Risk Econometrics Exam

Review of the paper: Fan Zhang Tsai and Wei (2008) Energy Economics

Ferniani Matteo

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1 Research Aim

In this research paper, the primary goal is to assess the Value at Risk (VaR) of crude oil prices and examine its spillover effects. The focus lies on identifying the most suitable GARCH model and distribution for accurate estimation. The study involves computing both upside and downside VaR using selected GARCH models and the Generalized Error Distribution (GED). To evaluate the efficacy of this approach in risk management, a comparative analysis will be conducted against traditional normal distribution based VaR and the Historical Simulation Approach (HSAF) with a specific emphasis on risk spillover effects, employing the Granger causality test. I analysed the various statistical tools and models that the paper used. Additionally, I delved further into the researchers' choices regarding the instruments, encompassing different types of models, distributions, and risk measures.

2 Tools and Motivations

The leptokurtic distribution and fat tail observed in oil price returns deviate significantly from the standard normal distribution. This discrepancy underscores the inadequacy of assuming a standard normal distribution, particularly in underestimating extreme risks associated with oil price fluctuations.

2.1 Distribution Analysis

An examination of the distribution of WTI oil returns against different distributions reveals that the Generalized Error Distribution (GED) proposed by Nelson (1990) is fitting for estimating residual series from GARCH-type models

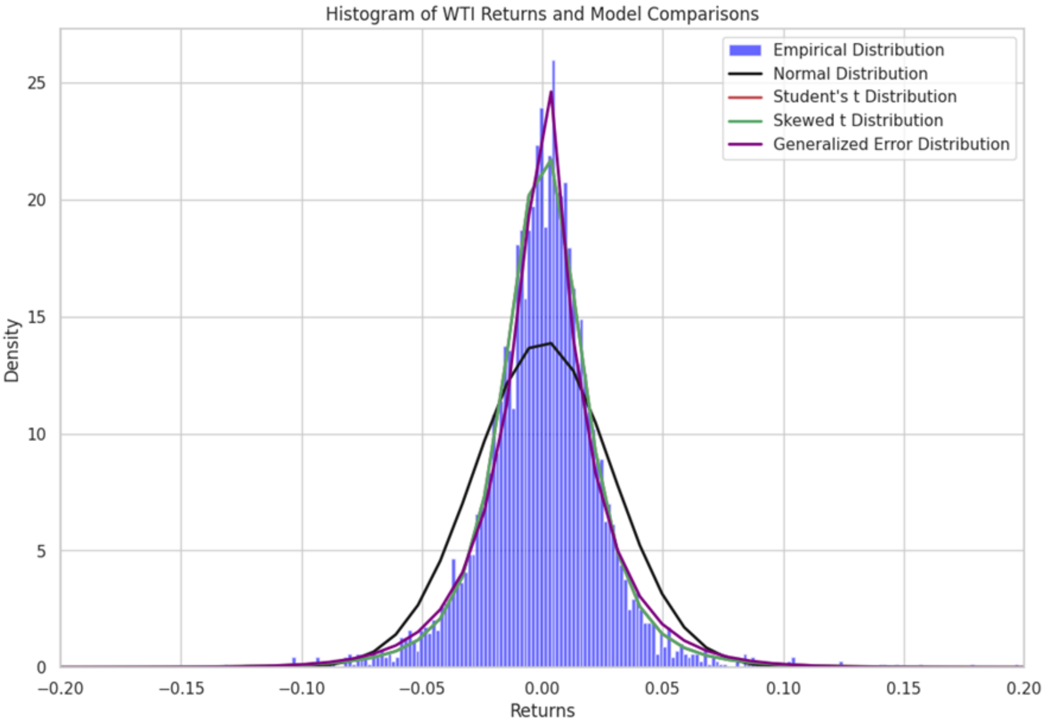

Figure 1: Histogram of WTI Returns and Distributions comparison

2.2 GED Probability Density Function

The probability density function (PDF) of the Generalized Error Distribution (GED) is given by:

$$f(x;\nu,\lambda) = \frac{\lambda}{2^{\frac{2\nu-1}{\nu}}\Gamma\left(\frac{1}{\nu}\right)} \exp\left(-\left(\frac{\lambda}{2}\right)^{\nu} |x|^{\nu}\right) \tag{1}$$

where:

$f(x;\nu,\lambda)$ is the PDF of the GED,
ν is the shape parameter,
λ is the scale parameter,
$\Gamma(\cdot)$ is the gamma function.

2.3 GARCH Models and Volatility Clustering

Crude oil price returns often exhibit a clustering phenomenon in volatility. This is evident in both the graphical representation of returns and squared return

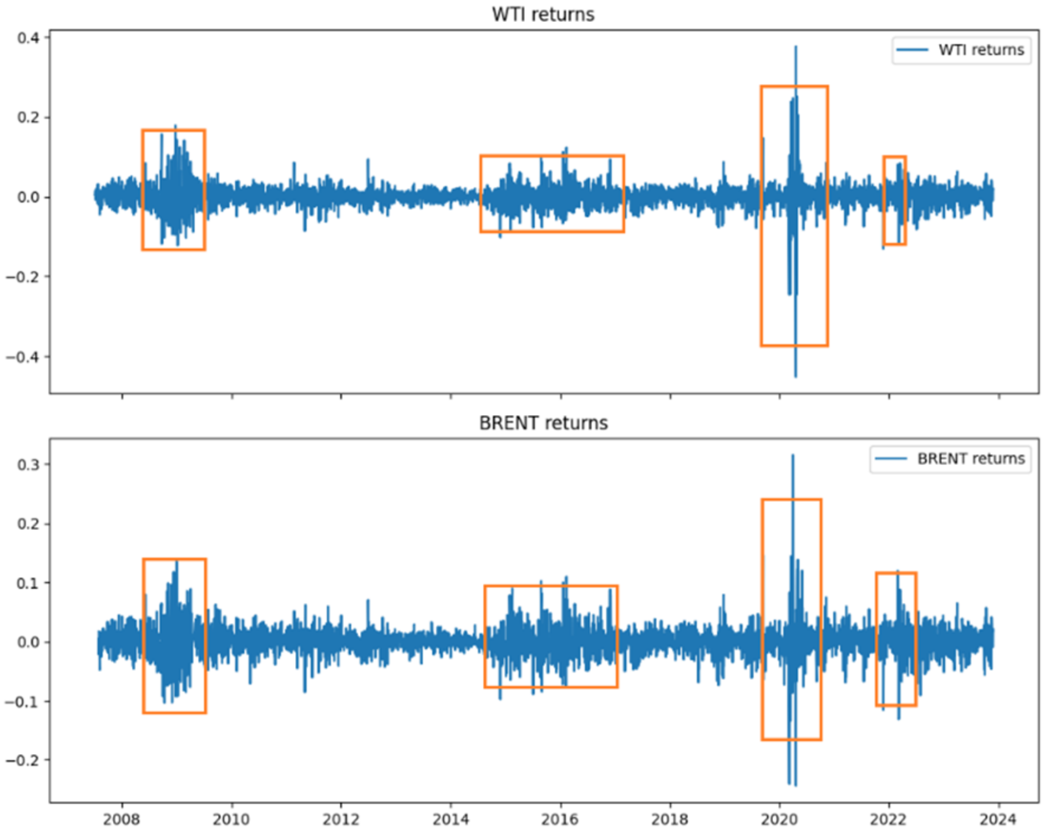

Figure 2: WTI and BRENT Returns

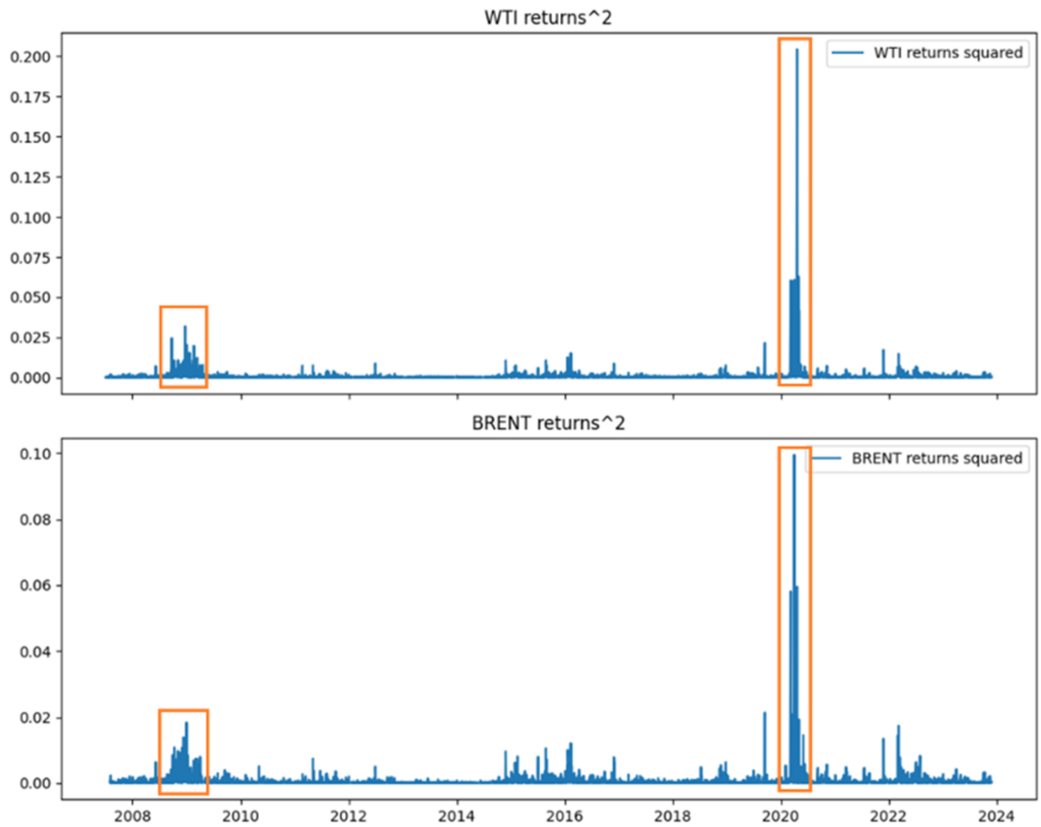

Figure 3: WTI and BRENT Squared Returns

To capture this phenomenon, the study employs the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model

2.4 GARCH(p, q) Model

The GARCH(p, q) model is defined as follows:

$$Y_t = \mu + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \cdot Y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \cdot h_{t-j}$$

where:

Y_t denotes the oil price return t,
μ is the mean return,
ϵ_t is the white noise error term,
h_t is the conditional variance at time t,
$\alpha_0, \alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_q$ are the GARCH model parameters.

This model allows for p autoregressive terms ($\alpha_i \cdot Y_{t-i}^2$) and q moving average terms ($\beta_j \cdot h_{t-j}$) in the volatility equation.

The model also includes constraints on the parameters to ensure its stability and validity:

$$\begin{aligned}
&p > 0, \quad q \geq 0, \\
&\alpha_0 > 0, \quad \alpha_i \geq 0 \quad (i = 1, 2, \ldots, p), \\
&\beta_j \geq 0 \quad (j = 1, 2, \ldots, q), \\
&\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1.
\end{aligned}$$

These constraints ensure that the autoregressive and moving average terms in the model, as well as the volatility, are well-defined and reflect the duration of return volatility.

2.5 Model Comparison

The paper's choice of the TGARCH model for WTI is scrutinized by fitting FIGARCH, GJR-GARCH, EGARCH, and TGARCH models. The comparison includes the conditional distribution for each model and the standardized residuals of the model's fit.

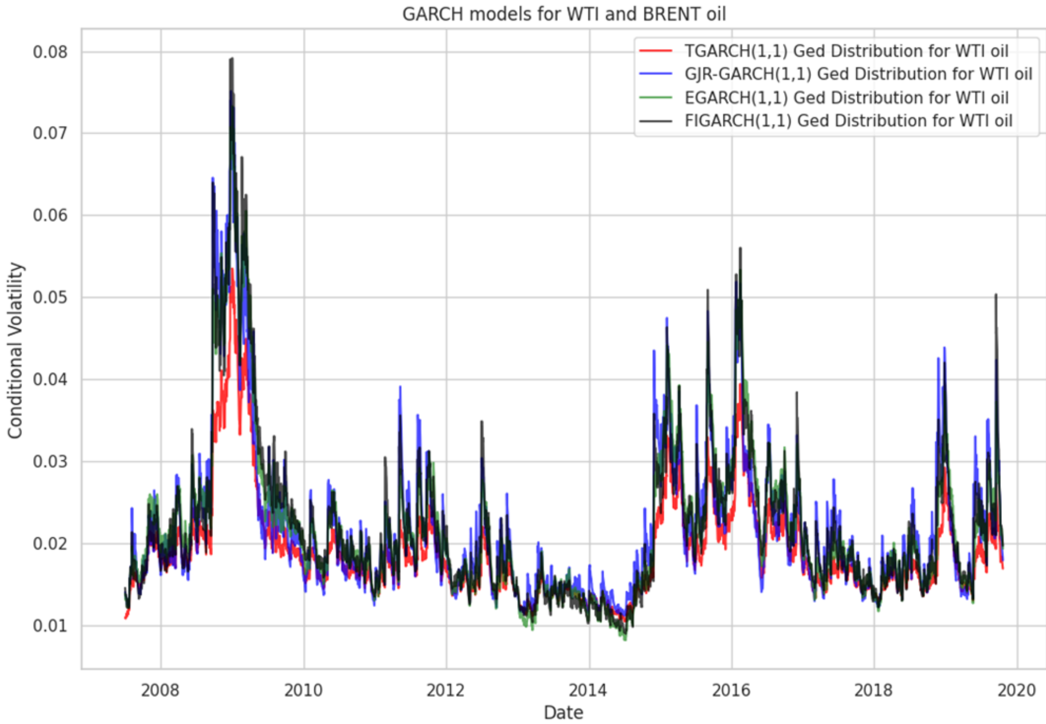

Figure 4: GARCH models for WTI oil

Just looking at the conditional volatility captured by the models we can notice strong volatility clustering and persistency (periods of high (or low) volatility tend to be followed by subsequent periods of high (or low) volatility).

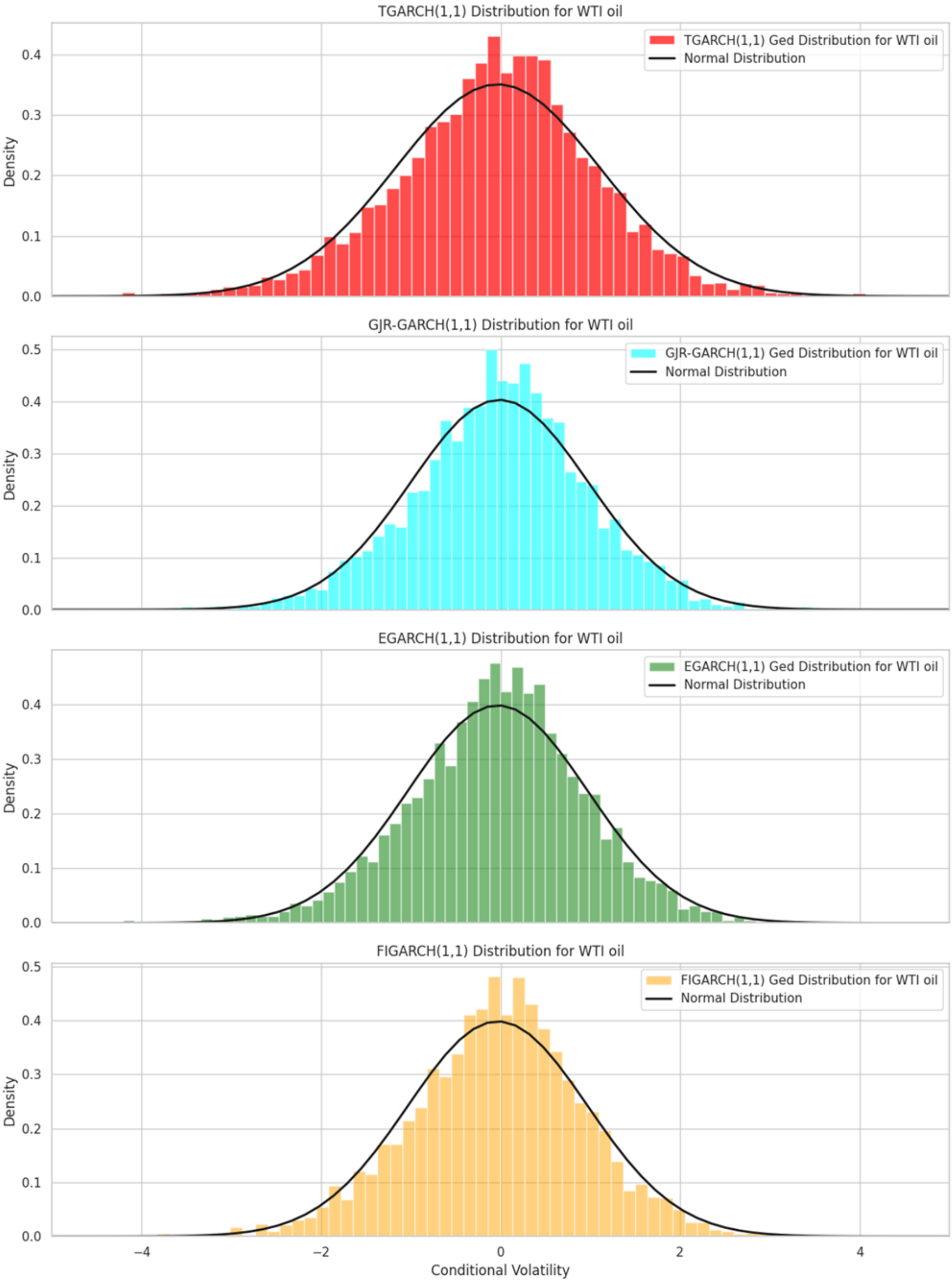

Figure 5: standardized residuals for GARCH models in WTI

The analysis, in conjunction with model summaries, identifies TGARCH and GJR-GARCH as the optimal models. Despite minimal differences, TGARCH, as chosen in the paper, is retained due to its incorporation of leverage features, crucial in capturing the asymmetric volatility caused by previous oil price return increases and decreases. After assessing various GARCH models for BRENT, including the sGARCH(1,1), GJR-GARCH, and TGARCH, the analysis indicates that there is no apparent leverage effect in the series. Despite testing these advanced models, the results did not reveal significant differences when compared to the standard GARCH model. In line with the paper's findings, where leverage features play a crucial role in capturing asymmetric volatility in the oil market, the decision was made to adhere to the simpler GARCH model recommended in the paper. This choice aligns with the notion that, in the absence of observable leverage effects, a more straightforward model suffices for accurate modelling of volatility dynamics in the BRENT market.

2.6 TGARCH(p, q) Model

The Threshold GARCH(p, q) model is defined as follows:

$$Y_t = \mu + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, h_t)$$

$$h_t = \begin{cases} \alpha_0 + \sum_{i=1}^{p} \alpha_i \cdot (Y_{t-i} - \gamma)^2 + \sum_{j=1}^{q} \beta_j \cdot h_{t-j}, & \text{if } Y_{t-1} > \gamma \\ \alpha_0 + \sum_{i=1}^{p} \alpha_i \cdot Y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \cdot h_{t-j}, & \text{if } Y_{t-1} \leq \gamma \end{cases}$$

where:

Y_t is the financial return at time t,

μ is the mean return,

ϵ_t is the white noise error term,

h_t is the conditional variance at time t,

$\alpha_0, \alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_q$ are the TGARCH model parameters,

γ is the threshold value.

This model allows for p autoregressive terms ($\alpha_i \cdot (Y_{t-i} - \gamma)^2$) and q moving average terms ($\beta_j \cdot h_{t-j}$) in the volatility equation, with a threshold at γ.

2.7 Risk Measures

$$\text{VaR-up}_{m,t} = \mu_{m,t} - z_{m,\alpha}\sqrt{h_{m,t}}, \quad (m = 1, 2)$$

$$\text{VaR-down}_{m,t} = -\mu_{m,t} + z_{m,\alpha}\sqrt{h_{m,t}}, \quad (m = 1, 2)$$

where:

VaR-up$_{m,t}$ and VaR-down$_{m,t}$ are the upper and lower Value at Risk for asset m at time t,
$\mu_{m,t}$ is the mean return for asset m at time t,
$z_{m,\alpha}$ is the critical value for the desired confidence level α,
$h_{m,t}$ is the conditional variance for asset m at time t.

These formulas help estimate the potential losses (downside risk) and gains (upside risk) for the given assets at a specific confidence level.

Recognizing the limitations of VAR, I introduced Expected Shortfall as an additional instrument

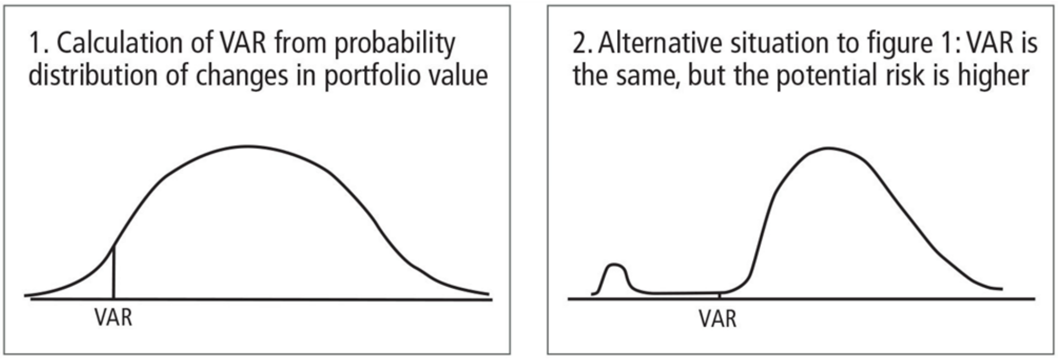

Figure 6: Source: Risk.net

2.8 Expected Shortfall

The formula for Expected Shortfall (ES) is given by:

$$\begin{aligned}\mathrm{ES}_\alpha &= -\frac{1}{1-\alpha}\int_{-\infty}^{\mathrm{VaR}_\alpha} x \cdot f(x)\,dx \\ &= -\frac{1}{1-\alpha}\sum_{i=1}^{n} p_i \cdot x_i,\end{aligned}$$

where:

ES_α is the Expected Shortfall at the confidence level α,
VaR_α is the Value at Risk at the confidence level α,
$f(x)$ is the probability density function (PDF) of the loss distribution,
p_i is the probability of observing loss x_i,
n is the number of loss scenarios.

This formula represents the expected value of losses beyond the VaR threshold at a specified confidence level.

Expected Shortfall addresses the drawbacks of VAR by considering the average loss over a specified period, conditional on the loss exceeding a certain percentile of the distribution. This measure provides better incentives for traders, preventing situations where portfolios meet VAR requirements but still pose significant risks. In conclusion, this comprehensive analysis contributes valuable insights into the distribution and volatility of oil price returns, emphasizing the importance of appropriate risk measures, particularly the adoption of Expected Shortfall, in capturing and managing risks effectively.

3 Results Obtained

Four main results can be observed.

3.1 Volatility Clustering

Significant volatility clustering is evident in both WTI and Brent returns. However, their volatility levels mostly fall within the same cluster during most sample periods, notwithstanding occasional periods where WTI returns appear more volatile.

3.2 Volatility Shock Decay and Leverage Effect

The decay of volatility shocks in both WTI and Brent returns is remarkably slow. Additionally, an asymmetric leverage effect is observed in WTI returns, while the volatility of Brent returns does not exhibit a leverage effect.

3.3 Comparison of VaR Models

At the 99% confidence level, both for upside and downside risks, the VaR model based on the Generalized Error Distribution (GED) method proves to be more robust and accurate than the one based on the standard normal distribution. However, at the 95% confidence level, both models perform equally and are deemed adequate.

3.4 Granger Causality in Risk

When examining Granger causality in risk, the results indicate that at the 95% or 99% confidence level, for both upside and downside risks, there is significant two-way Granger causality in risk for both WTI and Brent returns. This suggests a substantial risk spillover effect. Furthermore, at the 99% confidence level, in scenarios where negative news leads to a decline in oil price returns, risk information from the WTI market can assist in forecasting the risk of the Brent market. However, this process is not mirrored when positive news drives upward return; in such cases, risk information from both markets can facilitate future risk forecasting for each other.

4 Replication of the Research and Potential Modifications to the Paper

I downloaded the data for WTI and BRENT crude oil from July 1, 2007, to November 30, 2023, in contrast to the period covered by the original paper: 1987 – 2006. Considering the significant financial crisis and the COVID-19 crisis, as we will later observe, the more recent period might exhibit higher volatility compared to the earlier one. The data has been sourced from Yahoo Finance.

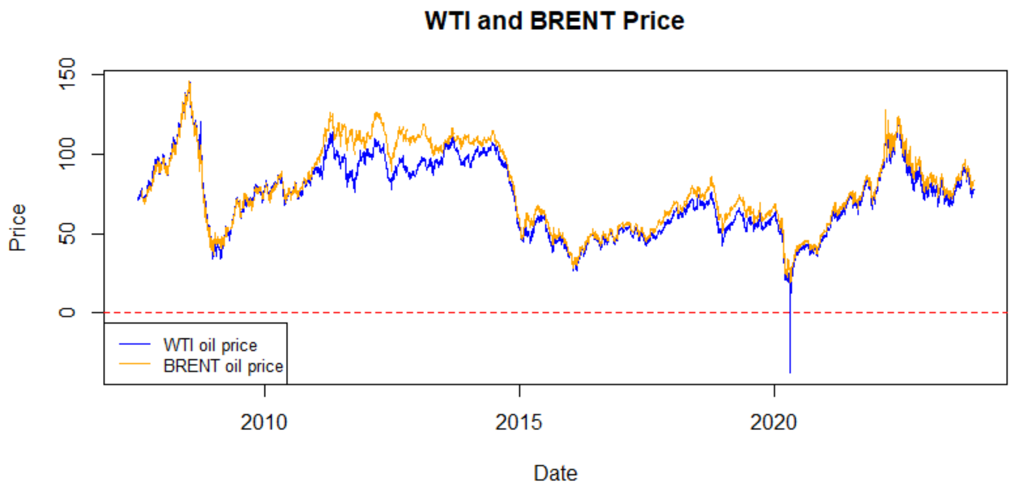

Figure 7: WTI and BRENT Price

Here, I generated plots depicting the price movements for both WTI and Brent oil. Notably, the WTI price exhibited a negative value, a seemingly implausible scenario in theory. However, attributable to the market downturn amid the COVID-19 crisis, futures briefly fell below 0. Consequently, I opted to exclude that particular observation from the dataset.

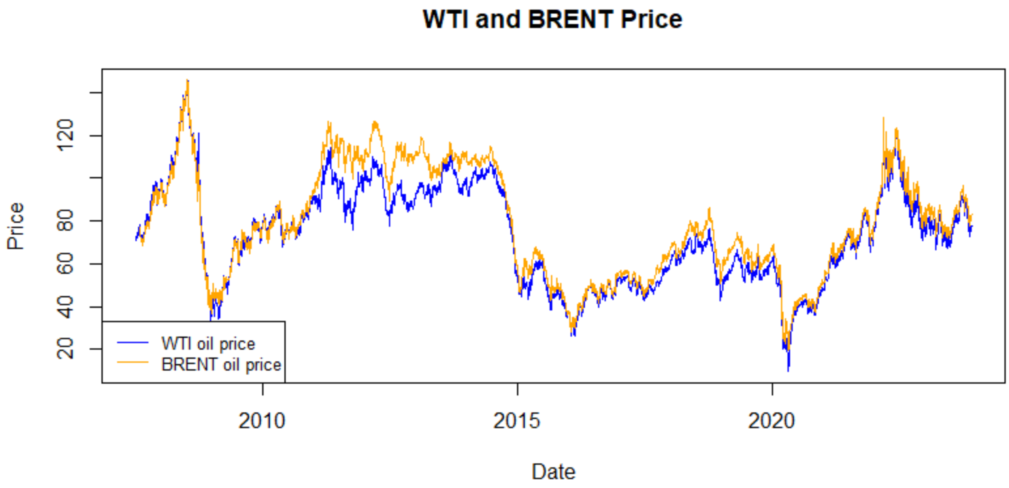

Figure 8: WTI and BRENT Price

The time series appears to be considerably volatile, with both oil types exhibiting aligned movements for the majority of the observed period. A preliminary observation suggests that WTI prices display higher volatility compared to Brent prices. Subsequently, I derived the returns, revealing volatility clustering, a phenomenon discussed earlier in the analysis.

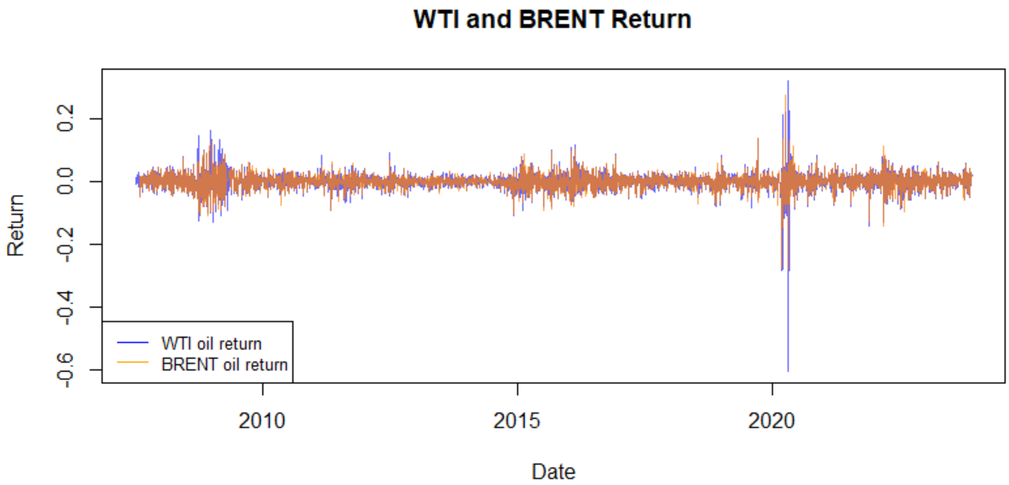

Figure 9: WTI and BRENT Price Returns

As analysed previously, the Generalized Error Distribution (GED) is identified as the most suitable for modelling financial returns, particularly in the context of fat-tailed and leptokurtic oil returns. To substantiate this assertion, I plotted the distributions, using the normal distribution as a benchmark, and conducted a thorough analysis of the series' statistics

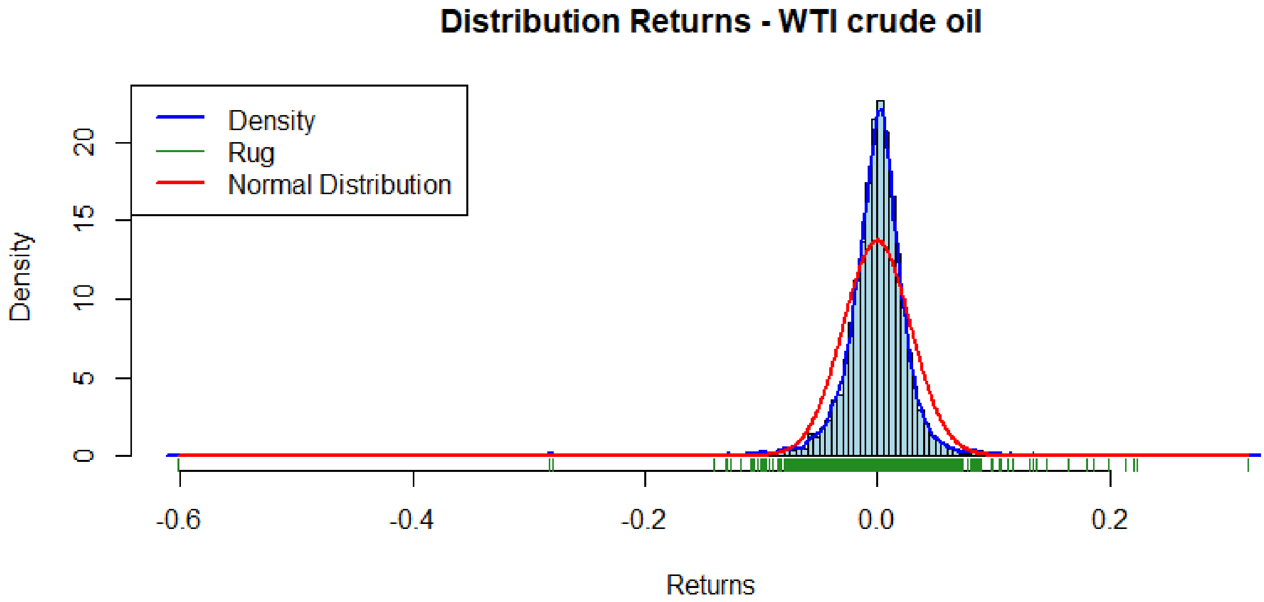

Figure 10: WTI Price Returns Distribution

We observe heavy tails and leptokurtosis in the data distribution. The "rug" provides assistance in highlighting each individual observation, facilitating a comprehensive understanding. The presence of non-normality is evident.

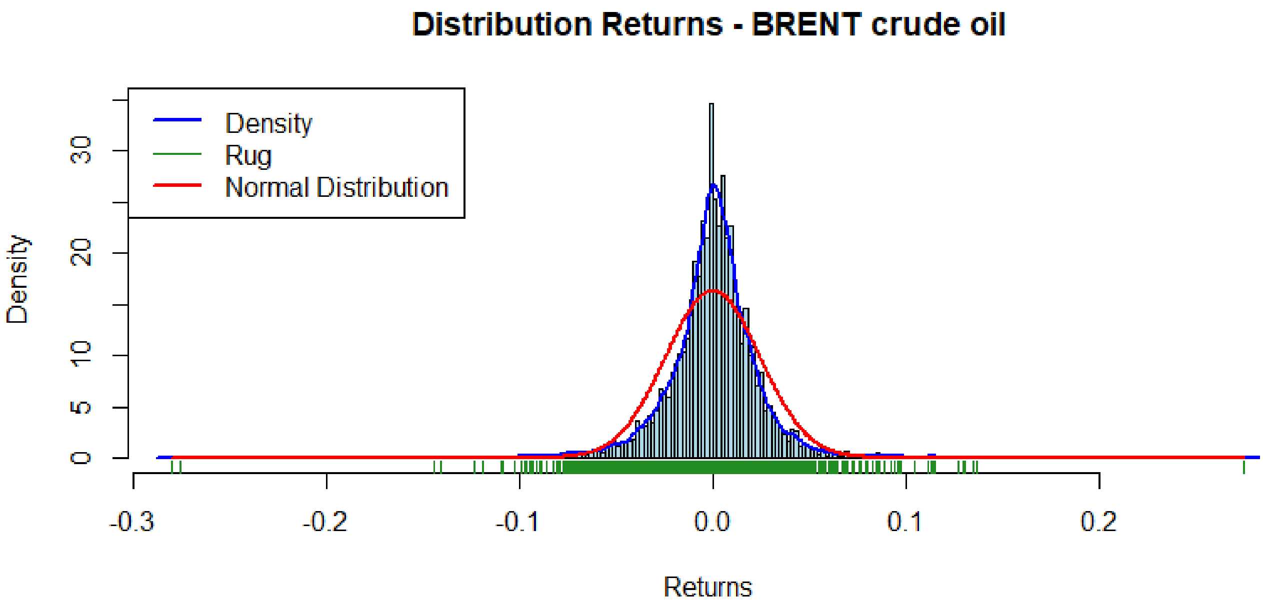

Figure 11: BRENT Price Returns Distribution

I divided the return series into two subsets: the in-sample "estimation" set, comprising 75% of the data, utilized for estimating risk measures, and the out-of-sample "forecast" set, constituting 25% of the data and employed for forecasting. The statistical measures provide insights in line with the graphical representation, encompassing excess kurtosis, skewness, non-normality, and autocorrelation. Notably, WTI returns exhibit higher volatility, increased kurtosis, and greater skewness compared to the observed patterns in the data (both estimation and forecast sets).

Subsequently, I applied the ARMA(1,1)TGARCH(1,1)-GED model to the WTI oil and the MA(1)GARCH(1,1)- GED model to the BRENT oil, mirroring the approach taken in the original paper [refer to the earlier discussion about the models]. The resulting conditional variance of these models is depicted below:

Table 1: WTI Estimation Set Statistics

Statistic	**Value**
Mean	0.000200636
Standard Deviation	0.024069565
Skewness	0.384564266
Excess Kurtosis	5.350476011
Max	0.178328937
Min	-0.122478397
JB test (p-value)	0.0
LB-Q Test	
Lag	p-value
1	0.000259
2	0.0010
3	0.001635
4	0.002747
5	0.001592
6	0.003384
7	0.006601
8	0.0120
9	0.0200
10	0.026816
ADF Test (p-value)	0.0

Skewness measures the asymmetry of the return distribution. A positive skewness suggests a right-skewed distribution, indicating that there may be more extreme positive returns.

Kurtosis measures the thickness of the tails of a distribution. In this case, the high excess kurtosis suggests the presence of fat tails in the return distribution, indicating the potential for extreme events.

The Jarque-Bera test assesses whether the sample data has the skewness and kurtosis matching a normal distribution. A p-value of 0.0 indicates a rejection of the null hypothesis, suggesting that the returns do not follow a normal distribution.

The Ljung-Box test checks for autocorrelation in the returns. The low p-values suggest that there is significant autocorrelation in the returns up to lag 7, indicating that past returns can help predict future returns.

The Augmented Dickey-Fuller (ADF) test assesses the stationarity of the time series. A p-value of 0.0 suggests rejecting the null hypothesis of non-stationarity, indicating that the returns are likely stationary over time.

Table 2: BRENT Estimation Set Statistics

Statistic	**Value**
Mean	0.000174219
Standard Deviation	0.021851431
Skewness	0.142652325
Excess Kurtosis	4.205714
Max	0.146130775
Min	-0.1036778
JB test (p-value)	0.0
LB-Q Test	
Lag	p-value
1	0.000003
2	0.000013
3	0.000041
4	0.000054
5	0.000076
6	0.000190
7	0.000348
8	0.000352
9	0.000202
10	0.000145
ADF Test (p-value)	4.46×10^{-23}

The returns of Brent crude oil exhibit skewness, although to a lesser extent than those of WTI.

A similar observation can be made regarding kurtosis, with Brent's excess kurtosis indicating the presence of thicker tails in its distribution, albeit to a lesser degree than observed in WTI returns.

The rejection of the normality assumption is confirmed by the Jarque-Bera test.

In terms of autocorrelation, the returns of Brent crude oil demonstrate a notable persistence in their patterns. The Ljung-Box test up to lag 10 reveals significant autocorrelation.

The Augmented Dickey-Fuller (ADF) test results support the conclusion that the returns are stationary.

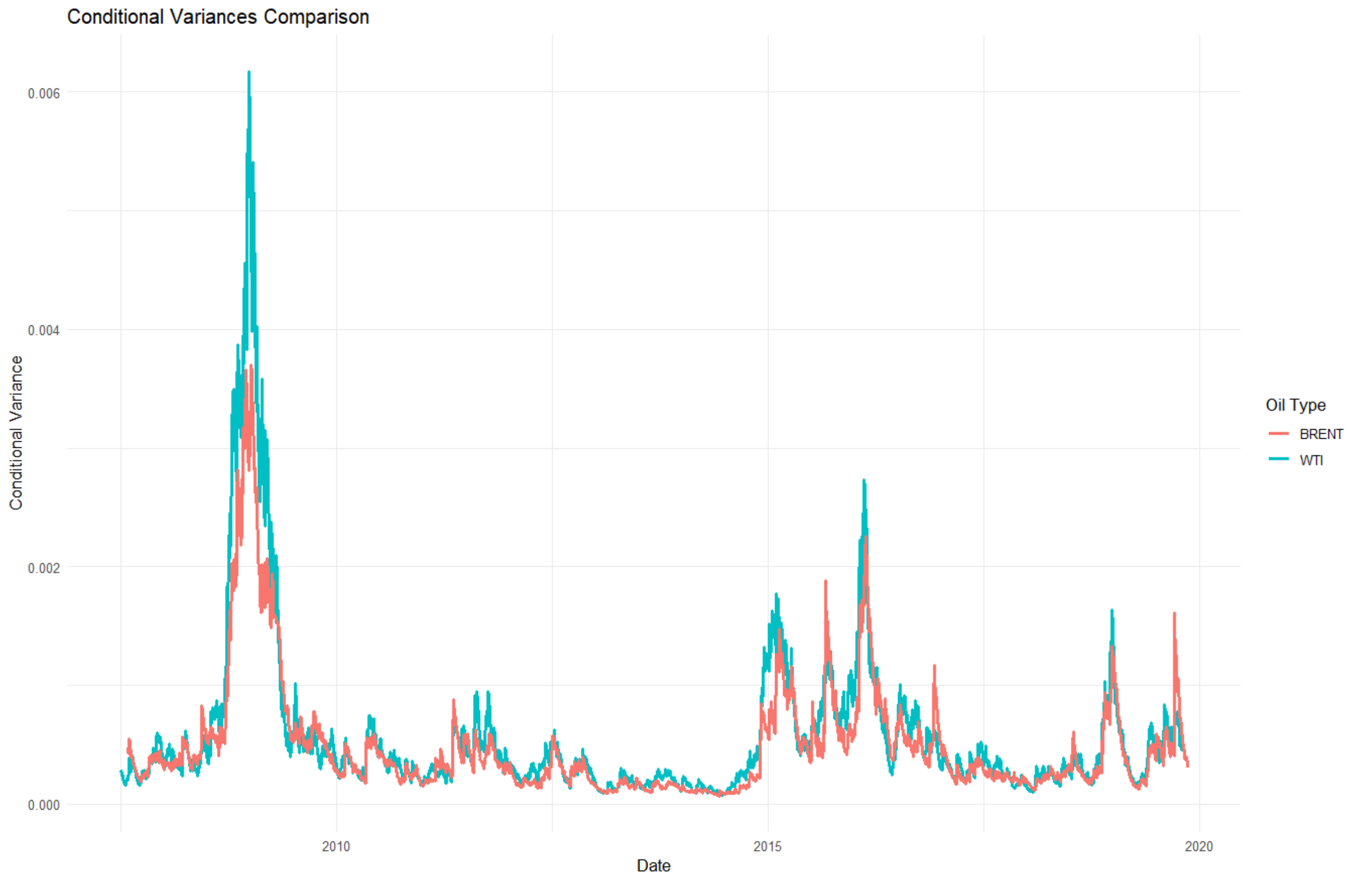

Figure 12: Conditional Variance comparison

In the WTI model, the coefficient of $h_{1,t-1}$ stands at 0.950865, implying that 95.08% of the current variance shock persists into the subsequent period. This slow decay contributes to the evident volatility clustering observed in the WTI return. While an investigation into the residual series in the mean equation reveals the absence of autocorrelation and volatility clustering, the non-rejection of the Ljung-Box Test on Standardized Squared Residuals, along with the result from the ARCH-LM test, indicates the persistence of volatility clustering.

In the Brent model, the coefficient of $h_{2,t-1}$ is 0.937034, signifying that 93.70% of the current variance shock is retained in the next period. Similar to the WTI return, the Brent return exhibits a slow decay in its volatility shock. Notably, the residual series no longer displays autocorrelation and volatility clustering, as confirmed by the results from the ARCH-LM test.

Comparing the volatility levels, the trends in the conditional variance series for WTI and Brent returns suggest closely aligned volatility levels, with occasional instances of slightly larger volatility in WTI returns and sharper volatility in Brent returns.

4.1 VaR model estimation for WTI and Brent oil price returns

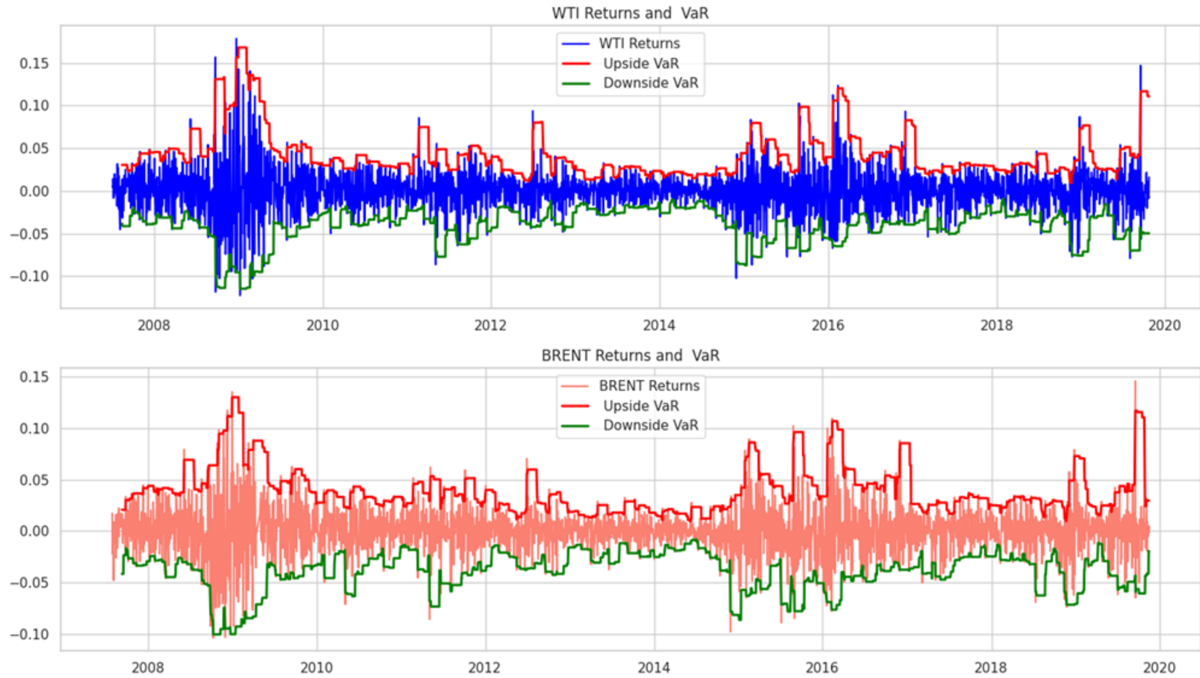

Figure 13: BRENT and WTI returns with 99% VaR

VaR modeling is crucial for comprehensively measuring both upside and downside risk, providing valuable insights for crude oil producers and concerned purchasers.

I replicated the estimation of VaR risk measures based on the GED distribution, considering the specified shape parameters of the models. Notably, the 99% quantile is significantly larger than that of the standard normal distribution, underscoring the presence of market fat tails in international crude oil price returns, a phenomenon observed earlier.

To validate the accuracy of the models, the paper conducted a back test using the method outlined by Kupiec (1995). The results affirmed that both TGARCH(1,1) and GARCH(1,1) models have effectively estimated VaRs for the two returns. This confirmation is further supported by the coherence between return trends and the VaRs chart.

Based on my chart, which incorporates modern-day data, I assert that my models remain adequately suited for the task, align with the paper's findings.

4.2 Expected Shortfall

As mentioned earlier, I proceeded to calculate an additional risk measure, the Expected Shortfall (ES), acknowledging its superiority over VaR in addressing the limitations by considering the average loss over a specified period, conditional on the loss exceeding a certain percentile of the distribution.

Subsequently, I visually presented the ES alongside the VaRs and the corresponding returns

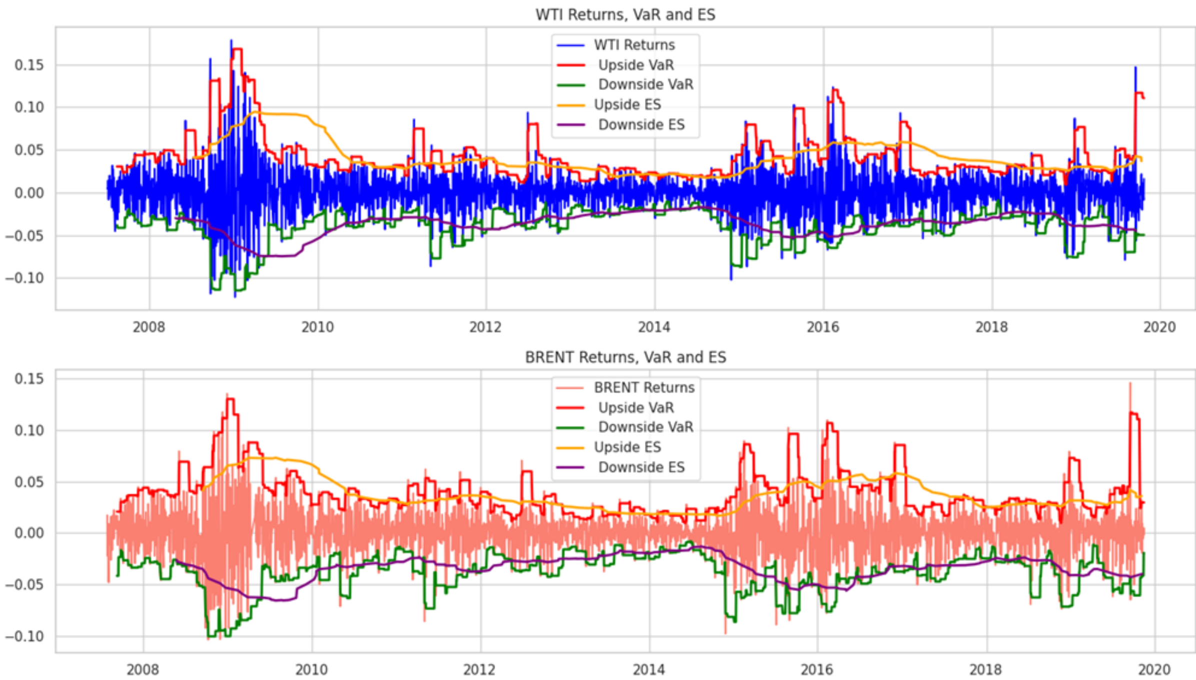

Figure 14: BRENT and WTI returns with 99% VaR and ES

Following that, I plotted the distribution chart to provide a comprehensive illustration

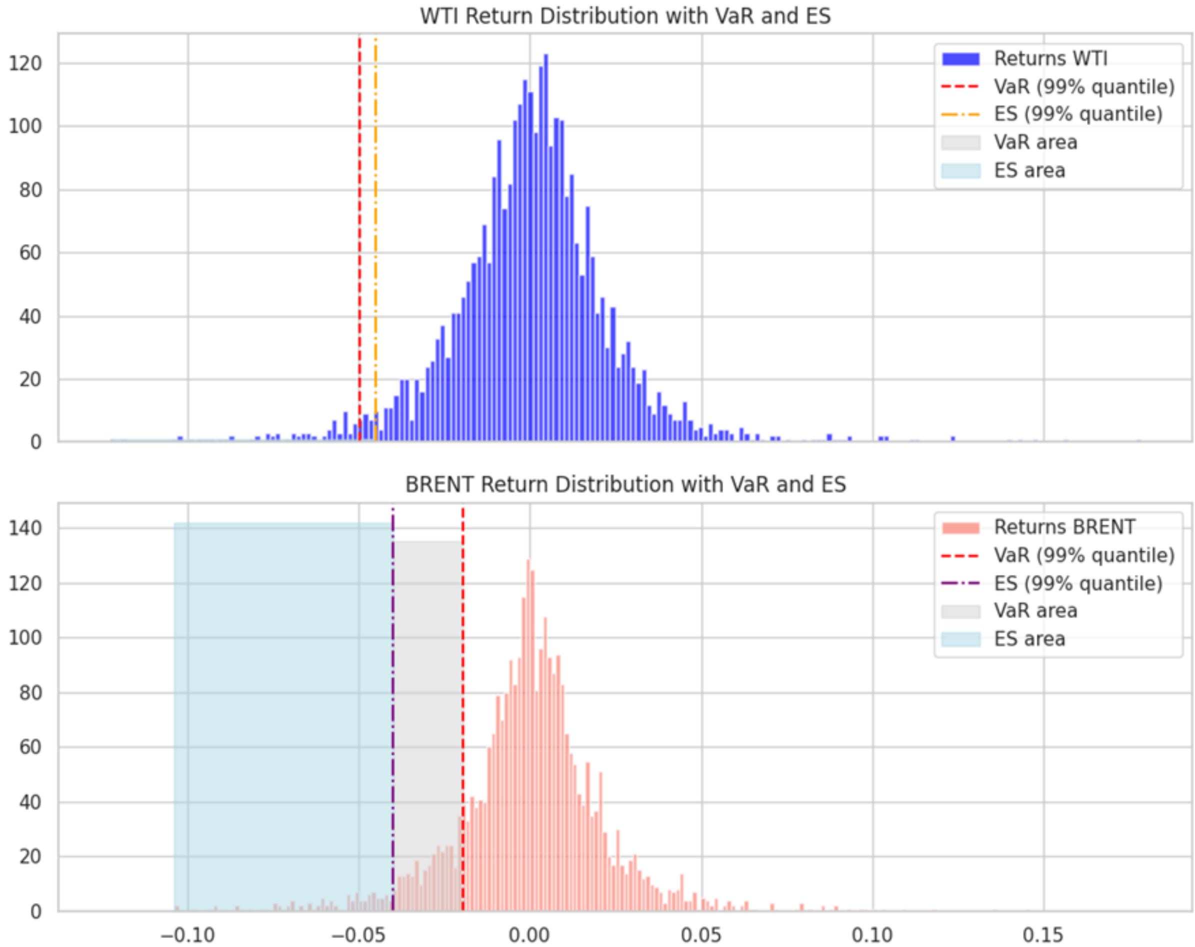

Figure 15: BRENT and WTI returns distributions with 99% VaR and ES

The paper demonstrated that the common assumption of residual series following a standard normal distribution is incorrect. By comparing the VaR under the normal distribution assumption with the GED-based VaR, it was revealed that VaR under the usual assumption would not be adequate.

Table 3: Statistics for Upside VaR WTI

Statistic	**Value**
Failure Time	107
Rate of Failure	0.034549564

Table 4: Statistics for Upside VaR BRENT

Statistic	**Value**
Failure Time	105
Rate of Failure	0.034539474

Table 5: Statistics for Downside VaR BRENT

Statistic	**Value**
Failure Time	99
Rate of Failure	0.032565789

We observe an increased failure rate, attributed to the examination of a more volatile period. As mentioned earlier in the analysis, the dataset I selected encompasses significant shocks such as the great financial crisis, recession, and the COVID-19 pandemic. The larger estimation and forecast sets are consequently subjected to higher volatility. Consequently, the failure rate in both estimating and forecasting VaR is elevated.

4.3 Forecasting performance analysis

The forecasting performance analysis reveals that VaR models based on the GED-GARCH and GED-TGARCH models effectively estimate and test return risk over the in-sample period. Additionally, to investigate the forecasting power of these VaR models, the paper forecasts the VaRs over the out-of-sample period at the 95% confidence level, comparing them with the corresponding real return series.

The researcher discovered that when VaR models are used for returns over the in-sample period to forecast the VaRs over the out-of-sample period, their forecasting power is expected to be reliable. The introduction of the popular HSAF method for developing VaR models and testing out-of-sample forecasting was also discussed. However, the results indicate that the forecasting performance of the HSAF method is not very good and cannot be accepted according to the backtesting method provided by Kupiec (1995). Therefore, it might be concluded that the GED-based VaR method outperforms the HSAF method in forecasting VaRs over the out-of-sample period.

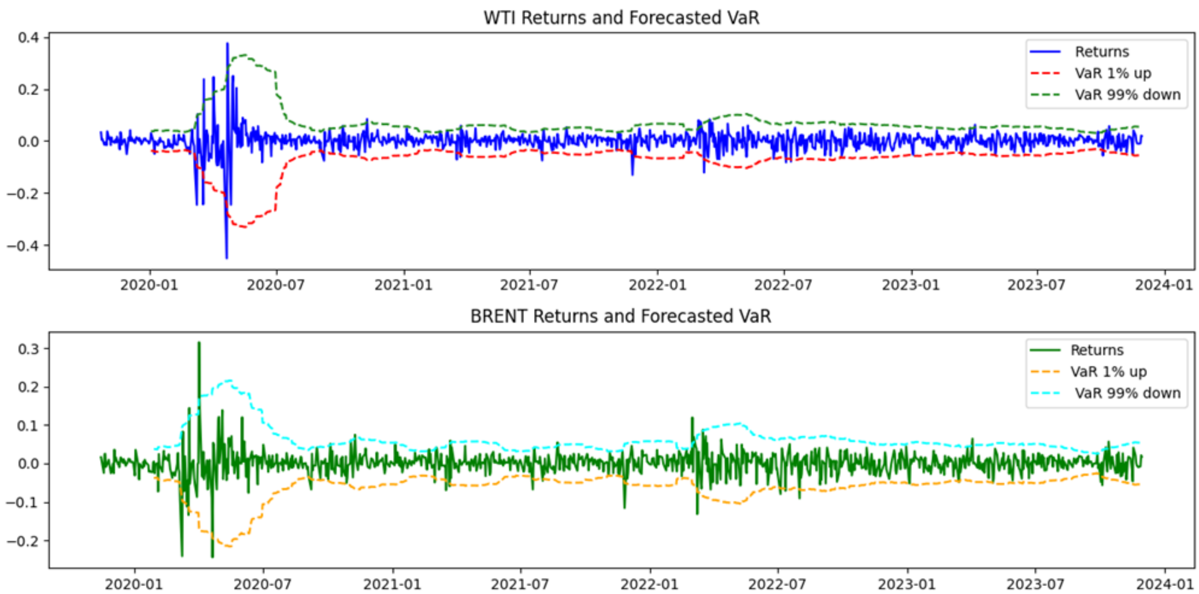

Figure 16: BRENT and WTI returns with 99% Forecasted VaR

4.4 Risk spillover effect test for WTI and Brent returns

The researchers conducted a risk spillover effect test for WTI and Brent returns, developing two statistics based on the concept of Granger causality in risk—specifically, the one-way and two-way risk spillover effects between WTI and Brent returns were examined.

The findings indicate that both WTI and Brent returns exhibit significant two-way Granger causality in risk, suggesting a notable risk spillover effect between the two oil markets. The process of economic globalization has transformed the oil market into a global and interactive stage.

In assessing the direction of risk spillover, the analysis revealed significant risk spillover effects from WTI to Brent returns, regardless of the confidence level (95% or 99%) and whether the returns are upside or downside. However, when examining risk spillover effects from Brent to WTI returns, complexities arise. At the 95% confidence level, there exists a risk spillover effect for downside returns but not for upside returns. Conversely, at the 99% confidence level, the reverse is observed—there is a risk spillover effect for upside returns but not for downside returns.

In conclusion, there is no downside risk spillover effect from Brent to WTI returns at the 99% confidence level. This implies that when negative news impacts oil price returns, the risk information of WTI returns can help forecast the risk of Brent returns, but this is not reciprocated in the opposite direction. The dominance of WTI in the international oil market is attributed to factors such as the use of the US dollar as the main invoicing currency and the significant influence of the NYMEX in America where WTI is primarily traded.