Problem 2: Material Nonlinearity (40 points)

The notched four-point bending test is a common alternative to the notched three-point bending test for fracture toughness testing in order to assess the ability of fractures to propagate in a material. A simplified geometry of the test is shown in Figure 1. The test is performed by applying two punctual forces P/2 at the top of the specimen, while the bottom part is maintained by two supports, to propagate the initial sharp crack.

In this problem, we will investigate different ways to model the mechanical behavior of a specimen made of wood (E=12 GPa and ν=0,35) subjected to this test using an elasto-plastic constitutive law in Abaqus.

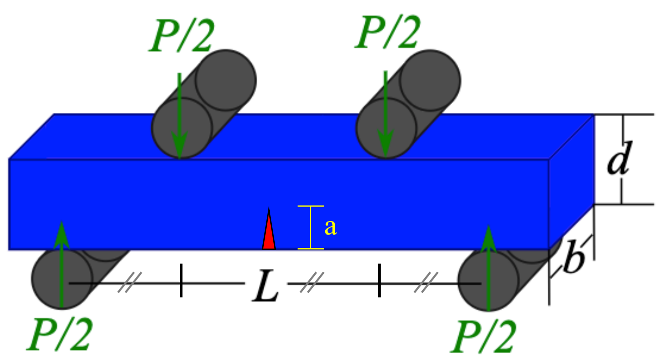

Figure 1. 3D sketch of notched four-point bending test.

In the following, we consider L=260mm, d=40 mm, b=15 mm, a=d/2 and the crack is modelled as approximated as a triangular crack with a maximum width of a/10.

(a) As a first step, simulate the structure depicted in Figure 1 with Abaqus using beam elements, as well as a three-point bending test for the same specimen (replacing the two loads at the top by one in the middle). The load(s) P is modelled as a point load with a maximum of 1 kN. The notch is ignored in this question.

(a.1) Which type of elements and beam theory would you choose for this analysis? Justify.

(a.2) For four-point bending test only, what is the displacement obtained for the node in the middle and maximum von Mises stress for different meshes? For which mesh size can we consider that we have obtained an accurate value?

(a.3) Plot the moment and shear force diagrams obtained from Abaqus. Based on those diagrams, what are the benefits of using four-point bending test compared to three-point?

(b) Simulate the structure in 2D and 3D (with the notch) and perform in this case as well a mesh convergence analysis. How do the displacements compare to (a) and to each other? Why?

(c) In a second step, you use an elasto-plastic constitutive law (with a von Mises yield function) for the 3D geometry considering a yield strength of 50MPa (no hardening or softening are considered here). What is the displacement (maximum one in the middle of the beam) required to reach plasticity?

(d) You now consider a hardening for the constitutive law with the values defined in Table 1. Plot the force (P)-(central) displacement curve of the specimen in 3D. How is this force-displacement diagram changing while refining the mesh? What is the value of displacement at which stress the first point reaches plasticity for the different meshes? Why?

Yield stress (MPa)	Plastic strain (-)
50	0
60	0.0235
70	0.0474
80	0.0935
90	0.1377
100	0.18

Table 1. Hardening rule (linear approximation between points)

(e) To obtain a more realistic response compared to the experimental results, you consider softening for the material with the values of Table 2. What problem are you facing and why? To overcome this problem, you consider a ramp of displacement applied at the top with a final value of 20% of the height. How is this force-displacement diagram changing while refining the mesh in this case? How can you explain this difference by observing the final deformation of the whole specimen and how does it differ from the deformation of the specimen with hardening.

Yield stress (MPa)	Plastic strain (-)
50	0
40	0.0235
30	0.0474
20	0.0935
10	0.1377

Table 2. Softening rule (linear approximation between points)