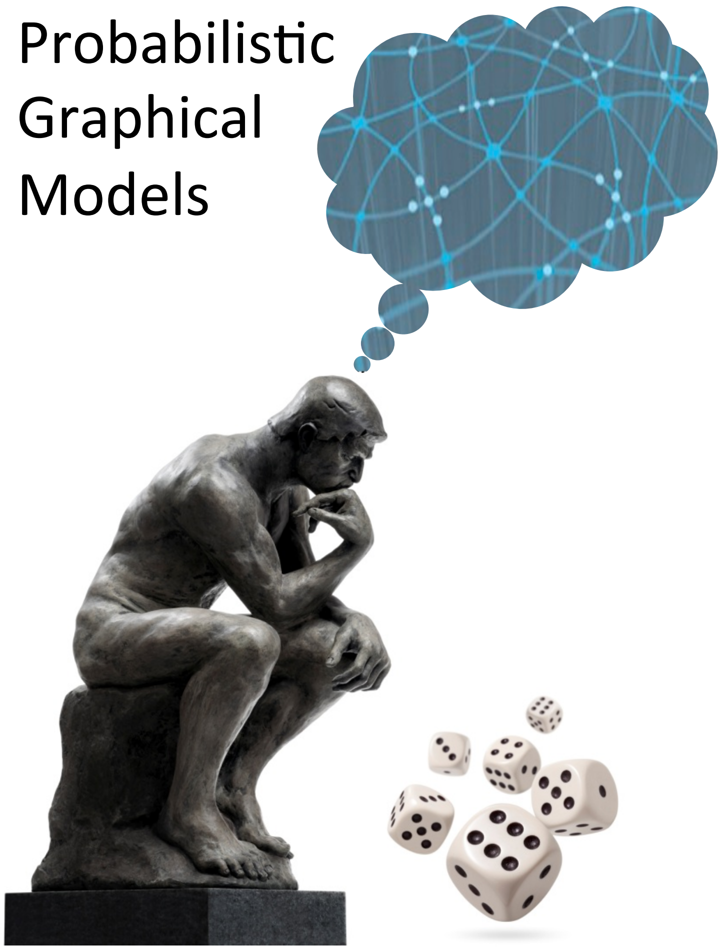
Probabilistic
Graphical
Models

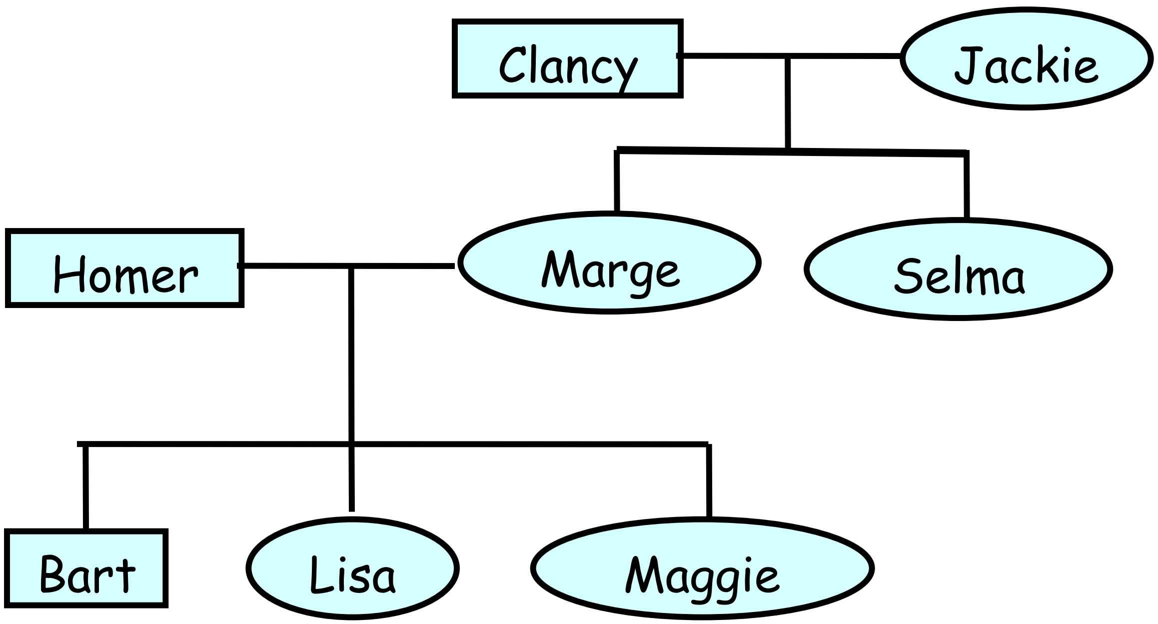


Representation

Template Models

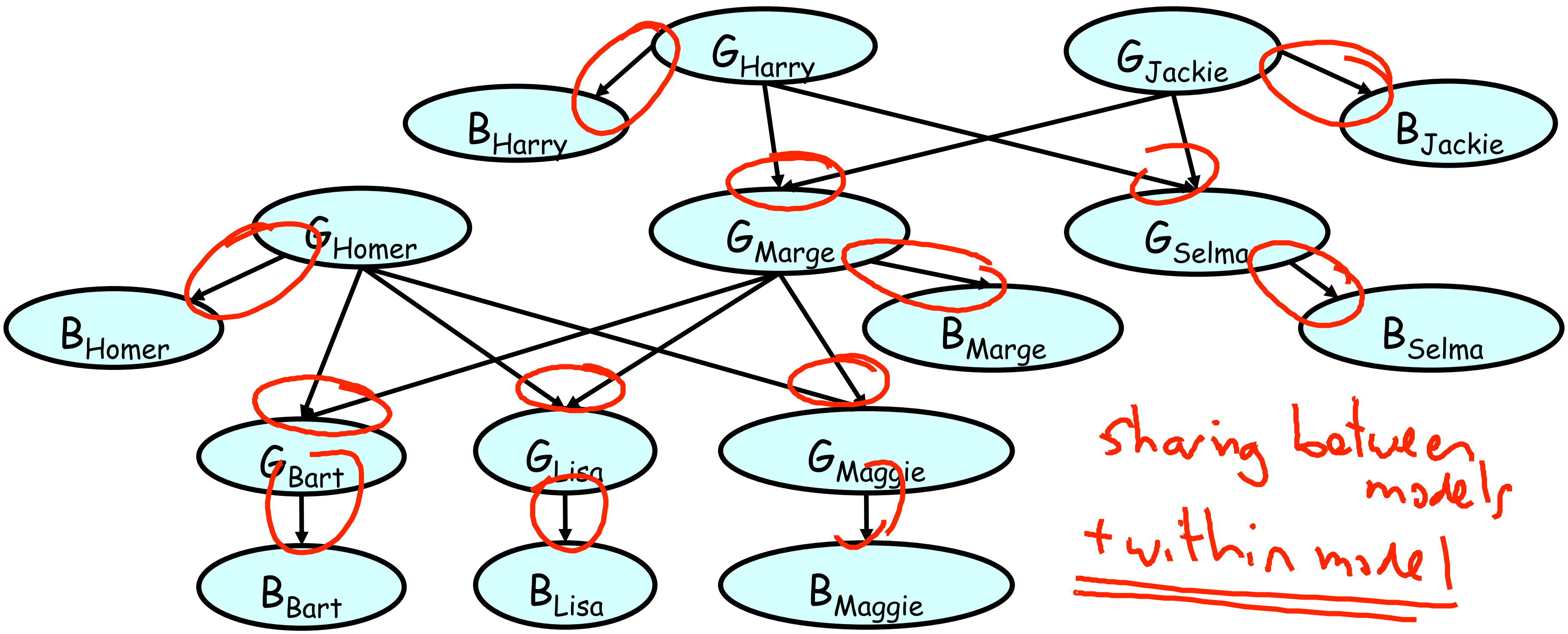
Overview

Genetic Inheritance



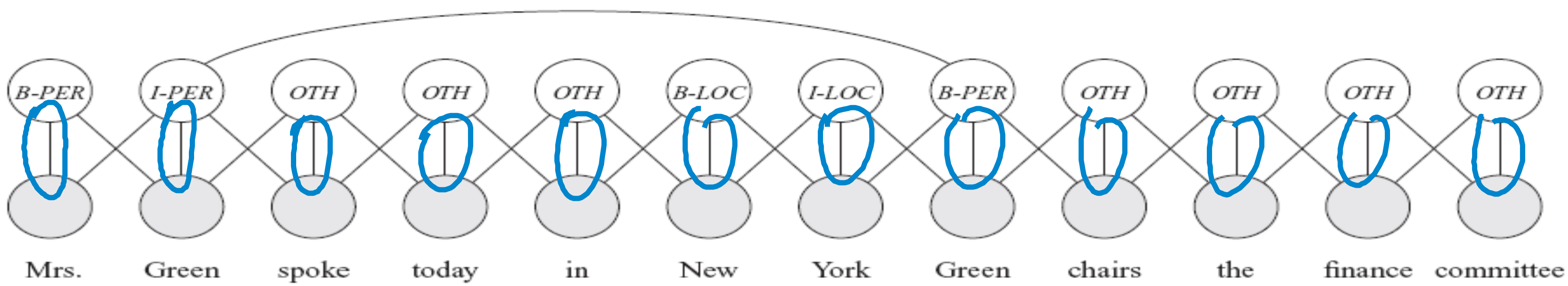
pedigree
trait

Genetic Inheritance



sharing between models
+ within model

NLP Sequence Models



KEY

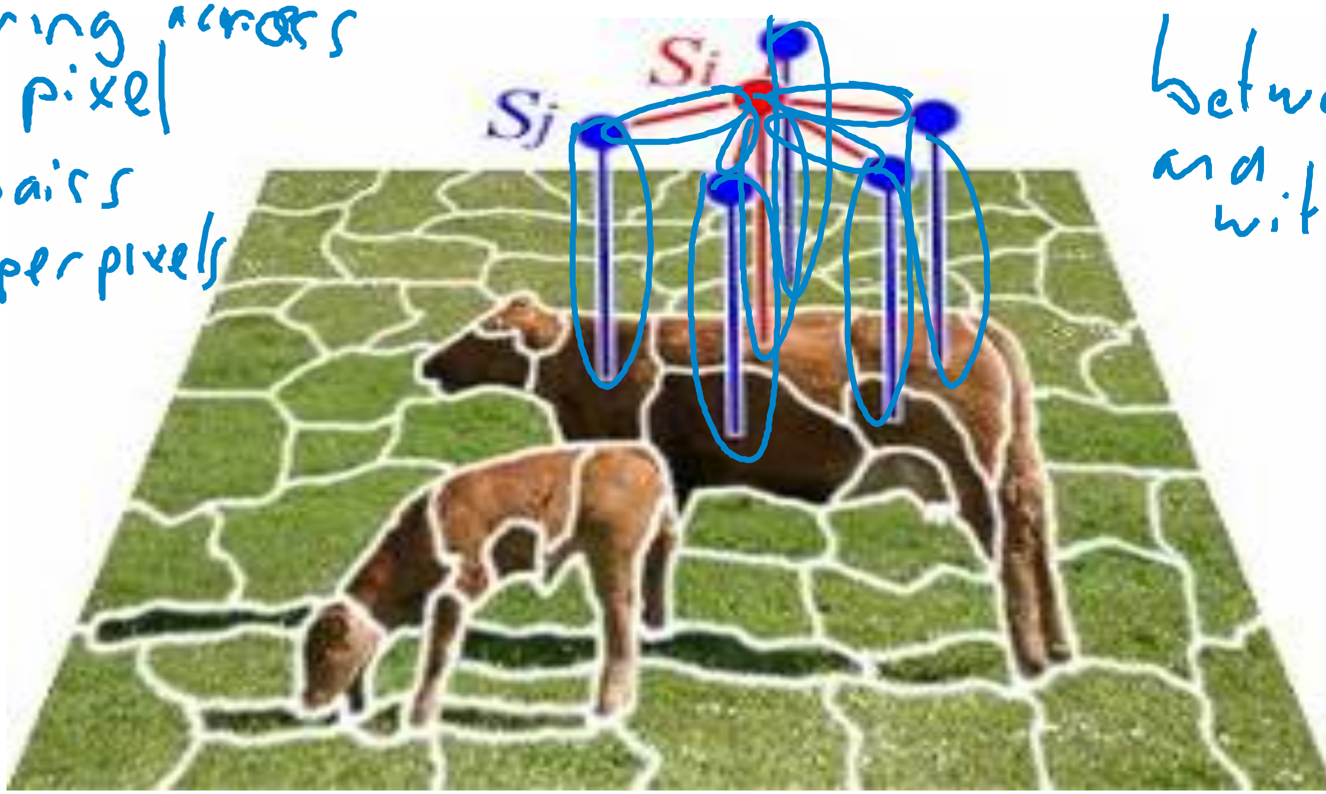
B-PER	Begin person name	*I-LOC*	Within location name
I-PER	Within person name	*OTH*	Not an entitiy
B-LOC	Begin location name		

Named entity recognition

Image Segmentation

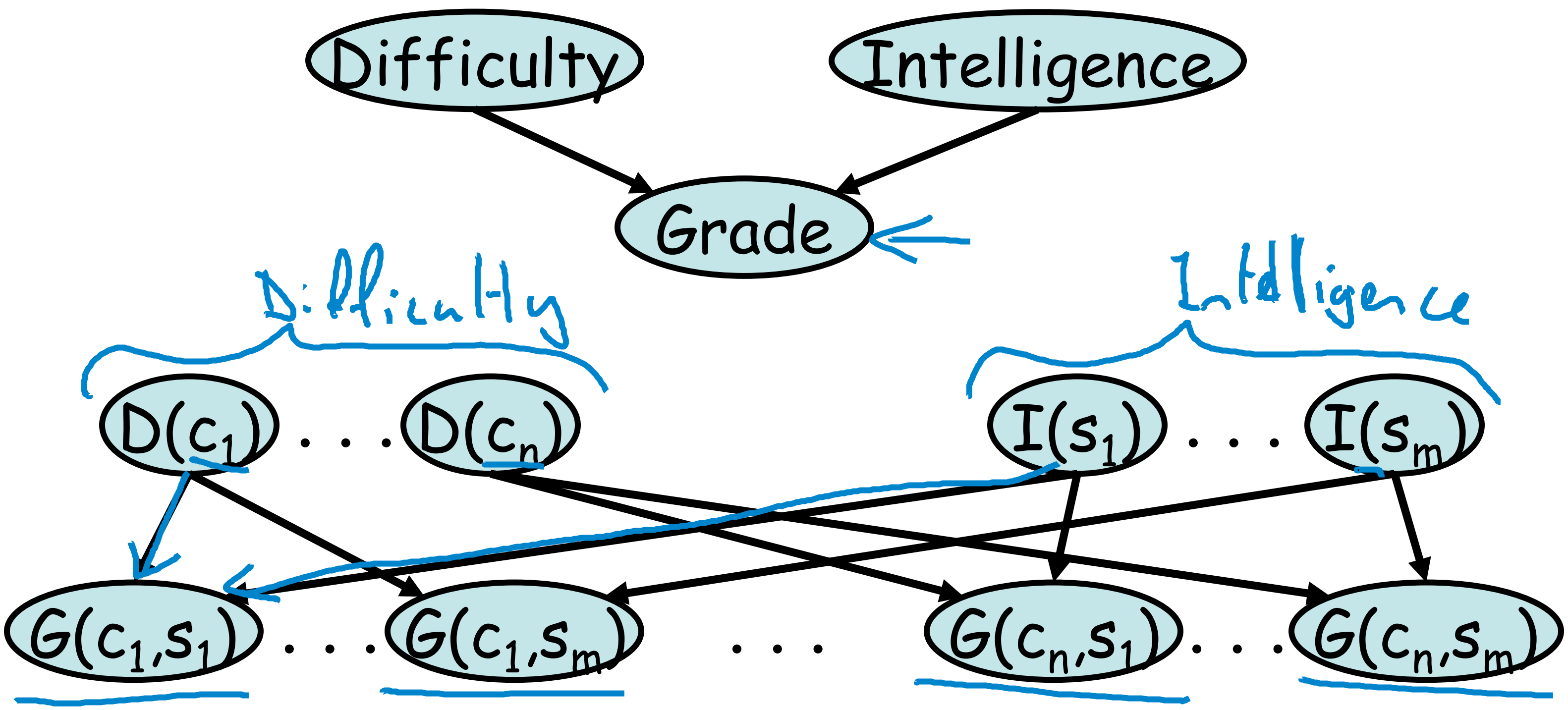
sharing across pixel

and pairs of superpixels



between and within

The University Example



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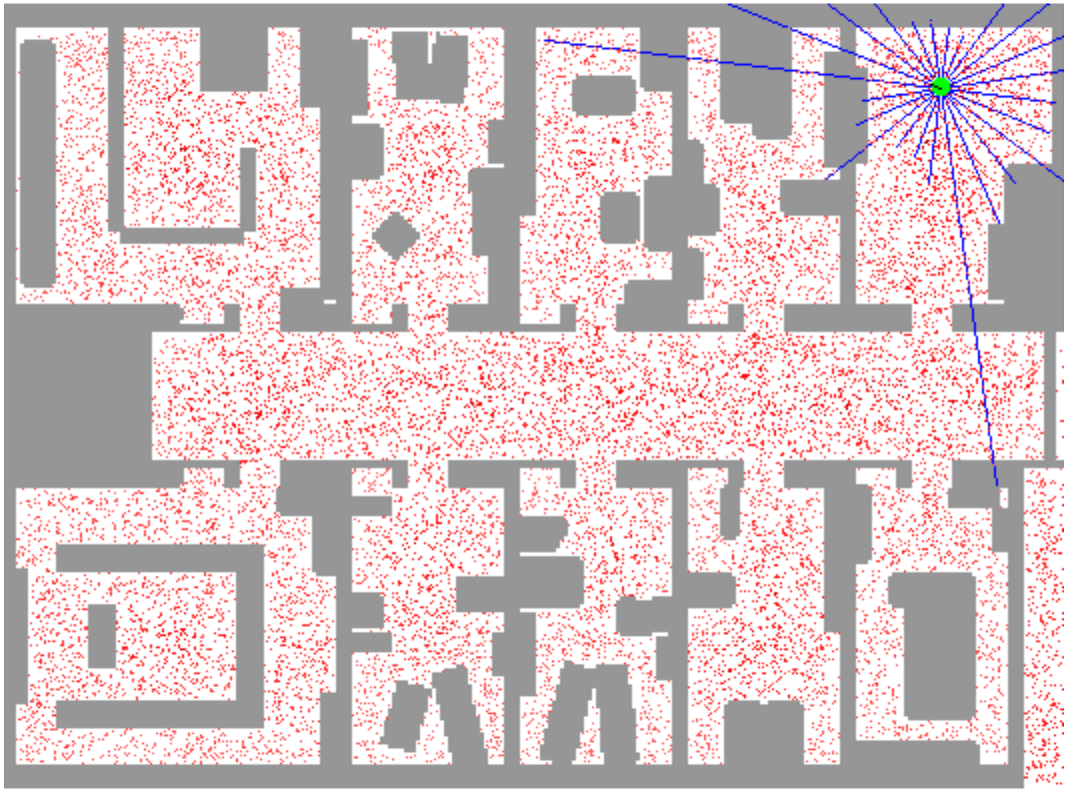
Robot Localization

time series

position at time t changes over time

robot dynamics are fixed



Fox, Burgard, Thrun

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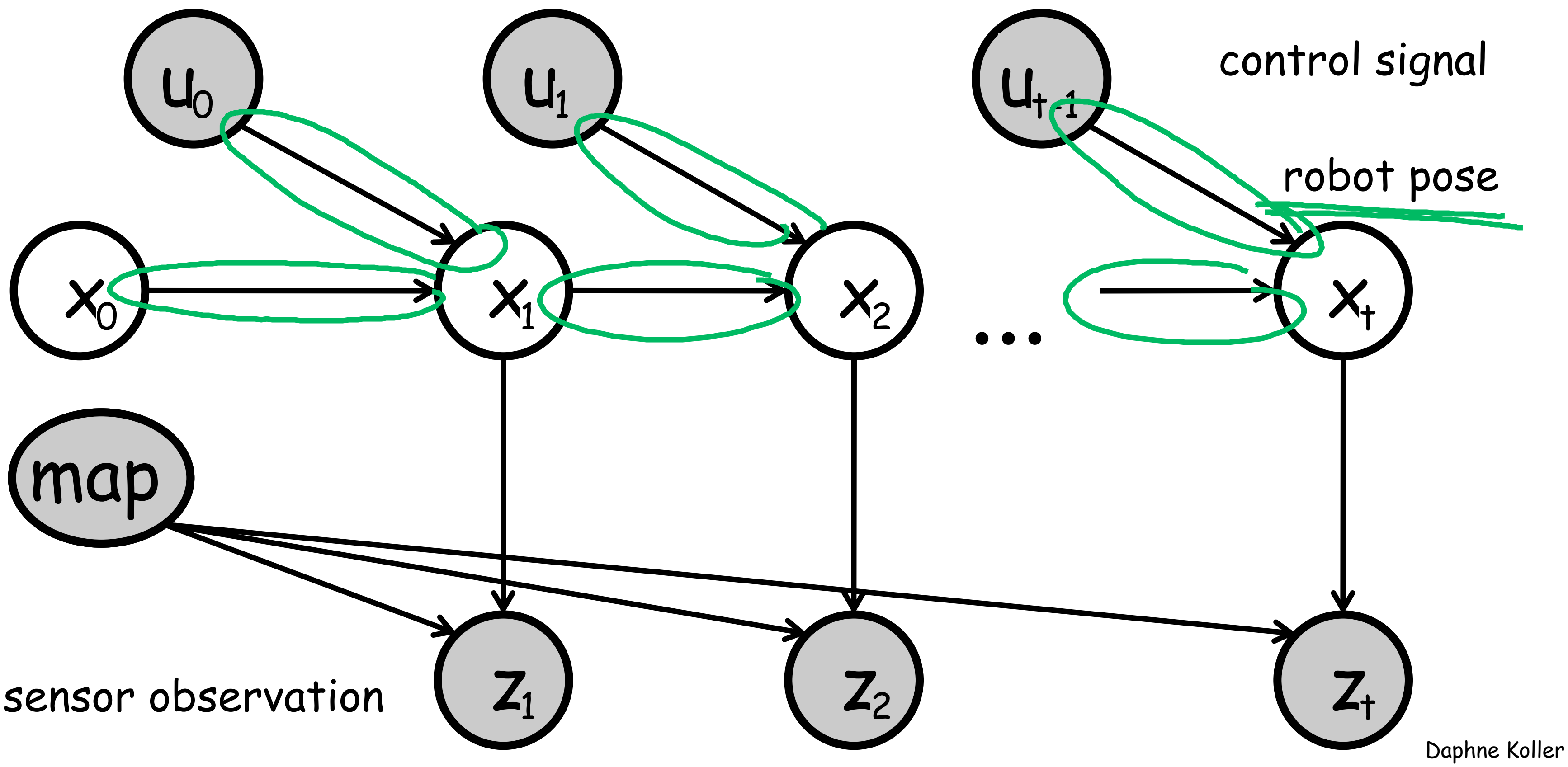
Robot Localization



Template Variables

- Template variable $X(U_1,\ldots,U_k)$ is instantiated (duplicated) multiple times
  - Location(t), Sonar(t)
  - Genotype(person), Phenotype(person)
  - Label(pixel)
  - Difficulty(course), Intelligence(student), Grade(course, student)

Template Models

- Languages that specify how ground variables inherit dependency model from template
- Dynamic Bayesian networks ← temporal
- Object-relational models people, courses, pixels, ...
  - Directed
    - Plate models
  - Undirected

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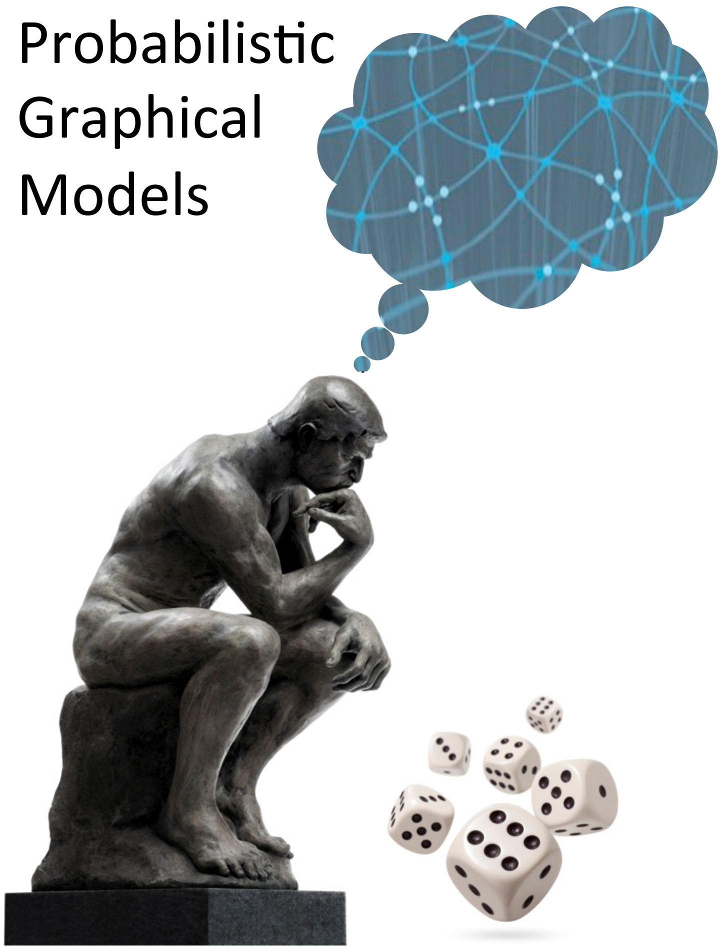


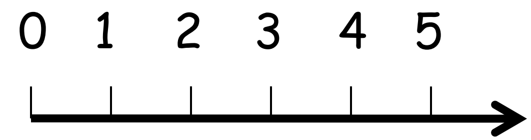
Representation

Template Models

Temporal Models

Distributions over Trajectories

discretize time



sensor

- Pick time granularity Δ
- $X^{(t)}$ – variable X at time $t\Delta$
- $X^{(t:t')} = \{X^{(t)}, \ldots, X^{(t')}\}$ $(t \leq t')$

- Want to represent $P(X^{(t:t')})$ for any t, t'

Markov Assumption

chain rule for probabilities

$$P(\boldsymbol{X}^{(0:T)}) = P(\boldsymbol{X}^{(0)}) \prod_{t=0}^{T-1} P(\boldsymbol{X}^{(t+1)} \mid \boldsymbol{X}^{(0:t)})$$

time flows forward

state at t+1 state at 0..t

$$(\underbrace{\boldsymbol{X}^{(t+1)}}_{\text{next step}} \perp \underbrace{\boxed{\boldsymbol{X}^{(0:t-1)}}}_{\text{past}} \mid \underbrace{\boldsymbol{X}^{(t)}}_{\text{present}})$$

forgetting

$$P(\boldsymbol{X}^{(0:T)}) = P(\boldsymbol{X}^{(0)}) \prod_{t=0}^{T-1} P(\boldsymbol{X}^{(t+1)} \mid \boldsymbol{X}^{(t)})$$

Is this true?

X = Location of robot probably not

$L^{t+1} \perp L^{t-1} \mid L^t$?

velocity

enrich state by adding V^t and other variables

(adding dependencies back in time - semi-Markov)

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Time Invariance

next time

current time

- Template probability model $P(\mathbf{X}' \mid \mathbf{X})$
- For all t:

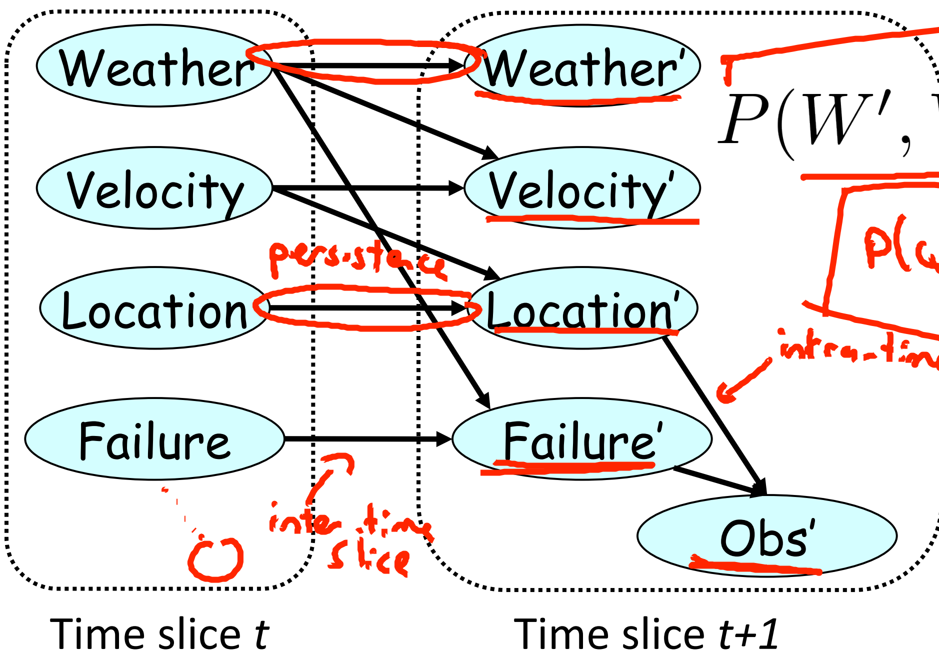
$$P(\boldsymbol{X}^{(t+1)} \mid \boldsymbol{X}^{(t)}) = P(\boldsymbol{X}' \mid \boldsymbol{X})$$

traffic time of day, day of week, football

enrich model by including

Template Transition Model

network fragment



cond dist t+1 given t

$$P(W', V', L', F', O' \mid W, V, L, F) =$$

$P(W'|W) \cdot P(V'|W,V) \cdot P(L'|L,V)$

$\underbrace{P(F'|F,W)}_{t \text{ to } t+1} \cdot \underbrace{P(O'|L',F')}_{t+1}$

$\cancel{P(W,V,L,F)}$

rapidly-acting

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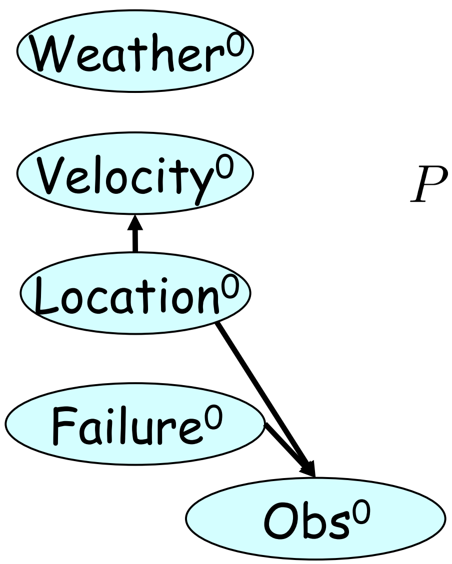
Initial State Distribution



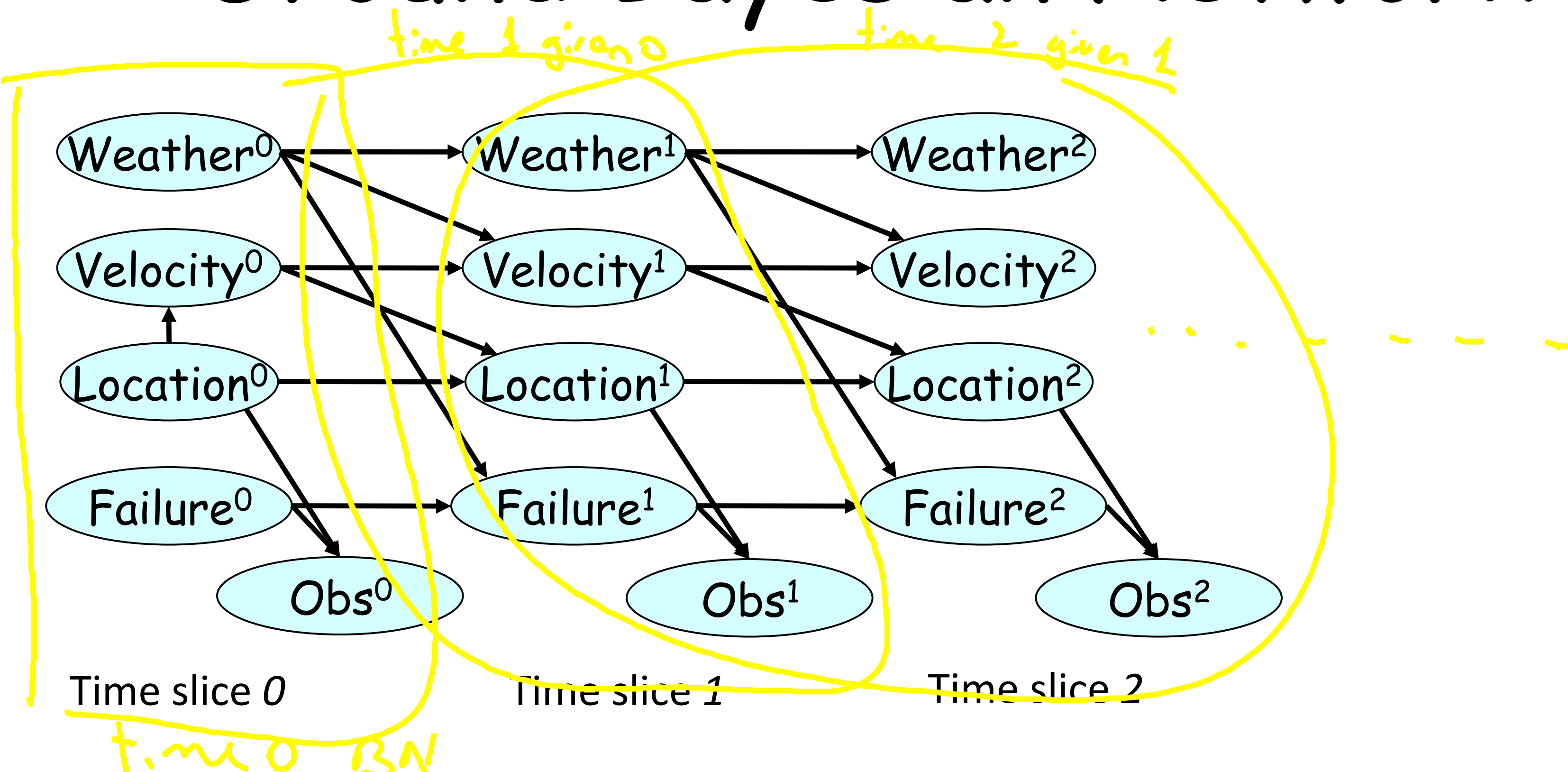
Time slice *0*

$$P(W^{(0)}, V^{(0)}, L^{(0)}, F^{(0)}, O^{(0)}) =$$

$$P(W^{(0)})P(V^{(0)} \mid L^{(0)})P(L^{(0)})P(F^{(0)})P(O^{(0)} \mid F^{(0)}, L^{(0)})$$

chain rule

Ground Bayesian Network



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2-time-slice Bayesian Network

template

- A transition model (2TBN) over $X_1,\ldots,X_n$ is specified as a BN fragment such that:
  - The nodes include $X_1',\ldots,X_n'$ and a subset of $X_1,\ldots,X_n$

    the time t vars that directly affect state at t+1
  - Only the nodes $X_1',\ldots,X_n'$ have parents and a CPD
- The 2TBN defines a conditional distribution

$$P(\boldsymbol{X}' \mid \boldsymbol{X}) = \prod_{i=1}^{n} P(X_i' \mid \mathbf{Pa}_{X_i'})$$

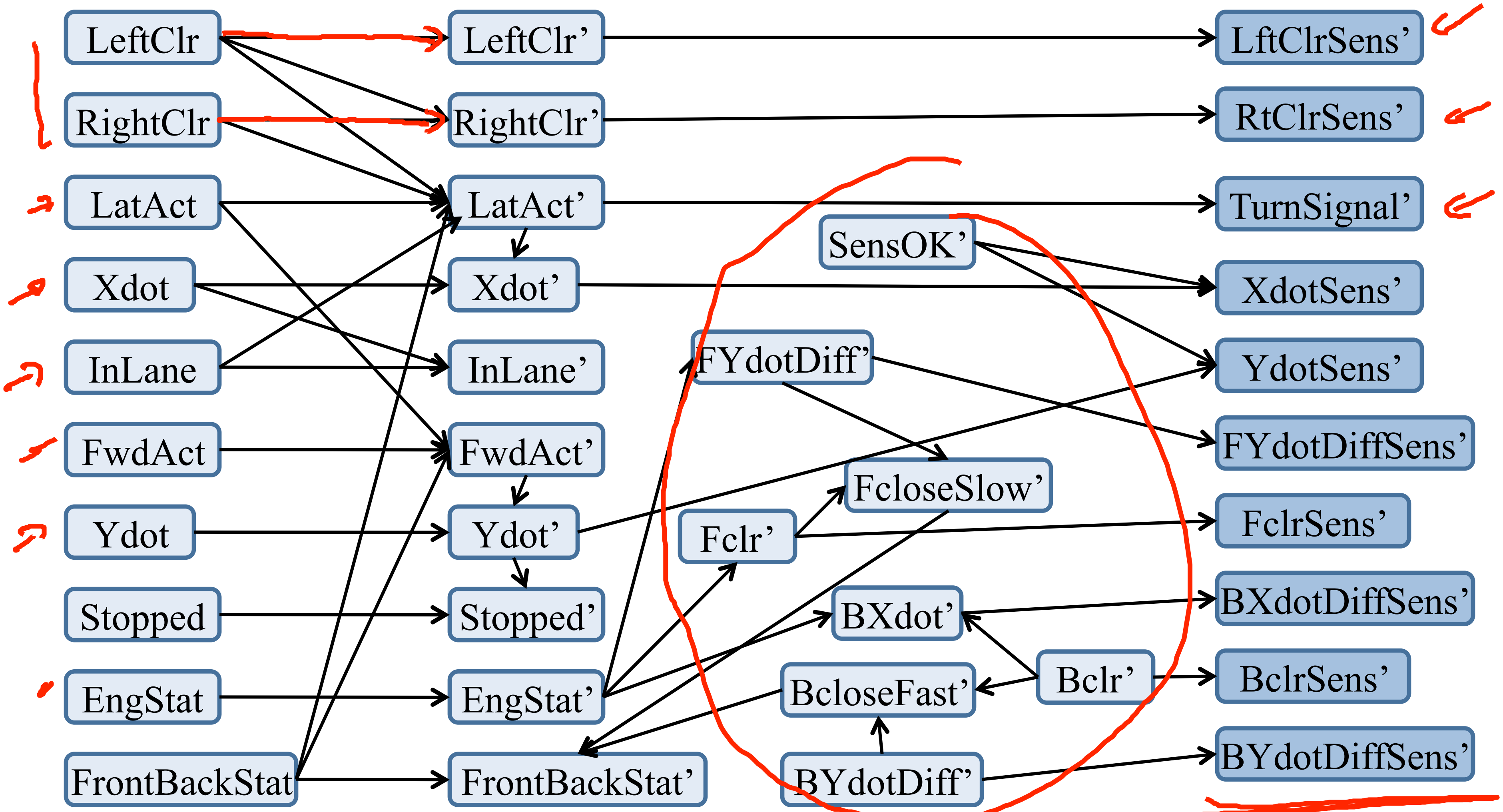
chain rule

Dynamic Bayesian Network

- A dynamic Bayesian network (DBN) over $X_1,\ldots,X_n$ is defined by a
  - $\underbrace{\text{2TBN } BN_{\rightarrow}}$ over $X_1,\ldots,X_n$ dynamics
  - time 0 – a Bayesian network $BN^{(0)}$ over $X_1^{(0)},\ldots,X_n^{(0)}$

Ground Network

- For a trajectory over 0,…,T we define a ground (unrolled network) such that
  - time 0 – The dependency model for $X_1^{(0)}, \ldots, X_n^{(0)}$ is copied from $BN^{(0)}$
  - trans.t. – The dependency model for $X_1^{(t)}, \ldots, X_n^{(t)}$ for all $t > 0$ is copied from $BN_{\rightarrow}$



Huang, Koller, Malik, Ogasawara, Rao, Russell, Weber, AAAI 94

Summary

- DBNS are a compact representation for encoding structured distributions over arbitrarily long temporal trajectories
- They make assumptions that may require appropriate model (re)design:
  - Markov assumption
  - Time invariance

Probabilistic
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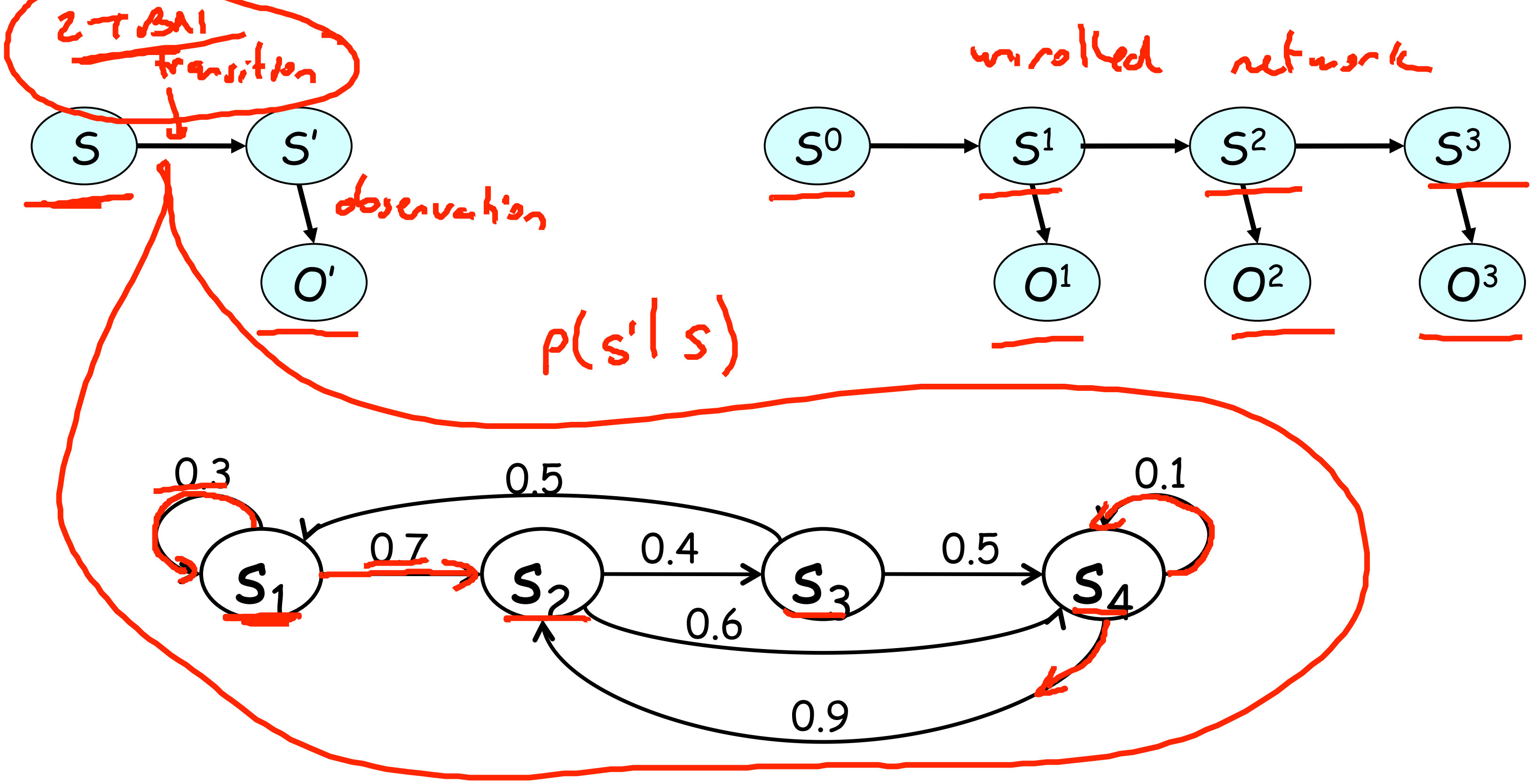
Representation

Template Models

Hidden Markov Models

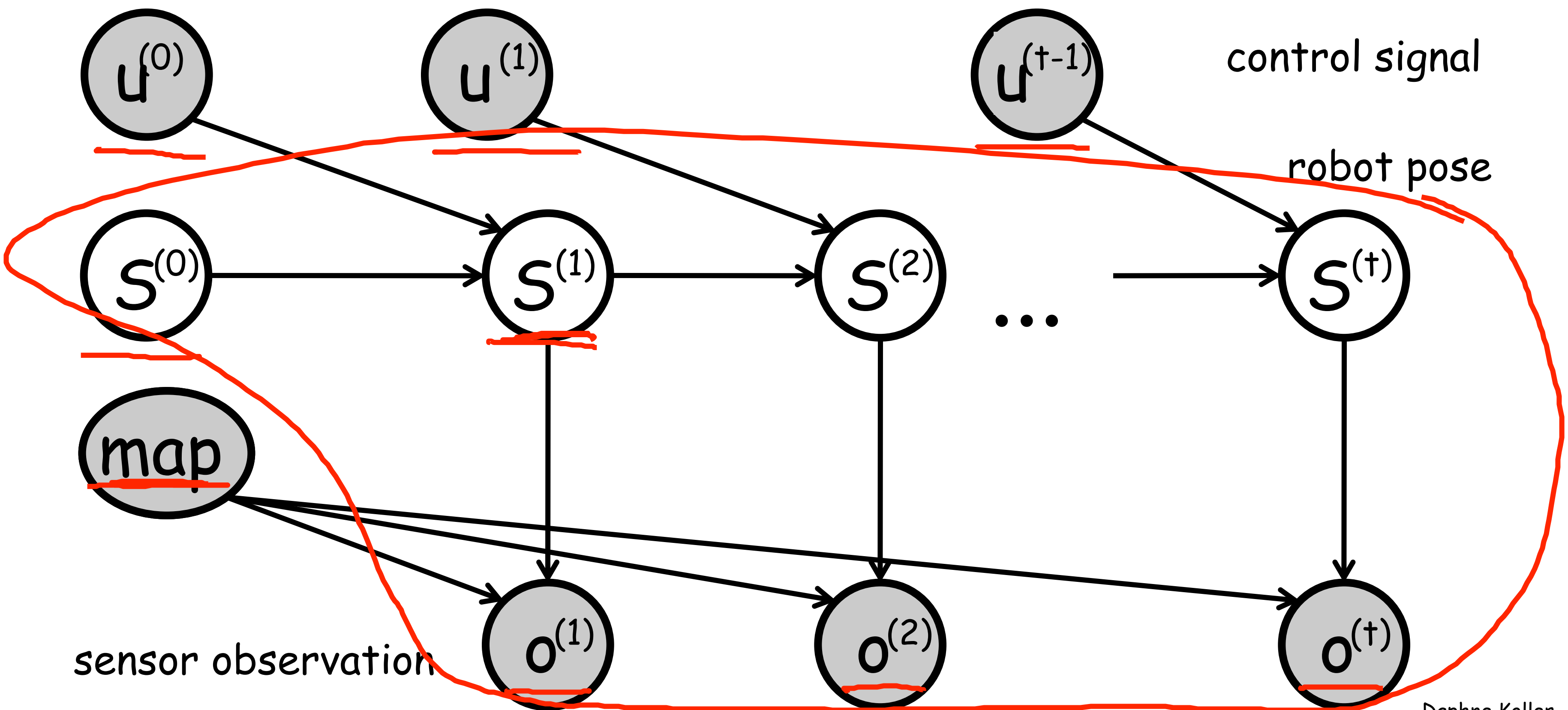
Hidden Markov Models



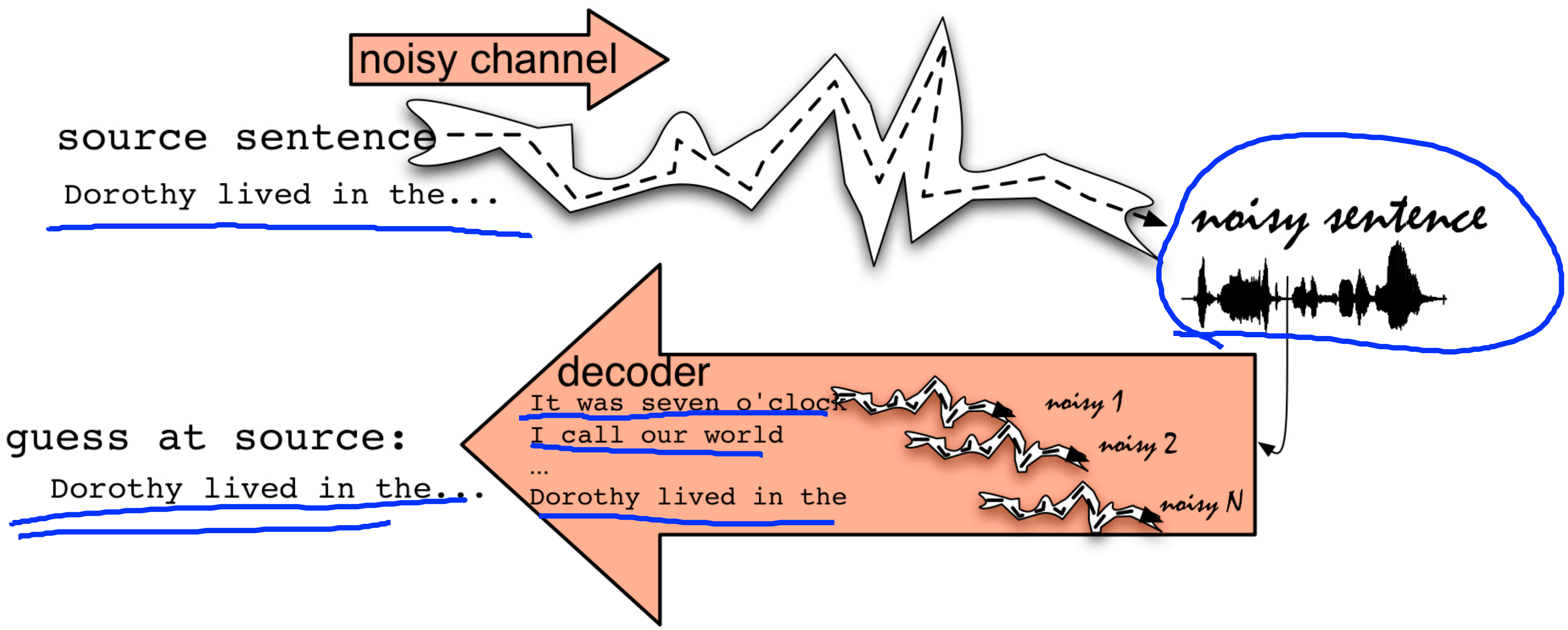
Numerous Applications

- Robot localization
- Speech recognition
- Biological sequence analysis
- Text annotation

Robot Localization



Speech Recognition



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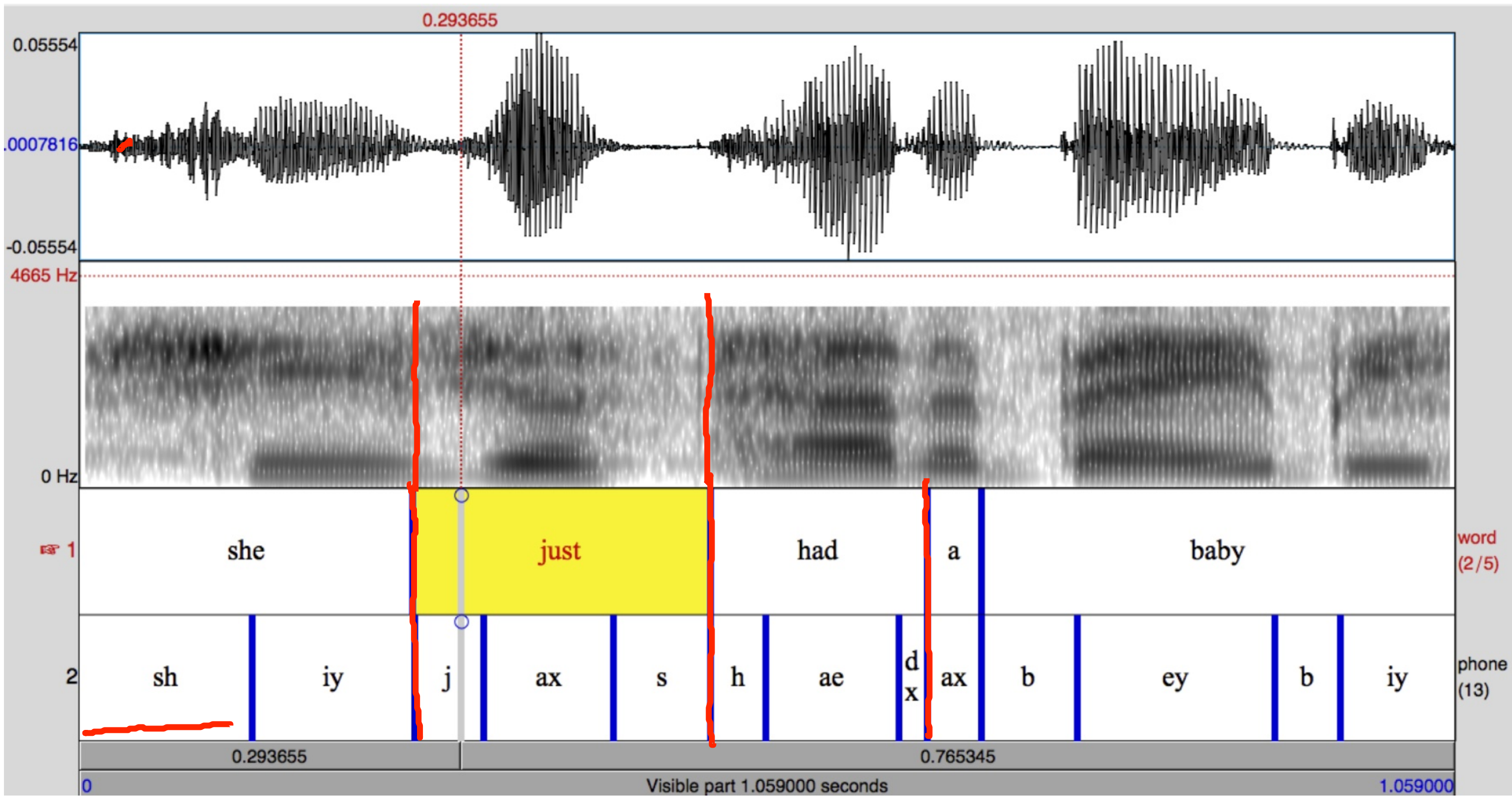
Segmentation of Acoustic Signal



phones

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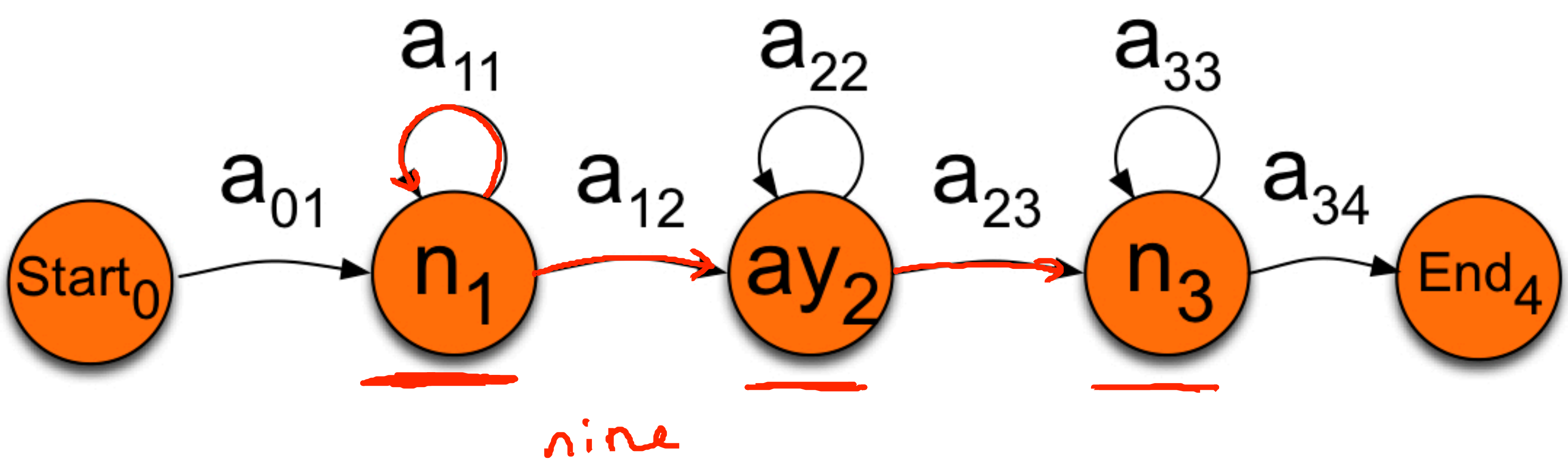
Phonetic Alphabet

AA	odd	AA D
AE	at	AE T
AH	hut	HH AH T
AO	ought	AO T
AW	cow	K AW
AY	hide	HH AY D
B	be	B IY
CH	cheese	CH IY Z
D	dee	D IY
DH	thee	DH IY
EH	Ed	EH D
ER	hurt	HH ER T
EY	ate	EY T
F	fee	F IY
G	green	G R IY N
HH	he	HH IY
IH	it	IH T
IY	eat	IY T
JH	gee	JH IY
K	key	K IY
L	lee	L IY
M	me	M IY
N	knee	N IY
NG	ping	P IH NG
OW	oat	OW T
OY	toy	T OY
P	pee	P IY
R	read	R IY D
S	sea	S IY
SH	she	SH IY
T	tea	T IY
TH	theta	TH EY T AH
UH	hood	HH UH D
UW	two	T UW
V	vee	V IY
W	we	W IY
Y	yield	Y IY L D
Z	zee	Z IY
ZH	seizure	S IY ZH ER

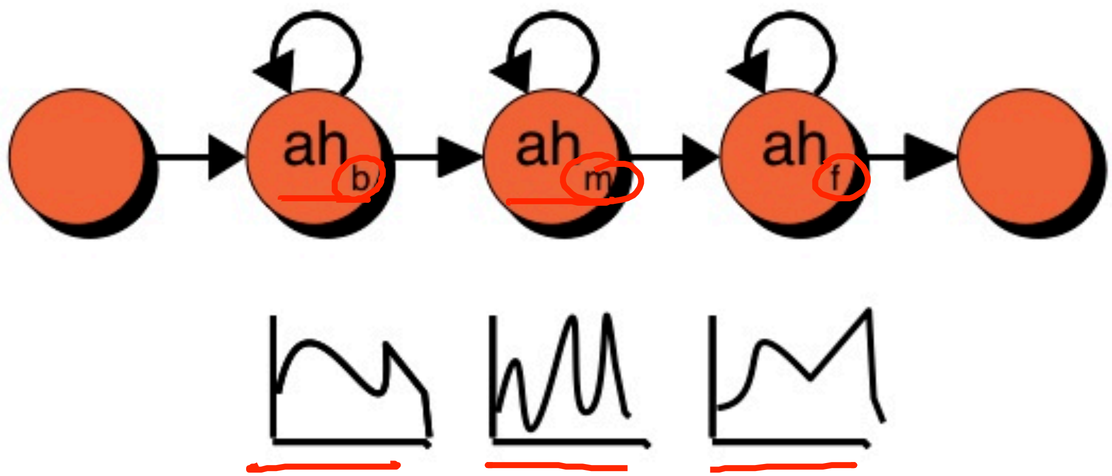
http://www.speech.cs.cmu.edu/cgi-bin/cmudict

The CMU Pronouncing Dictionary

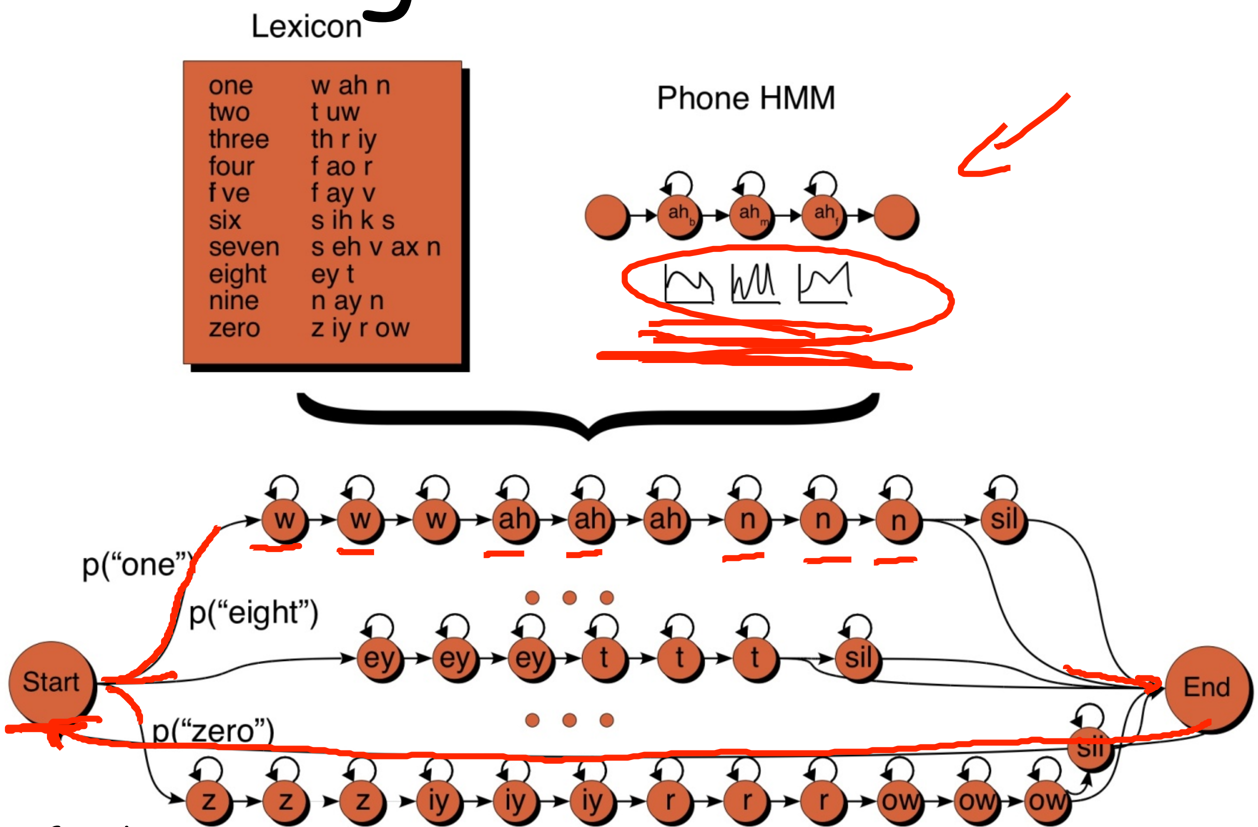
Word HMM



Phone HMM



Recognition HMM



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Summary

- HMMs can be viewed as a subclass of DBNs
- HMMs seem unstructured at the level of random variables
- HMM structure typically manifests in sparsity and repeated elements within the transition matrix
- HMMs are used in a wide variety of applications for modeling sequences

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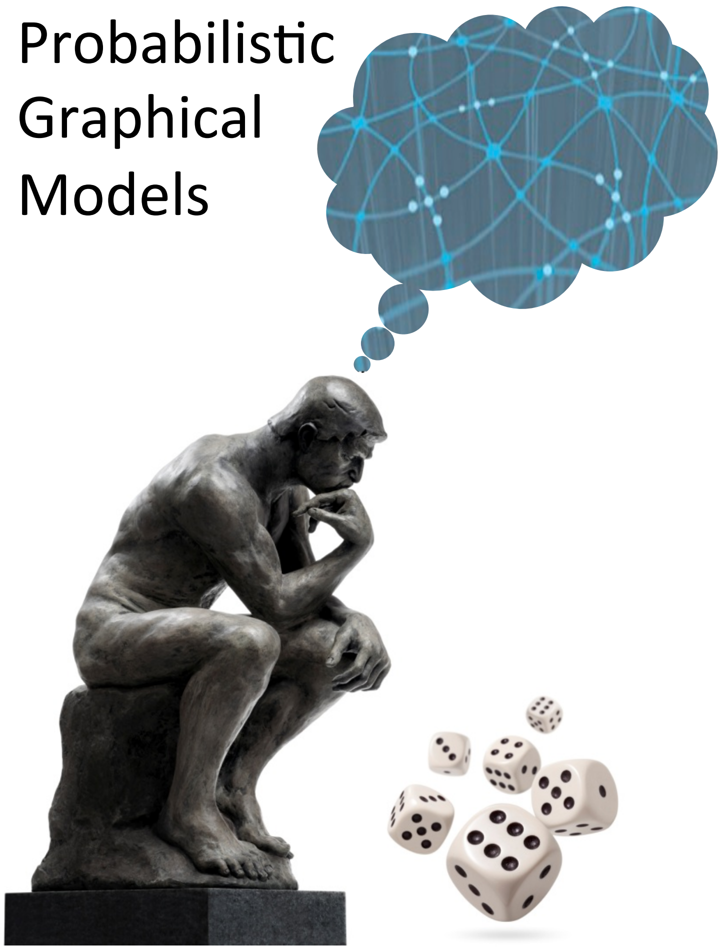
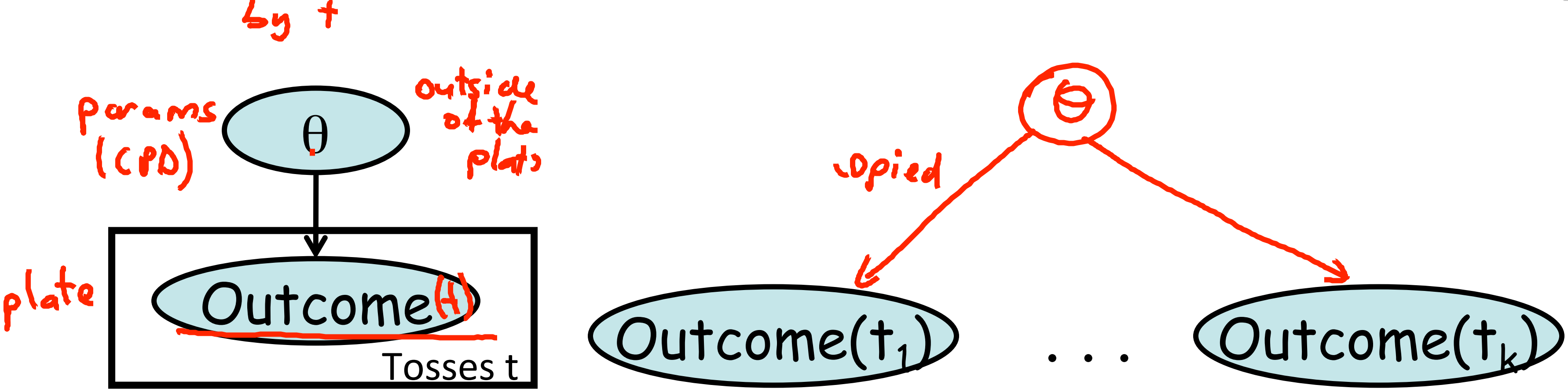


Representation

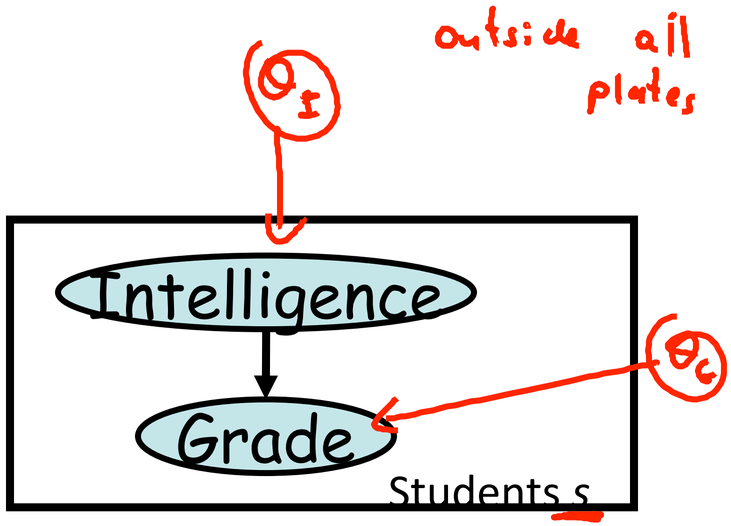
Template Models

Plate Models

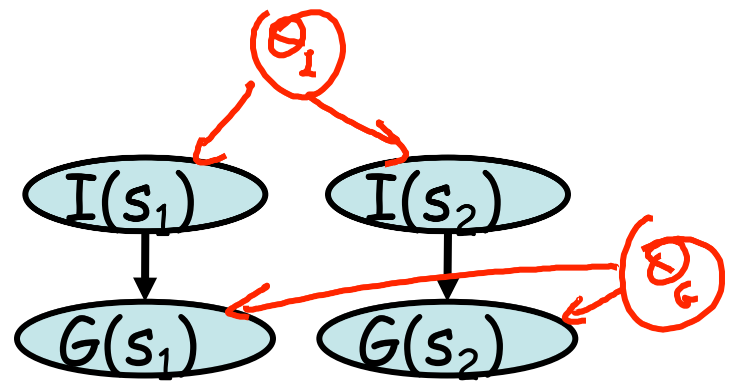
Modeling Repetition



$\underbrace{\{t_1, \ldots, t_k\}}$



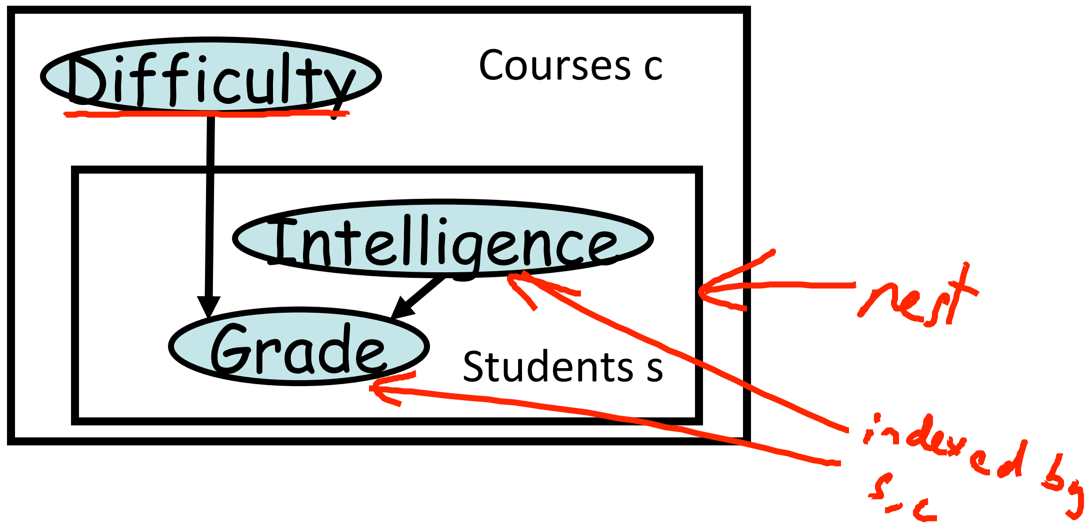
outside all plates
θ_I
Intelligence
Grade
θ_G
Students s

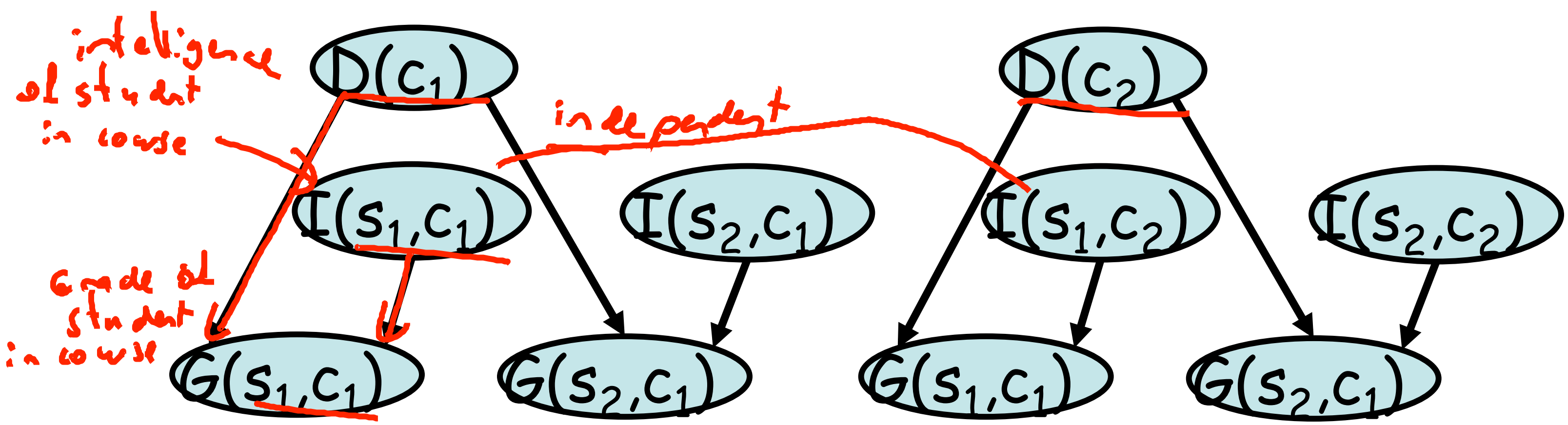


θ_I
$I(s_1)$
$I(s_2)$
$G(s_1)$
$G(s_2)$
θ_G

Nested Plates

courses c
students s





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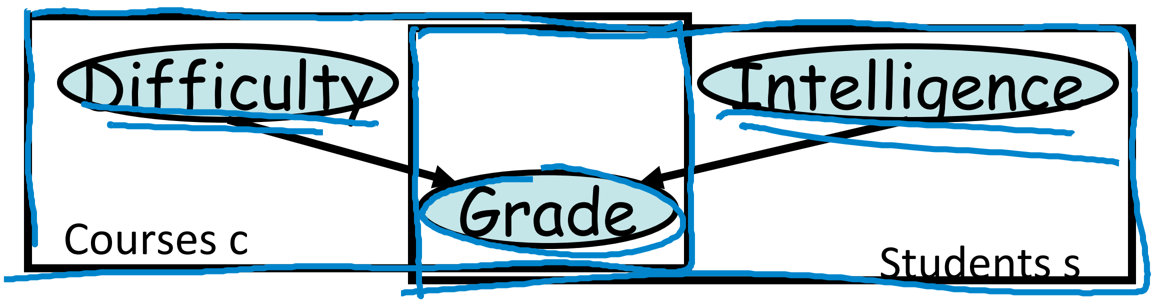
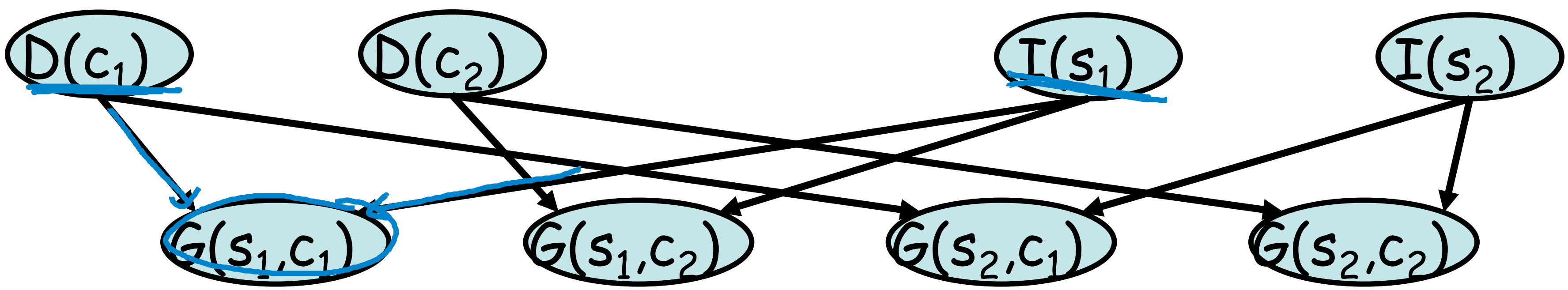
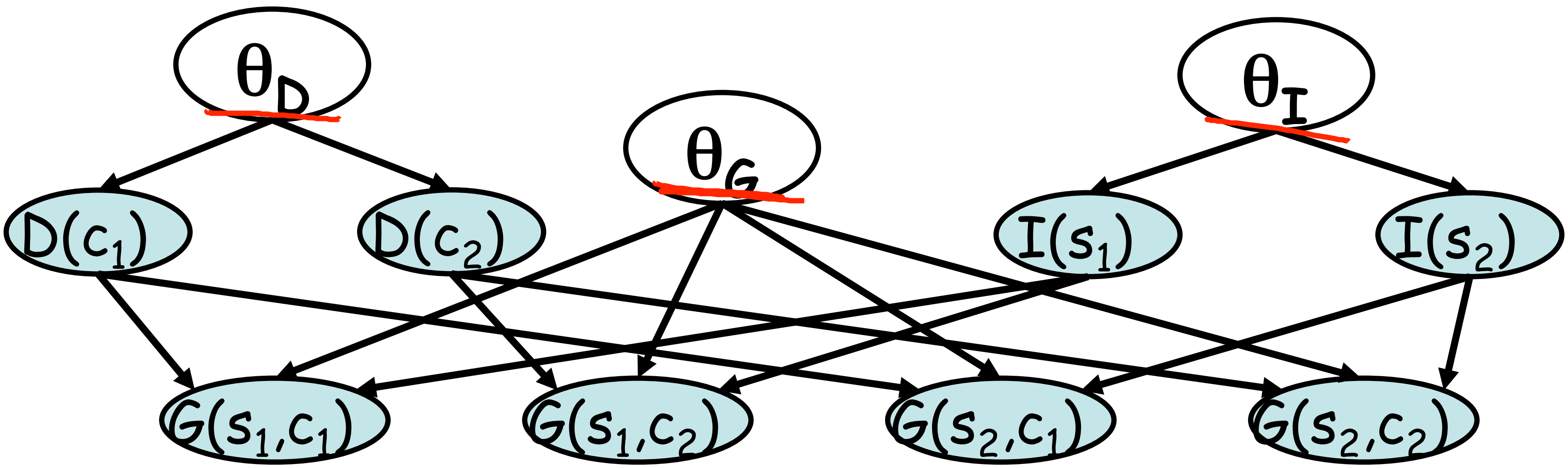
Overlapping Plates





Explicit Parameter Sharing



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Collective Inference

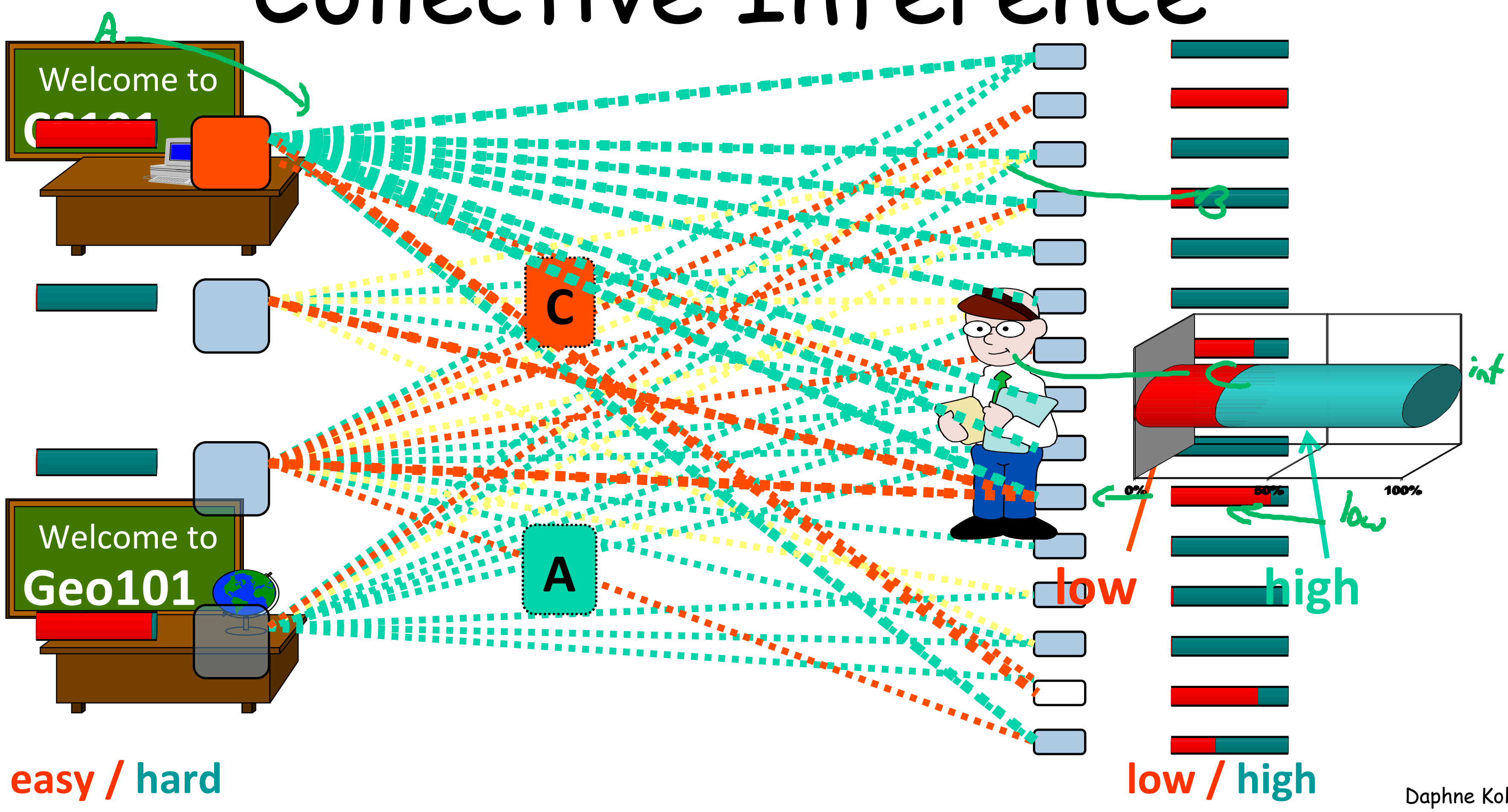

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Plate Dependency Model

indexed

- For a template variable $A(U_1,\ldots,U_k)$:
  - Template parents $B_1(U_1),\ldots,B_m(U_m)$

$U_i \subseteq \{U_1,\ldots,U_k\}$

I(s) D(c)

G(s,c)

G(s,c)

Honors(s)

unbounded # of parents

aggregator CPD

template

  - CPD $P(A \mid B_1,\ldots, B_m)$

Ground Network

Let $A(U_1,\ldots,U_k)$ with parents $B_1(\mathbf{U}_1),\ldots,B_m(\mathbf{U}_m)$

- for any instantiation $u_1,\ldots,u_k$ to $U_1,\ldots,U_k$ we would have:

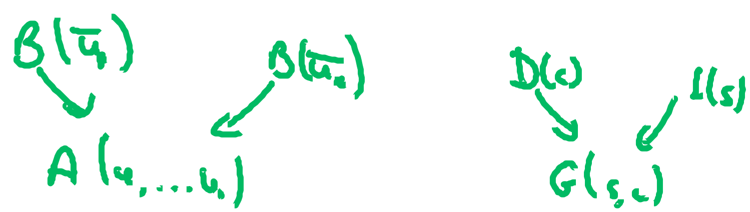

Plate Dependency Model

Let $A(U_1,\ldots,U_k)$ with parents $B_1(\mathbf{U}_1),\ldots,B_m(\mathbf{U}_m)$

- For each i, we must have $\mathbf{U}_i \subseteq U_1,\ldots,U_k$
  - No indices in parent that are not in child

$G(m) \rightarrow G(c) \leftarrow G(p)$

Summary

$X^{t-1} \rightarrow X^{t}$

- Template for an infinite set of BNs, each induced by a different set of domain objects
- Parameters and structure are reused within a BN and across different BNs
- Models encode correlations across multiple objects, allowing collective inference
- Multiple "languages", each with different tradeoffs in expressive power