

# **ECON 6100 - Applied Bayesian Statistics**

## **U.S. small, large cap reactions to the Treasuries spread**

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### **Abstract**

There is a vast array of financial instruments available in the U.S. investment market, catering to all types of investors. Generally speaking, investing and risk tolerance are related. You or others acting on your behalf may switch from one asset class to another due to our aversion to risk. Furthermore, there are many different kinds of securities available within each financial market and asset class. We shall categorize investment into two groups to make things simpler: money markets and capital markets. Short-term, very liquid debt securities are traded on money markets. However, because debt securities have longer holding periods and equity securities are more susceptible to market volatility, capital markets are generally associated with riskier asset types. I'll go into great detail about equities securities in this empirical research, with a particular emphasis on the U.S. stock market. The basic idea that common stock represents ownership in a firm is one that all investors are familiar with. It is interesting to observe, nevertheless, that the average person is only familiar with twenty to twenty-five of the S&P 500 index companies. However, this index, which is the primary stock benchmark, includes the top 500 companies. Size does matter because this is a market-cap-weighted index, but the reason there are five hundred companies included instead of just twenty-five is intentional. Additionally, the Russell 2000 index, also referred to as "Small Caps," is made up of companies ranked from 1000 to 3000 according to market capitalization. This methodology is similar to that of the S&P 500. Through my research, I hope to shed light on the crucial role that "smaller" companies play in the long-term health of the U.S. stock market and how they perform better than larger stocks in some phases of the U.S. business cycle. I'll support this theory using historical return data from the main U.S. stock indexes as well as a key U.S. macroeconomic indicator discussed in our Bayesian Stats class this semester since the introduction of the Russell 2000 in October 1987.

## Research Question

The question at hand is how I'm going to connect the notion that smaller caps might perform better than larger caps under specific conditions. The intricacy of stocks necessitates numerous hours of fundamental and technical analysis using various methodologies to ascertain a company's performance in relation to its industry and the market at large. These assessments usually yield information about whether a company is overvalued or undervalued, as well as the causes for the difference and projections for the stock's performance in the near future. In the end, though, the matter comes down to figures. To get clear answers, Wall Street examines the financial statements of every business, big or small. But why, over the course of, say, fifty years, do corporations that appear to be in good condition underperform for extended periods of time? Alternatively, even if the outlook for smaller companies is generally bleaker than that of their larger counterparts, what is the rationale behind investing in smaller ones? The macro aspect of the equation enters the picture at this point.

I would contend that the microeconomic factors—that is, the industry and firm fundamentals—are more significant in a distinct research study. Put simply, this indicates that you can profit from whatever business cycle that the United States is going through. But that applies to stock picking. Today's talk is on entire indexes made up of the top 3000 U.S. firms that move as a single entity, or as two if we divide them into large and small caps (with mid-caps in between). Macroeconomics is the key to indexes. The movement of these indexes is significantly influenced by the different stages of the business cycle.

We experience phases of expansion, peak, contraction, and trough throughout a business cycle. In the end, it's a notion akin to what I alluded to previously in relation to stock analysis. Again, it all comes down to the numbers in macroeconomics, even though there are a lot of economic evaluations that must be done. In this case, it concerns multiple economic sectors within the United States, and we could say the same thing about every other nation on the planet. These macro variables don't always move in unison because they represent distinct economic sectors in certain circumstances, as we will see later, but they nonetheless give analysts and economists vital information that helps them comprehend a nation's economy in the present and, most crucially, predict its near future. After comprehending this phase, we go to the stock indexes themselves, including the U.S. Russell 2000 and S&P 500. Once more, not all of these macro factors are always applicable to the stock market, particularly in certain situations. But once more, the business cycle is reflected in the economic indicators, and the stock indexes typically reflect this as well.

Before diving into the topic, I would like to mention insights from a couple of literature papers or articles that I gathered information from before starting my research.

“Whereas Switzer, Fan (2007) came to the result that the high returns of small caps could be country-specific (Switzer, 2010). Based on the results of Fama, French (1993), that smaller and therefore riskier firms achieve higher returns than larger companies, Pandev, Sehgal (2016) identified

several factors which caused the higher risk. With their investigation Pandev, Sehgal (2016) support the existence of anomaly caused by company size. A recent study by Norland (2020) resumes for the US an outperformance of large versus small caps during economic expansion, whereas small caps outperformed large caps during economic downtrend.”

“Smaller caps have offered a better risk/return performance in the post-GFC era (2008-2020) in EU and Germany.” - Fahling, E., Ghiani, M., Simmert, D. (2020). Scientific Research

“Small-cap firms outperform large caps over the year subsequent to an economic trough. In the year prior to the business cycle peak, however, small caps tend to lag.” – US and Canada. Switzer, L. (2010). ScienceDirect

“Based on the results, the following assumptions can be made. First, one could assume that small cap indices are normally young companies and therefore more growth potential is seen with small caps. At the same time these companies also represent a higher risk of default. However, this assumption only applies to the US-market, as the Sharpe ratio demonstrates. With regard to Europe, the small cap indices clearly have beaten the large cap indices. The SDAX also clearly outperformed the DAX with regard to the German indices. By contrast, the American large cap index outperformed the small cap index in terms of volatility. In contrast, in terms of return the two indices are quite close to each other, which illustrates once again with the comparison of return and risk.” - Fahling, E., Ghiani, M., Simmert, D. (2020). Scientific Research

I will revisit some of these assertions and thoughts towards the end of my research paper, aiming to link my analyses with some already existing great ideas on the matter.

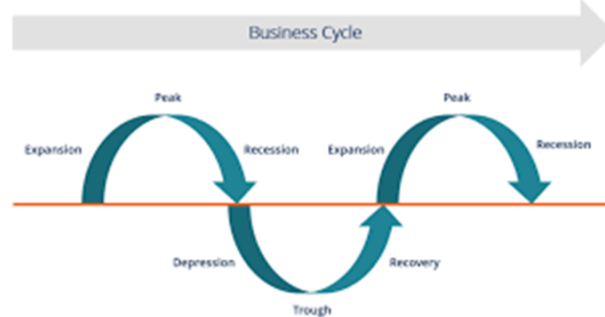
## Variables

When discussing macroeconomics in the U.S., we typically observe five key areas, which I refer to as the overall position, employment, goods and services, housing, and international endeavors. I will now list these five areas, mentioning the selected variables for each:

- Overall position:
  - Goldman Sachs Commodity Index
  - Treasuries spread (10-year Note – T-Bill)
  - Consumer sentiment index
  - Real GDP growth
  - YoY Core CPI
  - YoY PCE
  - YoY all commodities PPI
- Employment:
  - Unemployment rate
- Goods and services:
  - Net Government investment (excluding defense)
  - Industrial production
  - Net private domestic investment
  - ISM manufacturing index
  - ISM non-manufacturing index
  - Advance retail sales
  - Total wholesale trade
- Housing:
  - Existing house sales
- International endeavors:
  - International trade balance

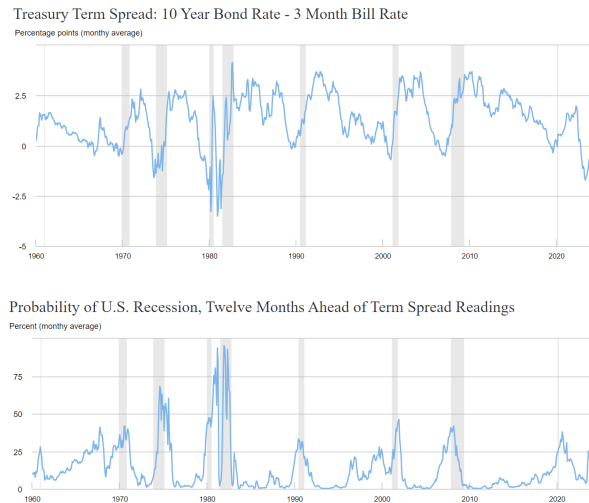
## Argument

The next stage is to thoroughly examine the U.S. business cycle and track the historical responses of larger caps (GSPC) and smaller caps (RUT) based on the phase. Every economy, as we all know, eventually peaks when consumers spend money and businesses make investments in the future. Although this period is advantageous, rising inflation and other economic issues mean that it cannot continue indefinitely. At this point, central banks—the Fed in our case—intentionally begin raising interest rates in an effort to slow down the economy, starting then the contractionary phase. We ultimately reach the trough, which might seem as a recession or a soft landing, where the economy needs to be stimulated. Although inflation has been contained at this time, there are still significant jobless rates, declining consumer expenditure, and declining business investment. At this point, the Fed would begin to lower interest rates to kick off the expansionary phase once again. After a certain time, the economy will reach its peak, and the US business cycle will resume.



I will simplify by exclusively focusing on the treasuries spread, as we have discussed in class how it provides answers regarding the current business cycle phase. In other words, it is itself a recession indicator proven historically.

Why is the yield-curve slope inverted such a strong indicator of impending recessions? The yield-curve slope condenses a wide range of factors that affect the state and trajectory of the economy into a single metric. Benzoni and Chyruk (2018) indicates that the mechanisms connecting changes in Treasury rates to the future state of the economy can be separated by breaking down the yield-curve slope into its expectations and risk-premia components. Specifically, we discover that a monetary policy easing is linked to a higher likelihood of a recession within the next year, as seen by either a fall in the projected real-rate spread or a lower level of real interest rates currently. On the other hand, depending on the cause of the downturn, a decrease in the slope of risk premia is linked to either a larger or reduced recession chance. A reduction in the real-rate risk-premium slope has been a sign of decreased recession probability, whereas a decline in the inflation risk-premium slope has been associated with an increased chance of a recession in recent years. This implies that not all steepenings are positive for the economy, and not all reductions in the yield-curve slope are good news either.



Besides the complex times we are living in, both in terms of the current year and many other aspects, the inflationary period and the reasons behind it, as well as the Fed tightening cycle, the spread has shown a particularly interesting behavior in historical U.S. economic data. I won't delve into today's situation, as it would require a separate research paper. However, I believe it all boils down to the uncertainty we have been experiencing since 2022, both in the economy and in the financial markets. Expectations continue to fluctuate, and as we approach summer 2023, we still have little insight into the Fed's plans regarding interest rates in the near future. I believe this instability is reflected in the duration of the negative spread since 2022. If you examine it closely, we can agree that almost every time it has turned negative, we have entered a recession. However, the spread has bounced back relatively quickly to positive territory. This is not the case nowadays, as it has been negative for almost two years with no sign of a recession.

However, going back to the "whys" in a simple way, it all makes sense. Money markets and the bond market are key components of economics. They represent the way governments and corporations raise money by selling debt. It may not be as flashy as the stock market, but it is more important when analyzing economies. Short-term bonds should never be riskier (have a higher yield) than long-term bonds if the economy is thriving. Think about it: if longer-duration bonds have a lower yield, it means people are willing to bet on the future rather than on the present.

## Methodology - Hypotheses

I am going to start by listing the various steps I am about to implement to come up with conclusions based on the returns of the S&P 500 and the Russell 2000 since 1987, comparing them to recessionary periods in the U.S., supported by the idea that the Treasury spread is a reliable macro indicator:

- Group three different sets of monthly stationary data: RUT returns MoM, GSPC returns MoM, and the U.S. treasuries spread (10-year T-Note minus 1-year T-Bill). Please note the T-bill can be modified in any span between three months one year.
- Process data and define recession given the Treasuries spread. That is, establishing 0 as the threshold. If data > 0, no recession / If data < 0, recession
- Bayesian analysis and components separately for the Russell 200 and the S&P 500:
  - Russell 2000:
    - \* Define data:  $x = \text{data}[\text{Treasuries Spread}] / y = \text{data}[\text{Russell 2000 returns}]$
    - \* Normalize data:  $x = (x - x.mean()) / x.std() / y = (y - y.mean()) / y.std()$
    - \* Define Bayesian Model, a Bayesian linear regression model where Russell 2000 returns are modeled as a linear function of the Treasury spread:
      - Define priors:  $\pi(\alpha_m, \alpha_s | y), \pi(\beta_m, \beta_s | y), \pi(\sigma_m, \sigma_s | y)$  - where  $m$  denotes mean and  $s$  denotes standard deviation
      - Define the number of samples
      - Sampling from priors
      - Define the likelihood function using the normal probability density function for a linear regression model:  $f(y|x, \alpha, \beta, \sigma) = \mathcal{N}(\alpha + \beta x, \sigma)$
      - Compute likelihood for each sample
      - Compute the posterior distribution using Bayes' theorem:

$$\pi(\alpha, \beta, \sigma | x, y) = \frac{f(y|x, \alpha, \beta, \sigma) \pi(\alpha, \beta, \sigma)}{\int f(y|x, \alpha, \beta, \sigma) \pi(\alpha, \beta, \sigma) d\alpha d\beta d\sigma}$$

- S&P 500:
  - \* Define data:  $x = \text{data}[\text{Treasuries Spread}] / y = \text{data}[\text{S\&P 500 returns}]$
  - \* Normalize data:  $x = (x - x.mean()) / x.std() / y = (y - y.mean()) / y.std()$
  - \* Define Bayesian Model, a Bayesian linear regression model where S&P 500 returns are modeled as a linear function of the Treasury spread:
    - Define priors:  $\pi(\alpha_m, \alpha_s | y), \pi(\beta_m, \beta_s | y), \pi(\sigma_m, \sigma_s | y)$  - where  $m$  denotes mean and  $s$  denotes standard deviation
    - Define the number of samples



- Sampling from priors
- Define the likelihood function using the normal probability density function for a linear regression model:  $f(y|x, \alpha, \beta, \sigma) = \mathcal{N}(\alpha + \beta x, \sigma)$
- Compute likelihood for each sample
- Compute the posterior distribution using Bayes' theorem:

$$\pi(\alpha, \beta, \sigma|x, y) = \frac{f(y|x, \alpha, \beta, \sigma)\pi(\alpha, \beta, \sigma)}{\int f(y|x, \alpha, \beta, \sigma)\pi(\alpha, \beta, \sigma)d\alpha d\beta d\sigma}$$

- Compare both results and make assumptions based on the U.S. business cycle

It is important to denote and explain the following parameters:

- $\alpha$ : The intercept of the linear regression model. It represents the average return of the stock index when the Treasury spread is zero
- $\beta$ : The slope of the linear regression model. It represents the change in the stock index returns for a one-unit change in the Treasury spread
- $\sigma$ : The standard deviation of the residuals. It represents the variability of the stock index returns not explained by the Treasury spread

We need to understand that plotting the returns of these two key indexes in the U.S. against the Treasury spread monthly contains a unique relationship. Some months in our period show bigger shocks, but, on average, the results are going to be similar, especially using a longer period in time when there's less room for monthly differences. Given that some years are red in the stock market, but most are green and in a "similar" range.

However, we see clear evidence of differences between the two, with Figure 1 representing the S&P 500 and Figure 2 representing the Russell 2000. The first notable observation is that Figure 2 shows us greater uncertainty, both for the good and the bad, allowing for less room in the tails of the distribution. The distribution becomes more peaked at the center and has thinner tails, making it less robust against outliers. On the other hand, Figure 1 shows us lesser uncertainty, both for the good and the bad, allowing for more values in the tails of the distribution. The distribution becomes less peaked at the center and has thicker tails, making it more robust against outliers. In our scenario, the outliers are the key economic indicators affecting the Treasury spread, causing it to fluctuate.

Smaller caps' response to positive shocks in the spread, indicating the economy is about to hit the trough phase and start the recovery phase in the business cycle, will be greater in terms of percentage monthly returns than that of the ones in the S&P 500. Moreover, Smaller caps' response to negative shocks in the spread, indicating the economy is about to hit the peak phase and start the contractionary phase in the business cycle, will also be greater in terms of percentage monthly returns (more negative) than that of the ones in the S&P 500.

Now, comparing our parameters:

- $\alpha$ : The S&P 500 shows more volatility in the  $\alpha$  posterior distribution because the Treasuries spread is not as important for it as it is for the Russell 2000. When the spread is zero, we are not experiencing either a positive or a negative shock.
- $\beta$ : The Russell 2000 reaches a higher density area because smaller caps in the U.S. experience a greater magnitude of response to the actual phase of the business cycle the economy is experiencing, or to changes in the Treasuries spread.
- $\sigma$ : The S&P 500 reaches a higher density area because larger caps in the U.S. depend less on the actual phase of the business cycle the economy is experiencing, or the Treasuries spread fluctuation.

Figure 1: S&P 500 Returns

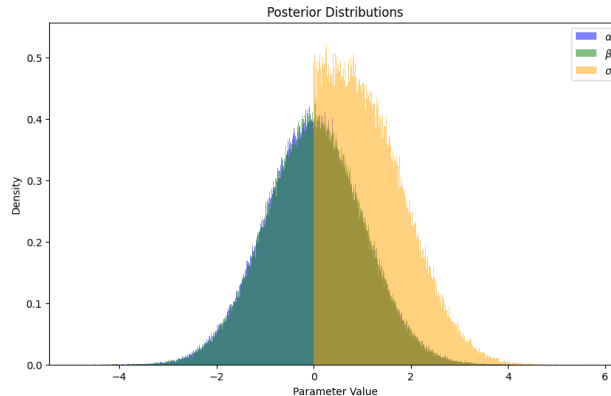
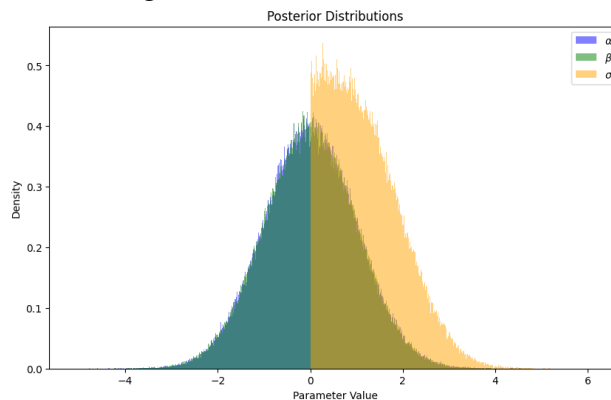


Figure 2: Russell 2000 Returns



## Conclusion

Everything makes logic, in my perspective, and it also fits historically. Because of their higher debt levels and generally weaker quality fundamentals than larger caps, smaller caps typically perform better in a context of lower interest rates. This pattern makes sense in a market where smaller companies do well at expansionary and peak stages of the business cycle. But it's important to keep in mind that the stock market, like other asset classes, is continually pricing for the future. Even if conditions may not be ideal, anticipation of rate decreases becomes apparent when the economy reaches its lowest point. Herein lies an opportunity window for smaller caps in the United States, particularly during the trough and the post-trough recovery phases. The stock market finally begins pricing in future rate hikes as we start to expand, usually around the peak. Larger caps show their strength at this point. Furthermore, both indexes typically experience downward pressure during contractionary phases.

We have also seen how the theory functions in other regions of the world, frequently across longer timeframes. For example, after the Global Financial Crisis, we have seen smaller caps outperforming in Germany and the European Union. This implies that smaller caps offer whole trends for prolonged periods outside our nation, in addition to windows of opportunity throughout the business cycle. However, smaller capitalization will never be able to match the technological brilliance of businesses like Apple, Microsoft, Nvidia, and others, thus in my opinion, this situation is unlikely to occur in the United States. American high-tech is unmatched. Nonetheless, there are opportunities to invest in U.S. smaller-cap companies during certain times.

## References

- Bloomberg Terminal
- Yahoo Finance - Stock Market Live, Quotes, Business & Finance News
- Investing.com - Stock Market Quotes & Financial News
- St. Louis Federal Reserve Bank FRED - Federal Reserve Economic Data — FRED — St. Louis Fed (stlouisfed.org)
- SPGlobal - US MGDG-research
- Switzer, L. (2010). The behavior of small cap vs. large cap stocks in recessions and recoveries: Empirical evidence for the United States and Canada. ScienceDirect. Retrieved from:  
<https://www.sciencedirect.com/science/article/pii/S0261560610000288>
- Fahling, E., Ghiani, M., Simmert, D. (2020). Small versus Large Caps-Empirical Performance Analyses of Stock Markets Indices in Germany, EU & US since Global Financial Crisis. Scientific Research. Retrieved from:  
<https://www.scirp.org/journal/paperinformation.aspx?paperid=104626>
- Mcdermott, J., D'Auria, D. (2014). What Do We (Really) Know About U.S. Small-Cap Investing? FPA. Retrieved from: [https://www.financialplanningassociation.org/sites/default/files/research/What%20Do%20We%20\(Really\)%20Know%20About%20US%20Small-Cap%20Investing.pdf](https://www.financialplanningassociation.org/sites/default/files/research/What%20Do%20We%20(Really)%20Know%20About%20US%20Small-Cap%20Investing.pdf)
- Bocock, J. (2008). Investing in Small Cap Companies. The Wall Street Transcript
- Emory, T., Lee, R. (1995). Economics. Social Sciences
- Kellner, T. (2006). Investing in Large & Small Cap Value Stocks.