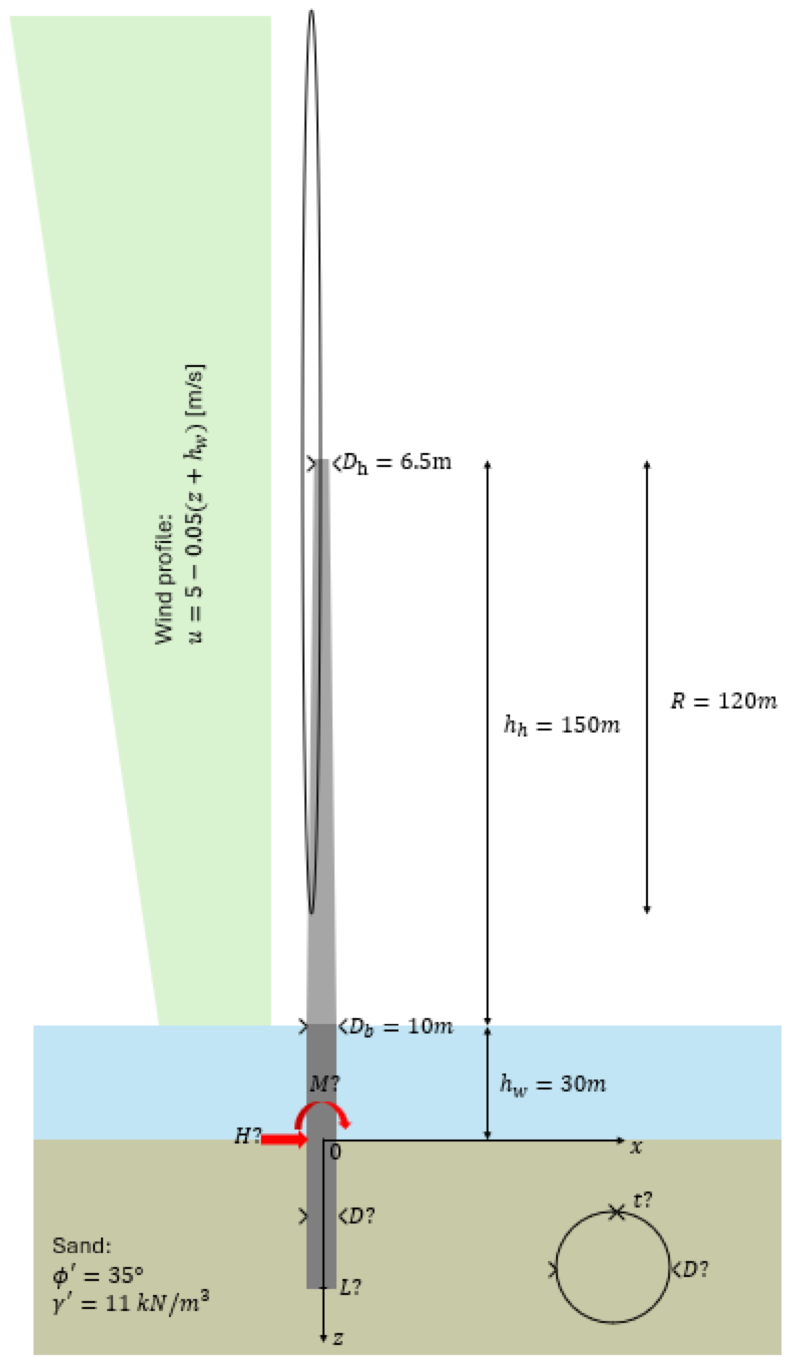
Soil Structure interaction, assignment

1. Case study

You are going to design the monopile foundation for an approximation of the theoretical NREL 15MW wind turbine presented in the following figure.



2. Loading

Environmental loadings should be factored by 1.35. The following forces should lead to the determination of H (the global horizontal force) and M (the overturning moment at the mudline).

a. Rotor thrust force

The thrust force on the blades is given by the simplified formula:

$$F_{rot} = \frac{1}{2}\rho_a A_R C_T U^2 \ [N]$$

Where ρ_a is the air volumetric weight, A_R the rotor swept area, C_T the thrust coefficient, and U the wind speed. The thrust coefficient is here assumed to be 0.5.

b. Tower drag force

The drag force on a 1m high portion of the tower is given by the simplified formula:

$$F_{tow} = \frac{1}{2}\rho_a D_{tow}(z) C_D U^2 \ \ [N/m]$$

Where D_{tow} is the varying diameter of the tower and C_D the drag coefficient assumed to be 0.4.

c. Force from the waves

Assuming linear waves within Airy theory, one can write the equation for the free surface:

$$\eta(x,t) = \frac{H_m}{2}\cos(\omega t - kx)$$

Where $H_m = 10m$ the wave height selected for this design case, $k = 2\pi/\lambda$ the angular wave number in radian per meter where $\lambda = 200m$ the wavelength selected for this design case, and $\omega = 2\pi/T$ the angular frequency in radian per second where $T = 15s$ is the wave period selected herein.

One can also write the horizontal particle velocity:

$$w(x,z,t) = \frac{H_m}{2}\omega\frac{\cosh\big(k(h_w - z)\big)}{\sinh(kh_w)}\cos(\omega t - kx)$$

To estimate the force applied by the wave, the Morison equation can be used to determine the force on a unit length of the monopile:

$$dF_w(z,t) = dF_D(z,t) + dF_I(z,t) = \frac{1}{2}\rho_w D C_D w(z,t)|w(z,t)| + C_m\rho_w A\dot{w}(z,t)$$

Where F_D and F_I the drag and inertia forces, $\rho_w = 1030\ kg/m^3$ the volumetric mass of sea water, $C_D = 0.4$ the drag coefficient, $C_m = 2$ the inertia coefficient, and $A = \pi D^2/4$ the monopile area.

One can thus determine the force applied on the monopile by the waves by integrating from the mudline to the sea surface (likewise for the moment at the mudline).

d. Other ignored loads

In this study, the following loads are ignored: weight of the structure, 1P and 3P loads, sea current, ice loads, accidental ship impact, earthquake loads, tsunami or rogue wave load, rotor load (including emergency brake).

3. Monopile design

The goal of this assignment is to design a monopile (hollow steel cylinder) foundation for the illustrated wind turbine under the previously explained loads. One should note that this implies an iterative process as the diameter of the monopile directly influences the global horizontal force and moment applied to the head of the monopile (mudline, at $z = 0$).

a. Numerical implementation of the governing equation

Referring to the SSI class, the static equilibrium of an active pile, neglecting axial forces is given by:

$$EI\frac{d^4y(z)}{dz^4} + k_y D y(z) = 0$$

The goal here is to solve this equation numerically, which can be done by using the finite difference method and applying adequate boundary conditions.

The pile should thus be discretised in an adequate number of elements. Thus, you should be able to implement a relation of the form $\underline{\underline{K}}\,\underline{y} = \underline{b}$ where $\underline{\underline{K}}$ of dimension $N \times N$ and $\underline{y}$ and $\underline{b}$ of dimension N, where N is either the number of nodes or the number of nodes plus four depending on what scheme you use.

b. Boundary conditions

To be able to solve this numerical problem (under defined), we need to define four boundary conditions:

- At the mudline the moment $M(0) = M = EI\frac{d^2y}{dz^2}$ and the shear $V(0) = H = EI\frac{d^3y}{dz^3}$.

- At the pile toe, often, null moment and shear are assumed and can be integrated in the matrix as the above: $M(L) = V(L) = 0$.

c. API p-y curve for sand

For the lateral soil resistance k_y, we are going to use the basic sand API curves:

$$p(y,z) = A(z)\, p_u(z) \tanh\left(\frac{k\, z}{A(z) p_u(z)}\, y\right)$$

$A(z)$ is a factor which for static condition is given by $A = \max\left(3 - 0.8\frac{z}{D}, 0.9\right)$.

k is the rate of increase with depth of initial modulus of subgrade reaction, it is taken here as 22 MN/m3.

p_u is the ultimate lateral bearing capacity of the sand which is given by:

$$p_u = \min(\,(C_1 z + C_2 D)\gamma' z \;\;,\;\; C_3 D \gamma' z\,)$$

Where γ' is the submerged soil weight taken as 11kN/m3.

Coefficient $C_1 = \frac{(\tan\beta)^2 \tan\alpha}{\tan(\beta-\phi')} + K_0 \left[\frac{\tan(\phi')\sin\beta}{\cos\alpha\tan(\beta-\phi')} + \tan\beta\,(\tan\phi' \sin\beta - \tan\alpha)\right]$

Coefficient $C_2 = \frac{\tan\beta}{\tan(\beta-\phi')} - K_a$

Coefficient $C_3 = K_a((\tan\beta)^8 - 1) + K_0 \tan\phi'\,(\tan\beta)^4)$

And the angles are given relative to the friction angle (ϕ'): $\alpha = \frac{\phi'}{2}$ and $\beta = 45 + \frac{\phi'}{2}$, while the coefficients of lateral earth pressure are $K_0 = 0.4$ and $K_a = \frac{1-\sin(\phi')}{1+\sin\phi'}$.

To obtain the k_y at a certain (z,y), one has to use the secant value: $Dk_y = p(y,z)/y$.

d. Monopile optimisation

Once this is implemented, the goal is to optimise the dimensions of the monopile (embedded length L and diameter D) assuming a wall thickness assumption: $t = \min(0.00635 + D/100, 0.09)$.

In this case, optimising means that **one needs to determine the (L,D) combination that minimises the use of steel, while respecting the ULS criteria of a displacement at the mudline inferior to a tenth of the pile diameter ($y(0) \leq 0.1D$).**

For the optimisation, it is possible to use the tools of the scipy.optimize library.

Once this is optimised, check that the wall thickness is sufficient: does the bending moment bring the monopile over its yield stress?

4. Summary of tasks

In the report (that should be no longer than 7 pages for the core text [with fontsize 12, 1.5 line spacing, acceptable margins, readable figures], the code should be included in annex) several points should be highlighted:

- Calculation of the forces;
- Determination of the p-y curve (and thus of k as a function of y) under the API for sand assumption;
- Finite difference method implementation under matrix form for the monopile including the definition of the boundary conditions;
- Determination of an adequate number of elements to discretize the monopile and explanation of the number of iterations used.
- Iteration and optimization process for the monopile diameter and embedded length (graph welcome);
- Final optimal value for the diameter and embedded length and associated deformed shape of the monopile as well as shear force and bending moment.
- For this final monopile, compare the results given by the theoretical formulae given during your class for a semi-infinite pile (noting that this is a combined horizontal force and moment case such that you have to determine the theoretical formula yourself by setting the right boundary conditions).

5. Additional remarks

This is a very simplified design case, where solely one loading condition is explored. Normally a large array of loading conditions are explored for not only ULS, SLS and FLS. A great focus is also normally given to the identification of eigen frequencies of the system to avoid resonance with waves, wind or rotation of the blades.

Interesting reads:

Subhamoy Bhattacharya. Design of Foundations for Offshore Wind Turbines. en. 1st ed. Wiley, April 2019. isbn: 978-1-119-12812-0 978-1-119-12813-7. doi: 10.1002/9781119128137.

“Recommandations pour la conception et le dimensionnement des fondations d’éoliennes offshore”. In: Revue Française de Géotechnique 157 (2018), p. 1. issn: 0181-0529, 2493-8653. doi: 10.1051/geotech/2019004.