

Neural Sliding Mode Control for Antilock Braking System

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Abstract—Antilock Braking System (ABS) is an active Advanced Driver-Assistance System (ADAS) that prevents *skidding* in extreme braking conditions by means of active brake control measures. A road vehicle is a complex system with highly nonlinear dynamics that pose a challenge in developing adequate ABS controllers. In this paper, we propose a hybrid ABS controller that can operate in varying road conditions, and robust to the nonlinearities and measurement uncertainties of vehicle dynamics. Sliding mode control (SMC) is robust in modeling the nonlinear dynamics of a vehicle but suffer from chattering effect which limit their practical application. The proposed controller combines SMC with a Recurrent Neural Network (RNN) uncertainty observer overcome this challenge and compensate vehicle model uncertainties. The simulation results verify the robustness and feasibility of the proposed hybrid controller.

I. INTRODUCTION

Advanced Driver Assistance Systems (ADAS) are features that vehicles are equipped with to improve safety and efficiency of modern transportation systems, they one of the most researched fields in road vehicles today. ABS is one of the oldest and an important ADAS application for safety and accident mitigation, it maximizes tire traction and ensure steerability during harsh braking conditions. ABS prevents the occurrence of *wheel lock* during braking, wheel lock increases breaking distance, leads to loss of steerability (lateral instability), and tyre wear. The underlying challenge with designing adequate ABS and other ADAS applications is the complex nonlinear vehicle dynamics.

Braking applies opposite tangential force to a wheel in motion which then begins to slip, where the forward velocity of the wheel is larger than the rolling velocity. In extreme conditions such as sudden braking, braking on slippery roads (wet, icy, etc.), the risk of rapid increase in wheel slip and wheel deceleration will lead to wheel lock-up. A locked wheel results in skidding and has no lateral stability, increasing the likelihood of an accident. ABS is a safety system that was invented to control the wheel slip to achieve adequate tyre friction to prevent wheel lock-up, ensuring steerability i.e. lateral stability and reduced stopping distance. ABS uses sensors to detect when the wheel tangential velocity is significantly lower than the vehicle velocity, actuates the braking hydraulic valves and by modulating braking torque to attain optimal slip condition, the wheel turns faster and adequate tyre friction achieved.

Designing an ABS is a challenging problem because vehicle dynamics is highly non-linear, with time-varying parameters, uncertainties, and sensor measurement noise. There also exists

a complex relationship between wheel slip and friction coefficient, a relationship which varies with road conditions, vehicle speed and tyre types. This application imposes a constraint of rapid response time, discouraging complex time-consuming control methodologies. Classic PID controllers have been applied in the design of ABS, but linearization-based controllers lack robustness to uncertainties such as varying road conditions, measurement noise, hence, offering limited practical applications. Other notable ABS control approaches include Lyapunov based optimal control, sliding-mode control (SMC) for robust control, gain-scheduling control for adaptive control, fuzzy logic for intelligent control, and hybrid controllers as a combinations of two or more control methods [1].

Hybrid control approaches aim to combine the advantages and compensate the drawbacks of the individual approaches. SMC has been studied extensively in the control of nonlinear systems with adequate performance as in the case of a vehicle ABS, it operates by defining a sliding surface from the dynamics of the system on which it drives the system on the sliding surface by means of an equivalent controller and a robust controller which drives the system from an initial state to the sliding surface. However, the high frequency chattering of the SMC can be problematic in practical applications. This work proposes a hybrid control approach, Neural Sliding Mode Control (NSMC), combining the robustness of SMC approach, and a Recurrent Neural Network (RNN) to overcome the chattering and system uncertainties.

The rest of the report is organized as follows. The formulation of the ABS from the quarter car model in Section II. Section III presents the SMC system design; the sliding surface, the equivalent and the robust controllers. Section IV outlines the structure of hybrid controller: RNN uncertainty observer, hybrid control approach. Section V outlines simulation results of the proposed hybrid controller and performance compared to a state-of-art approach. Section VI concludes the report.

II. ABS FORMULATION

ABS is a closed-loop electronic control system as in Fig. 1 that requires a mathematical model that describes the dynamics of tyre, vehicle, road, and the braking system with sufficient accuracy [2]. The ABS in this paper adopts a simplified longitudinal vehicle model that constitutes the dynamics of the tyre, vehicle, braking torque and road dynamics, neglecting external forces like aerodynamic effects, rolling resistance, and suspension and steering system dynamics. This model is sufficient because braking control limited to the longitudinal vehicle dynamics is self consistent and meaningful.

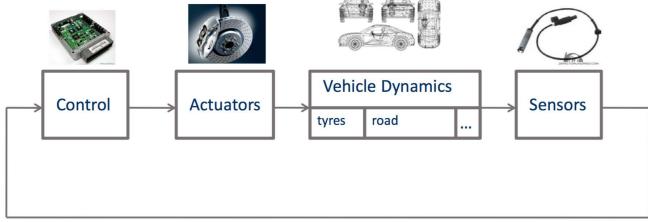


Fig. 1: Antilock braking system (ABS) components

The simplified model is the single corner model otherwise known as the quarter-car model, it considers the vehicle/tyre/road dynamics at one wheel; wheel rotational dynamics, linear vehicle dynamics, braking torque, road reaction force and the interactions between them. Modern vehicles today possess all-wheel brakes therefore this model is adequate to implement the ABS. From the single corner model in Fig. 2, applying Newtons equations we obtain the longitudinal dynamics during braking in Eqn. 1

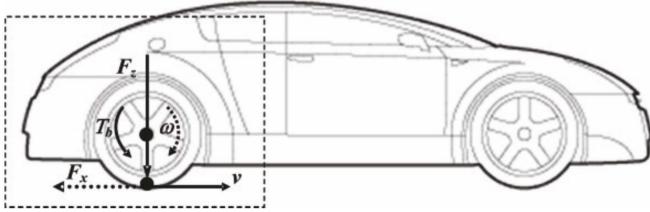


Fig. 2: Single corner model

$$\begin{aligned} J\dot{\omega} &= -T_b + F_x R \\ m\dot{v} &= -F_x \end{aligned} \quad (1)$$

where; T_b is the *braking torque*, ω is the *wheel angular speed*, v is the *longitudinal velocity*, F_x is the *longitudinal tyre road contact force*, J is the *moment of inertia of the wheel*, m is the *single corner mass* (1/4 of the car), R is the *wheel radius*.

The tyre is the source of all the forces that affect the dynamics and performance of the vehicle (neglecting aerodynamic forces), and tyre dynamics describes the relationship of forces acting on the vehicle from the road. The three forces acting on the tyre-road road contact patch; longitudinal force F_x that determines the braking (and traction) performance, lateral force F_y affects lateral stability (steerability), and vertical load F_z . F_x and F_y are significantly dependent on F_z , and can be approximated with a *linear relationship* (2) in control problems.

$$\begin{aligned} F_x &= \mu_x(\lambda, \alpha, \gamma, F_z) \approx \mu_x(\lambda, \alpha, \gamma)F_z \\ F_y &= \mu_y(\lambda, \alpha, \gamma, F_z) \approx \mu_y(\lambda, \alpha, \gamma)F_z \end{aligned} \quad (2)$$

where, γ is the *camber angle*, α is the *wheel side-slip angle*, λ is the *wheel longitudinal slip*. In this work the *Burckhardt friction model* was adopted as the *tyre model* to obtain the longitudinal friction coefficient μ_x ;

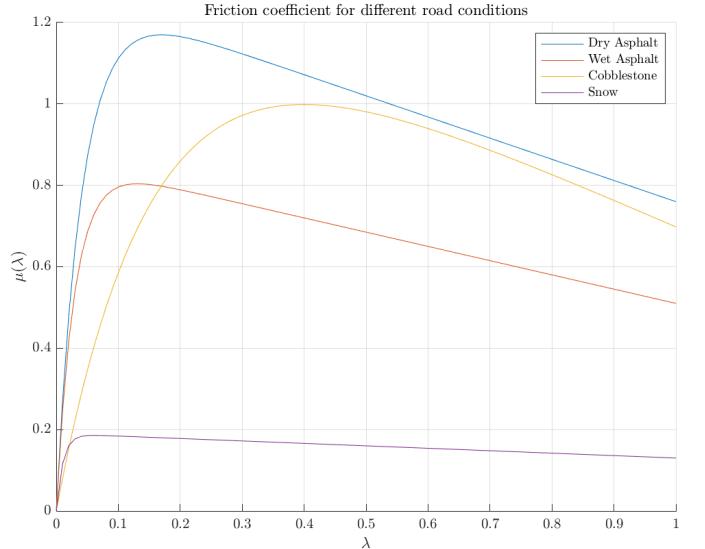


Fig. 3: Wheel longitudinal slip λ and longitudinal friction coefficient μ relationship under different road conditions

$$\mu(\lambda, \theta_r) = \theta_{r,1}(1 - e^{-\lambda\theta_{r,2}}) - \lambda\theta_{r,3} \quad (3)$$

where; $\theta_r = (\theta_{r,1}, \theta_{r,2}, \theta_{r,3})$ are the three parameters of the model.

Wheel slip is fundamental in the longitudinal control of a vehicle (braking/traction) because the road friction coefficient is dependent on it. Although wheel slip measurement is challenging as it requires the estimation of vehicle longitudinal speed, the control of wheel slip is simple and dynamically robust. Wheel slip is determined by Eqn. 4, and the relationship between wheel slip and the road friction coefficient under different road conditions is shown in Fig. 3. When $\lambda = 0$ (0%) wheel is in *free rolling*, while $\lambda = 1$ (100%) is the *wheel lock* condition where $\approx 20\%$ of braking/traction capability, and steerability is lost.

$$\lambda = \frac{v - \omega R}{v} \quad (4)$$

During braking, the vehicle is decelerated by the road/wheel friction force and goal of an ABS is to regulate the wheel slip to maximize the coefficient of friction $\mu(\lambda)$ during braking operation. Substituting (2) in (1) and rewriting to obtain $(\dot{\omega}, \dot{v})$, the derivative of wheel slip (4) is given as:

$$\dot{\lambda} = -\frac{1}{v} \left(\frac{1-\lambda}{m} + \frac{R^2}{J} \right) F_z \mu(\lambda) + \frac{R}{Jv} T_b \quad (5)$$

It can be rewritten as:

$$\dot{\lambda} = f(\lambda, t) + g(t)u(t)$$

where; $f(\lambda, t)$ denotes the *nonlinear dynamic function*, $g(t)$ is a *gain*, and $u(t) = T_b(t)$ is the control action i.e. *braking torque*. Eqn. (5) shows that the system is nonlinear, contains vehicle parameters with uncertainties and may contain sensor measurement noise. We formulate the lambda-derivative model as a combination of the nominal conditions (where all the parameters of the system are well known) for $f(\lambda, t)$ and

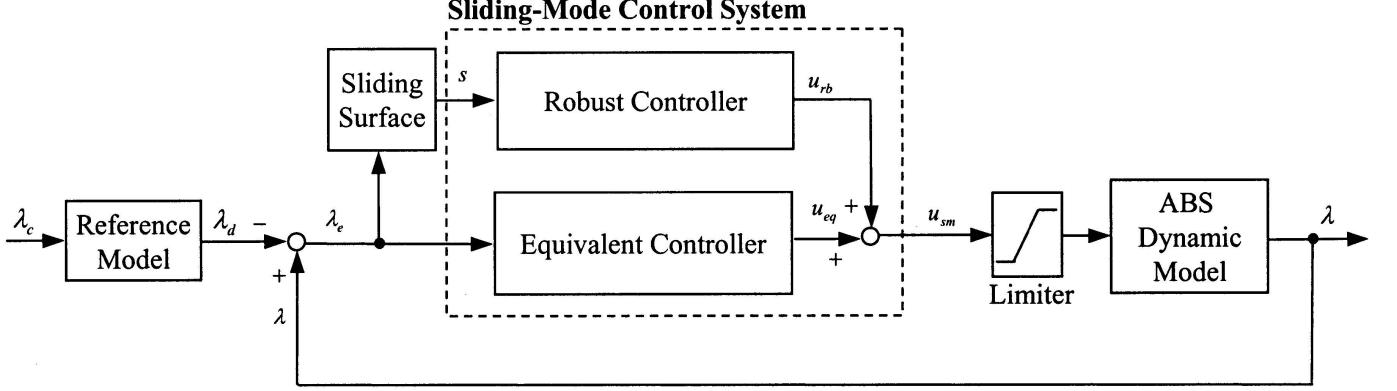


Fig. 4: SMC scheme for ABS

$g(t)$, and the uncertainties and measurement noise are lumped together;

$$\begin{aligned} \dot{\lambda} &= [f_n(\lambda, t) + \Delta f(\lambda, t)] + [g_n(t) + \Delta g(t)]u(t) + n(t) \\ &= f_n(\lambda, t) + g_n(t) + d(\lambda, t) \end{aligned} \quad (6)$$

where; $f_n(\lambda, t)$ and $g_n(t)$ are the *nominal values* of $f(\lambda, t)$ and $g(t)$, $\Delta f(\lambda, t)$ and $\Delta g(t)$ are the *uncertainties*, $n(t)$ is the *measurement noise*, and $d(\lambda, t)$ is the *lumped uncertainty*, defined as $d(\lambda, t) = \Delta f(\lambda, t) + \Delta g(t) + n(t)$. We assume $|d| \leq \rho$, where ρ is a positive constant.

III. SMC SYSTEM DEFINITION

Sliding-mode control (SMC) is a robust Lyapunov based method to control nonlinear systems in the presence of uncertainties, the sliding mode is defined by a performance index i.e. *sliding surface* according to the system dynamics and SMC guarantees the system dynamics stability in finite time once in the sliding mode. The goal of the SMC is determine a control law that reduces the system order dynamics for the slip to track the reference value. Tracking error defined as:

$$\lambda_e(t) = \lambda(t) - \lambda_{ref} \quad (7)$$

where; λ_{ref} is the *reference slip value* verified by experiments to provide adequate braking performance and stability in all road conditions (Fig. 3). The sliding surface is designed first, it describes the system dynamics which satisfies the desired closed-loop performance. The control law is then designed to drive the system to the sliding surface and remain there as the state variable tends to zero along the sliding surface i.e. *sliding motion*. The sliding surface state variable is defined as:

$$s(t) = \lambda_e(t) + k \int_0^t \lambda_e(\tau) d\tau \quad (8)$$

where; $k > 0$ is a *gain*. The SMC control law is defined as:

$$u_{sm}(t) = u_{eq}(t) + u_{rb}(t) \quad (9)$$

The equivalent controller $u_{eq}(t)$ governs the sliding motion, it moves the system state to the sliding surface and then along it once reached, defined as:

$$u_{eq}(t) = g_n^{-1}(t)[-f_n(\lambda, t) - k\lambda_e(t)] \quad (10)$$

The robust controller $u_{rb}(t)$ is designed to compensate the system uncertainties during sliding motion, given as:

$$u_{rb}(t) = g_n^{-1}(t)[-psgn(s(t))] \quad (11)$$

where; $sgn(.)$ is the *sign function*, k and ρ are *control gains*.

It has been proven in [3] that by Lyapunov analysis, the SMC defined in (9) guarantees the finite time convergence of the system, $\dot{s}\dot{s} \leq 0$. Thus, the control law (9) guarantees the stability of the ABS control system.

IV. HYBRID CONTROL APPROACH (NSMC)

A hybrid control system is proposed to improve tracking performance by estimating the lumped uncertainty $d(\lambda, t)$ in (6) online, and adding it to the control effort. SMC produces robust ABS performance, however, the impact of the uncertainty bound ρ limits their practical application; ρ chosen too large leads to large chattering that wears the bearing and can lead to instability, too small then the control system may not be robust enough to uncertainties and external noise.

As shown in Fig. 5, the hybrid control system consists of ideal controller, and compensation controller. The ideal controller control effort incorporates the RNN uncertainty observer $d(\lambda, t)$ online estimation into the control effort. The compensation controller estimates the error bound and compensate the uncertainty observer errors.

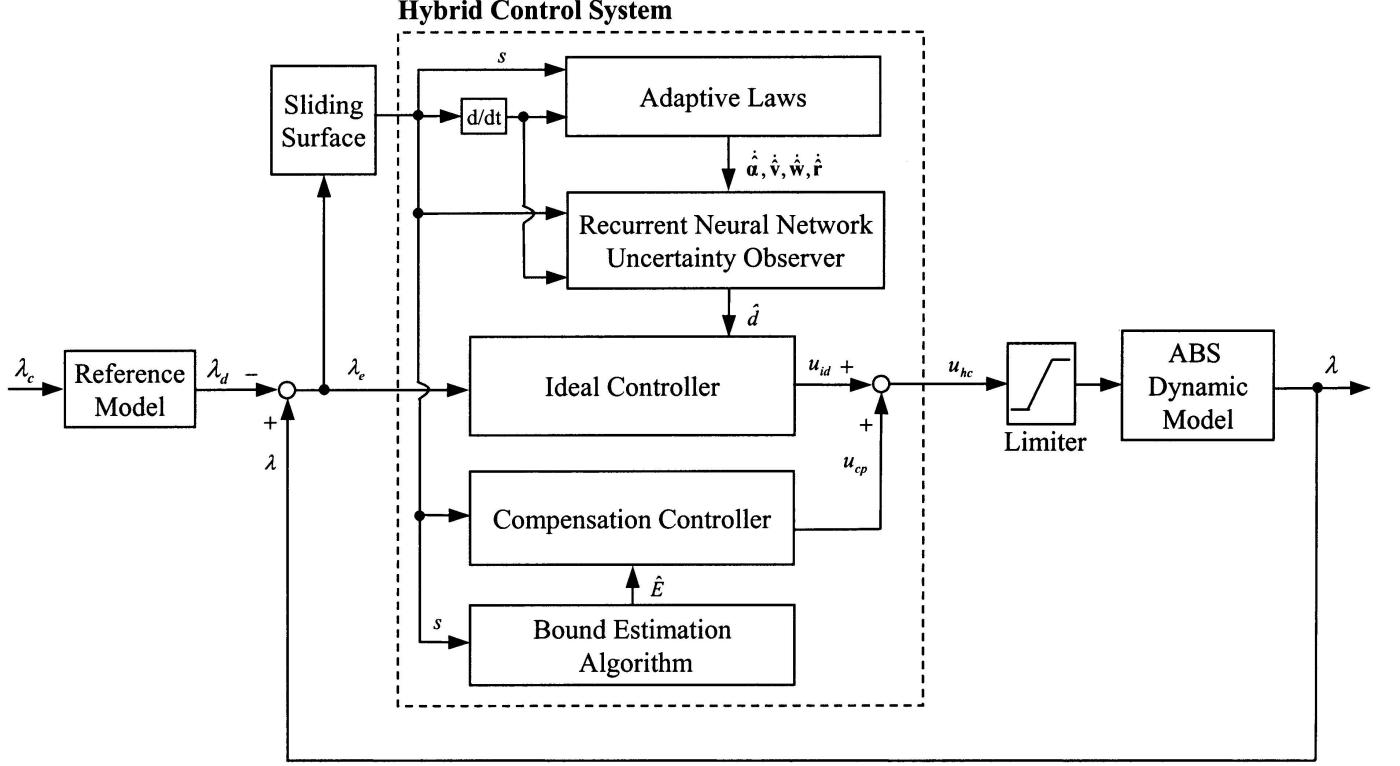


Fig. 5: Hybrid control (NSMC) scheme for ABS

A. RNN Formulation

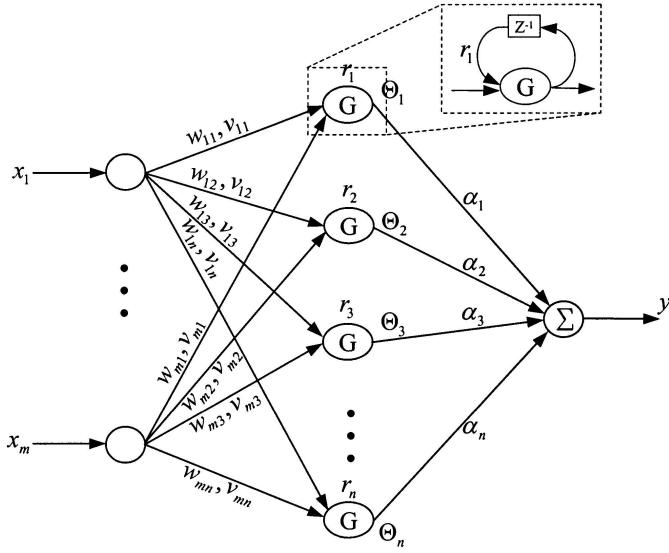


Fig. 6: Structure of a RNN

A RNN can be described as a dynamic mapping network because it possesses an internal feedback loop that enables capturing system dynamic response with feedback through delays. As shown in Fig. 6, a RNN typically consists of three layers; an input layer, a hidden layer with a feedback unit and an output layer. The RNN here uses the Radial Basis Function (RBF) as activation functions and maps as follows:

$$y(N) = \sum_{k=1}^n \alpha_k \Theta_k(\|x_i(N) - v_{ik}\|, \sigma_{ik}, r_k, \Theta_k(N-1)) \quad (12)$$

where; $x = x_i, i = 1, 2, \dots, m$ and y are the input variables and output variables respectively, N is the number of iterations, α_k are the weights of the k^{th} neuron output, Θ_k is the output of the k^{th} neuron, v_{ik} and σ_{ik} are the center and width of the RBF respectively, r_k is the internal feedback gain, and $\|\cdot\|$ denotes the Euclidean norm. The output at each k^{th} neuron is given as:

$$\text{net}_k(N) = \sum_{i=1}^m w_{ik}^2 [x_i(N) + \Theta_k(N-1)r_k - v_{ik}]^2 \quad (13)$$

and

$$\Theta_k(N) = e^{-\text{net}_k(N)} \quad (14)$$

where; $w_{ik} = 1/\sigma_{ik}$ is the inverse radius of the RBF. All the parameters of the hidden layer collected as vectors, \mathbf{v} , \mathbf{w} , \mathbf{r} :

$$\begin{aligned} \mathbf{v} &= [v_{11} \dots v_{m1} \quad v_{12} \dots v_{m2} \quad v_{1n} \dots v_{mn}]^T \\ \mathbf{w} &= [w_{11} \dots w_{m1} \quad w_{12} \dots w_{m2} \quad w_{1n} \dots w_{mn}]^T \\ \mathbf{r} &= [r_1 \dots r_n]^T \end{aligned} \quad (15)$$

The output of the RNN re-written:

$$y(x, \mathbf{v}, \mathbf{w}, \mathbf{r}, \boldsymbol{\alpha}) = \boldsymbol{\alpha}^T \boldsymbol{\Theta}(x, \mathbf{v}, \mathbf{w}, \mathbf{r}) \quad (16)$$

where; $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_n]^T$ and $\boldsymbol{\Theta} = [\Theta_1, \Theta_2, \dots, \Theta_n]^T$.

B. Hybrid Control Approach

The hybrid control approach includes the system uncertainty estimated by the RNN uncertainty observer in the model (6) to modify the control effort from (9) to (21). The optimal lumped uncertainty that satisfies the control effort (9) is denoted as d^* , such that:

$$d = d^* + \Delta = \alpha^* \Theta^* + \Delta \quad (17)$$

where; Δ is the approximation error, α^* and Θ^* are the optimal parameters of α and Θ . The RNN uncertainty observer estimates the system lumped uncertainty, \hat{d} , with an estimation error \tilde{d} given as:

$$\tilde{d} = d - \hat{d} = d^* - \hat{d} + \Delta = \tilde{\alpha}^T \hat{\Theta} + \hat{\alpha}^T \tilde{\Theta} + \tilde{\alpha}^T \Theta + \Delta \quad (18)$$

where; $\tilde{\alpha} = \alpha^* - \hat{\alpha}$ and $\tilde{\Theta} = \Theta^* - \hat{\Theta}$. Nonlinear RBF Taylor linearization is used to design the adaptive laws that online tune the center, radii and feedback gain of the RNN uncertainty observer. These adaptive laws aim to minimize Θ :

$$\tilde{\Theta} = \mathbf{A}^T \tilde{\mathbf{v}} + \mathbf{B}^T \tilde{\mathbf{w}} + \mathbf{C}^T \tilde{\mathbf{r}} + \mathbf{H} \quad (19)$$

where; $\mathbf{A} = [\partial \Theta_1 / \partial \mathbf{v} \dots \partial \Theta_n / \partial \mathbf{v}]|_{\mathbf{v}=\hat{\mathbf{v}}}$, $\mathbf{B} = [\partial \Theta_1 / \partial \mathbf{w} \dots \partial \Theta_n / \partial \mathbf{w}]|_{\mathbf{w}=\hat{\mathbf{w}}}$, $\mathbf{C} = [\partial \Theta_1 / \partial \mathbf{r} \dots \partial \Theta_n / \partial \mathbf{r}]|_{\mathbf{r}=\hat{\mathbf{r}}}$, \mathbf{H} is the vector of higher order terms, $\tilde{\mathbf{v}} = \mathbf{v}^* - \hat{\mathbf{v}}$, $\tilde{\mathbf{w}} = \mathbf{w}^* - \hat{\mathbf{w}}$; \mathbf{v}^* , \mathbf{w}^* , \mathbf{r}^* are the optimal parameter vectors of \mathbf{v} , \mathbf{w} , \mathbf{r} ; $\hat{\mathbf{v}}$, $\hat{\mathbf{w}}$, $\hat{\mathbf{r}}$ are the estimated vectors of \mathbf{v}^* , \mathbf{w}^* , \mathbf{r}^* . Substituting (19) in (18), estimation error \tilde{d} re-written:

$$\tilde{d} = \tilde{\alpha}^T \hat{\Theta} + \tilde{\mathbf{v}} \mathbf{A} \hat{\alpha} + \tilde{\mathbf{w}} \mathbf{B} \hat{\alpha} + \tilde{\mathbf{r}} \mathbf{C} \hat{\alpha} + \epsilon \quad (20)$$

where; $\epsilon = \hat{\alpha}^T \mathbf{H} + \tilde{\alpha}^T \Theta + \Delta$ is assumed bounded by $|\epsilon| \leq E$.

The proposed hybrid control is governed by:

$$u_{hc}(t) = u_{id}(t) + u_{cp}(t) \quad (21)$$

The ideal controller $u_{id}(t)$ is given as:

$$u_{id}(t) = g_n^{-1}(t) [-f_n(\lambda, t) - k\lambda_e(t) - \hat{d}] \quad (22)$$

and the compensation controller $u_{cp}(t)$ is given as:

$$u_{cp}(t) = -g_n^{-1}(t) \hat{E} sgn(s(t)) \quad (23)$$

where; \hat{E} is the estimated value of the approximation bound E .

The adaptive laws chosen for online tuning of the control parameters are:

$$\begin{aligned} \dot{\hat{\alpha}} &= -\dot{\hat{\alpha}} = \eta_1 \dot{s}(t) \hat{\Theta} \\ \dot{\hat{\mathbf{v}}} &= -\dot{\hat{\mathbf{v}}} = \eta_2 \dot{s}(t) \mathbf{A} \hat{\alpha} \\ \dot{\hat{\mathbf{w}}} &= -\dot{\hat{\mathbf{w}}} = \eta_3 \dot{s}(t) \mathbf{B} \hat{\alpha} \\ \dot{\hat{\mathbf{r}}} &= -\dot{\hat{\mathbf{r}}} = \eta_4 \dot{s}(t) \mathbf{C} \hat{\alpha} \\ \dot{\hat{E}}(t) &= -\dot{\hat{E}}(t) = \eta_5 |s(t)| \end{aligned} \quad (24)$$

It has been proven in [3] that by Lyapunov analysis the proposed hybrid control law (21) and adaptive laws (24) guarantee the system asymptotic stability. In the next section we analyse the simulation results of the proposed ABS hybrid controller under different road conditions and compare with the state-of-the-art PID wheel slip control approach.

V. SIMULATION RESULTS

Numerical simulation was carried out with MATLAB/Simulink software to demonstrate the performance of the hybrid controller. The simulation parameters are as follows: wheel radius, $R = 0.3\text{ m}$; single corner mass (quarter-car), $m = 225\text{ kg}$; wheel moment of inertia, $J = 1\text{ Nm}$; initial vehicle speed, $v = 100\text{ km/h}$; maximum braking torque, $T_{b_{max}} = 1500\text{ Nm}$; and, simulation time $t = 7\text{ s}$. The simulation setup also includes a state machine that captures the activation and de-activation of the ABS under specific conditions, critical for practical applications. The reference slip value the ABS controller must track is $\lambda_{ref} = 0.1\text{ (10\%)}$.

The performance of the hybrid control for ABS was tested under the different road conditions (3) and the results are shown in Fig. 7. The NSMC implemented comprises a RNN of 5 neurons with chosen parameters; $k = 50$, $\eta_1 = 3$, $\eta_2 = \eta_3 = \eta_4 = 0.01$ and $\eta_5 = 0.1$, and is capable of adequately tracking the reference slip in various road conditions, reaching steady state on average in approximately $\approx 0.5\text{ s}$. Under cobblestone road condition however, the vehicle with the ABS has a greater stopping distance due to to a higher friction coefficient (3) for a locked wheel ($\lambda = 1$) than at the desired wheel slip ($\lambda = \lambda_{ref}$). The results also show that the hybrid controller eliminates chattering phenomenon that arises with SMC.

Comparative Analysis with PID Wheel Slip Controller (PID-WSC)

Fig. 8 presents the performance of the proposed NSMC compared to the classical PID-WSC, results show comparable stopping distance; NSMC performs better on dry asphalt and snow, and poorer on cobblestone. However, PID-WSC performs better in the cobblestone because it attains a higher peak and a slower settling time, thereby taking advantage of the higher friction coefficient in this road condition (3).

The PID-WSC results in significantly larger overshoot and longer steady-state time than the hybrid controller. Also, the case of the snow road condition the PID-WSC exhibits an undesirable behaviour; falling to zero after its peak before rising to the desired value. The NSMC outperforms the PID-WSC with a better λ_{ref} tracking response; small overshoots and fast settling time.

In conclusion, NSMC produces a better performance overall because tracking response is the most important criteria of this work. Further, the braking distance performance of the NSMC can be improved with an adaptive slip model for different road conditions instead of a fixed reference value.

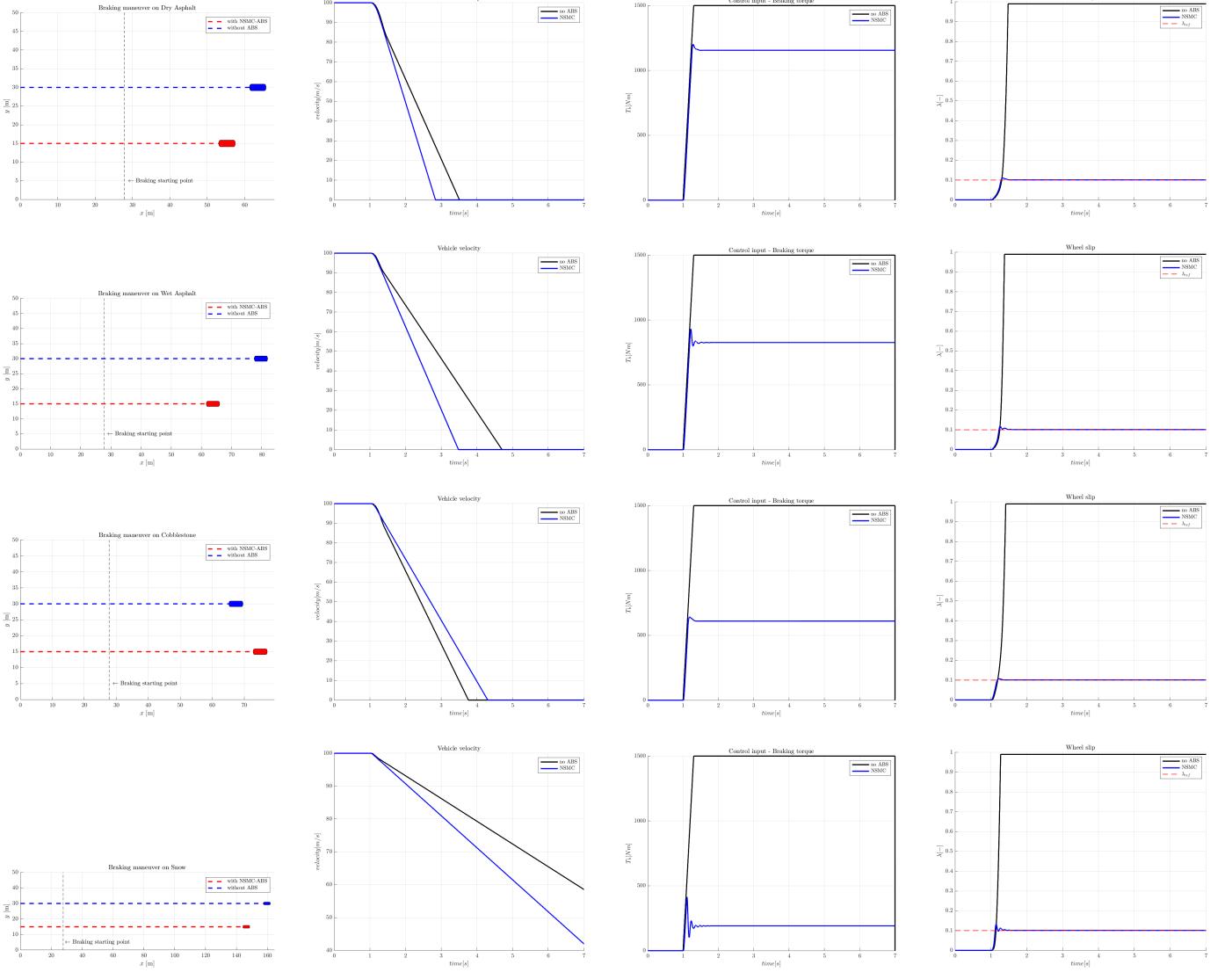


Fig. 7: Simulation results for different road conditions: | Braking distance | Vehicle velocity | Braking torque | Wheel slip |

VI. CONCLUSION

The ABS problem is formulated the single corner model and the longitudinal wheel slip is identified as the single most important parameter for braking control. SMC provides robust control for the ABS, but suffers high frequency chattering effect. First he sliding surface is designed based from wheel slip dynamics, and then the control law that guarantees asymptotic stability of the system comprising equivalent and robust control efforts. Then, a hybrid controller capable of compensating the system uncertainties and dispel the chattering effects of SMC is proposed. By adopting a RNN uncertainty observer, the control scheme is extended to include the estimated uncertainty and error bound.

Simulation results show that the hybrid control scheme, NSMC provides robust ABS performance in various road conditions. Comparative analysis shows it's effectiveness in eliminating chattering of SMC, and providing overall better response with lower overshoot and faster settling time

(≈ 0.5 s). This work shows that neural network integrated control approaches can compensate uncertainties of nonlinear dynamics and un-modeled dynamics from limited-accuracy mathematical models.

Further research could extend implementation to a full vehicle model, include load transfer and other vehicle dynamics. Longitudinal velocity estimation methods can also be included in the simulation as it was assumed to known in this work. Neural networks can be explored for slip-friction model estimation for varying road conditions, to create an adaptive reference slip model and overcome challenge of ABS performance on cobblestone in the case of constant λ_{ref} . This work shows the capacity of NN in highly nonlinear systems with uncertainties and could be further investigated for other nonlinear systems.

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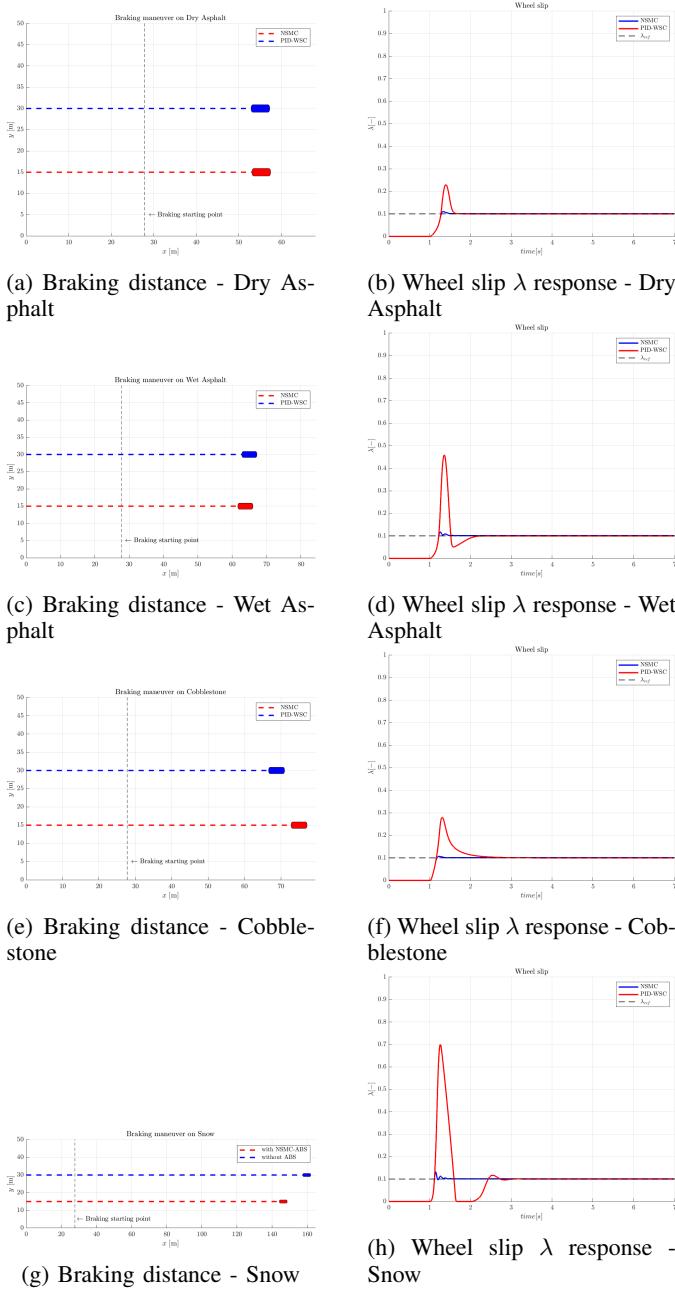


Fig. 8: Comparative analysis of NSMC vs. PID-WSC, simulation results for different road conditions.

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