Expressivity and Complexity in Probabilistic Team Semantics

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Dependence Logic

$$\forall x \, \exists y \, \forall z \, \exists w \, R(x, y, z, w)$$

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$$\begin{pmatrix} \forall x \, \exists y \\ \forall z \, \exists w \end{pmatrix} R(x, y, z, w)$$

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$$\exists f \, \exists g \, \forall x \, \forall z \, R(x, f(x), z, g(z))$$

$$\forall x \, \exists y \, \forall z \, \exists w \, R(x, y, z, w)$$

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$$\exists f \, \exists g \, \forall x \, \forall z \, R(x, f(x), z, g(z))$$

$$\forall x \, \exists y \, \forall z \, \exists w / x \, R(x, y, z, w)$$

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$$\forall x \, \exists y \, \forall z \, \exists w / x \, R(x, y, z, w)$$

$$\forall x \, \exists y \, \forall z \, \exists w \; (=(z, w) \wedge R(x, y, z, w))$$

Relational databases

- Databases are tables, or relations

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- Reduce data redundancy and improve data integrity

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Databases = teams.

Probabilistic conditional independence

- $X \perp\!\!\!\perp Y \mid Z$

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- $X \perp\!\!\!\perp Y \mid Z$

In the finite case:

- uniform probability distributions = teams
- probability distributions = multiteams

The thesis: Expressivity and Complexity in Probabilistic Team Semantics

Four papers

- Article I: On Elementary Logics for Quantitative Dependencies
- Article II: Logics with Probabilistic Team Semantics and the Boolean Negation
- Article III: The Implication Problem for Functional Dependencies and Variants of Marginal Distribution Equivalences
- Article IV: Axiomatization of Implication for Probabilistic Independence and Unary Variants of Marginal Identity and Marginal Distribution Equivalence

Article III and IV are single authored and I and II jointly with supervisors and others.

All published in peer-reviewed venues.

Two themes

The thesis is about probabilistic team semantics.

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Without "logic": Articles III and IV are on axiomatization and complexity of combinations of groups of atoms:

- Functional dependence
- Marginal identity and marginal distribution equivalence
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Focus is on complete axiomatizations and corresponding complexity results.

Two themes

The thesis is about **probabilistic team semantics**.

Without "logic": Articles III and IV are on axiomatization and complexity of combinations of groups of atoms:

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- Marginal identity and marginal distribution equivalence
- Probabilistic independence

Focus is on complete **axiomatizations** and corresponding **complexity** results.

With "logic": Articles I and II are about adding "logics" to some atoms:

- First-order logic, FO
- First-order team logic, FOT

Focus is on **expressivity** and **complexity** of model-checking and satisfiability.