LGCIV2041: Numerical Analysis of Civil Engineering Structures – Assignment

Handout: 17.02.2024 <u>**Due:** Sunday 05.04.2025</u>

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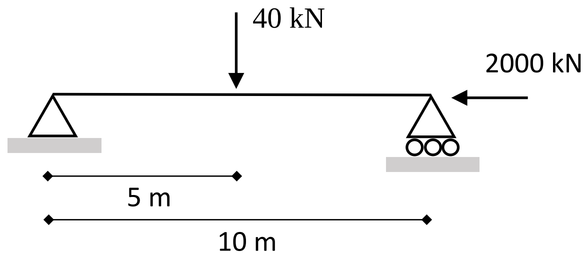
<u>Instructions</u>

- Assignment is graded to 100 points.
- The assignment is composed of 2 problems.
- The assignment should be solved in groups of 2 students.
- Please respect the due date indicated above, beyond which assignments will be accepted with penalty.
- Hand in your solution <u>together with the scripts and your input files</u>.

Problem 1. Finite Elements (FEs) in Linear and Geometrically Nonlinear Analyses (60 points)

Consider the simply supported beam, 10 m long, with rectangular cross section and shear coefficient k = 5/6. It is subjected, at mid span, to a downward force of 40 kN, and to a compressive horizontal load of 2000 kN at the roller support. Consider an axial rigidity EA = 4×10^7 kN, a flexural rigidity EI = 4×10^6 kN.m^2, and a Poisson's ratio $\nu = 0.2$.



(a) (7.5 points) Derive the exact solution for the transverse displacement field $u_{y0}(x)$ and for the rotational field $\theta(x)$. Then, compute the response of the beam with the Python script distributed for the first exercise session ("Python script to study frames.py"), i.e. using Euler-Bernoulli finite elements (FEs). Use a mesh of 2 elements as well as a mesh of 20 elements. Compare the three results and comment.

(b) (7.5 points) Implement, in the same Python script, the stiffness matrix corresponding to a Timoshenko finite element, assuming a linear approximation both for the rotations $\theta(x)$ and the transverse displacements $u_{y0}(x)$. Plot the transverse displacements and rotations for a mesh of 2, 8, 20, and 200 FEs, together with the exact solution. Compare the response obtained with the four meshes, and with respect to the exact solution. Discuss whether the FE responses satisfy the compatibility conditions, namely: is the displacement field continuous within each element and between elements?, does it satisfy the support conditions?

(c) (7.5 points) Consider the response obtained in (b) with 8 FEs. Show the evolution of the bending moment and shear force along the beam. Discuss if the FE response satisfies the equilibrium conditions, namely: locally (within each element), between elements, and the natural boundary conditions.

(d) (7.5 points) Consider the following alternative beam lengths: L = 2 m, 20 m, and 200 m. For each length, and considering always a mesh of 200 FEs, plot in the same graph the transverse displacement along the beam for: Timoshenko FEs, Euler-Bernoulli FEs, and the exact solution. Comment on the results, discussing which numerical results you would "trust" as an engineer if you did not have access to the exact solution.

(7.5-points bonus: Redo the analyses of question (d) with Abaqus and compare the results.)

(e) (7.5 points) Compute analytically and show the Timoshenko stiffness matrix considering selective reduced integration, as discussed in the lecture. Implement it in the Python script and plot again the transverse displacement for the same cases and mesh of question (d). Comment on the results obtained.

(f) (22.5 points) Adapt the Python script to compute the geometrically nonlinear response of the beam using a corotational formulation. Consider Euler-Bernoulli FEs. Show the transverse displacements for a mesh of 2 elements as well as for a mesh of 20 elements. Compare the results with those of the geometrically linear case, in (a), and explain physically the differences in the results. PS. You can implement a classical Newton-Raphson method and/or a displacement-control method, as discussed in the lecture and exercise session, and shown in the shared Python code.

Problem 2: Abaqus (40 points)

Coming soon…