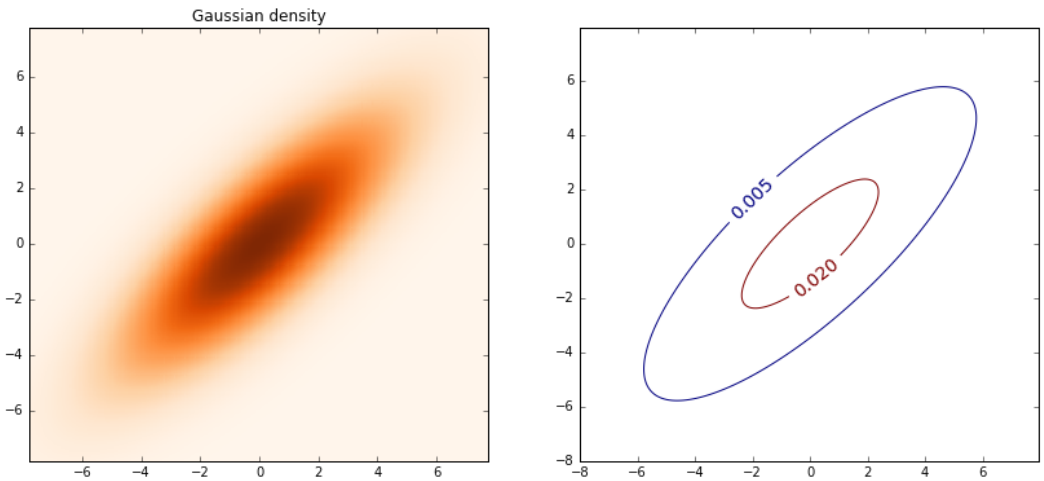
In [3]:
```
import scipy.stats as sps
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d.axes3d import Axes3D
%pylab inline
```

```
Populating the interactive namespace from numpy and matplotlib
```

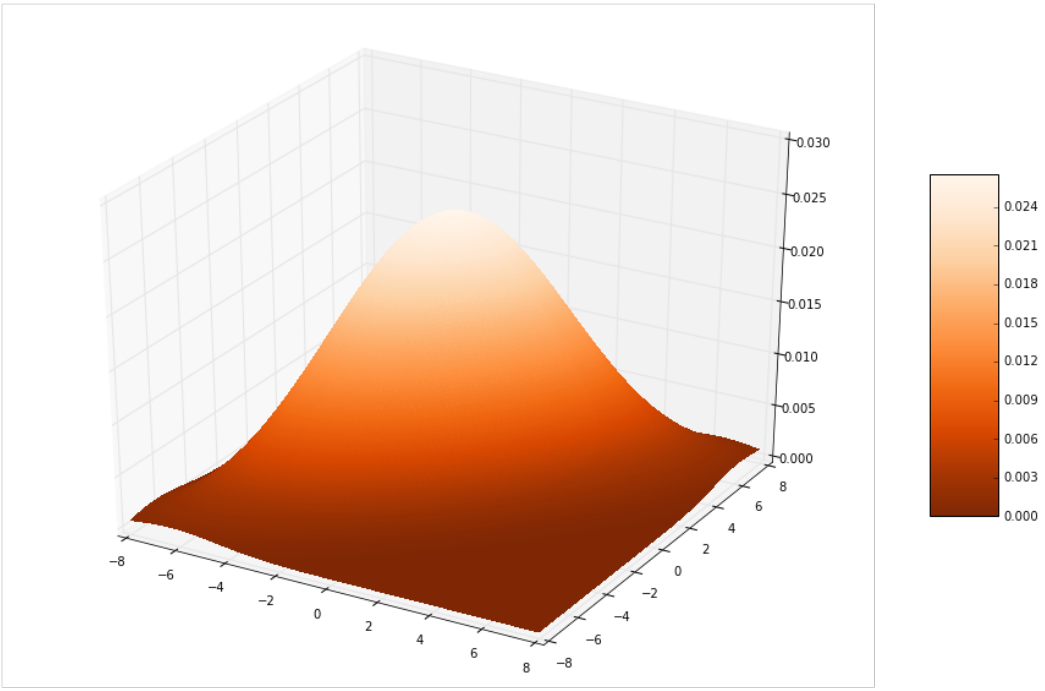
Построим график плотности для случайного вектора

$$\xi = (\xi_1, \xi_2) \sim N(a, \Sigma), a = (0, 0), \Sigma = \begin{pmatrix} 10 & 8 \\ 8 & 10 \end{pmatrix}.$$

In [4]:
```
grid = np.mgrid[-8:8:0.05, -8:8:0.05]
density = np.array([[sps.multivariate_normal.pdf((grid[0, i, j], grid[1, i,
j]), mean=[0, 0], cov=[[10, 8], [8, 10]])
                     for i in range(grid[0].shape[0])]
                    for j in range(grid[0].shape[1])])

plt.figure(figsize=(14, 6))

subplot(1, 2, 1)
plt.pcolormesh(grid[0], grid[1], density, cmap='Oranges')
plt.xlim((np.min(grid[0]) + 0.2, np.max(grid[0]) - 0.2))
plt.ylim((np.min(grid[1]) + 0.2, np.max(grid[1]) - 0.2))
plt.title('Gaussian density')

subplot(1, 2, 2)
CS = plt.contour(grid[0], grid[1], density, [0.005, 0.02])
plt.clabel(CS, fontsize=14, inline=1, fmt='%1.3f')
plt.xlim((np.min(grid[0]), np.max(grid[0])))
plt.ylim((np.min(grid[1]), np.max(grid[1])))

plt.show()
```

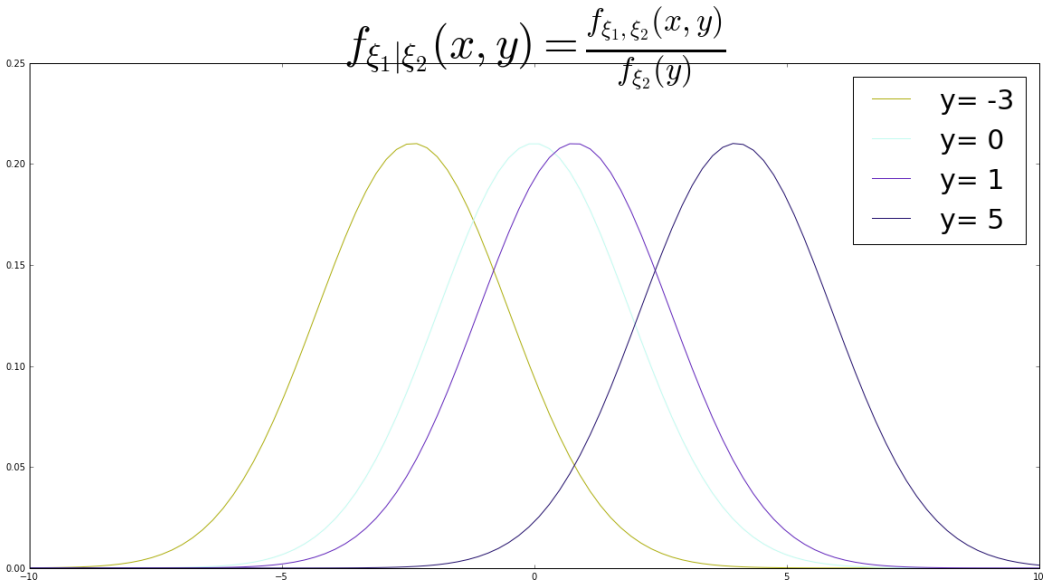


In [5]:
```
fig = plt.figure(figsize=(16, 10))
ax = fig.gca(projection='3d')
surf = ax.plot_surface(grid[0], grid[1], density, rstride=1, cstride=1, cmap
='Oranges_r',
                       linewidth=0, antialiased=False)
ax.set_zlim(0, 0.03)
fig.colorbar(surf, shrink=0.5, aspect=5)
plt.show()
```



Теперь для каждого $y \in \{-3, 0, 1, 5\}$ построим график $f_{\xi_1|\xi_2}(x|y)$

In [41]:
```
grid = np.linspace(-10, 10, 100)
plt.figure(figsize=(20, 10))
plt.title(r'$f_{\xi_1|\xi_2}(x, y) = \frac{f_{\xi_1, \xi_2}(x, y)}{f_{\xi_2}
(y)}$', fontsize = 50)

for y in [-3, 0, 1, 5]:
    multiv_norm = sps.multivariate_normal.pdf(np.array([grid, np.linspace(y,
 y, 100)]).T,
                                              mean=[0, 0], cov=[[10, 8], [8,
 10]])
    y_norm = sps.norm.pdf(np.linspace(y, y, 100), loc=0, scale=math.sqrt(10)
)
    plt.plot(grid, multiv_norm / y_norm, color=random.rand(3,1), label='y= '
 + str(y))

plt.legend(fontsize = 30)
plt.show()
```



Графики для $f_{\xi_1|\xi_2}(x,y) = \frac{f_{\xi_1,\xi_2}(x,y)}{f_{\xi_2}(y)}$, где $y \in \{-3, 0, 1, 5\}$

...

Теперь график $E(\xi_1|\xi_2 = y)$

Подберем $\alpha, \beta : \alpha\xi_1 + \beta\xi_2 \perp\!\!\!\perp \xi_2$

$$Cov(5\xi_1 - 4\xi_2, \xi_2) = 5Cov(\xi_1, \xi_2) - 4Cov(\xi_2, \xi_2) = 0$$

$$E(\xi_1|\xi_2) = \frac{1}{5}E(5\xi_1 - 4\xi_2 + 4\xi_2|\xi_2) = \frac{4}{5}\xi_2$$

In [42]:
```
plt.figure(figsize=(20, 10))
x = np.linspace(-10,10,1000)
plt.plot(x, 4*x/5, label = r'$E(\xi_1 | \xi_2)$')
plt.plot(x, np.zeros(1000), label = r'$E\xi_1$')
plt.legend(fontsize = 30)
plt.show()
```