LGCIV2042: Dynamics of Structures **(Due : 6/11/2025)**

Assignment : Linear SDoF systems and frequency-domain analysis

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Instructions:
Read all the instructions below carefully before you start working on the assignment, and before you submit it.

- The assignment is graded to 100 points and consists of a report.
- Each group of two students is asked to submit a report of the assignment on November 6^{th}, 2025
- The report cannot be longer than 25 pages. All figures should be large enough, with readable axes and legends.
- Please respect the due date indicated above, beyond which the assignment report will be accepted with penalty.
- You are free to use the programming language that suits you. Additional files (codes, additional plots, etc.) should be sent by email to the teaching team along with the report in one single *zip* file. Code files should be clear, commented and ready to be run and checked.
- You don't need to detail the theoretical background if you refer to the course's slides.
- A folder called *Assignment files* is given on Moodle and contains all the information that you need to complete this assignment.

During this project, groups of two students will work on a physical model built at the LEMSC laboratory. This model consists of a steel plate column on top of which several masses can be attached, along with a RecoVib triaxial accelerometer. The base of the column is welded to a steel plate which is itself attached to a 50kg-capacity uniaxial QuakeLogic shake table. During this project, you will derive the dynamic properties of this small-scale structure, and try to predict its response to several types of excitations.

Detailed plans of the model are available on Moodle. The picture below shows a 3D-schematic of the structure. All the parts of the structure are made of the same steel ($\rho = 7900$ kg/m^3 and $E = 210$ GPa). The ground motion will be applied in the direction parallel to the weak axis of inertia of the column.

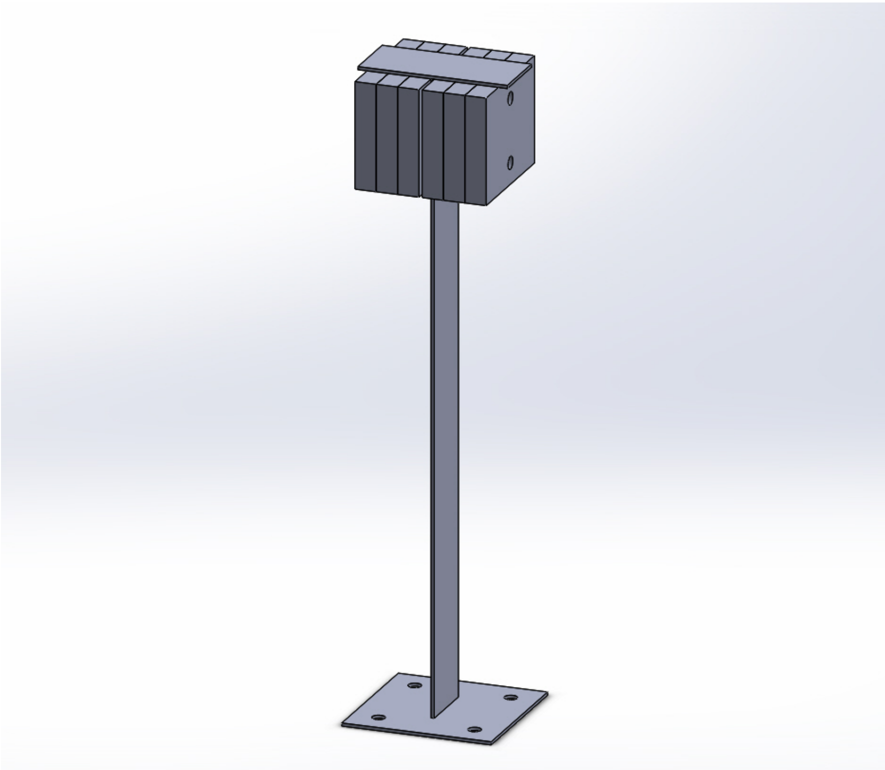

Figure 1: 3D view of the column fixed to the shake table.

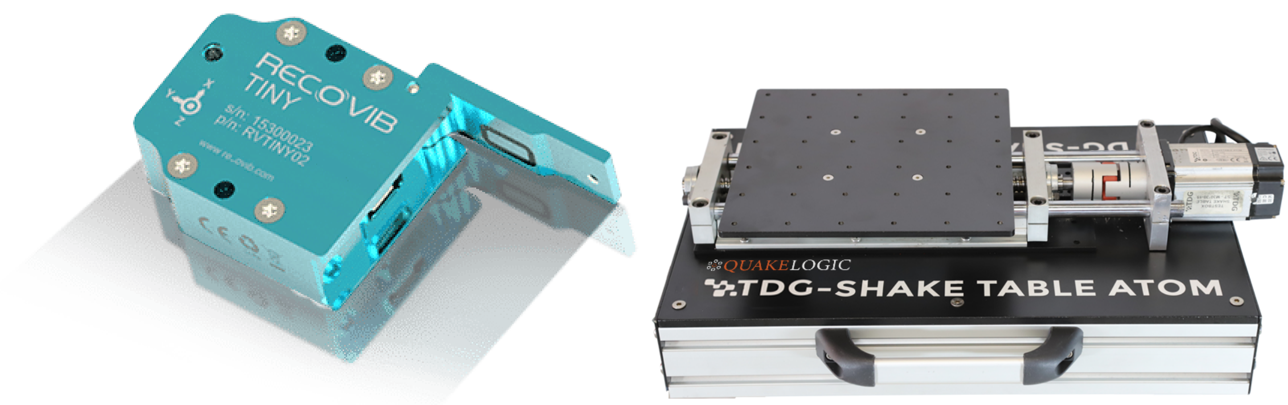

Figure 2: RecoVib Triaxial accelerometer (left) and QuakeLogic uniaxial shake table (right).

Part 1 - Dynamic properties and free vibration of the model (25 Points)

Throughout this project, different values of mass will be used for different groups. The mass of the system can be changed by removing or adding steel blocks (dim. 1 cm $\times$ 5 cm $\times$ 6 cm) at the head of the column. The number of blocks equals $2 \times (1 + (N-1) \mod 3)$ where N is your group number. As an example, if your group number is $N = 9$: you have to work with $2 \times (1 + (8) \mod 3) = 6$ blocks.

(1.1) Propose a model for the structure and compute its dynamic properties: stiffness, natural frequency and period. Explain clearly your assumptions and calculations.

The head of the column is pushed horizontally a few centimeters (u_0) to the right and quickly released. The acceleration record from the RecoVib is given on Moodle.

(1.2) Derive the displacement history from the experimental acceleration record (plot it in 1.4). Then, compute the initial displacement u_0 of the column's head. Justify.

(1.3) Write down the equation of motion of the undamped system with its initial conditions found in (1.2). Solve the differential equation analytically.

(1.4) Plot the response in terms of acceleration, velocity and displacement from the questions (1.2) and (1.3) for 20s. Identify and discuss the sources of the energy loss. If needed, adjust the natural frequency of your model for the rest of the project.

(1.5) Compute the damping coefficient based on the recorded acceleration. Develop the method. Compare this value with the damping coefficient of steel structures.

(1.6) Using the damping coefficient calculated in (1.5), write down the equation of motion of the damped system. Solve the differential equation analytically and plot the response of the structure in terms of acceleration, velocity and displacement during 20s. Compare with the experimental response in (1.2) and comment.

Part 2 - Response to ground motion (30 Points)

From now on, use the natural frequency chosen from (1.4) and the damping ratio derived from (1.5).

For the 10^{th} of October, send your natural frequency ω_n from (1.4) to basile.payen@uclouvain.be.

The physical model is now subjected to five different ground motions $\ddot{u}_g(t) = 0.1g \sin(k \cdot \omega_n t)$ where g is the acceleration of gravity, ω_n the natural frequency computed previously, and $k = 0.5$, 0.75, 1, 1.5 and 2 is the ratio between the excitation and natural frequency. On week 5, you will perform an experimental campaign using the shake table to obtain accelerometer records for those different k.

(2.1) Using the accelerometer record, extract the amplitude of the steady-state acceleration response of the structure, plot it as a function of the ratio k and explain the curve obtained.

(2.2) For an arbitrary value of excitation frequency $\bar{\omega}$, compute the analytical expression of the steady-state displacement response of the structure. From the displacement response, compute the amplitude of the acceleration as a function of $\bar{\omega}$. Compare and comment with experimental values from (2.1). Comment the results for an extremely low and an extremely high frequency.

The structure is subjected to a two-sided step impulse as shown in the figure below. Impulse amplitude and period vary from one group to the other (see table below).

(2.3) Using Duhamel's integral, derive the displacement response of the structure and plot it for 40s. Plot your result and comment.

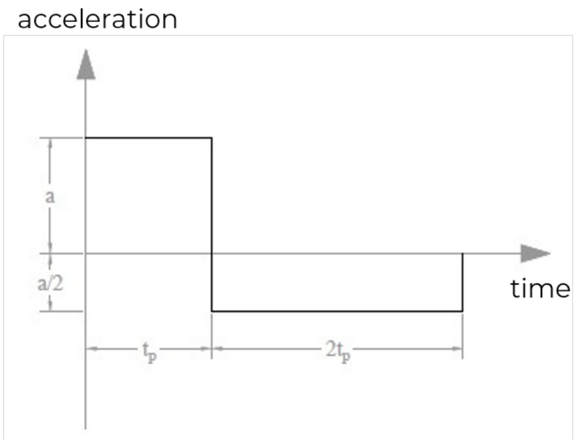

Figure 3: Two-sided acceleration impulse.

Group	Impulse period t_p [s]	Impulse amplitude a [g]
1	0.1	0.05
2	0.2	0.075
3	0.3	0.1
4	0.4	0.05
5	0.1	0.075
6	0.2	0.1
7	0.3	0.05
8	0.4	0.075
9	0.1	0.1
10	0.2	0.05

Table 1: Impulse period and amplitude per group.

Part 3 - Numerical time-domain integration (20 Points)

(3.1) For the ground motions described in Part 2 ($\ddot{u}_g(t) = 0.1g\sin(\omega_n t)$), program the Central Difference Method to compute the response $U(t)$ and $\ddot{U}(t)$ of the structure during 40s. Use different time step: $\Delta t = T_n/50$; $T_n/25$; $T_n/5$; $T_n/2$. Compare with the experimental data and your answer from (2.2). Discuss the influence of the choice of the time step.

Part 4 - Frequency-domain analysis (25 Points)

For these four questions, you can use any tool of your choice to compute the FFT (Fast Fourier Transform). Specify it.

(4.1) Plot on the same graph the FFT of both acceleration record of question (1.2) and the undamped free-vibration acceleration response computed in (1.3). Compare both results and comment.

(4.2) Plot on the same graph the FFT of the damped free-vibration acceleration response computed analytically in (1.6) and the responses with a damping coefficient of 2%, 5% and 10%. Compare results and comment.

(4.3) For each $k = 0.5, 0.75, 1, 1.5$ and 2, plot on the same graph the FFT of the acceleration record from (2.1), and plot the corresponding FFT of the acceleration response computed analytically. Compare and comment. *Reminder:* ($\ddot{u}_g(t) = 0.1g\sin(k \cdot \omega_n t)$)

(4.4) Repeat question (4.3), but compute the FFT using only the first 20s of the acceleration response of (2.1). Comment on differences you observe with respect to question (4.3).