Abstract

This article proposes a system for generating possible *University Classes Schedules*. It uses multi-agent negotiation to find satisfactory solutions to the problem, while trying to consider *personal preferences* of the represented people and institutions.

1 Implementation

1.1 University Classes

A class is an en event, that brings together a *group of students*, and a *professor* in certain *classroom* in order to learn/teach the specified *discipline*. It happens periodically, usually weekly, at the established *day of week* and *time*.

A *discipline* should describe an atomic (not dividable) educational activity. For example, if the students are required to take a normal class and also do some specific laboratory practice, then two disciplines should be created, one of them describing the required lab equipment.

```
data Discipline = Discipline { disciplineId               :: String
                             , disciplineMinutesPerWeek   :: Int
                             , disciplineRequirements     :: Set Requirement
                             }
                deriving (Typeable, Show, Eq, Ord)
newtype Requirement = Requirement String deriving (Show, Eq, Ord)
```

For inner usage, the classes are divided into

- *abstract* — without day and time;
- *concrete* — with full time information.

```
class (Ord c, Show c, Typeable c) ⇒
   AbstractClass c where classDiscipline :: c → Discipline
                         classGroup      :: c → GroupRef
                         classProfessor  :: c → ProfessorRef
                         classRoom       :: c → ClassroomRef
                         classNumber     :: c → Word
class (AbstractClass c, DiscreteTime time) ⇒
  ConcreteClass c time | c → time
    where classDay    :: c → Day
          classBegins :: c → time
          classEnds   :: c → time
data Class     = ∀c time.ConcreteClass c time ⇒ Class c
data SomeClass = ∀c     .AbstractClass c      ⇒ SomeClass c
```

The “System.Time.Day” is redefined, dropping the “Sunday”.

$$
\begin{aligned}
&\textbf{data } \mathit{Day} = \mathit{Monday} \mid \mathit{Tuesday} \mid \mathit{Wednesday}\\
&\qquad\qquad \mid \mathit{Thursday} \mid \mathit{Friday} \mid \mathit{Saturday}\\
&\quad \textbf{deriving } (\mathit{Eq}, \mathit{Ord}, \mathit{Enum}, \mathit{Bounded}, \mathit{Ix}, \mathit{Read}, \mathit{Show})
\end{aligned}
$$

The classes are negotiated by the interested parties: 1) students / groups, 2) professors, 3) classrooms. Each negotiation participant has a *timetable*, holding a schedule for one week, that repeats throughout the academic period. The *timetable* is actually a table: the columns represent days of week; the rows — discrete time intervals. Actual timetable structure may vary, as can be seen in figure 1.

	Mon	Tue	Wed	Thu	Fri	Sat
08:30 – 09:00						
09:00 – 09:30						
09:30 – 10:00						
10:00 – 10:30						
10:30 – 11:00						
11:00 – 11:30						
11:30 – 12:00						
⋮ ⋮						

(a) Timetable without recesses.

	Mon	Tue	Wed	Thu	Fri	Sat
08:30 – 09:10						
09:15 – 09:55						
10:05 – 10:45						
10:50 – 11:30						
11:40 – 12:20						
12:25 – 13:05						
13:15 – 13:55						
⋮ ⋮						

(b) Timetable with recesses.

Figure 1: Possible *timetable* structures.

$$
\begin{aligned}
&\textbf{class } (\mathit{Ord}\ t, \mathit{Bounded}\ t, \mathit{Show}\ t, \mathit{Typeable}\ t) \Rightarrow \mathit{DiscreteTime}\ t \textbf{ where}\\
&\quad \mathit{toMinutes} \quad :: t \to \mathit{Int}\\
&\quad \mathit{fromMinutes} :: \mathit{Int} \to t
\end{aligned}
$$

$\textbf{data } \mathit{SomeTime} = \forall t.(\mathit{DiscreteTime}\ t) \Rightarrow \mathit{SomeTime}\ t$

$\mathit{someTimeMinutes}\ (\mathit{SomeTime}\ t) = \mathit{toMinutes}\ t$

$$
\begin{aligned}
&\textbf{instance } \mathit{Eq}\ \ \mathit{SomeTime} \textbf{ where } (\equiv) \qquad = (\equiv) \qquad \text{`}\mathit{on}\text{`}\ \mathit{someTimeMinutes}\\
&\textbf{instance } \mathit{Ord}\ \mathit{SomeTime} \textbf{ where } \mathit{compare} = \mathit{compare}\ \text{`}\mathit{on}\text{`}\ \mathit{someTimeMinutes}
\end{aligned}
$$

$\textbf{class } (\mathit{DiscreteTime}\ \mathit{time}) \Rightarrow \mathit{Timetable}\ \mathit{tt}\ e\ \mathit{time} \mid \mathit{tt} \to \mathit{time}$

$$
\begin{aligned}
& && , \; tt \rightarrow e \\
& && , \; e \rightarrow time \\
\textbf{where} \; & listEvents :: tt \rightarrow [e] \\
& eventsOn :: tt \rightarrow Day \rightarrow [e] \\
& eventsAt :: tt \rightarrow time \rightarrow [(Day, e)] \\
& eventAt :: tt \rightarrow Day \rightarrow time \rightarrow Maybe\ e
\end{aligned}
$$

One should distinguish the resulting timetables, shown in figure 1 and the timetable, held an agent during the negotiation. The first one is immutable and is the result of agent's participation in the negotiation. The set of such timetables, produced by every the participant, is the **university schedule** for given academic period.

During the negotiation, an agent's inner timetable gets changed on the fly, in order to record agreements made. This means that we are dealing with *side effects*, that need to be explicitly denoted in Haskell. The following definition leaves it free to choose the monad abstraction for those effects.

$$
\begin{aligned}
& \textbf{class}\ (DiscreteTime\ time, Monad\ m) \Rightarrow \\
& \quad TimetableM\ tt\ m\ e\ time \mid tt \rightarrow time \\
& \qquad\qquad\qquad\qquad , \; tt \rightarrow e \\
& \qquad\qquad\qquad\qquad , \; e \rightarrow time \\
& \quad \textbf{where}\ putEvent :: tt \rightarrow e \rightarrow m\ tt \\
& \qquad\quad delEvent :: tt \rightarrow e \rightarrow m\ tt \\
& \qquad\quad ttSnapshot :: (Timetable\ ts\ x\ time) \Rightarrow tt \rightarrow m\ ts
\end{aligned}
$$

1.2 Negotiating Agents

As it was mentioned before, the schedule is formed in a negotiation between *professors*, *groups* and *classrooms*. To distinguish those three types of participants, agent's role is introduced. The role: 1) identifies the kind of person/entity, represented by the agent; 2) defines agent's reaction on the messages received; 3) defines agent's goal.

A *representing agent* is a computational entity, that represents a *real person or object* in it's virtual environment. In current case, it represents one's interests in a *negotiation*. Such an agent must

(1) pursue the *common goal* — it must consider the common benefits, while being egoistic enough to achieve it's own goal;

(2) respond to the messages received in correspondence with (1);

(3) initiate conversations (send messages, that are not responses), driven by (1);

(4) become more susceptible (less egoistic) with passage of time.

$$
\begin{aligned}
&\textbf{data}\ \mathit{NegotiationRole} = \mathit{GroupRole}\\
&\qquad\qquad\qquad\qquad\quad \mid \mathit{FullTimeProfRole}\\
&\qquad\qquad\qquad\qquad\quad \mid \mathit{PartTimeProfRole}\\
&\qquad\qquad\qquad\qquad\quad \mid \mathit{ClassroomRole}\\
&\quad \textbf{deriving}\ (\mathit{Show}, \mathit{Typeable})
\end{aligned}
$$

1.2.1 Common Goal

Agent's own *goal* represents its egoistical interests. They may (and will) contradict another agent's interests, thus creating *incoherence*. The general rule in this case is to strive for solutions, benefiting the whole schedule. Because the schedule doesn't yet exist as a whole during the negotiation, an agent should consider instead the benefits, obtained by itself and the rest of the agents.

The *common goal* is incorporated in the *contexts* mechanism, and is discussed in Section 1.3.7.

1.2.2 Messaging

Is this section really needed?

1.3 Coherence

The coherence mechanism is based on [**?**]. It uses the *contexts* as means of separating (and further prioritizing) different *cognitive aspects*. The contexts used are based on *BDI* agent architecture.

The *combined coherence* is used as a measure of goal achievement. It's combined of coherence values, calculated by agent's contexts.

1.3.1 Information and Relations

The coherence is calculated over an *information graph*, that represents some aspect of agent's knowledge. The nodes of the graph are some *pieces of information* and the edges represent some *relations* between theese pieces.

$$
\begin{aligned}
&\textbf{newtype}\ \mathit{IGraph} = \mathit{IGraph}\ (\mathit{Set\ Information})\\
&\mathit{graphNodes} :: \mathit{IGraph} \to [\mathit{Information}]\\
&\mathit{graphNodes}\ (\mathit{IGraph\ inf}) = \mathit{Set.toList\ inf}\\
&\mathit{graphJoin} :: \mathit{IGraph} \to [\mathit{Information}] \to \mathit{IGraph}\\
&\mathit{graphJoin}\ (\mathit{IGraph\ inf})\ \mathit{new} = \mathit{IGraph}\ (\mathit{inf} \cup \mathit{Set.fromList\ new})\\
&\mathit{fromNodes} :: [\mathit{Information}] \to \mathit{IGraph}\\
&\mathit{fromNodes} = \mathit{IGraph} \circ \mathit{Set.fromList}\\
&\mathit{relationOn} :: \mathit{IRelation\ a} \to \mathit{IGraph} \to \mathit{RelValue\ a}\\
&\mathit{relationOn\ rel}\ (\mathit{IGraph\ inf}) = \bot \quad \text{-- TODO}
\end{aligned}
$$

The proposed system makes use of the following information:

1. **Personal knowledge**, known only by one actor.

   (a) **Capabilites**: information about what an agent can do, what kind of arrangments it can make.

   (b) **Obligations**: information about *strong restrictions*, imposed over the agent.

   (c) **Preferences**: information about *weak restrictions*.

2. **Shared knowledge**, obtained in the negotiation.

   (a) **Others' capabilities** — information about the counterpart agents, that are known to be (un-) capable of doing something.

   (b) **Classes proposals**:

      i. **Abstract** — has no specific time assigned.
         Concrete — has a specific time defined.

      ii. **Complete** — references all three representing agents: a *group*, a *professor* and a *classroom*.
         Partial — references less then three representing agents.

   (c) **Classes decisions**:

      i. **Class acceptance** — a mark for *accepted classes proposals*. Only *complete* proposals can be accepted; all the three mentioned agents must accept it, or none.

      ii. **Class rejection** — a mark for *ignored classes proposals*, a result of *yield* decision, discussed in Section **??**.

```
data InformationScope = Personal | Shared
  -- "Ord" instance is mainly needed to create "Set"s.
class (Typeable i, Eq i, Ord i) ⇒ InformationPiece i
    where type IScope i :: InformationScope
class (InformationPiece i, Personal∼IScope i) ⇒ PersonalInformation i
class (InformationPiece i, Shared∼IScope i)   ⇒ SharedInformation i
    where sharedBetween :: i → Set AgentRef
  -- ——————————————————
instance Eq  SomeClass where
    (SomeClass a) ≡ (SomeClass b) = cast a ≡ Just b
        -- TODO
instance Ord SomeClass
instance InformationPiece SomeClass where
    type IScope SomeClass = Shared
instance SharedInformation SomeClass
  -- ——————————————————
instance Eq  Class
```

```
instance Ord Class
instance InformationPiece Class where type IScope Class = Shared
instance SharedInformation Class
  -- ----------------------------------------
data Information = ∀i.InformationPiece i ⇒ Information i
collectInf :: (Typeable a) ⇒ Information → Maybe a
collectInf (Information i) = cast i
instance Eq Information where
  (Information i1) ≡ (Information i2) =
    case cast i1 of Just x    → x ≡ i2
                    _         → False
instance Ord Information where
  (Information i1) ⋚ (Information i2) = ⊥
  -- ----------------------------------------
newtype Needs = Needs (Set Discipline)
    deriving (Eq, Ord, Show, Typeable)
newtype CanTeach = CanTeach (Set Discipline)
    deriving (Eq, Ord, Show, Typeable)
instance InformationPiece Needs
instance InformationPiece CanTeach
```

The *binary relations* connect some information pieces, assigning to the edge some value. The *whole graph relations*, on the other side, are applied to the graph as a whole and produce a single value.

The relations used, as well as the information in the graph, depend on the *context*.

```
data RelValBetween a = RelValBetween {
    relBetween    :: (Information, Information)
  , relValBetween :: a
  }
type RelValsBetween a = Map (IRelation a) [RelValBetween a]
newtype RelValWhole a = RelValWhole a
unwrapRelValWhole (RelValWhole a) = a
type RelValsWhole a = Map (IRelation a) (RelValWhole a)
  -- ----------------------------------------
class InformationRelation r where relationName :: r a → String
class InformationRelation r ⇒
  BinaryRelation r where
    binRelValue :: (Num a) ⇒ r a → Information → Information → Maybe a
class InformationRelation r ⇒
  WholeRelation r where
```

$$wholeRelValue :: r\ a \rightarrow IGraph \rightarrow a$$

-- ―――――――――――――――――――――――――――

$$\textbf{data}\ IRelation\ a = \forall r.BinaryRelation\ r \Rightarrow RelBin\ (r\ a)$$
$$\quad\quad\quad\quad\quad\quad\quad \mid \forall r.WholeRelation\ r \Rightarrow RelWhole\ (r\ a)$$

$$relName\ (RelBin\ a) \quad = relationName\ a$$
$$relName\ (RelWhole\ a) = relationName\ a$$

$$\textbf{instance}\ Eq\ (IRelation\ a)\ \textbf{where}\ (\equiv) = (\equiv)\ `on`\ relName$$

$$\textbf{instance}\ Ord\ (IRelation\ a)\ \textbf{where}\ compare = compare\ `on`\ relName$$

$$\textbf{type}\ RelValue\ a = Either\ [RelValBetween\ a]\ (RelValWhole\ a)$$

1.3.2 Contexts

In order to use contexts for information *coherence assessment*, the concepts of *context-specific information graph* and *assessed information* are introduced. The context-specific graph holds the information, already known/accepted by the agent, and is relevant for the context in question. The assessed one is *assumed* during the evaluation process.

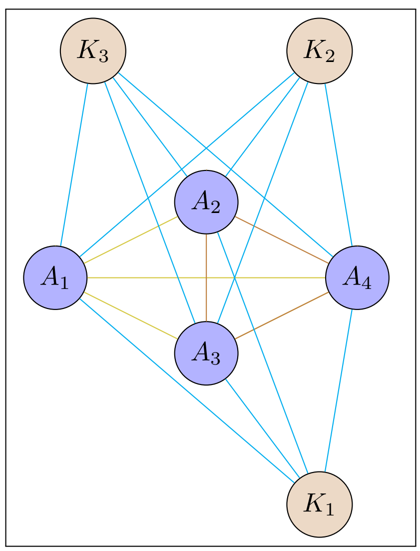

Figure 2: Binary relations within an information graph. One can distinguish the relations between the assessed information pieces and the relations between assessed and the known ones.

To assess some information, it's propagated through the contexts, in the *specified order*, that stands for contexts priority. Each context should have a *coherence threshold* specified; after the assessed information's coherence has been estimated, it's compared against the threshold and either `Success` or `Failure` is returned, along with the evaluated coherence value. The information, that has successfully passed a context, is propagated further; otherwise the failure is returned.

```
class Context (c :: * → *) a where
   contextName         :: c a → String
   contextInformation  :: c a → IO IGraph
   contextRelations    :: c a → IO [IRelation a]
   contextThreshold    :: c a → IO a
   combineBinRels      :: c a → RelValsBetween a   → Maybe (CBin a)
   combineWholeRels    :: c a → RelValsWhole a     → Maybe (CWhole a)
   combineRels         :: c a → CBin a → CWhole a → a
newtype CBin a   = CBin a
newtype CWhole a = CWhole a
data AssessmentDetails a  -- TODO
data SomeContext a = ∀c.Context c a ⇒ SomeContext (c a)
   -- ────────────────────────────────
type AnyFunc₁ res = ∀a.a → res a
mapEither :: AnyFunc₁ r → Either a b → Either (r a) (r b)
mapEither f (Left a)  = Left $ f a
mapEither f (Right a) = Right $ f a
assessWithin' ::      (Context c a) ⇒
                      [Information]
   →                  c a
   →                  IO (Maybe a, AssessmentDetails a)
assessWithin' inf c = do
   contextInf  ← contextInformation c
   contextRels ← contextRelations c
   let assumed = contextInf `graphJoin` inf
       (bins, whole)    = partitionEithers
                          $ (λr → mapEither ((,) r) $ r `relationOn` assumed)
                          <$> contextRels
       assessed = do  rBin    ← c `combineBinRels`   Map.fromList bins
                      rWhole ← c `combineWholeRels` Map.fromList whole
                      return  $ combineRels c rBin rWhole
   return (assessed, ⊥)
   -- ────────────────────────────────
data AssessedCandidate a = AssessedCandidate {
       assessedAt       :: SomeContext a
   ,   assessedVal      :: Maybe a
   ,   assessedDelails  :: AssessmentDetails a
   }
data Candidate a = Success {assessHistory :: [AssessedCandidate a]
```

$$
\begin{array}{lll}
 & & , \mathit{candidate} :: [\mathit{Information}] \\
 & & \} \\
 | & \mathit{Failure} & \{ \mathit{assessHistory} :: [\mathit{AssessedCandidate}\ a] \\
 & & , \mathit{candidate} :: [\mathit{Information}] \\
 & & \}
\end{array}
$$

-- ——————————————————

$$
\begin{array}{l}
\mathit{assessWithin} :: (\mathit{Context}\ c\ a, \mathit{Ord}\ a) \Rightarrow \\
\qquad \mathit{Candidate}\ a \to c\ a \to \mathit{IO}\ (\mathit{Candidate}\ a) \\
\mathit{assessWithin}\ f@\mathit{Failure}\ \{\ \}\ \_ = \mathit{return}\ f \\
\mathit{assessWithin}\ (\mathit{Success}\ \mathit{hist}\ c)\ \mathit{cxt} = \mathbf{do} \\
\quad (\mathit{mbA}, \mathit{details}) \quad \leftarrow c\ `\mathit{assessWithin}'`\ \mathit{cxt} \\
\quad \mathit{threshold} \quad \leftarrow \mathit{contextThreshold}\ \mathit{cxt} \\
\quad \mathbf{let}\ \mathit{ac} = \mathit{AssessedCandidate}\ (\mathit{SomeContext}\ \mathit{cxt})\ \mathit{mbA}\ \mathit{details} \\
\quad \mathit{return}\ \$\ \mathbf{if}\ \mathit{mbA} > \mathit{Just}\ \mathit{threshold} \\
\qquad\qquad \mathbf{then} \quad \mathit{Success}\ (\mathit{ac} : \mathit{hist})\ c \\
\qquad\qquad \mathbf{else} \quad \mathit{Failure}\ (\mathit{ac} : \mathit{hist})\ c
\end{array}
$$

Some contexts might also be capable of *splitting* information graphs into *valid candidates* – the sub-graphs, that are *valid* at the context. The candidates can be assessed by the rest of the contexts.

$$
\begin{array}{l}
\mathbf{class}\ (\mathit{Context}\ c\ a) \Rightarrow \mathit{SplittingContext}\ c\ a\ \mathbf{where} \\
\quad \mathit{splitGraph} :: c\ a \to \mathit{IGraph} \to \mathit{IO}\ [\mathit{Candidate}\ a]
\end{array}
$$

1.3.3 Capabilities

The capabilities context handles question "Am I able to do it?". It's main purpose is to discard immediately any proposal that would never be accepted.

- *Group*: "Am I interested in the discipline?"
- *Professor*: "Am I qualified to teach the disciple?"
- *Classroom*: "Do I suit the disciple?", "Do I have the capacity required?"

An agent should mark any other agent, that has declined some proposal for *capabilities* reasons, describing the reason. It should further avoid making same kind of proposals to the uncapable agent.

$$
\begin{array}{l}
\mathbf{data\ family}\ \mathit{Capabilities}\ (r :: \mathit{NegotiationRole}) :: * \to * \\
\mathbf{data\ instance}\ \mathit{Capabilities}\ \mathit{GroupRole}\ a = \mathit{GroupCapabilities}\ \{ \\
\quad \mathit{needsDisciplines} :: [\mathit{Discipline}] \\
\quad \} \\
\mathbf{data\ instance}\ \mathit{Capabilities}\ \mathit{FullTimeProfRole}\ a = \mathit{FullTimeProfCapabilities}\ \{ \\
\quad \mathit{canTeachFullTime} :: [\mathit{Discipline}]
\end{array}
$$

```
  }
  -- ------------------------------------
data CanTeachRel a = CanTeachRel
instance InformationRelation CanTeachRel where
  relationName _ = "CanTeach"
instance BinaryRelation CanTeachRel where
  binRelValue _ a b =
    let v ds c = if classDiscipline c ∈ ds then 1 else 0
    in case collectInf a of
      Just (CanTeach ds) → let
        r1 = case collectInf b of Just (SomeClass c) → Just $ v ds c
        r2 = case collectInf b of Just (Class c)     → Just $ v ds c
        in r1 <|> r2
      _                    → Nothing
  -- ------------------------------------
data NeedsDisciplineRel a = NeedsDisciplineRel
instance InformationRelation NeedsDisciplineRel where   -- TODO
instance BinaryRelation NeedsDisciplineRel where        -- TODO
  -- ------------------------------------
    -- Every capability must be coherent. 0*X = 0
combineBinRelsStrict _ bRels | null bRels = Nothing
combineBinRelsStrict _ bRels = Just ∘ CBin ∘ product
                               ∘ concatMap (map relValBetween)
                               $ Map.elems bRels
combineWholeRelsStrict _ wRels | null wRels = Nothing
combineWholeRelsStrict _ wRels = Just ∘ CWhole ∘ product
                                 ∘ map unwrapRelValWhole
                                 $ Map.elems wRels
combineRelsStrict _ (CBin b) (CWhole w) = b * w
  -- ------------------------------------
instance (Num a) ⇒ Context (Capabilities GroupRole) a where
  contextName _        = "Capabilities"
  contextInformation = return ∘ fromNodes ∘ (:[])
                       ∘ Information ∘ Needs
                       ∘ Set.fromList ∘ needsDisciplines
  contextRelations _  = return [RelBin NeedsDisciplineRel]
  contextThreshold _  = return 0
  combineWholeRels  = combineWholeRelsStrict
  combineBinRels    = combineBinRelsStrict
  combineRels       = combineRelsStrict
instance (Num a) ⇒ Context (Capabilities FullTimeProfRole) a where
  contextName _        = "Capabilities"
```

$$
\begin{aligned}
\mathit{contextInformation} &= \mathit{return} \circ \mathit{fromNodes} \circ (:[\,]) \\
&\circ \mathit{Information} \circ \mathit{CanTeach} \\
&\circ \mathit{Set.fromList} \circ \mathit{canTeachFullTime} \\
\mathit{contextRelations}\ \_ &= \mathit{return}\ [\mathit{RelBin}\ \mathit{CanTeachRel}] \\
\mathit{contextThreshold}\ \_ &= \mathit{return}\ 0
\end{aligned}
$$

$$
\begin{aligned}
\mathit{combineWholeRels} &= \mathit{combineWholeRelsStrict} \\
\mathit{combineBinRels} &= \mathit{combineBinRelsStrict} \\
\mathit{combineRels} &= \mathit{combineRelsStrict}
\end{aligned}
$$

1.3.4 Beliefs

The beliefs is a *splitting* context, that uses as it's internal knowledge: 1) *state of the timetable*, that represents *best* candidate, generated until now; 2) *interesting* proposals, both generated by agent itself and received from the others, that are preserved throughout agent's lifetime.

Assessing yields one of three values

$$
\begin{cases}
-1 & \text{if two proposals intersect in time} \\
0 & \text{if both proposals have the same } \textit{abstract} \text{ part} \\
1 & \text{otherwise}
\end{cases}
$$

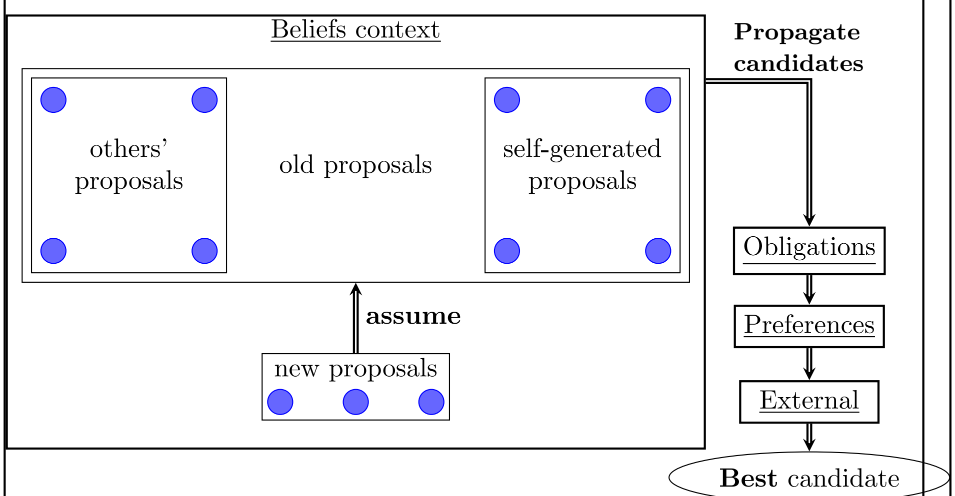

Figure 3: Assessing proposal coherence, starting from *Beliefs* context.

The assessment of *concrete proposals* (containing concrete classes) in the graph consists in

1. *assuming* the proposal information;
2. *splitting* the assumed information graph into valid candidates;
3. *propagating* of the candidates through the rest of the contexts;
4. comparing the *best candidate* with the previous *best*.

The proposal is called *interesting* and is accepted (and the assumed graph becomes the new information graph of *beliefs* context) if it's assumption causes better candidate generation. It's rejected otherwise (and the assumed graph is discarded).

Splitting is a process of extraction of *acceptable* sub-graphs, that compares the coherence values at graph's edges against a threshold. The splitting can be achieved with one of two following strategies:

1. *Joining* proposals while validness is preserved.
2. *Partitioning* of proposals until validness is achieved.

First strategy is used in this project, due to less memory consumption (it doesn't have to generate or store big invalid graphs, that would be present at the first steps of the second strategy).

The splitting is implemented as follows:

Let $C = \{c\}$ be a set of *class proposals*.
$A_i = \{a_i\}$ be a set of *acceptable candidates*, composed of i proposals.
$A = \bigcup_i A_i$ be a set of *acceptable candidates*.

1. Each single candidate is acceptable: $A_1 = \{[c] \mid \forall\ c \in C\}$.
2. Form A_2 by extending each candidate $[c'] = a_1 \in A_1$ with $c \in C$, if and only if c' and c do not intersect. If $A_1 \neq \emptyset$, then try to form A_2.

$\vdots$

i. Form A_i by extending each candidate $[c'_1, \ldots, c'_{i-1}] = a_{i-1} \in A_{i-1}$ with $c \in C$, if and only if $\forall c' \in a_{i-1},\ c'$ and c do not intersect. If $A_i \neq \emptyset$, then try to form A_{i+1}.

$\vdots$

n. $A_n = \emptyset \implies$ all the *acceptable candidates* were generated. Done.

```haskell
data Beliefs a = Beliefs { knownProposals :: IORef IGraph
                         , bestCandidate  :: IORef (Candidate a, a)
                         }

data TimeConsistency a = TimeConsistency

instance InformationRelation TimeConsistency where
  relationName _ = "TimeConsistency"

instance BinaryRelation TimeConsistency where
  binRelValue _ i1 i2 = do
    Class c1 <- collectInf i1
    Class c2 <- collectInf i2

    let sameParticipant = classGroup c1     == classGroup c2
                       || classProfessor c1 == classProfessor c2
                       || classRoom c1      == classRoom c2
        sameDay = classDay c1 == classDay c2
        timeIntersects x y  = classBegins x <= classBegins y
                           && classEnds x   >= classBegins y

        intersect = sameParticipant
                 && sameDay
                 && (timeIntersects c1 c2 || timeIntersects c2 c1)

    return $ if intersect then 0 else 1

instance (Num a) => Context Beliefs a where
  contextName _       = "Beliefs"
  contextInformation = readIORef . knownProposals
  contextRelations _  = return [RelBin TimeConsistency]
  contextThreshold _  = return 0

  combineWholeRels  = combineWholeRelsStrict
  combineBinRels    = combineBinRelsStrict
  combineRels       = combineRelsStrict

instance (Num a) => SplittingContext Beliefs a where
  splitGraph b gr = do
    iGraph <- readIORef $ knownProposals b
    let cNodes = catMaybes $ collectInf <$> graphNodes gr
        consistent x y = binRelValue TimeConsistency x y == Just 1
        extendCandidate Failure { } = []
        extendCandidate Success { candidate = inf } = do
          c <- cNodes
          [Success { assessHistory = [], candidate = graphNodes gr ++ [c]}
            | all (consistent c) inf
           ]

        a1 = Success [] . (:[]) <$> cNodes

    return $ fix (\f (acc, last) -> let ext = concatMap extendCandidate last
                                    in if null ext
                                       then acc
```

$$\textbf{else}\ f\ (acc + ext, ext)$$
$$)\ (a1, a1)$$

1.3.5 Obligations

Obligations determine the rest *strong restrictions* over the classes. Possible obligations might depend on agent's role and are usually determined by the institution. For example: maximum classes per day, lunch recess, lower/upper class time limit, two classes must/cannot follow etc.

1.3.6 Preferences

Preferences determine *weak restrictions*, that are intended to be set by the represented person (the institution in case of the classroom).

The context should disminus its influence over time to avoid possible over-restrictions due to conflicting personal interests.

1.3.7 External

External contexts take into account the *opinions* of the agents that are referenced by the solution candidate. It is responsible for *common goal* assessment. The assessment must be *objective* — it must give no preference to agent's own interests.

1.3.8 Decision

1.4 Agent

Here follows *agents* implementation.

$$\textbf{class}\ \mathit{AgentComm}\ \mathit{ag}\ \textbf{where}$$
$$\textbf{class}\ (\mathit{AgentComm}\ \mathit{ag}) \Rightarrow \mathit{CommAgentRef}\ \mathit{ref}\ \mathit{ag}\ \textbf{where}$$
$$\begin{aligned} &\mathit{agRef} :: \mathit{ag} \to \mathit{ref}\ \mathit{ag} \\ &\mathit{agComm} :: \mathit{ref}\ \mathit{ag} \to \mathit{ag} \end{aligned}$$
$$\textbf{data}\ \mathit{AgentRef} = \forall \mathit{ref}\ \mathit{ag}. \mathit{CommAgentRef}\ \mathit{ref}\ \mathit{ag} \Rightarrow \mathit{AgentRef}\ (\mathit{ref}\ \mathit{ag})$$

-- ————————————————

$$\begin{aligned} &\textbf{data}\ \mathit{GroupRef} &&= \mathit{GroupRef}\ \mathit{String}\ \textbf{deriving}\ (\mathit{Show}, \mathit{Eq}, \mathit{Ord}) \\ &\textbf{data}\ \mathit{ProfessorRef} &&= \mathit{ProfessorRef}\ \mathit{String}\ \textbf{deriving}\ (\mathit{Show}, \mathit{Eq}, \mathit{Ord}) \\ &\textbf{data}\ \mathit{ClassroomRef} &&= \mathit{ClassroomRef}\ \mathit{String}\ \textbf{deriving}\ (\mathit{Show}, \mathit{Eq}, \mathit{Ord}) \end{aligned}$$