

University of Stuttgart

Institute of Biomedical Genetics
RNA Biology and Bioinformatics

Succinct de Bruijn graphs

Alex Bowe, Taku Onodera, Kunihiko Sadakane
and Tetsuya Shibuya



De Bruijn Graph

- De Bruijn Graph – DBG
 - cool data structure for DNA sequencing
 - pre-step in DNA assembly
- “De Bruijnizing” algorithm
 1. Chop the reads into **k-mers** (strings of length k)
 2. Build the **directed graph** based on the overlaps
 3. Call the k-mers **nodes**
 4. Call the overlaps **edges**

TACGACGTGACT:

TAC

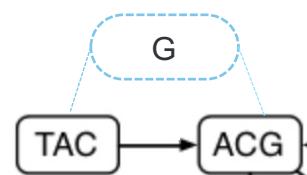
ACG

CGA

GAC

G

etc



Storage of DBG

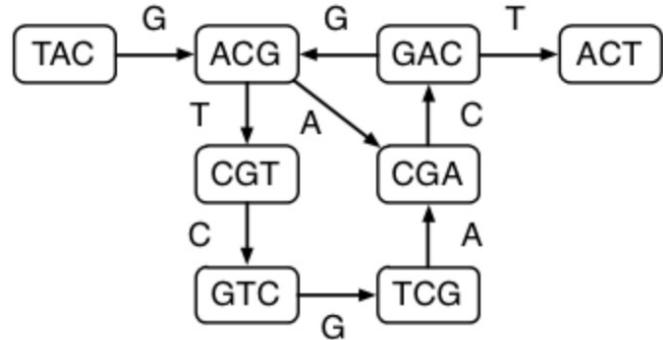
- How would you normally store DBG
 - Naive implementation: dictionary
 - Pros: Can be easily controlled/efficient **query operations**
 - Cons: **A lot of space** especially in case of metagenomics
- Optimize space

❑ Rules:

- Keep efficient query operations
- Compression allowed while saving the k-mers

✓ Solution:

- Information-theoretic lower bound - Z
- Or close to Z (let me explain)



From	To
TAC	ACG
ACG	CGT, CGA
CGT	GTC
GTC	TCG
TCG	CGA
CGA	GAC
GAC	ACT

Information Theoretic Lower Bound - OPT

- Given: information-theoretic lower bound: Z
- 3 ways to keep efficient query operations:
 - Implicit: $Z + \mathcal{O}(1)$ bits
 - Succinct:** $Z + o(Z)$ bits
 - Compact: $\mathcal{O}(Z)$ bits
- Succinct data structures:*
 - use space “close” to theoretical lower bound
 - allow efficient query operations
 - with efficient “dynamicity” also allow addition/removal of nodes

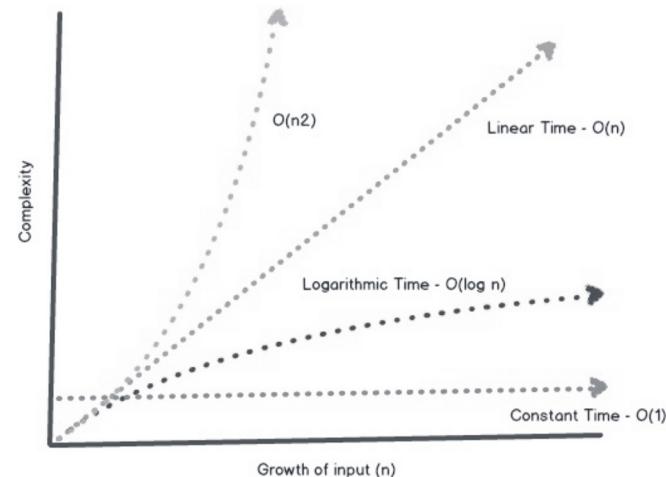
Information Theoretic Lower Bound:

Round 6.449 to one character after .

Rule: Max rounding would be 0.099.

$$Z = 6.4$$

$o(Z)$ unit is any function that grows slower than a linear function, e.g.: \sqrt{Z} , $lg(Z)$, etc.



Succinct de Bruijn Graphs

Alexander Bowe¹, Taku Onodera², Kunihiko Sadakane¹, and Tetsuo Shibuya²

¹ National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku,
Tokyo 101-8430, Japan
`{alex,sada}@nii.ac.jp`

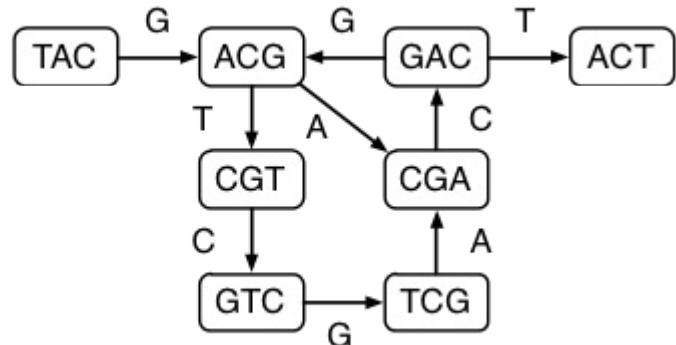
² Human Genome Center, Institute of Medical Science,
University of Tokyo 4-6-1 Shirokanedai, Minato-ku, Tokyo 108-8639, Japan
`{tk-ono,tshibuya}@hgc.jp`

SDBG – Succinct de Bruijn Graph

- Goal:
 - represent de Bruijn graph succinctly $Z + o(Z)$
 - enable efficient query operations
- Succinct de Bruijn graph:
 - DNA sequence of length: N
 - number of edges: m
 - $4m + o(m)$
- Comparisons:
 - distributed hash table(compressed dictionary): 336 GB
 - sparse bitvector representation:
 - $\mathcal{O}(mk)$ w/ encoding and compression: 32 GB
 - ***succinct representation:*** 2.5 GB

Construction

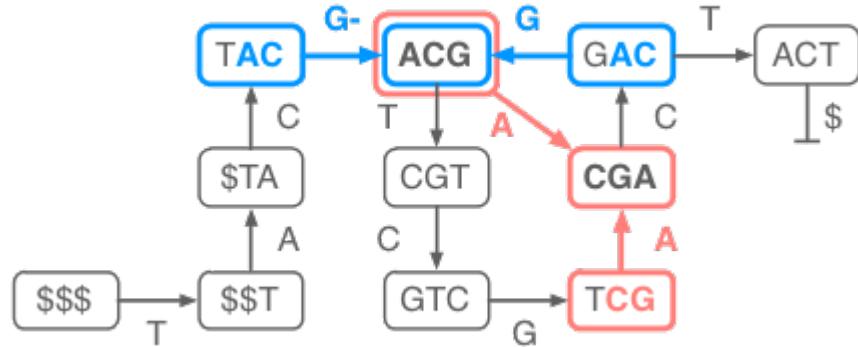
- Take every $\langle \text{node}, \text{edge} \rangle$
 - Sort them based on reverse(colex order) of node label
 - Remove duplicates
-
- Additional steps:
 - Use padding to know the start and end
 - Rules for padding:
 - k "\$"s to the beginning
 - 1\$ to the end



<i>Node</i>	<i>W</i>
C G A	C
G A C	G
G A C	T
T A C	G-
G T C	G
A C G	A
A C G	T
T C G	A-
C G T	C

Construction

- W – edge vector(1 char)
- disambiguate identically labelled incoming edges:
 - A and $A-$, G and $G-$
- each outgoing edge is stored contiguously
- Bit-vector L :
 - introduce a bit vector(0 or 1):
 - represent whether an edge is the *last* edge exiting a node
- Bit-vector F :
 - indicates the first and last occurrences of characters from alphabet $\{\$, A, C, G, T\}$
 - works due to colex ordering



	i	L	Node	W
F	0	1	\$	T
	1	1	C	C
	2	1	C	G
\$	3	0	T	T
A	1	0	A	G
C	3	1	C	T
G	7	1	C	G
T	10	0	G	G
	7	1	A	A
	8	1	C	T
	9	1	C	G
	10	1	T	T
	11	1	A	A
	12	1	C	C

Rank and Select

RANK and SELECT have been used before on binary alphabets.

RANK

\$	A	A	C	C	C	G	G	G	T	T		
F	0	1	2	3	4	5	6	7	8	9	10	11

SELECT

\$	A	A	C	C	C	G	G	G	T	T		
F	0	1	2	3	4	5	6	7	8	9	10	11

- Returns the number of elements equal to q up to position x

$$rank_q(x) = |\{l, l \in [0 \dots x], S[l] = q\}|$$

$$rank_A(7) = 2$$

(num of A's are there until position 7)

$$rank_C(5) = 3$$

(num of C's are there until position 5)

- Returns the position of the x^{th} occurrence of q .

$$select_q(x) = \min\{l \in [0 \dots x]: rank_q(l) = x\}$$

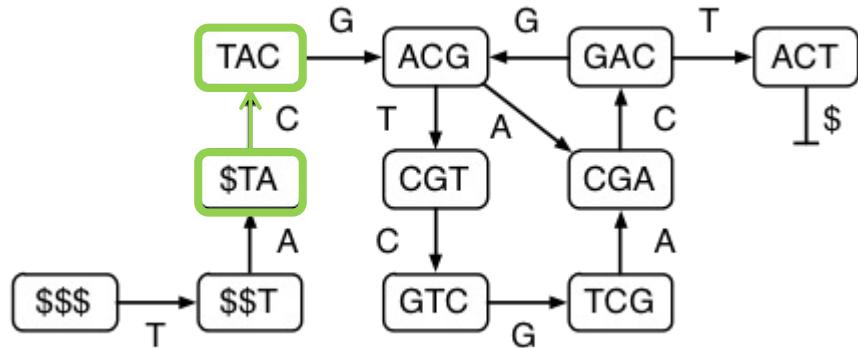
$$select_T(1) = 10$$

(position of 1'st occurrence of T)

$$select_G(3) = 9$$

(position of 3'rd occurrence of G)

$$forward(v, c) = w$$



	<i>i</i>	<i>L</i>	<i>Node</i>	<i>W</i>
<i>F</i>	0	1	\$ \$ \$	T
	1	1	C G A	C
	2	1	T A C	G
	3	0	G A C	T
	4	1	G A C	G
	5	1	T A C	G
	6	1	G T C	G
	7	0	A C G	A
	8	1	A C G	T
	9	1	T C G	A
	10	1	\$ \$ T	A
	11	1	A C T	\$
	12	1	C G T	C

1. rank_C = 2

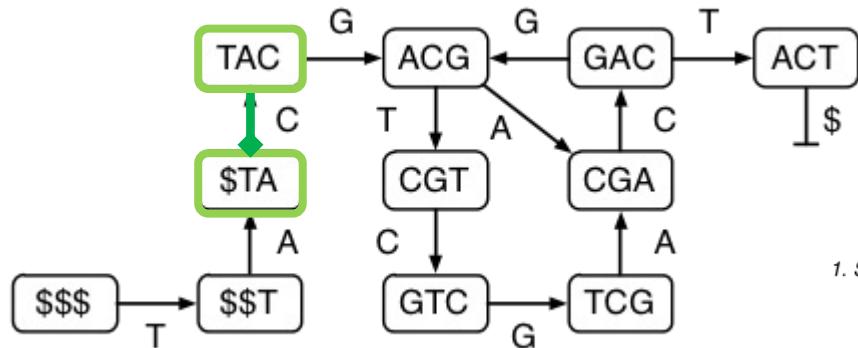
2. Starting position
of C = 3

3. Rank to base = 3

4. Select to (3 + 2)th position (i.e. the second C)

- Find second occurrence of **C** in *Node* column
- Ignore the first **C** since that has *L* equal 0
- $O(forward) = 1$ time unit
 - W access - $\mathcal{O}(1)$
 - rank over W - $\mathcal{O}(1)$
 - rank and select over L - $\mathcal{O}(1)$
 - accessing F - $\mathcal{O}(1)$

$$\text{backward}(w, c) = v \text{ (inverse_forward)}$$



1. Starting position
of $C = 3$

F	i	L	Node	W
\$ 0	0	1	\$ \$ \$	T
A 1	1	1	C G A	C
C 3	2	1	\$ T A	G
G 7	3	0	G A C	T
T 10	4	1	G A C	G
	5	1	T A C	G
	6	1	G T C	A
	7	0	A C G	T
	8	1	A C G	A
	9	1	T C G	-
	10	1	\$ \$ T	A
	11	1	A C T	\$
	12	1	C G T	C

4. $\text{select}_C(2) = 2$

- Find second occurrence of **C** in W column
- We ignore $L[\text{index}(C)] == 0$
- $O(\text{backward}) = 1$ time unit
 - Node access - $\mathcal{O}(1)$
 - rank over Node - $\mathcal{O}(1)$
 - rank and select over L - $\mathcal{O}(1)$
 - accessing W - $\mathcal{O}(1)$

2. Rank to base = 3

3.1. Rank to current edge = 5,

3.2. $5 - 3 = 2$, so we are at the **second C**

$$label(v) = dna_sequence$$

i	L	$Node$	W
0	1	\$	T
1	1	A	C
2	1	A	C
3	0	C	G
4	1	C	T
5	1	C	G-
6	1	C	G
7	0	G	A
8	1	G	T
9	1	G	A-
10	1	T	A
11	1	T	\$
12	1	T	C

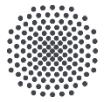
- What if we need to print out the sequence of Node(e.g. $node_8$)
- Execute backward function k times
- Function backward takes $\mathcal{O}(1)$ time
- Repeat the backward function k times $\Rightarrow \mathcal{O}(k)$

Operations and Time Complexity

Operation	Description	Complexity
$\text{outdegree}(v)$	Return number of outgoing edges from node v .	$\mathcal{O}(1)$
$\text{outgoing}(v, c)$	From node v , follow the edge labeled by symbol c .	$\mathcal{O}(1)$
$\text{label}(v)$	Return (string) label of node v .	$\mathcal{O}(k)$
$\text{indegree}(v)$	Return number of incoming edges to node v .	$\mathcal{O}(1)$

Lesson and Conclusion

- Using “creative counting” with two operations *rank* and *select* the authors created a memory efficient data structure
- The total space has been reduced to the “closer” to OPT space units
 - i.e.: 300 GB → 2.5 GB
- In principle fairly straight forward to implement
- Tools and libraries(C++ based) like: [MegaHIT](#), [BOSS](#), [DynaBOSS](#) and [cosmo](#) have already implemented the data structure
 - For [MegaHIT](#) it took **12** hrs to build a graph from 100 GB Metagenomic reads
 - For slightly further future we might use [DynaBOSS](#)



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Thank you!