

Connectionist and Evolutionary Systems: ACO

Final Project: UCSP

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Abstract

The University Classes Schedule Problem (UCSP) consists in finding all the required disciplines for each group at some academic period. It doesn't really matter whether the disciplines are chosen by the students or assigned by the institution. Anyway, the primary task for the "ants" is to encounter valid configurations of classes, such that provide exactly the required time of each required discipline for each group. The secondary task is to encounter the solution, that provides the best satisfaction by the represented persons and the institution.

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1 Problem

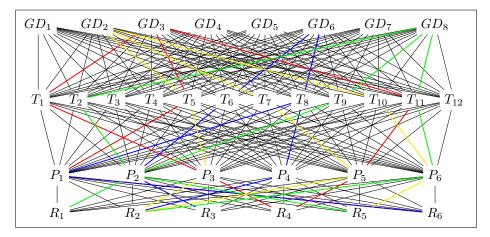


Figure 1: Problem graph schematic, representing Groups, Disciplines, Time/day, Professors, ClassRooms.

1.1 Classes

A *class* is an event, that links together the following types of entities, denoted as *roles*:

- 1. group-discipline pairs
- 2. day/time
- 3. professors
- 4. classrooms

Each of the roles must have a finite and non-empty domain, therefore ensuring finite number of unique permutations.

```
-- Used as kind (see data type promotion)

data Role = Groups \mid DayTime \mid Professors \mid Classrooms deriving Typeable

-- 'Role' kind container

data Role' (r :: Role) = Role' deriving Typeable
```

1.2 Graph Nodes

The problem graph nodes are <u>different</u> permutations of role domains. They are grouped into layers, depending on the corresponding role.

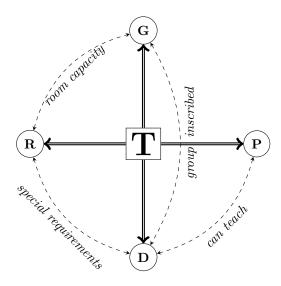


Figure 2: Class structure.

The nodes at some layer have exactly the same underlying size and it's the power of it's domain set.

```
type family Role Value (r :: Role) :: *
newtype Node (r :: Role) = Node (String, [Role Value r])
nodeId\ (Node\ (id,\_)) = id
instance Eq \pmod{r} where (\equiv)
                                                              `on` nodeId
                                                 = (\equiv)
instance Ord (Node r) where compare = compare 'on' node Id
class HasDomain\ a\ v\mid a\rightarrow v
                            :: a \to Set v
   where domain
           domainPower :: a \rightarrow Int
type RoleDomain \ r = HasDomain \ (Role' \ r) \ (RoleValue \ r)
mkNodes :: RoleDomain \ r \Rightarrow String \rightarrow Role' \ r \rightarrow [Node \ r]
\mathit{mkNodes}\ \mathit{name} = \mathit{map}\ \mathit{Node}
                  \circ zip (map ((name++) \circ show) [1..])
                   \circ permutations \circ Set.toList \circ domain
```

1.2.1 Timetable

A timetable holds schedule for one week, that repeats throughout the academic period. The timetable is actually a table: the columns represent days of week; the rows — discrete time intervals. Actual timetable structure may vary, as can be seen in figure 3.

	Mon	Tue	Wed	Thu	Fri	Sat
08:30 - 09:00						
09:00 - 09:30						
09:30 - 10:00						
10:00 - 10:30						
10:30 - 11:00						
11:00 - 11:30						
11:30 - 12:00						
: :						

(a) Timetable without recesses.

	Mon	Tue	Wed	Thu	Fri	Sat
08:30 - 09:10						
09:15 - 09:55						
10:05 - 10:45						
10:50 - 11:30						
11:40 - 12:20						
12:25 - 13:05						
13:15 - 13:55						
: :						

(b) Timetable with recesses.

Figure 3: Possible timetable structures.

```
class (Eq t, Ord t, Enum t, Bounded t) \Rightarrow
   DiscreteTime\ t\ \mathbf{where}\ timeQuantum::t
                                                          \rightarrow Int
                                  to Minutes
                                                    :: t \longrightarrow Int
                                  from Minutes :: Int \rightarrow Maybe \ t
class (Discrete Time t, Enum d, Bounded d) \Rightarrow
   Timetable tt \ t \ d \ ev \mid tt \rightarrow t
                            , tt \rightarrow d
                            ,tt \rightarrow ev
   where listEvents :: tt
                                               \rightarrow [((d,t),ev)]
             newTTable :: [((d, t), ev)] \rightarrow tt
             eventsOn :: tt \rightarrow d
                                           \rightarrow [(t, ev)]
                                            \rightarrow [(d, ev)]
             eventsAt :: tt \rightarrow t
                           :: tt \to d \to t \to Maybe \ ev
             eventAt
```

1.3 Graph Edges

The edges are possible routes, that can be taken by an "ant". They connect nodes, belonging to different layers.

```
\begin{aligned} \forall a \in \text{Layer}_A \\ \forall b \in \text{Layer}_B \\ \text{if Layer}_A \text{ and Layer}_B \text{ are neighbors} \\ \exists \text{ an edge between } a \text{ and } b. \end{aligned}
```

A selection of some sub-route, connecting some nodes A_i and B_j (from some layers A and B) means that the ant "proposes" a (partial) solution, that is described by the nodes' underlying values. The "ant" agent must be capable of selecting exactly one node of each role. The selection order doesn't matter.

A complete route (through all the layers) describes a *solution candidate*: some schedule, that holds a list of *classes*.

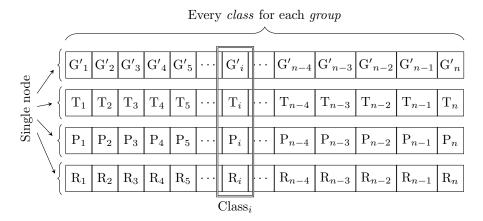


Figure 4: Route decomposition.

2 Formalization

Let's denote

 N_G — number of groups;

 N_P — number of professors;

 N_R — number of classrooms;

 N_D — number of disciplines;

 N_T — number of *time periods* per week: number of *time periods* per day × number of *days*; N_d^g — number of time periods of discipline d, assigned for group g;

$$G = \{g_i\}_{i=1}^{N_G}$$
 — set of groups;

$$D = \{d_i\}_{i=1}^{N_D}$$
 — set of disciplines;

$$P = \{p_i\}_{i=1}^{N_P}$$
 — set of professors;

$$R = \{r_i\}_{i=1}^{N_R}$$
 — set of classrooms;

$$D_g = \{d \mid N_d^g \neq 0\}_{d \in D}$$
 — set of disciplines, assigned to group g ;

$$N_{\Sigma} = \sum_{g \in G} \sum_{d \in D_g} N_d^g$$
 — total number of classes time periods per week.

Problem Dimensions 2.1

Groups and Disciplines

Let G' be a list of pairs $\langle \text{group, discipline} \rangle$ of length N_{Σ} , such that $\forall \langle g, d \rangle \in G' \implies \text{count}_{G'}(\langle g, d \rangle) = N_d^g$. There are $N_{\Sigma}!$ unique permutations.

2.1.2 **Professors and Classrooms**

With no optimization applied, exists $\binom{N_{\Sigma}+N-1}{N_{\Sigma}-1}$ (combinations with repetitions), where $N = N_P$ or N_R .

Some invalid instances can be discarded, such that, for example, don't have enough professors capable of teaching some discipline; or classrooms configurations that won't fit all the students etc.

2.1.3 Day and Time

In general case, any day and time may be assigned for any class period, including repetitions, that yields $\binom{N_{\Sigma}+N_{T}-1}{N_{\Sigma}-1}$ possible combinations.

This number may be diminished by

- joining class periods;
- requiring a minimum entropy.

Total combinations (worst case):

$$\binom{N_{\Sigma} + N_P - 1}{N_{\Sigma} - 1} \binom{N_{\Sigma} + N_R - 1}{N_{\Sigma} - 1} \binom{N_{\Sigma} + N_T - 1}{N_{\Sigma} - 1} N_{\Sigma}!$$
 (1)

2.2 Assessing Candidates

$$\eta = \eta(\lbrace r_i \rbrace_{i=1}^{n-1}, r_n) = \begin{cases} 0 & \text{if any restriction is broken} \\ \operatorname{pref}(\lbrace r_i \rbrace_{i=1}^n) & \text{otherwise} \end{cases}$$
(2)

where r_i is some sub-route.

2.2.1 Restrictions

There are two kinds of restrictions: over time and over capabilities.

Time restriction require the schedule to be *time consistent*: no group, professor and classroom can have two different classes, assigned at the same day/time. The capabilities represent:

Group: Disciplines needed (searched).

Professors: Known disciplines (that can be taught).

Classrooms: Special requirements (labs etc.); students capacity.

Note: group capabilities are incorporated into nodes generation.

2.2.2 Preferences

Preferences create an order over *valid candidates*, that permits the algorithm to optimize them. The preferences might vary for each entity (group, professor, classroom), but they all must have a form of function:

$$\operatorname{pref}'[E] : \langle \operatorname{discipline}, \operatorname{day/time} \rangle \mapsto [0, 1]$$

The preference value for a *complete route*:

$$\operatorname{pref}(r) = \frac{\operatorname{pref}'[G](r) + \operatorname{pref}'[P](r) + \operatorname{pref}'[R](r)}{3}$$

3 Implementation

3.1 Entities

Here follows definition of the input data, as stated in Section 2.

$$\begin{tabular}{ll} \textbf{data} \ \textit{Discipline} = \textit{Dicipline} \ \{ \ \textit{disciplineId} & :: String \\ & , \ \textit{disciplineTime} :: Int \\ & , \ \textit{disciplineReqs} \ :: Set \ \textit{Requirement} \\ & \} \\ \end{tabular}$$

newtype Requirement = Requirement String **deriving** (Show, Eq, Ord)

 $instance \ Show \ Discipline \ where \ show = discipline Id$

```
instance Eq
                 Discipline \ \mathbf{where} \ (\equiv)
                                                          'on' disciplineId
                                             = (\equiv)
instance Ord Discipline where compare = compare 'on' disciplineId
data \ Group = Group \{ groupId \}
                                           :: String
                       ,\,group Size
                                           :: Int
                        , \, group Disciplines :: Set \,\, Discipline
instance Show Group where show
                                          = groupId
                                                      `on`\ groupId
                 Group where (\equiv)
instance Eq
                                          = (\equiv)
instance Ord Group where compare = compare 'on' groupId
\mathbf{data}\ \mathit{Professor} = \mathit{Professor}\ \{\mathit{professorId} :: \mathit{String}
                               , can Teach :: Set Discipline
instance Show Professor where show
                                             = professorId
instance Eq
                Professor \ \mathbf{where} \ (\equiv)
                                             = (\equiv)
                                                         'on' professorId
instance Ord Professor where compare = compare 'on' professorId
data \ Classroom = Classroom \{ roomId \}
                                                  :: String
                                , roomCapacity :: Int
                                , roomEquipment :: Set Requirement
instance Show Classroom where show
                                              = roomId
instance Eq Classroom where (\equiv)
                                              = (\equiv)
                                                          'on' roomId
instance Ord Classroom where compare compare on roomId
```

3.1.1 Timetable

Timetable is defined over Mon-Sat, from 8:00 till 22:00 with 30 minutes discretization.

```
 \begin{array}{l} \textbf{newtype} \ Time = Time \ Int \\ \textbf{deriving} \ (Eq,Ord) \\ \\ time Q = 30 \\ time Min = 60*8 \\ time Max = 60*22 \\ time DMin = 0 \\ time DMax = (time Max - time Min) `quot` time Q \\ \textbf{instance} \ Enum \ Time \ \textbf{where} \\ \end{array}
```

```
fromEnum\ (Time\ t) = t
  toEnum\ i = \mathbf{if} \quad i \geqslant timeDMin
                    i \leq timeDMax
              then Time i
              else error $ "wrong discrete time: " + show i
instance Bounded\ Time\ where\ minBound=Time\ timeDMin
                                   maxBound = Time \ timeDMax
instance DiscreteTime Time where
  toMinutes\ (Time\ t) = timeMin + timeQ * t
  timeQuantum = 30
  from Minutes m = \mathbf{if} \ m \geqslant time Min
                       \wedge m \leq timeMax
                       \wedge m 'rem' timeQ \equiv 0
                      then Just \circ Time \$ (m - timeMin) `quot` timeQ
                      else Nothing
  -- redefined 'System.Time.Day' — no 'Sunday'
\mathbf{data} \ Day = Monday \mid Tuesday \mid Wednesday
           | Thursday | Friday | Saturday
  deriving (Eq, Ord, Enum, Bounded, Ix, Read, Show)
type DaySchedule = Map Time Class
newtype WeekSchedule = WeekSchedule (Map Day DaySchedule)
group With' :: (Ord \ k) \Rightarrow (a \rightarrow k) \rightarrow (a \rightarrow v) \rightarrow [a] \rightarrow Map \ k \ [v]
group With' f g es =
  let groupIn []
                       = id
      groupIn(x:xs) = Map.insertWith(++)(f x)[g x]
  in es 'groupIn' Map.empty
instance Timetable WeekSchedule Time Day Class where
  listEvents (WeekSchedule ws) = do
     (day, classes)
                       \leftarrow Map.assocs \ ws
     (time, class')
                       \leftarrow Map.assocs\ classes
     return\ ((day, time), class')
  newTTable = WeekSchedule \circ Map.map\ Map.fromList
                                 \circ group With' (fst \circ fst)
                                                (first snd)
```

3.1.2 Classes

A Class entity links a discipline, group, professor, classroom and some day-time.

```
\label{eq:data_class} \begin{array}{l} \mathbf{data} \ \mathit{Class} = \mathit{Class} \ \{ \mathit{classDiscipline} :: \mathit{Discipline} \\ , \mathit{classGroup} \quad :: \mathit{Group} \end{array}
```

```
, classProfessor :: Professor
                          , classRoom
                                          :: Classroom
                                          :: Day
                          . classDay
                          , classBegins
                                          :: Time
     type instance RoleValue\ DayTime = (Day, Time)
     type instance Role Value Groups
                                            = (Group, Discipline)
     type instance Role Value Professors = Professor
      type instance RoleValue\ Classrooms = Classroom
     class RoleExtra (r :: Role) where
                  :: Role' \ r \rightarrow Int
        roleIx
        roleName :: Role' \ r \rightarrow String
        mbRole :: Role' r \rightarrow PartClass \rightarrow Maybe (Role Value r)
        classRole :: Role' \ r \rightarrow Class \rightarrow RoleValue \ r
     instance RoleExtra Groups
                                      where roleIx _
                                                          = 0
                                              roleName \_ = "Groups"
                                              mbRole \ \_r = (,) < $>
                                                             mbGroup \ r < *>
                                                             mbDiscipline r
                                              classRole = classGroup \&\&\&
                                                             classDiscipline
     instance RoleExtra DayTime
                                     where roleIx _
                                              roleName \_ = "DayTime"
                                              mbRole \_ = mbDayTime
                                              classRole = classDay \&\&\&
                                                             classBegins
     instance RoleExtra Professors where roleIx _
                                              roleName \_ = "Professors"
                                              mbRole = mbProfessor
                                              classRole \_ = classProfessor
     instance RoleExtra Classrooms where roleIx _
                                                          =3
                                              roleName \_ = "Classrooms"
                                              mbRole \ \_ \ = mbRoom
                                              classRole = classRoom
     instance (RoleExtra\ r) \Rightarrow Show\ (Role'\ r) where show = roleName
   Meanwhile a PartClass stands for a partially defined Class and a Route —
for a sequence of PartClasses.
     data \ PartClass = PartClass \ \{ \ mbDiscipline :: Maybe \ Discipline \}
                                   , mbGroup
                                                  :: Maybe Group
```

, mbProfessor :: Maybe Professor

```
:: Maybe Classroom
                                 , mbRoom
                                 , mbDayTime :: Maybe (Day, Time)
toFullClass \ r = \mathbf{do}
                               \leftarrow mbDiscipline \ r
                         di
                               \leftarrow mbGroup \ r
                               \leftarrow mbProfessor r
                               \leftarrow mbRoom \ r
                         (d,t) \leftarrow mbDayTime\ r
                         return $ Class di g p cr d t
data Route = Route \{ routeParts \}
                                               :: [PartClass]
                                               ::!(Maybe (Node Groups))
                         , mbGroupsNode
                         , mbDayTimeNode :: !(Maybe (Node DayTime))
                         , mbProfessorsNode :: !(Maybe (Node Professors))
                         , mbRoomsNode
                                               :: !(Maybe (Node Classrooms))
                         , assessHistory
                                               :: ![InUnitInterval]
hasDisciplines = isJust \circ mbGroupsNode
hasGroups
                = isJust \circ mbGroupsNode
hasProfessors = isJust \circ mbProfessorsNode
hasRooms
                = is Just \circ mbRoomsNode
hasDayTime = isJust \circ mbDayTimeNode
emptyRoute = Route [] Nothing Nothing Nothing []
class UpdRoute\ (r :: Role) where updRoute :: Node\ r \rightarrow Route \rightarrow Route
updRoute' upd (Node (\_, xs)) r =
     \mathbf{do}(pc, x) \leftarrow routeParts\ r\ 'zip'\ xs
         [upd\ pc\ x]
instance UpdRoute Groups where
   updRoute \ n \ r = r \quad \{
     mbGroupsNode = Just n,
     route Parts \\
                      = updRoute' (\lambda pc (g, d) \rightarrow pc \{ mbGroup = Just g \})
                                                         , mbDiscipline = Just d
                                                         }) n r
{\bf instance}\ {\it UpdRoute}\ {\it DayTime}\ {\bf where}
  updRoute \ n \ r = r \quad \{
     mbDayTimeNode = Just n,
     routeParts = updRoute' (\lambda pc \ x \rightarrow pc \ \{ mbDayTime = Just \ x \}) \ n \ r
instance UpdRoute Professors where
  updRoute \ n \ r = r  {
     mbProfessorsNode = Just n,
     routeParts = updRoute' (\lambda pc \ x \rightarrow pc \ \{ mbProfessor = Just \ x \}) \ n \ r
```

```
} instance UpdRoute\ Classrooms\ where updRoute\ n\ r=r\ \{ mbRoomsNode=Just\ n, routeParts=updRoute'\ (\lambda pc\ x \to pc\ \{mbRoom=Just\ x\})\ n\ r }
```

3.2 Relations

3.2.1 Restrictions

Classes must be time consistent for each group, professor and classroom.

```
timeConsistent :: Route \rightarrow Bool
  timeConsistent \ r =
     let test :: (Ord \ a) \Rightarrow (Route \rightarrow Bool) \rightarrow (PartClass \rightarrow a) \rightarrow Maybe Bool
         test\ b\ sel = \mathbf{if}\ b\ r then timeConsistent'\ (routeParts\ r)\ sel
                                    <|> Just False
                                 else Nothing
         bs = [test\ hasGroups\ mbGroup]
               , test\ has Professors\ mb Professor
               , test\ has Rooms\ mbRoom
     in hasDayTime\ r \land fromMaybe\ False\ (foldr\ (<|>)\ Nothing\ bs)
  timeConsistent' :: (Ord \ a) \Rightarrow [PartClass] \rightarrow (PartClass \rightarrow a)
                                 \rightarrow Maybe\ Bool
  timeConsistent'\ pcs\ select = foldr\ f\ Nothing\ byRole
     where byRole = groupWith select pcs
       f \ xs \ acc = (\lor) < \$ > acc < * > timeIntersect \ xs
  mbAllJust :: [Maybe \ a] \rightarrow Maybe \ [a]
  mbAllJust \ l = inner \ l \ []
     where inner (Just x : xs) acc = inner xs (x : acc)
              inner []
                                   acc = Just \ acc
                                   _{-} = Nothing
              inner _
  timeIntersect :: [PartClass] \rightarrow Maybe Bool
  timeIntersect = fmap \ hasRepetitions \circ mbAllJust \circ map \ mbDayTime
  hasRepetitions (x:xs) = x \in xs \lor hasRepetitions xs
  hasRepetitions []
                          = False
Obligations:
  data Obligation (r :: Role) = Obligation {
     obligationName :: String
     , assessObligation :: RoleValue \ r \rightarrow PartClass \rightarrow Maybe \ Bool
```

```
professorCanTeach :: Obligation \ Professors \\ professorCanTeach = Obligation \ "Can teach" \\ \$ \lambda p \ c \rightarrow fmap \ (\in canTeach \ p) \ (mbDiscipline \ c) \\ roomSatisfies :: Obligation \ Classrooms \\ roomSatisfies = Obligation \ "Room \ Capacity \ and \ Special \ Requirements" \\ \$ \lambda r \ c \rightarrow \mathbf{do} \ gr \leftarrow mbGroup \ c \\ di \leftarrow mbDiscipline \ c \\ return \ \$ \ roomCapacity \ r \geqslant groupSize \ gr \\ \land \ all \ (\in roomEquipment \ r) \\ (disciplineReqs \ di)
```

3.2.2 Preferences

3.2.3 Assessment

```
then mean $ concatMap assess preferences
                       else 0
  where satisfies (ByRole \ r \ os) = \mathbf{case} \ r \ 'mbRole' \ pc \ \mathbf{of}
              Just \ rr \rightarrow \ all \ (from Maybe \ False
                                 \circ (\$pc) \circ (\$rr)
                                 \circ \ assessObligation
                                ) os
              Nothing \rightarrow True
           assess (ByRole \ r \ ps) = fromMaybe [] $
              do dt \leftarrow mbDayTime\ pc
                  di \leftarrow mbDiscipline \ pc
                  rv \leftarrow mbRole \ r \ pc
                  return \$ map (from UnitInterval
                                   \circ (\$(rv, di, dt))
                                   \circ uncurry3
                                   \circ assessPreference
                                  ) ps
\eta:: SomeObligations \rightarrow SomePreferences \rightarrow Route \rightarrow InUnitInterval
\eta obligations preferences route = from Just \circ in UnitInterval $
  if timeConsistent\ route \land notElem\ 0 assessed then mean assessed
                                                         else 0
  where assessed = from UnitInterval \circ assess Part obligations preferences
                      <$> routeParts route
 ACO
\mathbf{data} \; ParamsACO = ParamsACO \; \{ \alpha \; :: Float \;
                                         , \beta :: Float
                                          , \mathcal{Q} :: Float
                                         , Q_0 :: Float
                                          , \rho \quad :: Float
type RelationsACO = (SomeObligations, SomePreferences)
\mathbf{data} \ Population ACO = \forall gen. Random Gen \ gen \Rightarrow
       GenPopulation Int GenUnique (IORef gen)
type GenUnique = Bool
data ACO = ACO \{ setup \}
                                            :: ParamsACO
                        , relations ACO :: Relations ACO
                        , population ACO :: Population ACO
```

3.3

newtype Pheromone = Pheromone Float deriving (Show, Eq. Ord)

pheromone Quantity (Pheromone n) = n

```
mapPheromone\ f\ (Pheromone\ n) = Pheromone\ (f\ n)
mapPheromone\ f\ (Pheromone\ x)\ (Pheromone\ y) = Pheromone\ (f\ x\ y)
instance\ Num\ Pheromone\ where\ (+) = mapPheromone\ (f\ x\ y)
(-) = mapPheromone\ (-)
(*) = mapPheromone\ (*)
abs = mapPheromone\ abs
signum = mapPheromone\ signum
fromInteger = Pheromone\ \circ\ fromInteger
```

3.3.1 Graph

The **problem graph** is defined by the nodes of each *role*; while the edges hold the *pheromone*. If the memory permits it, the graph should hold all the permutations of *roles* domains.

```
type NodeSet \ r = Set \ (Node \ r)
type PheromoneBetween = Map (AnyNode, AnyNode) Pheromone
type Pheromone Cache = Map (AnyNode, AnyNode) (IORef Pheromone)
data \ Graph = Graph \{ \ groupsNodes \}
                                              :: NodeSet\ Groups
                          temporal Nodes
                                             :: NodeSet\ DayTime
                          professorsNodes :: NodeSet Professors
                          classroomsNodes::NodeSet\ Classrooms
                           pheromone Cache :: Pheromone Cache
currentPheromone :: Graph \rightarrow IO \ PheromoneBetween
currentPheromone = mapM \ readIORef \circ pheromone Cache
phKey(x, y) = (x 'min' y, x 'max' y)
  -- Lazy update
updPheromone :: Graph
                \rightarrow (AnyNode, AnyNode)
                \rightarrow (Pheromone \rightarrow Pheromone)
                \rightarrow IO ()
updPheromone \ q \ k \ upd =
  case phKey\ k 'Map.lookup' pheromoneCache\ g of
     Just ref
                    \rightarrow modifyIORef ref upd

ightarrow \mathit{error} \$ "no pheromone cache for " + \mathit{show} \ \mathit{k}
\mathbf{data}\ ExecACO = ExecACO\ \{\ exACO\ ::\ ACO\ \}
                               , exGraph :: Graph
                               , exRuns :: IORef Int
data AnyNode = \forall r. (Typeable \ r, RoleExtra \ r) \Rightarrow
      AnyNode (Role' r) (Node r)
```

```
nodeRoleIx (AnyNode r _)
                               = roleIx r
nodeId'
           (AnyNode \_ n)
                               = nodeId n
nodeId'' = nodeRoleIx \&\&\& nodeId'
anyNode n = AnyNode Role' n
instance Eq AnyNode where (\equiv)
                                         = (\equiv)
                                                     'on' nodeId''
instance Ord AnyNode where compare compare on nodeId"
instance Show AnyNode where
  show (AnyNode \ r \ n) = "Node-" + show \ r ++ ":" + nodeId \ n
routeNodes :: Route \rightarrow [AnyNode]
routeNodes \ r = mapMaybe \ (\$r) \ [packNode \circ mbRoomsNode]
                                 , packNode \circ mbProfessorsNode
                                 , packNode \circ mbDayTimeNode
                                 , packNode \circ mbGroupsNode
  where packNode :: (Typeable \ r, RoleExtra \ r) \Rightarrow
                      Maybe\ (Node\ r) \rightarrow Maybe\ AnyNode
          packNode = fmap (AnyNode Role')
```

3.3.2 Evaluation

Route probabilistic evaluation function:

```
ACO \rightarrow PheromoneBetween \rightarrow [Route]
\xi ::
                [(InUnitInterval, Route)]
\xi aco ph rs =
   first (from Just \circ in UnitInterval \circ (/psum))
       <$>
                    zip ps rs'
   where (rs', ps) = unzip \$ map p rs
             psum = sum ps
             p \ r = \mathbf{let} \ (r', \eta \prime) = assessRoute' \ r
                       in (r', \tau^{\alpha} \cdot \eta \prime^{\beta})
             assessRoute' \ r = \mathbf{let} \ v = uncurry \ \eta \ (relationsACO \ aco) \ r
                in \{r \in assessHistory = v : assessHistory r\}
                    , from \ Unit Interval \ v
             \tau = \mathbf{case} \ routeNodes \ r \ \mathbf{of}
                x: y: \_ \rightarrow maybe \ \mathcal{Q}_0 \ pheromone Quantity
                                phKey(x,y)'Map.lookup'ph
                           \rightarrow Q_0
```

Pheromone secretion for each neighboring nodes pair in a route:

$$\Delta \tau_r :: ACO \rightarrow Route \rightarrow [((AnyNode, AnyNode), Pheromone)]$$

 $\Delta \tau_r \ aco \ r =$

```
let edgs = lPairs \$ routeNodes \ r
hist = assessHistory \ r
w = \mathcal{Q} / sum \ (map \ from UnitInterval \ hist)
weight = Pheromone \circ (*w) \circ from UnitInterval
in if length \ edgs \not\equiv length \ hist
then error "[BUG] wrong assess history length"
else edgs \ 'zip' \ map \ weight \ hist
lPairs \ (x0:x1:xs) = (x0,x1): lPairs \ (x1:xs)
lPairs \ \_
= []
```

Pheromone update (secretion and vaporization):

```
\widetilde{\Delta \tau} :: ExecACO \rightarrow [Route] \rightarrow IO \ ()
\widetilde{\Delta \tau} \ ExecACO \ \{ exACO = aco, exGraph = graph \} \ rs = forM_rs \ update
\gg vaporize
\mathbf{where} \ update \ r = sequence_\$ \ \mathbf{do} \ (i, ph) \leftarrow \Delta \tau_r \ aco \ r
[updPheromone \ graph \ i \ (+ \ ph \cdot \rho)]
vaporize = sequence_\$ \ \mathbf{do} \ ref \leftarrow Map.elems \ \$
pheromoneCache \ graph
[modifyIORef' \ ref \ (\cdot(1-\rho))] \ -- \text{ strict}
```

3.3.3 Execution

```
\label{eq:continuous} \begin{split} \textbf{type} \ StopCriteria &= ExecACO \rightarrow IO \ Bool \\ execACO :: ExecACO \rightarrow StopCriteria \rightarrow IO \ [Route] \\ execACO \ ex@ExecACO \ \{ \ exACO = aco, \ exGraph = graph \} \ stop = result \\ \textbf{where} \end{split}
```

1. Generate initial population by selectting randomly some nodes at *Group-Discipline* layer:

```
initial Population = \mathbf{case} \ population ACO \ aco \ \mathbf{of} \ Gen Population \ size \ unique \ gen Ref \ \rightarrow \mathbf{do} \ gen \leftarrow read IORef \ gen Ref \ 
\mathbf{let} \ rand' = \mathbf{if} \ unique \ \mathbf{then} \ rand Chooses Unique \ 
\mathbf{else} \ rand Chooses Unique \ 
\mathbf{else}
```

2. \forall layer, route **do**:

- (a) Generate all the possible *route* continuations by updating the *route* with *layer*'s nodes.
- (b) Assess continuations with ξ and select the node according to the assessed probabilities.

```
nextRoute :: (UpdRoute \ r, RandomGen \ gen) \Rightarrow
                  PheromoneBetween \rightarrow NodeSet \ r \rightarrow gen
                Route \rightarrow (gen, Route)
nextRoute \ ph \ nset \ gen \ r =
  let candidates = (`updRoute`r) < \$ > Set.toList nset
       evCandidates = \xi \ aco \ ph \ candidates
  in second snd $ randCoiceWithProb gen fst evCandidates
nextRoutes \_acc \_g[] = (g, acc)
nextRoutes\ ph\ acc\ nset\ gen\ (r:rs) =
  let (g', next) = nextRoute \ ph \ nset \ gen \ r
  in nextRoutes ph (next : acc) nset g' rs
routesIO = \mathbf{do} \ ph \leftarrow currentPheromone \ graph
                  g\theta \leftarrow getStdGen
                  p\theta \leftarrow initial Population
                  let next :: (RandomGen gen, UpdRoute r) \Rightarrow
                                (Graph \rightarrow NodeSet \ r) \rightarrow gen
                            \rightarrow [Route] \rightarrow (gen, [Route])
                                = nextRoutes ph [] \circ (\$graph)
                      (g1, p1) = next temporalNodes g0 p0
                      (g2, p2) = next \ professorsNodes \ g1 \ p1
                      (g3, p3) = next \ classroomsNodes \ g2 \ p3
                  setStdGen q3
                  return p3
```

3. Update pheromone and counter:

$$updateStates \ rs = \mathbf{do} \ \widetilde{\Delta \tau} \ ex \ rs$$
 $exRuns \ ex \ `modifyIORef' \ (+1)$

4. Return best routes if stop criteria applies, go to 1 otherwise.

$$result = \mathbf{do} \ routes \leftarrow routesIO$$
 $updateStates \ routes$
 $stop' \leftarrow stop \ ex$
 $\mathbf{if} \ stop' \ \mathbf{then} \ return \ routes$
 $\mathbf{else} \ execACO \ ex \ stop$

```
randChoice \ gen \ xs =
  if null xs then error "randChoice: empty list"
             else first (xs!!) $ randomR (0, length xs - 1) gen
randChoices\ gen\ count\ xs =
  if length xs < count
  then randChoices gen (length xs) xs
  else swap \$ foldr \ rand \ ([], gen) \ [1 \dots count]
        where rand = (acc, q) = first (:acc) \$ randChoice q xs
randUniqueIndices\ gen\ count\ length =
  let ixSet = Set.fromList [1..length]
  in if count > length then error "randUniqueIndices: count > length"
                         else inner gen ixSet count []
  where inner\ g = 0\ acc = (g, acc)
          inner g \ s \ c \ acc = \mathbf{let} \ (i, g') = randomR \ (0, Set.size \ s - 1) \ g
                                          = Set.elemAt\ i\ s
                                   s' = Set.delete \ v \ s
                               in inner q' s' (c-1) (v:acc)
randChoosesUnique\ gen\ count\ xs =
  if length \ xs < count
    then randChoosesUnique gen (length xs) xs
    else second\ (map\ (xs!!))\ \$\ randUniqueIndices\ gen\ count\ (length\ xs)
randCoiceWithProb :: (RandomGen gen) \Rightarrow
                        gen \rightarrow (a \rightarrow InUnitInterval) \rightarrow [a] \rightarrow (gen, a)
randCoiceWithProb\ gen\ probOf\ xs =
  let (r, g') = random \ gen
      accumulating \ acc \ \_[] = acc
      accumulating acc f(x:xs) = \mathbf{case} f acc x \mathbf{of}
         \textit{Just acc'} \rightarrow \textit{accumulating acc'} \ \textit{f xs}
      f(p, \_) x = \mathbf{let} \ p' = p + fromUnitInterval \ (probOf \ x)
                    in if p' > r then Nothing
                                   else Just (p', x)
  in (g', snd \$ accumulating (0, head xs) f (tail xs))
```

3.3.4 Creation

Here follows creation of an 'ExecACO' instance.

```
newExecACO :: (RoleDomain\ Groups \ , RoleDomain\ DayTime \ , RoleDomain\ Professors \ , RoleDomain\ Classrooms \ ) <math>\Rightarrow ACO \rightarrow IO\ ExecACO
```

```
newExecACO \ aco = \mathbf{do}
  let gs = mkNodes "G" (Role' :: Role' Groups)
      ts = mkNodes "T" (Role' :: Role' DayTime)
      ps = mkNodes "P" (Role' :: Role' \ Professors)
      rs = mkNodes "R" (Role' :: Role' Classrooms)
      ks = concat
                       [ pairs gs ts
                        , pairs ts ps
                        , pairs\ ps\ rs
      pairs xs \ ys = \mathbf{do} \ x \leftarrow xs
                          y \leftarrow ys
                          return \$ (anyNode x, anyNode y)
  cache \leftarrow sequence \$ \ \mathbf{do} \ k \leftarrow ks
                            [(,) k < \$ > newIORef (Pheromone Q_0)]
  let graph = Graph (Set.fromList gs)
                        (Set.fromList ts)
                        (Set.fromList ps)
                        (Set.fromList rs)
                        (Map.fromList\ cache)
  countRef \leftarrow newIORef~0
  return \ \$ \ ExecACO \ aco \ graph \ countRef
```