

# Power Analysis for Gift Exchange Experiment

```
suppressMessages({
  library(tidyverse)
  library(fixest)
  library(pwr)
})

DVERSION_GIFTEX <- "2024-04-30"

set.seed(42)

cost <- function(e) {
  case_when(
    e == 0.1 ~ 0,
    e == 0.2 ~ 1,
    e == 0.3 ~ 2,
    e == 0.4 ~ 4,
    e == 0.5 ~ 6,
    e == 0.6 ~ 8,
    e == 0.7 ~ 10,
    e == 0.8 ~ 12,
    e == 0.9 ~ 15,
    e == 1.0 ~ 28,
    TRUE ~ NA
  )
}

grounds <- read_csv(
  sprintf("../data/generated/giftex_%s_rounds.csv", DVERSION_GIFTEX),
  show_col_types = FALSE
) %>%
```

```

mutate(
  experiment = factor(ifelse(
    experiment == "fgiftex",
    "Business Framing", "Neutral Framing"
  ), c("Neutral Framing", "Business Framing"))
)

part <- grounds %>%
  group_by(experiment, session_code, group_id) %>%
  summarise(
    payoff_1 = sum((100 - wage)*effort),
    payoff_2 = sum(wage - cost(effort)),
    .groups = "drop"
  ) %>%
  pivot_longer(
    c(payoff_1, payoff_2), values_to = "payoff", names_to = "player_id",
    names_prefix = "payoff_", names_transform = as.integer
  )

```

## Descriptive statistics of pretest data to standardize the power tests

```

mn_wage_start <- mean(grounds$wage[grounds$round == 1])
sd_wage <- sd(grounds$wage)
mn_effort_start <- mean(grounds$effort[grounds$round == 1])
sd_effort <- sd(grounds$effort)

sprintf(
  "Mean wage start (SD all): %.2f (%.2f)", mn_wage_start, sd_wage
)

```

```
[1] "Mean wage start (SD all): 50.00 (17.68)"
```

```

sprintf(
  "Mean effort start (SD all) sent: %.2f (%.2f)", mn_effort_start, sd_effort
)

```

```
[1] "Mean effort start (SD all) sent: 0.47 (0.04)"
```

```
# Some pretest regressions to see how rounds affect our DVs:
```

```
ols_wage <- lm(wage ~ round, data = grounds)
summary(ols_wage)
```

Call:

```
lm(formula = wage ~ round, data = grounds)
```

Residuals:

Min	1Q	Median	3Q	Max
-30.861	-12.397	3.245	11.961	30.676

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	43.9333	3.4072	12.894	< 2e-16 ***
round	2.8212	0.5491	5.138	1.42e-06 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.77 on 98 degrees of freedom

Multiple R-squared: 0.2122, Adjusted R-squared: 0.2042

F-statistic: 26.4 on 1 and 98 DF, p-value: 1.417e-06

```
ols_effort <- lm(effort ~ round, data = grounds)
summary(ols_effort)
```

Call:

```
lm(formula = effort ~ round, data = grounds)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.192727	-0.011818	-0.003636	0.002500	0.104545

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.490000	0.008766	55.90	<2e-16 ***
round	0.002727	0.001413	1.93	0.0564 .

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.04058 on 98 degrees of freedom  
Multiple R-squared: 0.03664, Adjusted R-squared: 0.0268  
F-statistic: 3.727 on 1 and 98 DF, p-value: 0.05644

```
ols_effort_wage <- lm(effort ~ wage, data = grounds)
summary(ols_effort_wage)
```

Call:

```
lm(formula = effort ~ wage, data = grounds)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-0.198503	-0.012253	0.001497	0.008372	0.098059

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.4641272	0.0139199	33.343	< 2e-16 ***
wage	0.0006875	0.0002245	3.062	0.00284 **

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0395 on 98 degrees of freedom  
Multiple R-squared: 0.08732, Adjusted R-squared: 0.07801  
F-statistic: 9.377 on 1 and 98 DF, p-value: 0.002837

```
summary(lm(effort ~ wage*experiment, data = grounds))
```

Call:

```
lm(formula = effort ~ wage * experiment, data = grounds)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-0.193794	-0.014617	0.005495	0.006206	0.096911

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
--	----------	------------	---------	----------

(Intercept)	4.973e-01	2.633e-02	18.887	<2e-16 ***
wage	-7.109e-05	5.464e-04	-0.130	0.897
experimentBusiness Framing	-3.653e-02	3.858e-02	-0.947	0.346
wage:experimentBusiness Framing	8.396e-04	6.684e-04	1.256	0.212

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.03942 on 96 degrees of freedom

Multiple R-squared: 0.1094, Adjusted R-squared: 0.08154

F-statistic: 3.93 on 3 and 96 DF, p-value: 0.01081

```
summary(feols(
  effort ~ wage*experiment | round,
  cluster = c("round", "group_id"),
  data = grounds
))
```

OLS estimation, Dep. Var.: effort

Observations: 100

Fixed-effects: round: 10

Standard-errors: Clustered (round & group\_id)

	Estimate	Std. Error	t value	Pr(> t )
wage	0.000102	0.000083	1.22499	2.8778e-01
experimentBusiness Framing	-0.000545	0.000010	-52.75127	7.7300e-07 ***
wage:experimentBusiness Framing	0.000279	0.000197	1.41511	2.2996e-01

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

RMSE: 0.036878 Adj. R2: 0.076097

Within R2: 0.087277

```
fe_wage <- feols(
  wage ~ experiment | round, cluster = c("round", "group_id"),
  data = grounds
)
summary(fe_wage)
```

OLS estimation, Dep. Var.: wage

Observations: 100

Fixed-effects: round: 10

Standard-errors: Clustered (round & group\_id)

```

              Estimate Std. Error t value Pr(>|t|)
experimentBusiness Framing    24.7    4.72028 5.23274 0.0063718 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 9.36763      Adj. R2: 0.684559
              Within R2: 0.634783

```

```
fixef(fe_wage)$round
```

```

      1      2      3      4      5      6      7      8      9     10
37.65 36.15 40.15 42.65 44.15 45.15 50.15 55.15 59.65 60.15

```

```

# Clearly not exponential but decreasing positive trend for rounds.
# Sticking to a linear trend for the estimation.

```

```
summary(lm(wage ~ round*experiment, data = grounds))
```

Call:

```
lm(formula = wage ~ round * experiment, data = grounds)
```

Residuals:

```

      Min       1Q   Median       3Q      Max
-21.4909  -3.9470   0.0758   4.7727  26.0667

```

Coefficients:

```

              Estimate Std. Error t value Pr(>|t|)
(Intercept)    40.1333     2.6333  15.241 < 2e-16 ***
round           1.2667     0.4244   2.985  0.0036 **
experimentBusiness Framing    7.6000     3.7241   2.041  0.0440 *
round:experimentBusiness Framing  3.1091     0.6002   5.180 1.22e-06 ***
---

```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 8.62 on 96 degrees of freedom

Multiple R-squared: 0.7695, Adjusted R-squared: 0.7623

F-statistic: 106.8 on 3 and 96 DF, p-value: < 2.2e-16

```
fe_effort <- feols(
  effort ~ experiment | round, cluster = c("round", "group_id"),
  data = grounds
)
summary(fe_effort)
```

```
OLS estimation, Dep. Var.: effort
Observations: 100
Fixed-effects: round: 10
Standard-errors: Clustered (round & group_id)
               Estimate Std. Error t value Pr(>|t|)
experimentBusiness Framing    0.022   0.016069 1.36911  0.24281
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 0.037      Adj. R2: 0.090854
                Within R2: 0.081208
```

```
fixef(fe_effort)$round
```

```
      1      2      3      4      5      6      7      8      9     10
0.459 0.479 0.509 0.499 0.499 0.499 0.499 0.499 0.499 0.499
```

```
# No round trend in pct_returned based on pretest data
```

## Equation-based Power Analysis based on Pretest Data

### Round Based Analysis

```
pwr.t.test(500, 0.1*mn_wage_start/sd_wage)
```

Two-sample t test power calculation

```
      n = 500
      d = 0.2828096
sig.level = 0.05
```

```
power = 0.9939178
alternative = two.sided
```

NOTE: n is number in \*each\* group

```
pwr.t.test(d = 0.1*mn_wage_start/sd_wage, power = 0.8)
```

Two-sample t test power calculation

```
n = 197.2325
d = 0.2828096
sig.level = 0.05
power = 0.8
alternative = two.sided
```

NOTE: n is number in \*each\* group

```
pwr.t.test(500, 0.1*mn_effort_start/sd_effort)
```

Two-sample t test power calculation

```
n = 500
d = 1.142636
sig.level = 0.05
power = 1
alternative = two.sided
```

NOTE: n is number in \*each\* group

```
pwr.t.test(d = 0.1*mn_effort_start/sd_effort, power = 0.8)
```

Two-sample t test power calculation

```
n = 13.05699
d = 1.142636
```



```

sig.level = 0.05
power = 0.8
alternative = two.sided

```

NOTE: n is number in *each* group

```

pr <- pwr.t.test(n = 500, power = 0.8)
sprintf("MDE Wage: %.2f", pr$d * sd_wage)

```

```
[1] "MDE Wage: 3.14"
```

```

sprintf("MDE Effort: %.2f", pr$d * sd_effort)

```

```
[1] "MDE Effort: 0.01"
```

## Participant and Dyad Based Analysis

```

mn_payoff_part <- mean(part$payoff)
sd_payoff_part <- sd(part$payoff)
sprintf("Mean (SD) of part payoff: %.2f (%.2f)", mn_payoff_part, sd_payoff_part)

```

```
[1] "Mean (SD) of part payoff: 368.07 (198.99)"
```

```

pwr.t.test(100, (0.1*mn_payoff_part)/sd_payoff_part)

```

Two-sample t test power calculation

```

n = 100
d = 0.1849716
sig.level = 0.05
power = 0.2557083
alternative = two.sided

```

NOTE: n is number in *each* group

```
pr <- pwr.t.test(n = 100, power = 0.8)
sprintf(
  "MDE Payoff part: %.2f (%.1f %% of mean)", pr$d * sd_payoff_part,
  100*(pr$d * sd_payoff_part)/mn_payoff_part
)
```

```
[1] "MDE Payoff part: 79.23 (21.5 % of mean)"
```

```
dyads <- part %>%
  group_by(experiment, session_code, group_id) %>%
  summarise(sum_payoff = sum(payload), .groups = "drop")

mn_payoff_dyads <- mean(dyads$sum_payoff)
sd_payoff_dyads <- sd(dyads$sum_payoff)
sprintf("Mean (SD) of dyad payoff: %.2f (%.2f)", mn_payoff_dyads, sd_payoff_dyads)
```

```
[1] "Mean (SD) of dyad payoff: 736.15 (70.22)"
```

```
pwr.t.test(50, (0.1*mn_payoff_dyads)/sd_payoff_dyads)
```

Two-sample t test power calculation

```
      n = 50
      d = 1.048414
sig.level = 0.05
  power = 0.9993813
alternative = two.sided
```

NOTE: n is number in *each* group

```
pr <- pwr.t.test(n = 50, power = 0.8)
sprintf(
  "MDE Payoff dyads: %.2f (%.1f %% of mean)", pr$d * sd_payoff_dyads,
  100*(pr$d * sd_payoff_dyads)/mn_payoff_dyads
)
```

```
[1] "MDE Payoff dyads: 39.73 (5.4 % of mean)"
```

## Simulation for regression based tests

```
if (file.exists("../data/generated/giftex_sim_results.csv")) {
  giftex_sim_results <- read_csv("../data/generated/giftex_sim_results.csv", show_col_type = FALSE)
} else {
  sim_data <- function(parms, runs = 50, rounds = 10) {
    cl <- function(val, vmin = 0, vmax = 100) {
      if (val > vmax) return(as.integer(vmax))
      if (val < vmin) return(as.integer(vmin))
      as.integer(round(val))
    }
    el <- function(x) {
      steps <- seq(0.1, 1, by = 0.1)
      steps[which.min(abs(steps - x))][1]
    }
    gr <- function(rd, exp, g, parms) {
      wage_start = ifelse(
        exp == "fgiftex",
        parms$wage_start + parms$wage_start_teffect,
        parms$wage_start
      )
      wage_grate = ifelse(
        exp == "fgiftex",
        parms$wage_grate + parms$wage_grate_teffect,
        parms$wage_grate
      )
      effort_start = ifelse(
        exp == "fgiftex",
        parms$effort_start + parms$effort_start_teffect,
        parms$effort_start
      )
      effort_grate = ifelse(
        exp == "fgiftex",
        parms$effort_grate + parms$effort_grate_teffect,
        parms$effort_grate
      )
      effort_wage = ifelse(
        exp == "fgiftex",
        parms$effort_wage + parms$effort_wage_teffect,
        parms$effort_wage
      )
    }
  }
}
```

```

tibble(
  experiment = factor(ifelse(
    exp == "fgiftex",
    "Business Framing", "Neutral Framing"
  ), c("Neutral Framing", "Business Framing")),
  group_id = g,
  round = rd,
  wage = cl(
    wage_start + (rd-1)*wage_grate + rnorm(1, 0, parms$wage_evar),
    0, 100
  ),
  effort = el(
    effort_start + effort_wage*wage + (rd-1)*effort_grate +
    rnorm(1, 0, parms$effort_evar)
  )
)
}
bind_rows(
  lapply(
    c("giftex", "fgiftex"),
    function(e) bind_rows(
      lapply(
        1:runs,
        function(g) bind_rows(lapply(1:rounds, gr, e, g, parms))
      )
    )
  )
)
}

run_giftex_sim <- function(te) {
  parms <- tibble(
    wage_start = mn_wage_start,
    wage_start_teffect = te$teffect_wage,
    wage_grate = coef(ols_wage)[2],
    wage_grate_teffect = te$teffect_wage_grate,
    wage_evar = sd_wage,
    effort_start = mn_effort_start,
    effort_start_teffect = te$teffect_effort,
    effort_grate = coef(ols_effort)[2],
    effort_grate_teffect = te$teffect_effort_grate,

```

```

    effort_wage = coef(ols_effort_wage)[2],
    effort_wage_teffect = te$teffect_effort_wage,
    effort_evar = sd_effort
  )
  smp <- sim_data(parms)
  ci_wage_fe <- confint(
    feols(wage ~ experiment | round, cluster = c("round", "group_id"), data = smp)
  )
  ci_wage_round_fe <- confint(
    feols(wage ~ experiment*round, cluster = c("round", "group_id"), data = smp)
  )
  ci_effort_fe <- confint(
    feols(effort ~ experiment | round, cluster = c("round", "group_id"), data = smp)
  )
  ci_effort_round_fe <- confint(
    feols(effort ~ experiment*round, cluster = c("round", "group_id"), data = smp)
  )
  ci_effort_wage_fe <- confint(
    feols(effort ~ experiment*wage | round, cluster = c("round", "group_id"), data = smp)
  )
  tibble(
    wage_teffect_lb = pull(ci_wage_fe[1]),
    wage_teffect_ub = pull(ci_wage_fe[2]),
    wage_round_teffect_lb = ci_wage_round_fe[4, 1],
    wage_round_teffect_ub = ci_wage_round_fe[4, 2],
    effort_teffect_lb = pull(ci_effort_fe[1]),
    effort_teffect_ub = pull(ci_effort_fe[2]),
    effort_round_teffect_lb = ci_effort_round_fe[4, 1],
    effort_round_teffect_ub = ci_effort_round_fe[4, 2],
    effort_wage_teffect_lb = ci_effort_wage_fe[4, 1],
    effort_wage_teffect_ub = ci_effort_wage_fe[4, 2],
  )
}

sim_power_giftex <- function(plan) {
  sim_results <- bind_rows(
    lapply(
      1:nrow(plan),
      function(x) {
        message(
          sprintf("Running giftex sim, plan row %d of %d...", x, nrow(plan)),

```

```

        appendLF = F
      )
      rv <- bind_cols(plan[x,], run_giftext_sim(plan[x,]))
      message("")
      rv
    }
  )
}
plan <- bind_rows(
  expand_grid(
    n = 1:100,
    tibble(
      teffect_wage = 1:5,
      teffect_effort = c(0.02, 0.04, 0.06, 0.08, 0.1)
    ),
    teffect_wage_grate = 0,
    teffect_effort_grate = 0,
    teffect_effort_wage = 0,
  ),
  expand_grid(
    n = 1:100,
    tibble(
      teffect_wage = 0,
      teffect_wage_grate = c(0.5, 1, 1.5, 2, 2.5),
      teffect_effort = 0,
      teffect_effort_grate = c(0.002, 0.004, 0.006, 0.008, 0.01),
      teffect_effort_wage = 0
    )
  ),
  expand_grid(
    n = 1:100,
    teffect_wage = c(0, 5),
    teffect_wage_grate = 0,
    teffect_effort = 0,
    teffect_effort_grate = 0,
    teffect_effort_wage = c(0.0002, 0.0004, 0.0006, 0.0008, 0.001)
  )
)

message(sprintf("Starting giftext power simulations (%d runs): %s", nrow(plan), Sys.time(

```

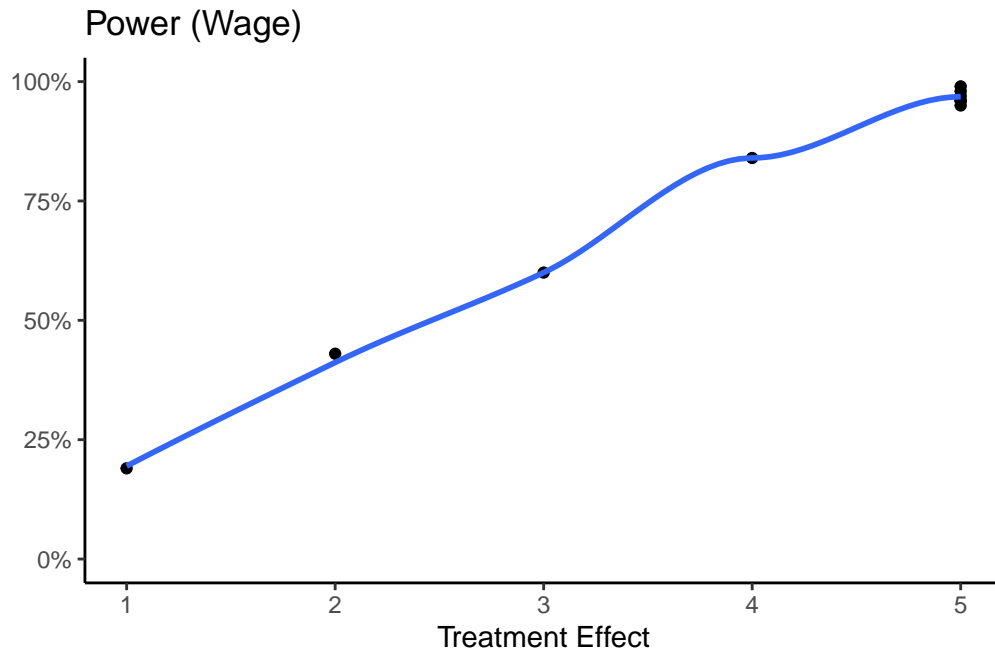
```

giftex_sim_results <- sim_power_giftex(plan)
write_csv(giftex_sim_results, "../data/generated/giftex_sim_results.csv")
message(sprintf("Done: %s", Sys.time()))
}

giftex_power <- giftex_sim_results %>%
  group_by(
    teffect_wage, teffect_wage_grate,
    teffect_effort, teffect_effort_grate,
    teffect_effort_wage
  ) %>%
  summarise(
    power_wage = mean(wage_teffect_lb > 0),
    power_wage_round = mean(wage_round_teffect_lb > 0),
    power_effort = mean(effort_teffect_lb > 0),
    power_effort_round = mean(effort_round_teffect_lb > 0),
    power_effort_wage = mean(effort_round_teffect_lb > 0),
    .groups = "drop"
  )

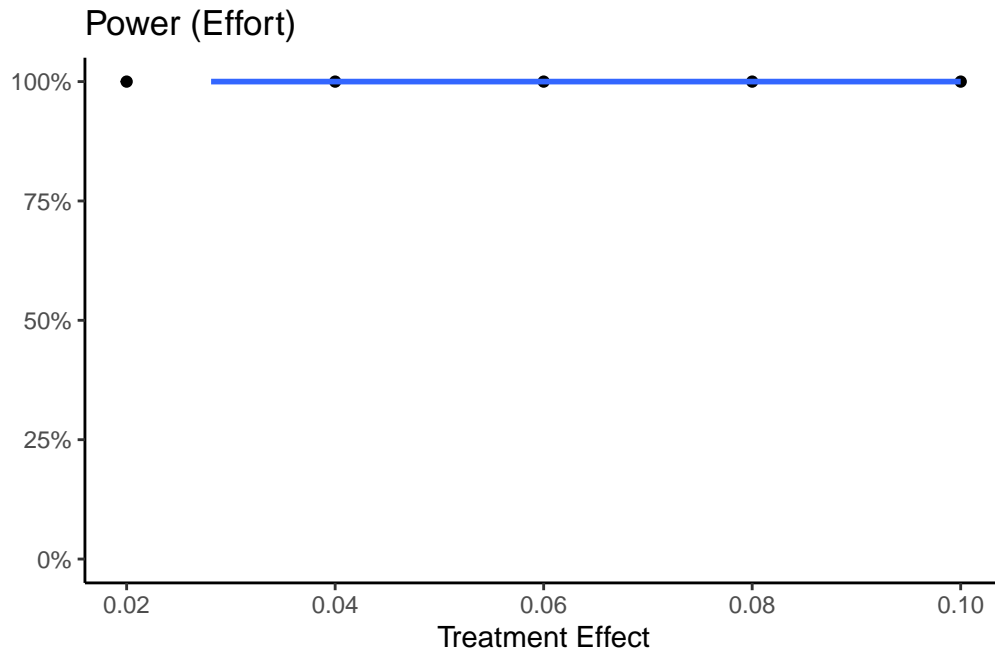
ggplot(
  giftex_power %>% filter(teffect_wage != 0),
  aes(x = teffect_wage, y = power_wage)
) + geom_point() + geom_smooth(se = FALSE) +
  scale_y_continuous(limits = c(0, 1), labels = scales::percent) +
  labs(title = "Power (Wage)", x = "Treatment Effect", y = "") +
  theme_classic()

```

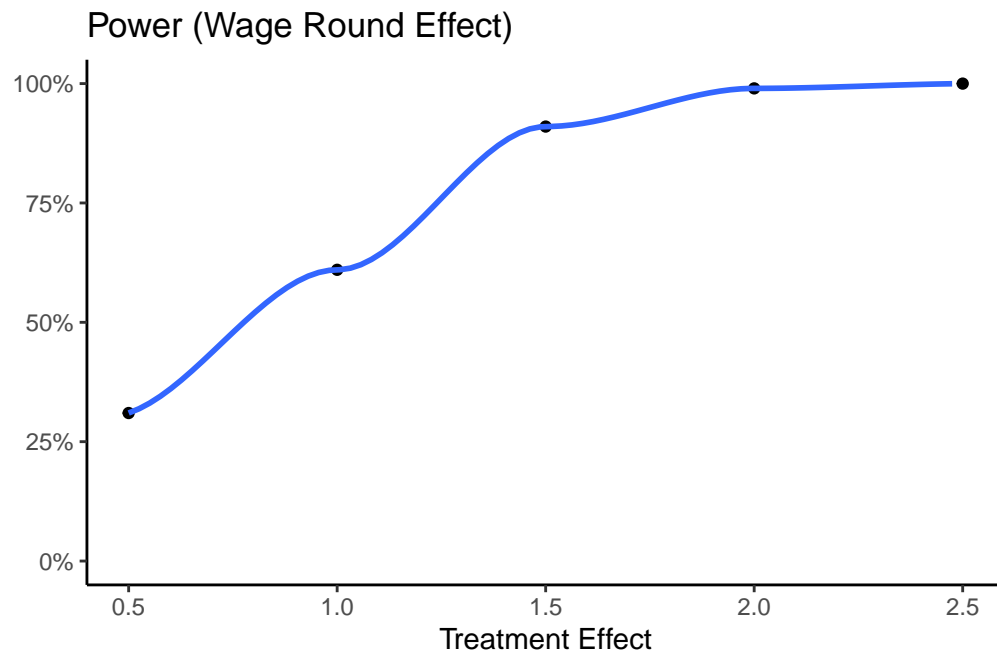


```
ggplot(  
  giftex_power %>% filter(teffect_effort != 0),  
  aes(x = teffect_effort, y = power_effort)  
) + geom_point() + geom_smooth(se = FALSE) +  
  scale_y_continuous(limits = c(0, 1), labels = scales::percent) +  
  labs(title = "Power (Effort)", x = "Treatment Effect", y = "") +  
  theme_classic()
```

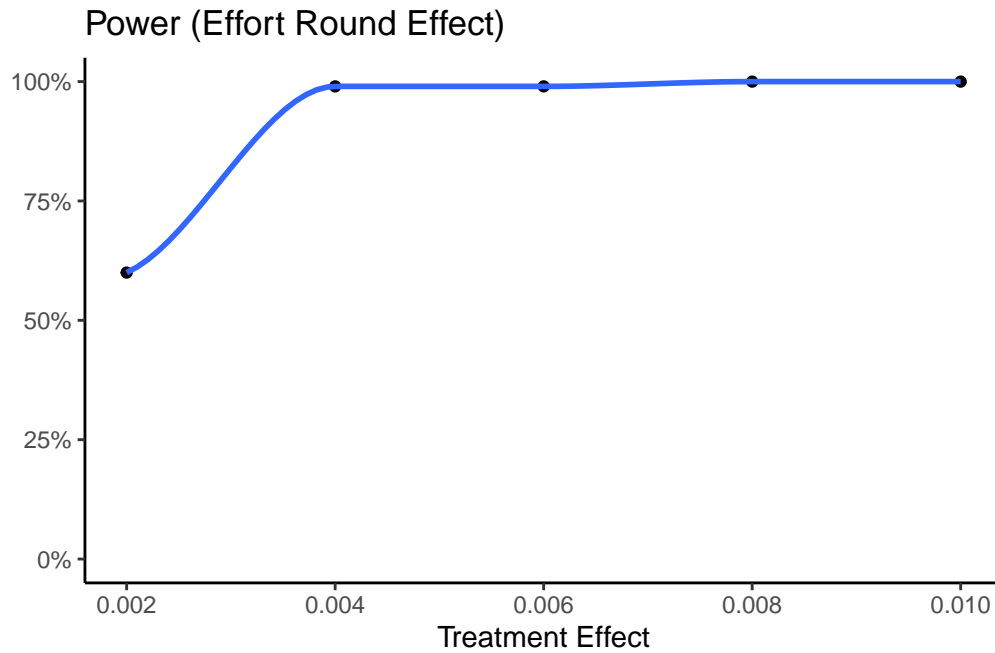




```
ggplot(  
  giftex_power %>% filter(teffect_wage_grate != 0),  
  aes(x = teffect_wage_grate, y = power_wage_round)  
) + geom_point() + geom_smooth(se = FALSE) +  
  scale_y_continuous(limits = c(0, 1), labels = scales::percent) +  
  labs(title = "Power (Wage Round Effect)", x = "Treatment Effect", y = "") +  
  theme_classic()
```



```
ggplot(  
  giftex_power %>% filter(teffect_effort_grate != 0),  
  aes(x = teffect_effort_grate, y = power_effort_round)  
) + geom_point() + geom_smooth(se = FALSE) +  
  scale_y_continuous(limits = c(0, 1), labels = scales::percent) +  
  labs(title = "Power (Effort Round Effect)", x = "Treatment Effect", y = "") +  
  theme_classic()
```



```
ggplot(
  giftex_power %>% filter(teffect_effort_wage != 0),
  aes(
    x = teffect_effort_wage, y = power_effort_wage,
    group = teffect_wage, color = teffect_wage
  )
) + geom_point() + geom_smooth() +
  scale_y_continuous(limits = c(0, 1), labels = scales::percent) +
  labs(title = "Power (Effort Wage Sensitivity)", x = "Treatment Effect", y = "") +
  theme_classic()
```

