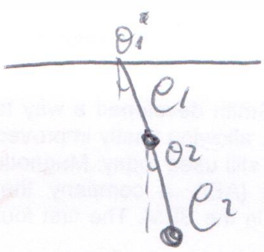
## ABSTRACT

this short article (:)) shows how to find the dynamic system equations of what we have called SPRINGED DOUBLE PENDULUM using a Lagrangian formulation of the mechanical problem.
the aim of this work is to build a simple application (written in python) that simulates the system integrating the four equations of the mechanical model.
~~This paperwork must be completed~~
~~A set of python scripts must be also~~
Besides this paperwork we have produced some python scripts that ~~also~~ ~~shows the~~ graphically shows the evolution of the systems and allows the user to force a given state to each one of the two pendulums by a mouseclick :)))

This was done during our 12 days holiday in Barcelona 30 jul - 10 aug 2017.




$$T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1$$

$x_1 = \ell_1 \sin\theta_1$

$y_1 = \ell_1 \cos\theta_1$

$x_2 = x_1 + \ell_2 \sin\theta_2 =$

$y_2 = y_1 + \ell_2 \cos\theta_2$

$$\begin{cases} x_2 = \ell_1 \sin\theta_1 + \ell_2 \sin\theta_2 \\ y_2 = \ell_1 \cos\theta_1 + \ell_2 \cos\theta_2 \end{cases}$$

$$T = \frac{1}{2}(m_1+m_2)\left(\dot\ell_1^2 + \ell_1^2\dot\theta_1^2\right) + \frac{1}{2} m_2\left(\dot\ell_2^2 + \ell_2^2\dot\theta_2^2\right) +$$
$$+ m_2\Big[ + \dot\ell_1\dot\ell_2 \cos(\theta_1 \mp \theta_2) \mp \dot\ell_1 \ell_2 \sin(\theta_1 \mp \theta_2) +$$
$$+ \ell_1\ell_2\dot\theta_2 \sin(\theta_1 \mp \theta_2) + \ell_1\ell_2\dot\theta_1\dot\theta_2 \cos(\theta_1 \mp \theta_2)\Big]$$

$$T = \frac{1}{2}(m_1+m_2)\left(\dot\ell_1^2 + \ell_1^2\dot\theta_1^2\right) + \frac{1}{2} m_2\left(\dot\ell_2^2 + \ell_2^2\dot\theta_2^2\right)$$
$$+ m_2\left(\ell_1\ell_2\dot\theta_1\dot\theta_2 + \dot\ell_1\dot\ell_2\right)\cos(\theta_1 \mp \theta_2) +$$
$$+ m_2\left(-\ell_1\dot\ell_2\dot\theta_1 + \dot\ell_1\ell_2\dot\theta_2\right)\sin(\theta_1 \mp \theta_2)$$

$\dot x_1 = \dot\ell_1 \sin\theta_1 + \ell_1\dot\theta_1\cos\theta_1$

$\dot y_1 = \dot\ell_1 \cos\theta_1 - \ell_1\dot\theta_1 \sin\theta_1$

$\dot x_2 = \dot x_1 + \dot\ell_2 \sin\theta_2 + \ell_2\dot\theta_2\cos\theta_2$

$\dot y_2 = \dot y_1 + \dot\ell_2 \cos\theta_2 - \ell_2\dot\theta_2 \sin\theta_2$

$$\frac{m_2}{2}\left(\dot x_2^2 + \dot y_2^2\right) = \frac{m_2}{2}\Big[\underbrace{\dot x_1^2}_{T_1} + \overbrace{\left(\dot\ell_2 s_{\theta_2} + \ell_2\dot\theta_2 c_{\theta_2}\right)^2}^{T_2} +$$
$$+ 2\dot x_1\left(\dot\ell_2 s_{\theta_2} + \ell_2\dot\theta_2 c_{\theta_2}\right)$$
$$+ \underbrace{\dot y_1^2}_{T_1} + \overbrace{\left(\dot\ell_2 c_{\theta_2} - \ell_2\dot\theta_2 s_{\theta_2}\right)^2}^{T_2}$$
$$+ 2\dot y_1\left(\dot\ell_2 c_{\theta_2} - \ell_2\dot\theta_2 s_{\theta_2}\right)\Big] =$$

$$T_1 = \frac{1}{2}(m_1+m_2)\left[\dot x_1^2 + \dot y_1^2\right] = \frac{1}{2}(m_1+m_2)\left[\dot\ell_1^2 + \ell_1^2\dot\theta_1^2\right]$$

$$T_2 = \frac{1}{2} m_2\left[\dot\ell_2^2 + \ell_2^2\dot\theta_2^2\right]$$

$$T_3 = \frac{2m_2}{2}\left(\dot\ell_1 s_{\theta_1} + \ell_1\dot\theta_1 c_{\theta_1}\right)\left(\dot\ell_2 s_{\theta_2} + \ell_2\dot\theta_2 c_{\theta_2}\right) +$$
$$+ \left(\dot\ell_1 c_{\theta_1} - \ell_1\dot\theta_1 s_{\theta_1}\right)\left(\dot\ell_2 c_{\theta_2} - \ell_2\dot\theta_2 s_{\theta_2}\right) =$$

$$= m_2\Big\{\dot\ell_1\dot\ell_2 s_{\theta_1} s_{\theta_2} + \ell_1\dot\ell_2\dot\theta_1 c_{\theta_1} s_{\theta_2} + \dot\ell_1\ell_2\dot\theta_2 s_{\theta_1} c_{\theta_2} +$$
$$+ \ell_1\ell_2\dot\theta_1\dot\theta_2 c_{\theta_1} c_{\theta_2} + \Big(\dot\ell_1\dot\ell_2 c_{\theta_1} c_{\theta_2} - \ell_1\dot\ell_2\dot\theta_1 s_{\theta_1} c_{\theta_2} +$$
$$- \dot\ell_1\ell_2\dot\theta_2 c_{\theta_1} s_{\theta_2} + \ell_1\ell_2\dot\theta_1\dot\theta_2 s_{\theta_1} s_{\theta_2}\Big)\Big\} =$$

$$= m_2\Big\{+\dot\ell_1\dot\ell_2 \cos(\theta_1 \mp \theta_2) \mp \ell_1\dot\ell_2\dot\theta_1 \sin(\theta_1 \mp \theta_2) + \dot\ell_1\ell_2\dot\theta_2 \sin(\theta_1 \mp \theta_2) +$$
$$+ \ell_1\ell_2\dot\theta_1\dot\theta_2 \cos(\theta_1 \mp \theta_2)$$

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$$\frac{\partial L}{\partial \dot{\ell}_1} = (m_1+m_2)\dot{\ell}_1 + m_2\dot{\ell}_2\cos(\theta_1-\theta_2) + m_2\ell_2\dot{\theta}_2\sin(\theta_1-\theta_2)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\ell}_1} = (m_1+m_2)\ddot{\ell}_1 + m_2\ddot{\ell}_2\cos(\theta_1-\theta_2) - m_2\dot{\ell}_2\sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2) +$$
$$+ m_2\dot{\ell}_2\dot{\theta}_2\sin(\theta_1-\theta_2) + m_2\ell_2\left[\ddot{\theta}_2\sin(\theta_1-\theta_2) + \dot{\theta}_2\cos(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2) \right]$$

$$\frac{\partial L}{\partial \ell_1} = m_1 g\cos\theta_1 - K_1\ell_1 + m_2 g\cos\theta_1 + \ldots = (m_1+m_2)g\cos\theta_1 - K_1\ell_1 +$$
$$+ m_2\ell_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2) - m_2\dot{\ell}_2\dot{\theta}_1\sin(\theta_1-\theta_2)$$

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$$\frac{\partial L}{\partial \dot{\theta}_1} = (m_1+m_2)\ell_1^2\dot{\theta}_1 + m_2\ell_1\left[\ell_2\dot{\theta}_2\cos(\theta_1-\theta_2) - \dot{\ell}_2\sin(\theta_1-\theta_2)\right]$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_1} = (m_1+m_2)\ell_1^2\ddot{\theta}_1 + 2(m_1+m_2)\ell_1\dot{\ell}_1\dot{\theta}_1 + m_2\dot{\ell}_1\left[\ell_2\dot{\theta}_2\cos(\theta_1-\theta_2) - \dot{\ell}_2\sin(\theta_1-\theta_2)\right]$$
$$+ m_2\ell_1\Big[\dot{\ell}_2\dot{\theta}_2\cos(\theta_1-\theta_2) + \ell_2\ddot{\theta}_2\cos(\theta_1-\theta_2) - \ell_2\dot{\theta}_2\sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2) +$$
$$-\left(\ddot{\ell}_2\sin(\theta_1-\theta_2) + \dot{\ell}_2\cos(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2)\right)\Big]$$

$$\frac{\partial L}{\partial \theta_1} = -m_1 g\ell_1\sin\theta_1 - m_2 g\ell_1\sin\theta_1 + \ldots = -(m_1+m_2)g\ell_1\sin\theta_1 - m_2(\ell_1\ell_2\dot{\theta}_1\dot{\theta}_2 + \dot{\ell}_1\dot{\ell}_2 \text{[illegible]})\sin(\theta_1-\theta_2) +$$
$$\boxed{+m_2(-\ell_2\dot{\ell}_2\dot{\theta}_1 + \dot{\ell}_1\ell_2\dot{\theta}_2)\cos(\theta_1-\theta_2)}$$

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$$\frac{\partial L}{\partial \dot{\ell}_2} = (\cancel{m_1+}m_2)\dot{\ell}_2 + m_2\dot{\ell}_1\cos(\theta_1-\theta_2) - m_2\ell_1\dot{\theta}_1\sin(\theta_1-\theta_2)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\ell}_2} = m_2\ddot{\ell}_2 + m_2\ddot{\ell}_1\cos(\theta_1-\theta_2) - m_2\dot{\ell}_1\sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2) - m_2\Big[\dot{\ell}_1\dot{\theta}_1\sin(\theta_1-\theta_2) +$$
$$+ \ell_1\left(\ddot{\theta}_1\sin(\theta_1-\theta_2) + \dot{\theta}_1\cos(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2)\right)\Big]$$

$$\frac{\partial L}{\partial \ell_2} = m_2\ell_2\dot{\theta}_2^2 + m_2\ell_1\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1-\theta_2) + m_2\dot{\ell}_1\dot{\theta}_2\sin(\theta_1-\theta_2) + m_2 g\cos\theta_2 - K_2\ell_2$$

$$\frac{\partial L}{\partial \dot\theta_2} = m_2 \ell_2^2 \dot\theta_2 + m_2 \ell_1 \ell_2 \dot\theta_1 \cos(\theta_1-\theta_2) + m_2 \dot\ell_1 \ell_2 \sin(\theta_1-\theta_2) =$$

$$= m_2 \ell_2^2 \dot\theta_2 + m_2 \ell_2 \left[\ell_1 \dot\theta_1 \cos(\theta_1-\theta_2) + \dot\ell_1 \sin(\theta_1-\theta_2)\right]$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_2} = 2 m_2 \ell_2 \dot\theta_2 \dot\ell_2 + m_2 \ell_2^2 \ddot\theta_2 + m_2 \dot\ell_2 \left[\ell_1 \dot\theta_1 \cos(\theta_1-\theta_2) + \dot\ell_1 \sin(\theta_1-\theta_2)\right]$$

$$+ m_2 \ell_2 \Big\{ \dot\ell_1 \dot\theta_1 \cos(\theta_1-\theta_2) + \ell_1 \left[\ddot\theta_1 \cos(\theta_1-\theta_2) - \dot\theta_1 \sin(\theta_1-\theta_2)(\dot\theta_1-\dot\theta_2)\right]$$

$$+ \ddot\ell_1 \sin(\theta_1-\theta_2) + \dot\ell_1 \cos(\theta_1-\theta_2)(\dot\theta_1-\dot\theta_2) \Big\}$$

$$\frac{\partial L}{\partial \theta_2} = m_2 \left(\ell_1 \ell_2 \dot\theta_1 \dot\theta_2 + \dot\ell_1 \dot\ell_2\right) \sin(\theta_1-\theta_2) \cdot \cancel{\theta_2} +$$

$$- m_2 \left(-\ell_1 \dot\ell_2 \dot\theta_1 + \dot\ell_1 \ell_2 \dot\theta_2\right) \cos(\theta_1-\theta_2) \cancel{=} m_2 g \ell_2 \sin\theta_2$$

$$(m_1+m_2)\ddot\ell_1 + m_2\cos(\theta_1-\theta_2)\ddot\ell_2 + m_2\ell_2\sin(\theta_1-\theta_2)\ddot\theta_2 - m_2\dot\ell_2(\dot\theta_1-\dot\theta_2)\sin(\theta_1-\theta_2) +$$
$$+ m_2\dot\ell_2\dot\theta_2\sin(\theta_1-\theta_2) + m_2\ell_2\dot\theta_2(\dot\theta_1-\dot\theta_2)\cos(\theta_1-\theta_2) - m_2\ell_2\dot\theta_1\dot\theta_2\cos(\theta_1-\theta_2) +$$
$$+ m_2\dot\ell_2\dot\theta_1\sin(\theta_1-\theta_2) - (m_1+m_2)g\cos\theta_1 + k_1\ell_1 = 0$$

$$(m_1+m_2)\ddot\ell_1 + m_2\cos(\theta_1-\theta_2)\ddot\ell_2 + m_2\ell_2\sin(\theta_1-\theta_2)\ddot\theta_2 + m_2\dot\ell_2\sin(\theta_1-\theta_2)\left[\cancel{-\dot\theta_1} + \dot\theta_2 + \dot\theta_2 + \cancel{\dot\theta_1}\right]$$
$$+ m_2\ell_2\dot\theta_2\cos(\theta_1-\theta_2)\left[\cancel{\dot\theta_1} - \dot\theta_2 - \cancel{\dot\theta_1}\right] - (m_1+m_2)g\cos\theta_1 + k_1\ell_1 = 0$$

$$\boxed{(m_1+m_2)\ddot\ell_1 + m_2\cos(\theta_1-\theta_2)\ddot\ell_2 + m_2\ell_2\sin(\theta_1-\theta_2)\ddot\theta_2 + 2m_2\dot\ell_2\dot\theta_2\sin(\theta_1-\theta_2) - m_2\ell_2\dot\theta_2^2\cos(\theta_1-\theta_2) - (m_1+m_2)g\cos\theta_1 + k_1\ell_1 = 0}$$

$\boxed{\text{EQ. } \ell_1}$

$$(m_1+m_2)\ell_1^2\ddot\theta_1 - m_2\ell_1\sin(\theta_1-\theta_2)\ddot\ell_2 + m_2\ell_1\ell_2\cos(\theta_1-\theta_2)\ddot\theta_2 + 2(m_1+m_2)\ell_1\dot\ell_1\dot\theta_1 +$$
$$+ m_2\dot\ell_1\ell_2\dot\theta_2\cos(\theta_1-\theta_2) - m_2\dot\ell_1\dot\ell_2\sin(\theta_1-\theta_2) + m_2\ell_1\dot\ell_2\dot\theta_2\cos(\theta_1-\theta_2) - m_2\ell_1\ell_2\dot\theta_2(\dot\theta_1-\dot\theta_2)\sin(\theta_1-\theta_2)$$
$$- m_2\ell_1\dot\ell_2(\dot\theta_1-\dot\theta_2)\cos(\theta_1-\theta_2) + m_2\ell_1\ell_2\dot\theta_1\dot\theta_2\sin(\theta_1-\theta_2) + m_2\dot\ell_1\dot\ell_2\sin(\theta_1-\theta_2) +$$
$$+ m_2\ell_2\dot\ell_1\dot\theta_1\cos(\theta_1-\theta_2) - m_2\dot\ell_1\ell_2\dot\theta_2\cos(\theta_1-\theta_2) + (m_1+m_2)g\ell_1\sin\theta_1 = 0$$

$$(m_1+m_2)\ell_1^2\ddot\theta_1 - m_2\ell_1\sin(\theta_1-\theta_2)\ddot\ell_2 + m_2\ell_1\ell_2\cos(\theta_1-\theta_2)\ddot\theta_2 +$$
$$+ m_2\cos(\theta_1-\theta_2)\left[\cancel{\dot\ell_1\dot\theta_2\ell_2} + \ell_1\dot\ell_2\dot\theta_2 - \ell_1\dot\ell_2(\dot\theta_1-\dot\theta_2) + \ell_2\dot\ell_1\dot\theta_1 - \cancel{\dot\ell_1\ell_2\dot\theta_2}\right] +$$
$$+ m_2\sin(\theta_1-\theta_2)\left[\cancel{-\dot\ell_1\dot\ell_2} - \ell_1\ell_2\dot\theta_2(\cancel{\dot\theta_1}-\dot\theta_2) + \cancel{\ell_1\ell_2\dot\theta_1\dot\theta_2} + \cancel{\dot\ell_1\dot\ell_2}\right] +$$
$$+ 2(m_1+m_2)\ell_1\dot\ell_1\dot\theta_1 + (m_1+m_2)g\ell_1\sin\theta_1 = 0$$

$$\boxed{(m_1+m_2)\ell_1^2\ddot\theta_1 - m_2\ell_1\sin(\theta_1-\theta_2)\ddot\ell_2 + m_2\ell_1\ell_2\cos(\theta_1-\theta_2)\ddot\theta_2 + m_2\cos(\theta_1-\theta_2)\left(2\ell_1\dot\ell_2\dot\theta_2 - \ell_1\dot\ell_2\dot\theta_1 + \ell_2\dot\ell_1\dot\theta_1\right) + m_2\sin(\theta_1-\theta_2)\,\ell_1\ell_2\dot\theta_2^2 + 2(m_1+m_2)\ell_1\dot\ell_1\dot\theta_1 + (m_1+m_2)g\ell_1\sin\theta_1 = 0}$$

$\boxed{\text{EQ } \theta_1}$

$$m_2\cos(\theta_1-\theta_2)\ddot\ell_1 - m_2\ell_1\sin(\theta_1-\theta_2)\ddot\theta_1 + m_2\ddot\ell_2 +$$
$$+ m_2\sin(\theta_1-\theta_2)\left[-\dot\ell_1(\dot\theta_1-\cancel{\dot\theta_2}) - \dot\ell_1\dot\theta_1 - \cancel{\dot\ell_1\dot\theta_2}\right] - m_2\ell_1\dot\theta_1(\dot\theta_1-\cancel{\dot\theta_2})\cos(\theta_1-\theta_2) +$$
$$- m_2\ell_2\dot\theta_2^2 - \cancel{m_2\ell_1\dot\theta_1\dot\theta_2} - m_2 g\cos\theta_2 + K_2\ell_2 = 0$$

$$\boxed{\begin{aligned}&m_2\cos(\theta_1-\theta_2)\ddot\ell_1 - m_2\ell_1\sin(\theta_1-\theta_2)\ddot\theta_1 + m_2\ddot\ell_2 - 2m_2\dot\ell_1\dot\theta_1\sin(\theta_1-\theta_2) - m_2\ell_1\dot\theta_1^2\cos(\theta_1-\theta_2)\\&- m_2\ell_2\dot\theta_2^2 - \cancel{m_2\ell_1\dot\theta_1\dot\theta_2} - m_2 g\cos\theta_2 + K_2\ell_2 = 0\end{aligned}}$$

$\boxed{\text{EQU } \ell_2}$

$$m_2\ell_2\ddot\ell_1\sin(\theta_1-\theta_2) + m_2\ell_2\ell_1\cos(\theta_1-\theta_2)\ddot\theta_1 + m_2\ell_2^2\ddot\theta_2 + 2m_2\ell_2\dot\theta_2\dot\ell_2 +$$
$$+ m_2\dot\ell_2\ell_1\dot\theta_1\cos(\theta_1-\theta_2) + m_2\dot\ell_2\dot\ell_1\sin(\theta_1-\theta_2) + m_2\ell_2\dot\ell_1\dot\theta_1\cos(\theta_1-\theta_2) +$$
$$- m_2\ell_2\ell_1\dot\theta_1(\dot\theta_1-\dot\theta_2)\sin(\theta_1-\theta_2) + m_2\ell_2\dot\ell_1(\dot\theta_1-\dot\theta_2)\cos(\theta_1-\theta_2)$$
$$- m_2(\ell_1\ell_2\dot\theta_1\dot\theta_2 + \dot\ell_1\dot\ell_2)\sin(\theta_1-\theta_2) + m_2(-\dot\ell_1\ell_2\dot\theta_1 + \dot\ell_1\ell_2\dot\theta_2)\cos(\theta_1-\theta_2) + m_2 g\ell_2\sin\theta_2 = 0$$

$$m_2\ell_2\sin(\theta_1-\theta_2)\ddot\ell_1 + m_2\ell_2\ell_1\cos(\theta_1-\theta_2)\ddot\theta_1 + m_2\ell_2^2\ddot\theta_2 + 2m_2\ell_2\dot\theta_2\dot\ell_2 +$$
$$+ m_2\cos(\theta_1-\theta_2)\left[\dot\ell_2\dot\ell_1\dot\theta_1 + \ell_2\dot\ell_1\dot\theta_1 + \ell_2\dot\ell_1\dot\theta_1 - \cancel{\ell_2\dot\ell_1\dot\theta_2} - \ell_1\dot\ell_2\dot\theta_1 + \cancel{\dot\ell_1\ell_2\dot\theta_2}\right] +$$
$$+ m_2\sin(\theta_1-\theta_2)\left[\cancel{\dot\ell_2\dot\ell_1} - \ell_2\ell_1\dot\theta_1^2 + \cancel{\ell_2\ell_1\dot\theta_1\dot\theta_2} - \cancel{\ell_1\ell_2\dot\theta_1\dot\theta_2} - \cancel{\dot\ell_1\dot\ell_2}\right] + m_2 g\ell_2\sin\theta_2 = 0$$

$$\boxed{\begin{aligned}&m_2\ell_2\sin(\theta_1-\theta_2)\ddot\ell_1 + m_2\ell_2\ell_1\cos(\theta_1-\theta_2)\ddot\theta_1 + m_2\ell_2^2\ddot\theta_2 + 2m_2\ell_2\dot\theta_2\dot\ell_2 +\\&+ 2m_2\ell_2\dot\ell_1\dot\theta_1\cos(\theta_1-\theta_2) - m_2\ell_1\ell_2\dot\theta_1^2\sin(\theta_1-\theta_2) + m_2 g\ell_2\sin\theta_2 = 0\end{aligned}}$$

$\boxed{\text{EQ } \theta_2}$

$$B\ddot q + C = 0$$

$$
B = \begin{bmatrix} m_1+m_2 & 0 & m_2\cos(\theta_1-\theta_2) & m_2\ell_2\sin(\theta_1-\theta_2) \\ 0 & (m_1+m_2)\ell_1^2 & -m_2\ell_1\sin(\theta_1-\theta_2) & m_2\ell_1\ell_2\cos(\theta_1-\theta_2) \\ m_2\cos(\theta_1-\theta_2) & -m_2\ell_1\sin(\theta_1-\theta_2) & m_2 & 0 \\ m_2\ell_2\sin(\theta_1-\theta_2) & m_2\ell_1\ell_2\cos(\theta_1-\theta_2) & 0 & m_2\ell_2^2 \end{bmatrix} \begin{bmatrix} \ddot{\ell}_1 \\ \ddot{\theta}_1 \\ \ddot{\ell}_2 \\ \ddot{\theta}_2 \end{bmatrix}
$$

$$
\bar{C} = \begin{bmatrix} 2m_2\dot{\ell}_2\dot{\theta}_2\sin(\theta_1-\theta_2) - m_2\ell_2\dot{\theta}_2^2\cos(\theta_1-\theta_2) - (m_1+m_2)g\cos\theta_1 + k_1\ell_1 \\ m_2\cos(\theta_1-\theta_2)\left[2\dot{\ell}_1\dot{\ell}_2\dot{\theta}_2 - \dot{\ell}_1\dot{\ell}_2\dot{\theta}_1 + \ell_2\dot{\ell}_2\dot{\theta}_1\right] + m_2\sin(\theta_1-\theta_2)\ell_1\ell_2\dot{\theta}_2^2 + 2(m_1+m_2)\ell_1\dot{\ell}_1\dot{\theta}_1 + (m_1+m_2)g\ell_1\sin\theta_1 \\ -2m_2\dot{\ell}_1\dot{\theta}_1\sin(\theta_1-\theta_2) - m_2\ell_1\dot{\theta}_1^2\cancel{(\dot{\ell}_2\dot{\theta}_2)}\cos(\theta_1-\theta_2) - m_2\ell_2\dot{\theta}_2^2\sin(\theta_1-\theta_2)\cancel{\dot{\ell}_1\dot{\theta}_1\dot{\theta}_2} - m_2 g\cos\theta_2 + k_2\ell_2 \\ 2m_2\ell_2\dot{\ell}_2\dot{\theta}_2 + 2m_2\ell_2\dot{\ell}_1\dot{\theta}_1\cos(\theta_1-\theta_2) - m_2\ell_1\ell_2\dot{\theta}_1^2\sin(\theta_1-\theta_2) + m_2 g\ell_2\sin\theta_2 \end{bmatrix}
$$

$$
\ddot{q} = -B^{-1}\cdot\bar{C}
$$

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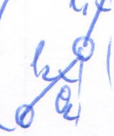
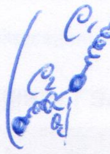
$$\begin{cases} (m_1+m_2)\, l_1^2\, \ddot{\theta}_1 + m_2 l_1 l_2 \cos(\theta_1-\theta_2)\, \ddot{\theta}_2 + m_2 \sin(\theta_1-\theta_2)\, l_1 l_2\, \dot{\theta}_2^2 + (m_1+m_2)\, g\, l_1 \sin\theta_1 = 0 \\ m_2 l_1 l_2 \cos(\theta_1-\theta_2)\, \ddot{\theta}_1 + m_2 l_2^2\, \ddot{\theta}_2 - m_2 l_1 l_2\, \dot{\theta}_1^2 \sin(\theta_1-\theta_2) + m_2 g\, l_2 \sin\theta_2 = 0 \end{cases}$$

~~The Cse the~~

These are the equations of the ~~simple~~ 2-DOF double pendulum



retrieved from the former model 4-DOF



forcing

$$\begin{cases} \varrho_1 = l_1 \\ \dot{\varrho}_1 = 0 \end{cases} \quad \begin{cases} \varrho_2 = l_2 \\ \dot{\varrho}_2 = 0 \end{cases}$$

because the pendulum lengths are fixed to $l_1$ and $l_2$.

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