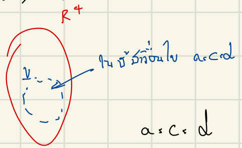
7.3 $V = \{(a,b,c,d) \mid a = c = d\}$




$a = c = d$

$(a,b,c,d) = (d, b, d, d)$

$= (d,0,d,d) + (0,b,0,0)$

$(a,b,c,d) = d(1,0,1,1) + b(0,1,0,0)$

$\hookrightarrow a_1 = d$

$a_2 = b$

$a_1, a_2$ หาค่าได้แสดงว่า แผ่ทั่ว

$\{(1,0,1,1), (0,1,0,0)\}$ แผ่ทั่ว $V$ ✓

อิสระไหม $(a_1 = 0, a_2 = 0)$

$(0,0,0,0) = a_1(1,0,1,1) + a_2(0,1,0,0)$

$(0,0,0,0) = (a_1, a_2, a_1, a_1)$

$a_1 = 0$

$a_2 = 0$

$\{(1,0,1,1), (0,1,0,0)\}$ เป็นฐานหลัก ✓

$\dim(V) = 2$ หรือ 2 มิติ ✗

$\hookrightarrow \{v_1, v_2\}$

$\{(1,0,1,1), (0,1,0,0)\}$

หา $(v)_S$, $[v]_S$

8.2 $v_1 = (2,-4)$, $v_2 = (3,8)$, $v = (1,1)$

$v = c_1 v_1 + c_2 v_2$

$(1,1) = c_1(2,-4) + c_2(3,8)$

$(1,1) = (2c_1 + 3c_2, -4c_1 + 8c_2)$

$2c_1 + 3c_2 = 1$

$-4c_1 + 8c_2 = 1$

$[A|B] = \left[\begin{array}{cc|c} 2 & 3 & 1 \\ -4 & 8 & 1 \end{array}\right]$

$\overset{R_1/2}{\sim} \left[\begin{array}{ccc} 1 & \frac{3}{2} & \frac{1}{2} \\ -4 & 8 & 1 \end{array}\right]$

$\overset{R_2+4R_1}{\sim} \left[\begin{array}{ccc} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 14 & 3 \end{array}\right]$

$R_2 + 4R_1$: $8 + \cancel{4}^{2}\left(\frac{3}{\cancel{2}_1}\right)$, $8 + 6 = 14$; $1 + \cancel{4}^{2}\left(\frac{1}{\cancel{2}}\right) = 3$

$\overset{R_2/14}{\sim} \left[\begin{array}{ccc} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & \frac{3}{14} \end{array}\right]$

$\overset{R_1 - \frac{3}{2}R_2}{\sim} \left[\begin{array}{ccc} 1 & 0 & \frac{5}{28} \\ 0 & 1 & \frac{3}{14} \end{array}\right]$

$R_1 - \frac{3}{2}R_2$: $\frac{1}{2} - \frac{3}{2}\left(\frac{3}{14}\right) \to \frac{1}{2} - \frac{9}{28}$

$= \frac{14 - 9}{28} = \frac{5}{28}$

Rank A = Rank [A|B] = n = 2

มีผลเฉลยเดียว

$c_1 = \frac{5}{28}$

$c_2 = \frac{3}{14}$

$(v)_S = (c_1, c_2) = \left(\frac{5}{28}, \frac{3}{14}\right)$ ✗

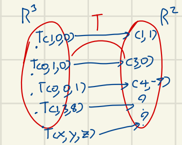
$[v]_S = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{28} \\ \frac{3}{14} \end{bmatrix}$ ✗

5. $T: R^3 \to R^2$ แปลงเชิงเส้น โดย $T(1,0,0) = (1,1)$

$T(0,1,0) = (3,0)$

$T(0,0,1) = (4,-7)$

หา $T(1,3,8)$ และ $T(x,y,z)$




$\{(1,0,0), (0,1,0), (0,0,1)\}$ เป็นฐานหลักมาตรฐาน

หา $T(1,3,8)$

$\hookrightarrow (1,3,8) = a_1(1,0,0) + a_2(0,1,0) + a_3(0,0,1) \to ①$

$a_1 = 1$

$a_2 = 3$ แทนในสมการ ①

$a_3 = 8$

$(1,3,8) = (1,0,0) + 3(0,1,0) + 8(0,0,1)$

$T(1,3,8) = T(1,0,0) + 3T(0,1,0) + 8T(0,0,1)$

$T(1,3,8) = (1,1) + 3(3,0) + 8(4,-7)$ ✗

$= (1 + 9 + 32, 1 - 56)$

$T(1,3,8) = (42, -55)$

หา $T(x,y,z) \to (x,y,z) = a_1(1,0,0) + a_2(0,1,0) + a_3(0,0,1)$

$a_1 = x$, $a_2 = y$, $a_3 = z$ แทนใน ①

$(x,y,z) = x(1,0,0) + y(0,1,0) + z(0,0,1)$

$T(x,y,z) = xT(1,0,0) + yT(0,1,0) + zT(0,0,1)$

$T(x,y,z) = x(1,1) + y(3,0) + z(4,-7)$

$T(x,y,z) = (x + 3y + 4z, x - 7z)$ ✗

(4.) $S = \{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$ เป็นฐานหลัก $R^3$ → แผ่ทั่ว → เป็นผลรวมเชิงเส้นของ $\underline{v}$ ใดๆได้

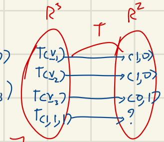
$\underline{v}_1 = (1,2,3)$, $\underline{v}_2 = (2,5,3)$, $\underline{v}_3 = (1,0,10)$

และให้ $T: R^3 \to R^2$ | $T(\underline{v}_1) = (1,0)$, $T(\underline{v}_2) = (1,0)$,

$T(\underline{v}_3) = (0,1)$

จงหา $T(1,1,1)$

$(1,1,1) = a_1(1,2,3) + a_2(2,5,3) + a_3(1,0,10)$ ①




$(1,1,1) = (a_1 + 2a_2 + a_3,\ 2a_1 + 5a_2,\ 3a_1 + 3a_2 + 10a_3)$

$a_1 + 2a_2 + a_3 = 1$
$2a_1 + 5a_2 = 1$
$3a_1 + 3a_2 + 10a_3 = 1$

$$\to [A|b] = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 5 & 0 & 1 \\ 3 & 3 & 10 & 1 \end{bmatrix}$$

$$\underset{R_3 - 3R_1}{\overset{R_2 - 2R_1}{\sim}} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & -3 & 7 & -2 \end{bmatrix}$$

$R_2 - 2R_1$: $5-2(2)$, $0-2(1)$, $1-2(1)$ → $1$, $-2$, $-1$

$R_3 - 3R_1$: $3-3(2)$, $10-3(1)$, $1-3(1)$ → $-3$, $7$, $-2$

$$\underset{R_3 + 3R_2}{\overset{R_1 - 2R_2}{\sim}} \begin{bmatrix} 1 & 0 & 5 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

$R_1 - 2R_2$: $1-2(-2)$, $1-2(-1)$ → $1+4$, $1+2$ → $5$, $3$

$R_3 + 3R_2$: $7+3(-2)$, $-2+3(-1)$ → $7-6$, $-2-3$ → $1$, $-5$

$$\underset{R_2 + 2R_3}{\overset{R_1 - 5R_3}{\sim}} \begin{bmatrix} 1 & 0 & 0 & 28 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

$R_1 - 5R_3$: $3-5(-5)$ → $3+25$ → $28$

$R_2 + 2R_3$: $-1+2(-5)$ → $-1-10$

rankA = rank(A|b) = n = 3
↳ มีผลเฉลยเดียว

$a_1 = 28$, $a_2 = -11$, $a_3 = -5$ } แทนใน ①

$(1,1,1) = 28(1,2,3) - 11(2,5,3) - 5(-3,1,2)$

$T(1,1,1) = 28T(1,2,3) - 11T(2,5,3) - 5T(-3,1,2)$

$T(1,1,1) = 28(1,0) - 11(1,0) - 5(0,1)$

$= (28-11,\ -5)$

$T(1,1,1) = (17, -5)$

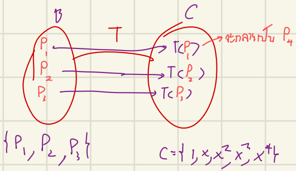
B.1 หา Matrix T เทียบกับฐาน B, B' หรือ $({}_BT_C)$ → ขอเปลี่ยนเป็น C นะครับ

↳ $T: P_2 \to P_4$ โดย $T(p(x)) = x^2 p(x)$

กล่าวคือ ${}_BT_C = [[T(\underline{v}_1)]_C, [T(\underline{v}_2)]_C, \ldots, [T(\underline{v}_n)]_C]$

โดย $B = \{P_1, P_2, P_3\}$ → เป็นฐานหลัก; $P_1 = 1 + x^2$, $P_2 = 1 + 2x + 3x^2$, $P_3 = 4 + 5x + x^2$

C เป็นฐานหลักมาตรฐานของ $P_4$ → $C = \{1, x, x^2, x^3, x^4\}$




หา $T(P_1) = T(1+x^2) = x^2(1+x^2)$
$= x^2 + x^4$

$T(1+2x+3x^2) = x^2(1+2x+3x^2)$
$= x^2 + 2x^3 + 3x^4$

$T(4+5x+x^2) = x^2(4+5x+x^2)$
$= 4x^2 + 5x^3 + x^4$

หา $[T(1+x^2)]_C = x^2 + x^4 = c_1 + c_2x + c_3x^2 + c_4x^3 + c_5x^4$

$c_1 = 0$, $c_2 = 0$, $c_3 = 1$, $c_4 = 0$, $c_5 = 1$

$$[T(P_1)]_C = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$[T(P_2)]_C = x^2 + 2x^3 + 3x^4 = c_1 + c_2x + c_3x^2 + c_4x^3 + c_5x^4$

$c_1 = 0$, $c_2 = 0$, $c_3 = 1$, $c_4 = 2$, $c_5 = 3$

$$[T(P_2)]_C = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$[T(P_3)]_C = 4x^2 + 5x^3 + x^4 = c_1 + c_2x + c_3x^2 + c_4x^3 + c_5x^4$

$c_1 = 0$, $c_2 = 0$, $c_3 = 4$, $c_4 = 5$, $c_5 = 1$

$$[T(P_3)]_C = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 5 \\ 1 \end{bmatrix}$$

${}_BT_C$ มีขนาดเมทริกซ์ $5 \times 3$

↳ $= [[T(P_1)]_C\ [T(P_2)]_C\ [T(P_3)]_C]$

$${}_BT_C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 4 \\ 0 & 2 & 5 \\ 1 & 3 & 1 \end{bmatrix}$$

13.2 หา ${}_BT_C$ มาหา $T(-3+5x-2x^2)$

สูตร

$3.\ [T(\underline{v})]_C = {}_BT_C\,[\underline{v}]_B$

$[T(P_n)]_C = {}_BT_C\,[P_n]_B$

$[T(-3+5x-2x^2)]_C = {}_BT_C\,[-3+5x-2x^2]_B$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 4 \\ 0 & 2 & 5 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} -\frac{25}{4} \\ \frac{5}{4} \\ \frac{1}{2} \end{bmatrix}$$

$5 \times 3$, $3 \times 1$ → คูณได้

ผลลัพธ์ $5 \times 1$

$$= \begin{bmatrix} 0 \\ 0 \\ -\frac{25}{4}+\frac{5}{4}+2 = \frac{-25+5+8}{4} = \frac{-12}{4} = -3 \\ 0+\frac{5}{2}+\frac{5}{2} = 5 \\ -\frac{25}{4}+\frac{15}{4}+\frac{2}{4} = \frac{-8}{4} = -2 \end{bmatrix}$$

$$[T(-3+5x-2x^2)]_C = \begin{bmatrix} 0 \\ 0 \\ -3 \\ 5 \\ -2 \end{bmatrix} \begin{matrix} \to C_1 \\ \to C_2 \\ \to C_3 \\ \to C_4 \\ \to C_5 \end{matrix}$$

$T(-3+5x-2x^2) = 0(1)+0(x)-3(x^2)+5(x^3)-2(x^4)$

$= -3x^2+5x^3-2x^4$

หา $[P_n]_B$

$-3+5x-2x^2 = c_1w_1+c_2w_2+c_3w_3$

$-3+5x-2x^2 = c_1(1+x^2)+c_2(1+2x+3x^2)+c_3(4+5x+x^2)$

$= c_1x^2+3c_2x^2+c_3x^2+2c_2x+5c_3x+c_1+c_2+4c_3$

$-3+5x-2x^2 = (c_1+3c_2+c_3)x^2+(2c_2+5c_3)x+c_1+c_2+4c_3$

$c_1+3c_2+c_3 = -2$

$2c_2+5c_3 = 5$

$c_1+c_2+4c_3 = -3$

$$[A|b] = \left[\begin{array}{ccc|c} 1 & 3 & 1 & -2 \\ 0 & 2 & 5 & 5 \\ 1 & 1 & 4 & -3 \end{array}\right]$$

$R_3-R_1$: $1-(3) = -2$, $4-(1) = 3$, $-3-(-2) = -3+2 = -1$

$$\sim \begin{bmatrix} 1 & 3 & 1 & -2 \\ 0 & 2 & 5 & 5 \\ 0 & -2 & 3 & -1 \end{bmatrix}$$

$R_3+R_2$

$$\sim \begin{bmatrix} 1 & 3 & 1 & -2 \\ 0 & 2 & 5 & 5 \\ 0 & 0 & 8 & 4 \end{bmatrix}$$

$\frac{R_3}{8}$, $\frac{R_2}{2}$

$$\sim \begin{bmatrix} 1 & 3 & 1 & -2 \\ 0 & 1 & \frac{5}{2} & \frac{5}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$$

$R_1-3R_2$: $1-3\left(\frac{5}{2}\right) = 1-\frac{15}{2} = \frac{2-15}{2} = \frac{-13}{2}$; $-2-3\left(\frac{5}{2}\right) = -2-\frac{15}{2} = \frac{-4-15}{2} = \frac{-19}{2}$

$$\sim \begin{bmatrix} 1 & 0 & -\frac{13}{2} & -\frac{19}{2} \\ 0 & 1 & \frac{5}{2} & \frac{5}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$$

$R_1+\frac{13}{2}R_3$, $R_2-\frac{5}{2}R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -\frac{25}{4} \\ 0 & 1 & 0 & \frac{5}{4} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$$

$R_1+\frac{13}{2}R_3$: $-\frac{19}{2}+\frac{13}{2}\left(\frac{1}{2}\right) = -\frac{19}{2}+\frac{13}{4} = \frac{-38+13}{4} = \frac{-25}{4}$

$R_2-\frac{5}{2}R_3$: $\frac{5}{2}-\frac{5}{2}\left(\frac{1}{2}\right) = \frac{5}{2}-\frac{5}{4} = \frac{10-5}{4} = \frac{5}{4}$

$c_1 = -\frac{25}{4}$

$c_2 = \frac{5}{4}$

$c_3 = \frac{1}{2}$

$$[-3+5x-2x^2]_B = \begin{bmatrix} -\frac{25}{4} \\ \frac{5}{4} \\ \frac{1}{2} \end{bmatrix}$$

นายธนวัฒน์ กมลา

6821651329 เลขที่ 28