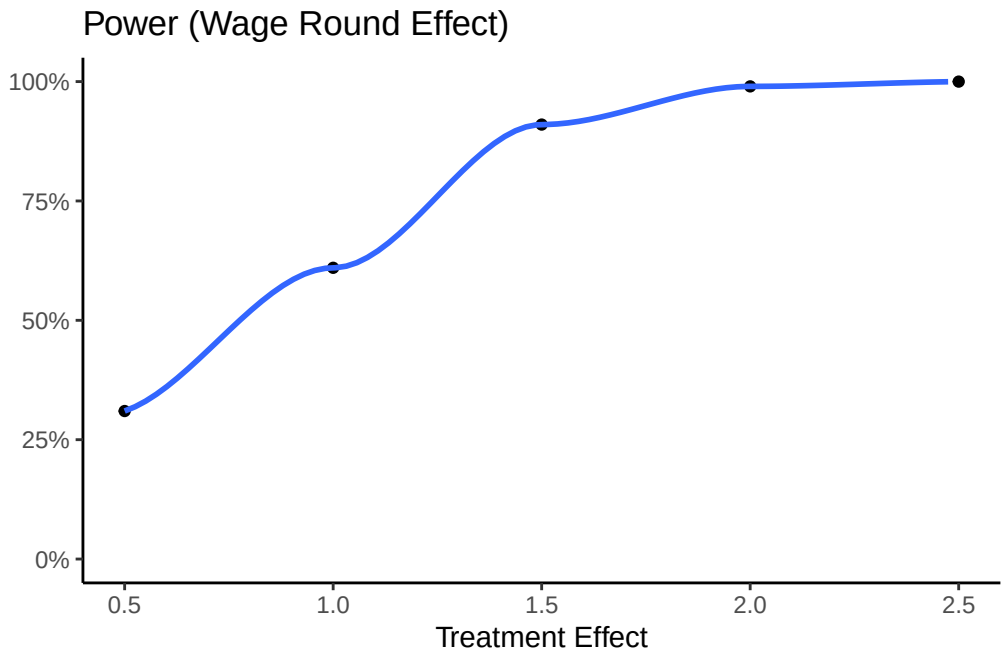
```r
giftex_power <- giftex_sim_results %>%
  group_by(
    teffect_wage, teffect_wage_grate,
    teffect_effort, teffect_effort_grate,
    teffect_effort_wage
  ) %>%
  summarise(
    power_wage = mean(wage_teffect_lb > 0),
    power_wage_round = mean(wage_round_teffect_lb > 0),
    power_effort = mean(effort_teffect_lb > 0),
    power_effort_round = mean(effort_round_teffect_lb > 0),
    power_effort_wage = mean(effort_round_teffect_lb > 0),
    .groups = "drop"
  )

ggplot(
  giftex_power %>% filter(teffect_wage != 0),
  aes(x = teffect_wage, y = power_wage)
) + geom_point() + geom_smooth(se = FALSE) +
  scale_y_continuous(limits = c(0, 1), labels = scales::percent) +
  labs(title = "Power (Wage)", x = "Treatment Effect", y = "") +
  theme_classic()
```



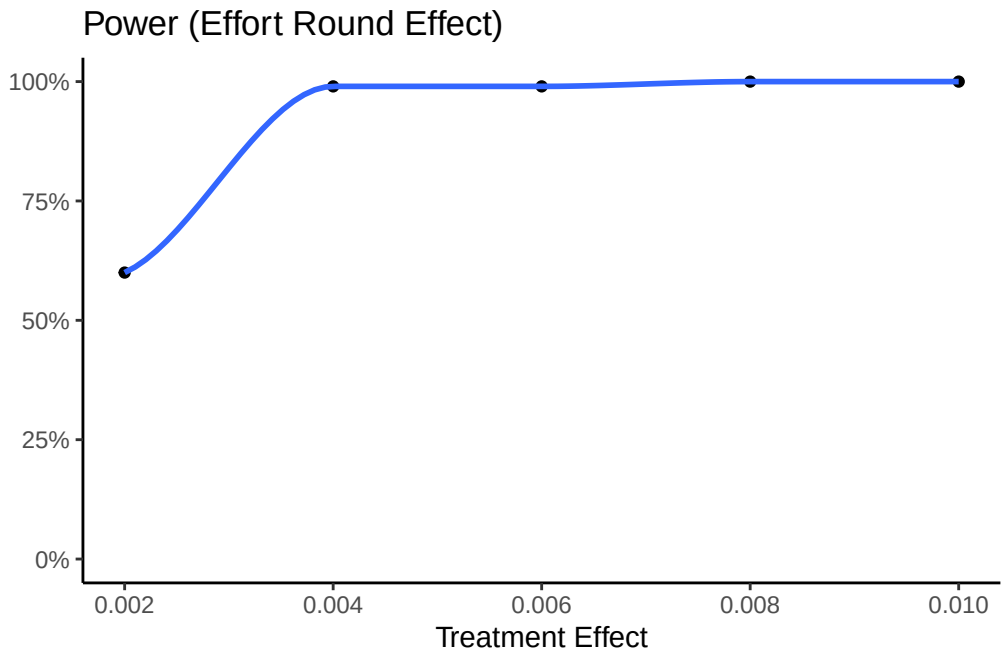
```r
ggplot(
  giftex_power %>% filter(teffect_effort != 0),
  aes(x = teffect_effort, y = power_effort)
) + geom_point() + geom_smooth(se = FALSE) +
  scale_y_continuous(limits = c(0, 1), labels = scales::percent) +
  labs(title = "Power (Effort)", x = "Treatment Effect", y = "") +
  theme_classic()
```



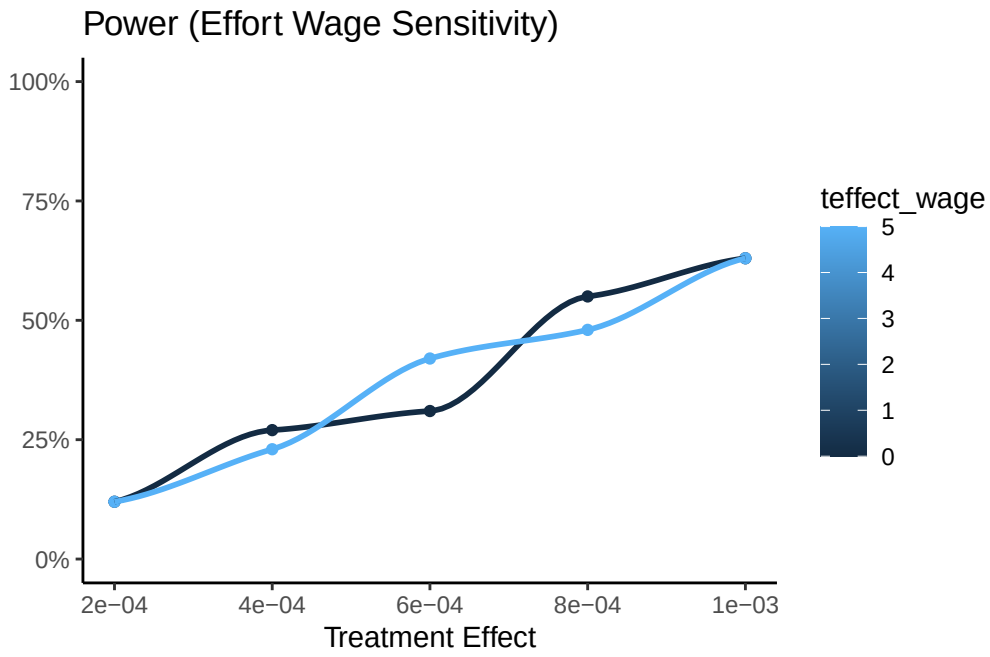

```r
ggplot(
  giftex_power %>% filter(teffect_wage_grate != 0),
  aes(x = teffect_wage_grate, y = power_wage_round)
) + geom_point() + geom_smooth(se = FALSE) +
  scale_y_continuous(limits = c(0, 1), labels = scales::percent) +
  labs(title = "Power (Wage Round Effect)", x = "Treatment Effect", y = "") +
  theme_classic()
```




```
ggplot(
  giftex_power %>% filter(teffect_effort_grate != 0),
  aes(x = teffect_effort_grate, y = power_effort_round)
) + geom_point() + geom_smooth(se = FALSE) +
  scale_y_continuous(limits = c(0, 1), labels = scales::percent) +
  labs(title = "Power (Effort Round Effect)", x = "Treatment Effect", y = "") +
  theme_classic()
```




```
ggplot(
  giftex_power %>% filter(teffect_effort_wage != 0),
  aes(
    x = teffect_effort_wage, y = power_effort_wage,
    group = teffect_wage, color = teffect_wage
  )
) + geom_point() + geom_smooth() +
  scale_y_continuous(limits = c(0, 1), labels = scales::percent) +
  labs(title = "Power (Effort Wage Sensitivity)", x = "Treatment Effect", y = "") +
  theme_classic()
```


Power (Effort Wage Sensitivity)
100%
75%
50%
25%
0%
2e−04
4e−04
6e−04
8e−04
1e−03
Treatment Effect
teffect_wage
5
4
3
2
1
0