

EXPRESSIVITY AND COMPLEXITY IN PROBABILISTIC TEAM SEMANTICS

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Dependence Logic

$$\forall x \exists y \forall z \exists w R(x, y, z, w)$$

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$$\forall x \exists y \forall z \exists w (= (z, w) \wedge R(x, y, z, w) Rxyzw)$$

Relational databases

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In the finite case:

- uniform probability distributions = teams
- probability distributions = multiteams

The thesis: Expressivity and Complexity in Probabilistic Team Semantics

Four papers

- Article I: On Elementary Logics for Quantitative Dependencies
- Article II: Logics with Probabilistic Team Semantics and the Boolean Negation
- Article III: The Implication Problem for Functional Dependencies and Variants of Marginal Distribution Equivalences
- Article IV: Axiomatization of Implication for Probabilistic Independence and Unary Variants of Marginal Identity and Marginal Distribution Equivalence

Article III and IV are single authored and I and II jointly with supervisors and others.

All published in peer-reviewed venues.

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Without “logic”: Articles III and IV are on axiomatization and complexity of combinations of groups of atoms:

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Focus is on complete **axiomatizations** and corresponding **complexity** results.

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With “logic”: Articles I and II are about adding “logics” to some atoms:

- First-order logic, FO
- First-order team logic, FOT

Focus is on **expressivity** and **complexity** of model-checking and satisfiability.