

---

# **Deep Energy-Based Generative Modeling and Learning**

---

**Yifei Xu**

**Ph. D. Defense**

Advisor: Ying Nian Wu

Department of Statistics

University of California, Los Angeles

March 3<sup>rd</sup> , 2022

# Self-introduction

---

徐亦飞 Yifei Xu



- 2003 – 2013 : Shanghai Experimental School (K-12)
  - Primary, Mid, High School (K-12)
- 2013 – 2017 : Shanghai Jiao Tong University
  - Bachelor of Engineering --- **Computer Science**
  - Zhiyuan College ACM Honored Class
  - Advisor: Liqing Zhang
- 2017 – Now : University of California, Los Angeles
  - Doctor of Philosophy --- **Statistics**
  - Advisor: Ying Nian Wu



# Knowledge Representation: Concepts and Models

Concept     $\longleftrightarrow$     Set     $\longleftrightarrow$     Model



# Energy-based Model

Concept     $\longleftrightarrow$     Set     $\longleftrightarrow$     Model

$$p_{\theta}(X) = \frac{1}{Z(\theta)} \exp f_{\theta}(X) p_0(X)$$



# Energy-based Model

Directly model the probability:

$$\log p_\theta(X) \propto f_\theta(X)$$

$$f_\theta(X): \mathbf{R}^D \rightarrow \mathbf{R}$$

Any differentiable function  
e.g. weight sum of a heuristic rule,  
Gabor filter on image, or neural network.

$$p_\theta(X) = \frac{1}{Z(\theta)} \exp f_\theta(X) p_0(X)$$

$$Z(\theta) = \int \exp f_\theta(X) dX$$

The normalization constant to ensure overall probability sum up to 1.

$$p_0(X) \sim N(0, I_D)$$

The white noise prior distribution.

# Discriminative, generative and descriptive

---



Discriminative Task  
 $p_{\theta}(C|X)$



Generative Task  
 $p_{\theta}(X|z)$

# Discriminative, generative and descriptive



Descriptive Model  
 $p_{\theta}(X)$



Discriminative Model  
 $p_{\theta}(C|X)$



Generative Model  
 $p_{\theta}(X|z)$

# Discriminative, generative and descriptive



Descriptive Model

$$p_{\theta}(X)$$

Discrimination

$$p_{\theta}(k|X) = \frac{\exp f_{\theta_k}(X)}{\sum_{l=1}^K \exp f_{\theta_l}(X)}$$



Generation

$$X \sim p_{\theta}(X)$$

# Advantage of EBM

---

## Simplicity

Directly model the probability.

## Stability

No need assisting network to ensure balance.

## Flexibility

Any bottom-up function can act as energy.

## Adaptivity

Avoid mode collapse and avoiding spurious modes from out-of-distribution samples.

## Compositionality

Models to be combined through product of experts or other hierarchical techniques.

# Everything to generate



# Learning Representing Controlling

# Learning

How to model a set?

Generation? Reconstruction?

Semi-supervised representation learning?

# Representing

How to represent a 3D shape?

Voxel? Point Cloud? Mesh?

A function itself can be a form of data representation?

How EBM works with VAE?

# Controlling

What is inverse optimal control?

How to control a vehicle driving on the road?

How to control if we do not even know what good is?

How to do control efficiently and accurately?

# Learning

**Generative PointNet: Energy-Based  
Learning on Unordered Point Sets**

# Representing

**Energy-based Implicit Function  
for 3D Shape Representation**

# Controlling

**Energy-based Continuous  
Inverse Optimal Control**

# Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

# Representing

Energy-based Implicit Function  
for 3D shape representation

# Controlling

Energy-based Continuous  
Inverse Optimal Control

# 0. Fundamental

- Train an EBM using MLE
- Sample-based Approximation

# Energy-based Model --- Training

- Maximum Likelihood Estimation:

$$l(\theta) = E_{q_{data}}[\log p_\theta(X)] \approx \frac{1}{n} \sum_{i=1}^n \log p_\theta(X_i)$$

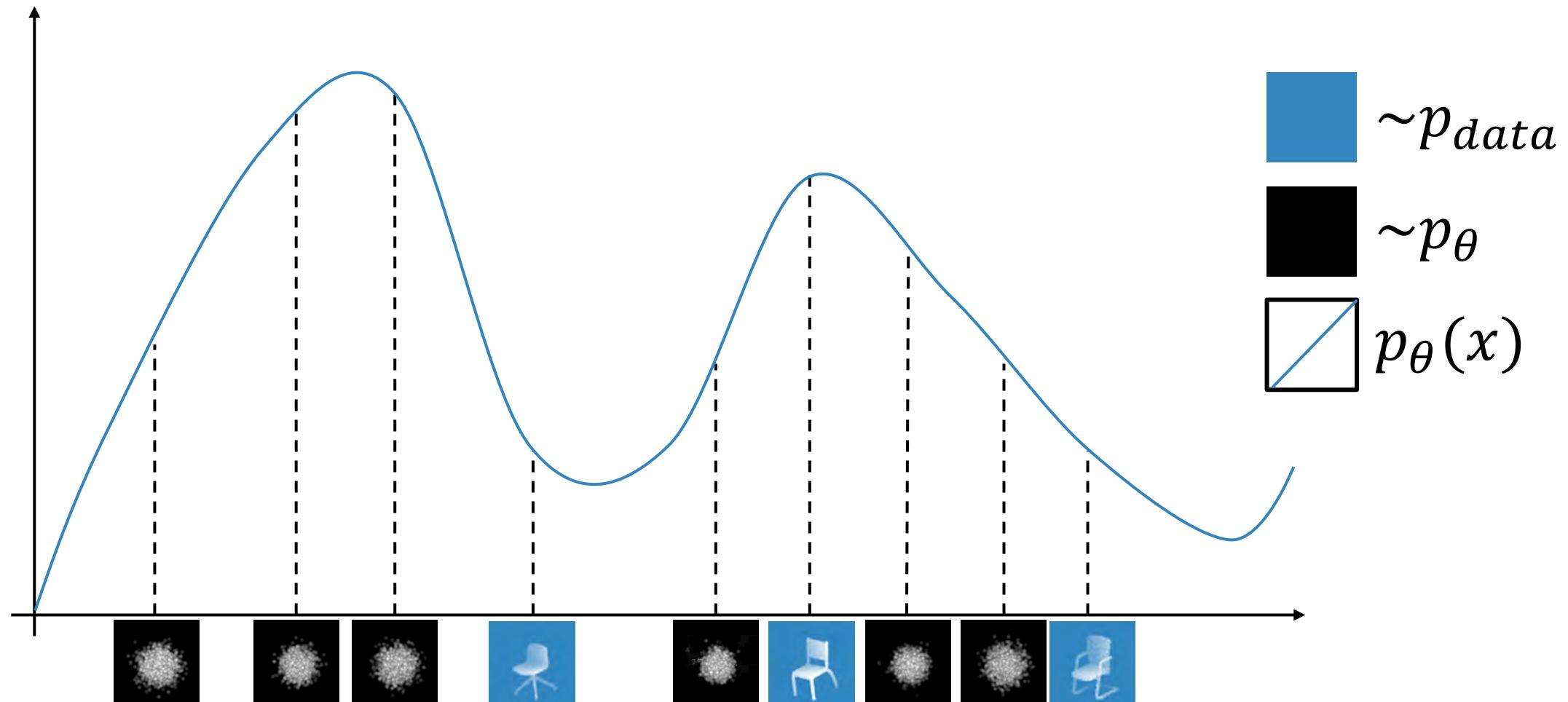
- Train model by gradient descent:

$$\frac{\partial}{\partial \theta} l(\theta) = E_{q_{data}} \left[ \frac{\partial}{\partial \theta} f_\theta(X) \right] - E_{p_\theta} \left[ \frac{\partial}{\partial \theta} f_\theta(X) \right] \approx \frac{1}{n} \sum_{i=1}^n \frac{\partial}{\partial \theta} f_\theta(X_i) - \frac{1}{n} \sum_{i=1}^n \frac{\partial}{\partial \theta} f_\theta(\tilde{X}_i)$$

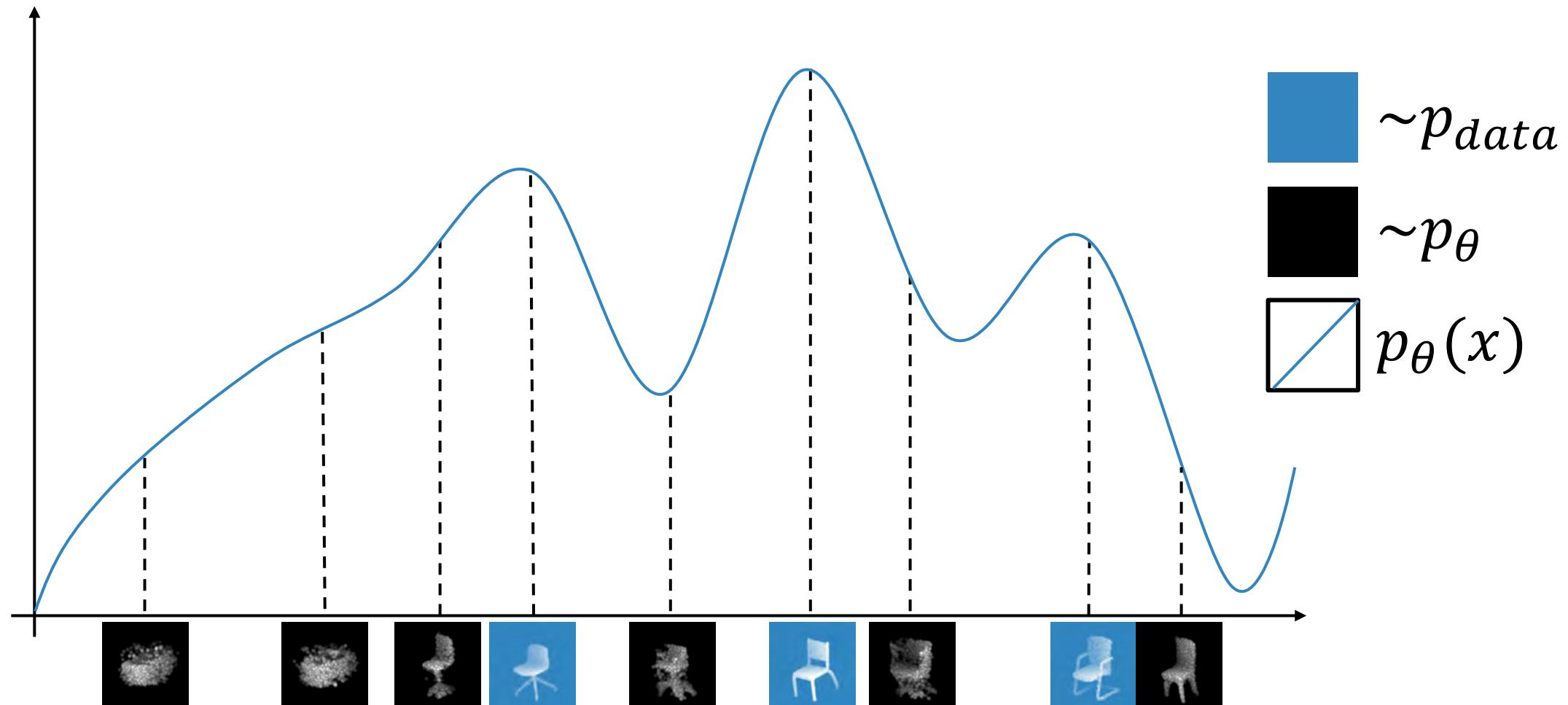
 $\sim p_{data}$  $\sim p_\theta$ 

Use MCMC sampling

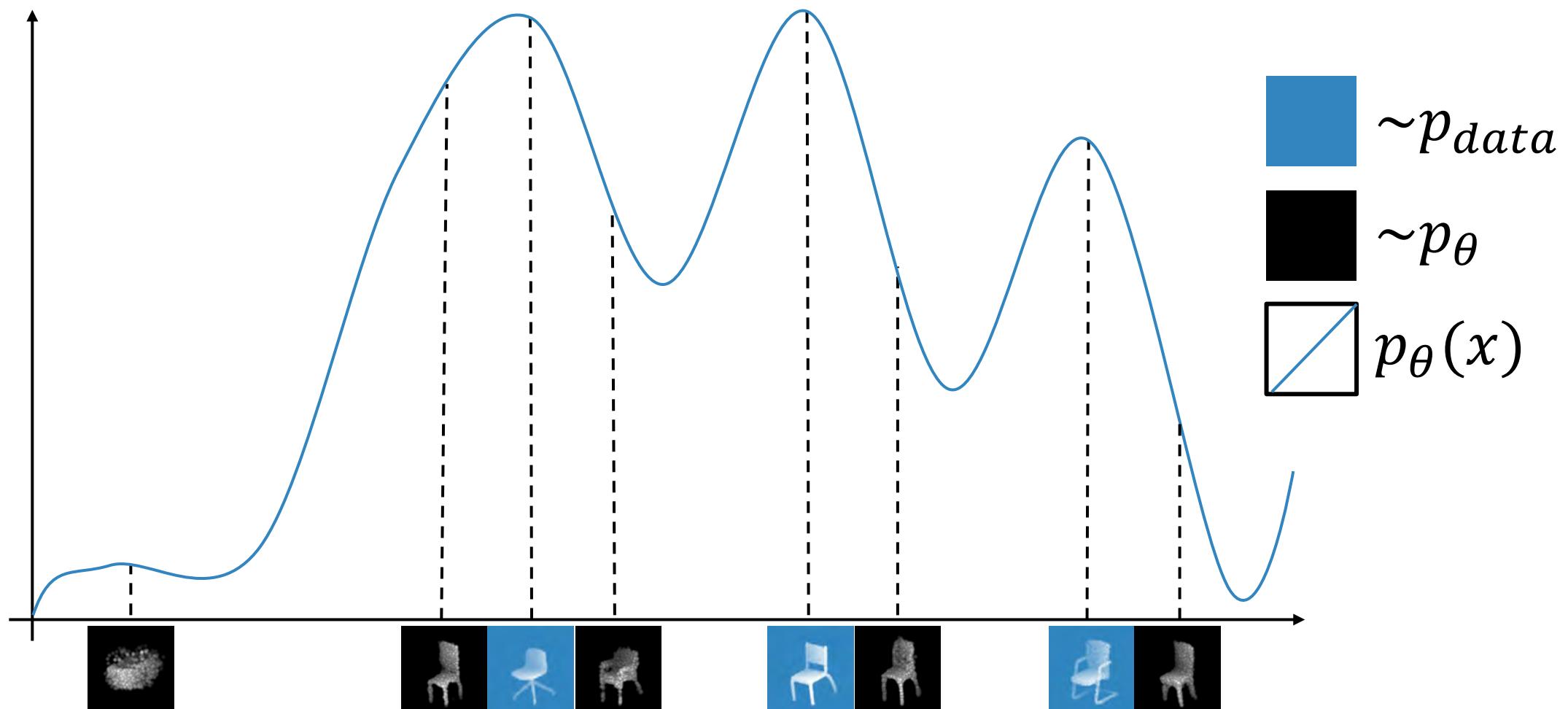
# Energy-Based Model --- Training



# Energy-Based Model --- Training



# Energy-Based Model --- Training



# 1. Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

# 2. Representing

Energy-based Implicit Function for 3D shape representation

# 3. Controlling

Energy-based Continuous Inverse Optimal Control

Current challenges?

Why EBM helps?

How to model and sample?

One more thing....

# 1. Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

$$p_{\theta} = \frac{1}{Z_{\theta}} \exp f_{\theta}$$

Energy-Based Model

on

# 2. Representing

Energy-based Implicit Function  
for 3D shape representation

# 3. Controlling

Energy-based Continuous  
Inverse Optimal Control



Point Clouds

# Point Cloud

Why Special?

**Unordered set**

$$\{x, y, z\} = \{y, z, x\}$$



Scene Reconstruction



Face ID



Lidar in AV

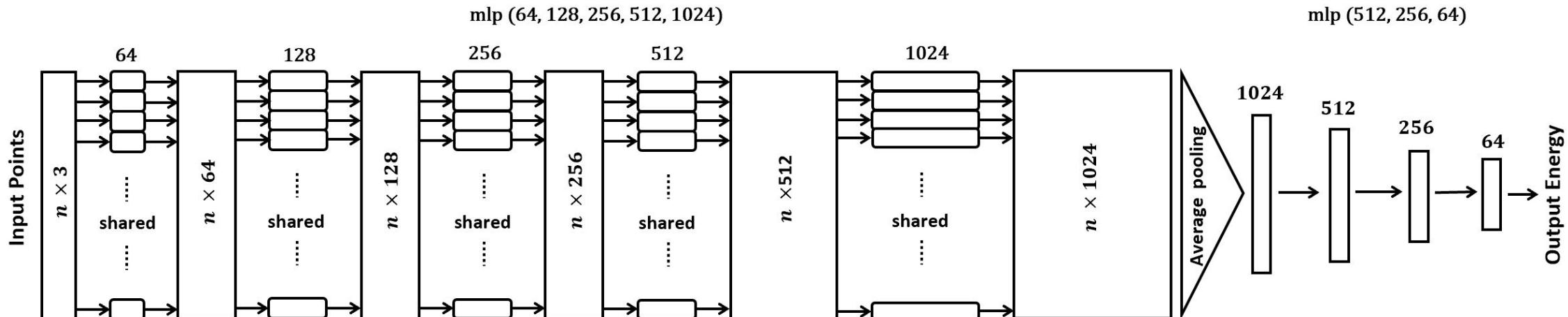
# Input-permutation-invariant Score Function

- Energy-Based Model on point cloud:

$$p_{\theta}(X) = \frac{1}{Z(\theta)} \exp f_{\theta}(X) p_0(X)$$

$Z(\theta)$ : Normalizing Constant;  $p_0(X)$ : prior distribution

$f_{\theta}(X)$  is parameterized by a bottom-up input-permutation-invariant neural network.



$$f_{\theta}(\{x_1, \dots, x_M\}) = g(\{h(x_1), \dots, h(x_m)\})$$

# Energy-based Model --- Sampling

- **Langevin Dynamics MCMC sampling:**

$$X_0 = N(0, \sigma^2)$$
$$X_{\tau+1} = X_\tau + \frac{\delta^2}{2} \frac{\partial}{\partial X} f_\theta(X_\tau) + \delta U_\tau$$

where  $U_\tau \sim N(0,1)$ ;

*Transformation*      *Noise*

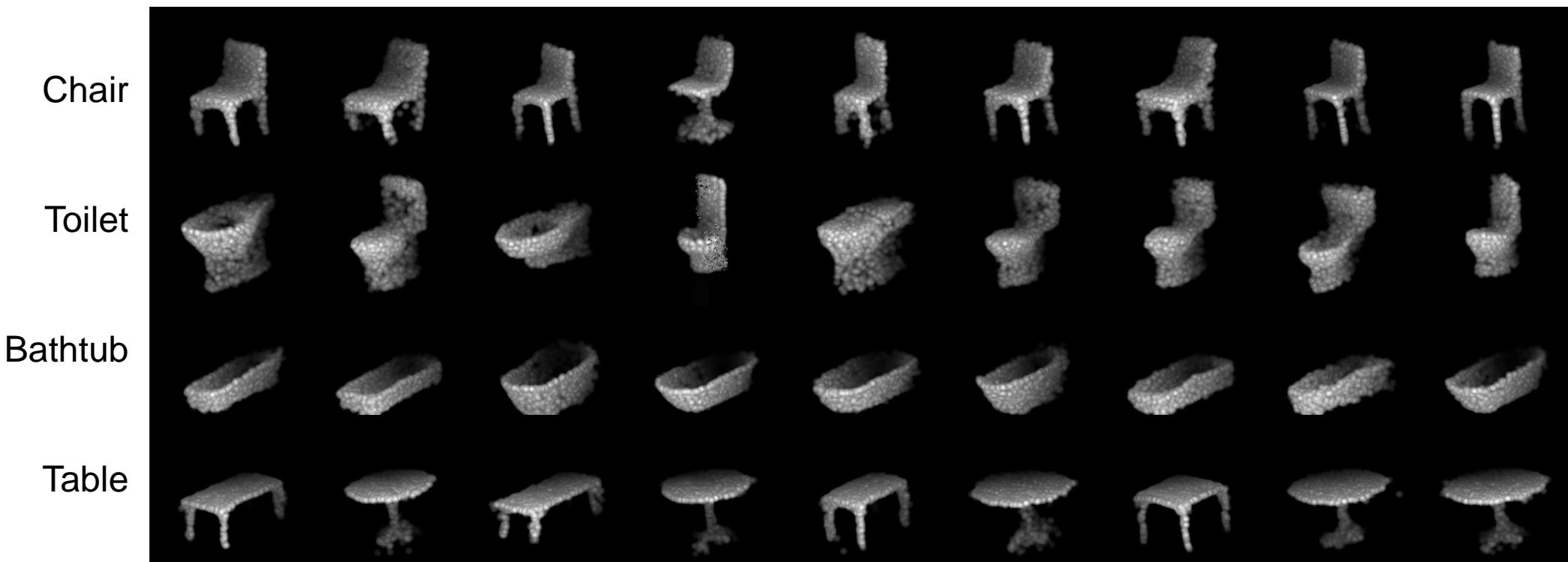
- **( $K$ -step) Short-run MCMC generator:**

Short-run MCMC procedure  $\xrightarrow{\text{regard as}}$   $K$ -layer generator model

$$\tilde{X} = M_\theta(Z, \xi), \quad Z \sim p_0(Z)$$

# Generation Results

- We synthesize 3D point clouds by short-run MCMC sampling from the learned model.

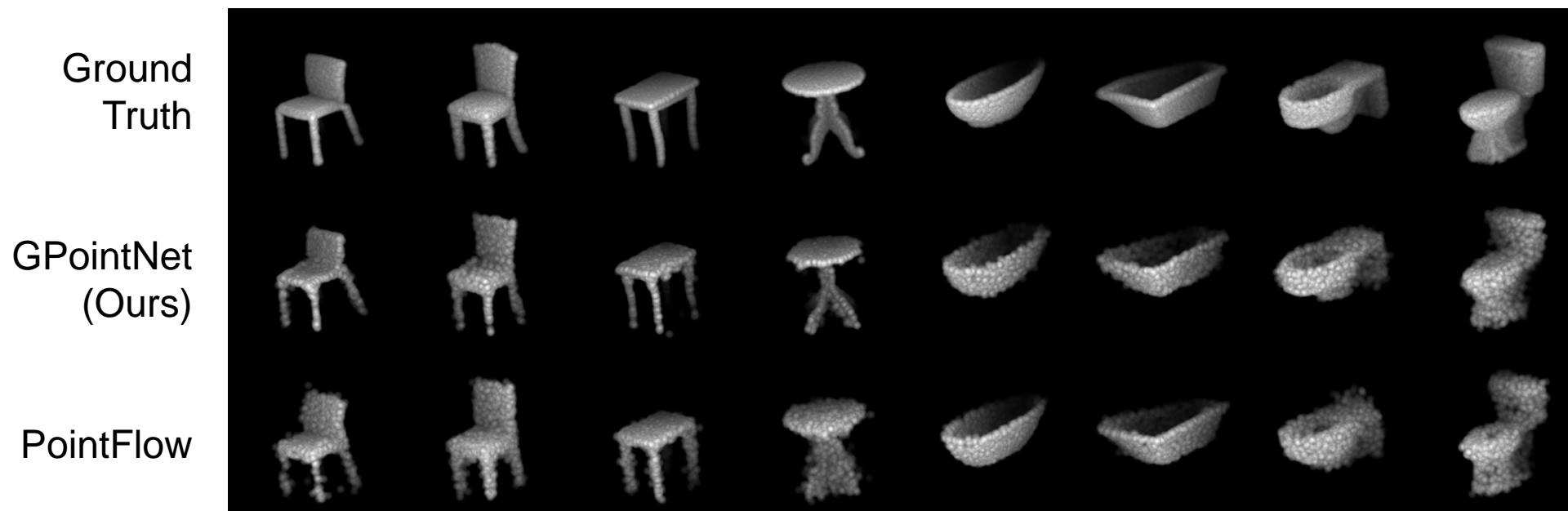


Lowest quantitatively score in **8 / 10** category

# Reconstruction Results

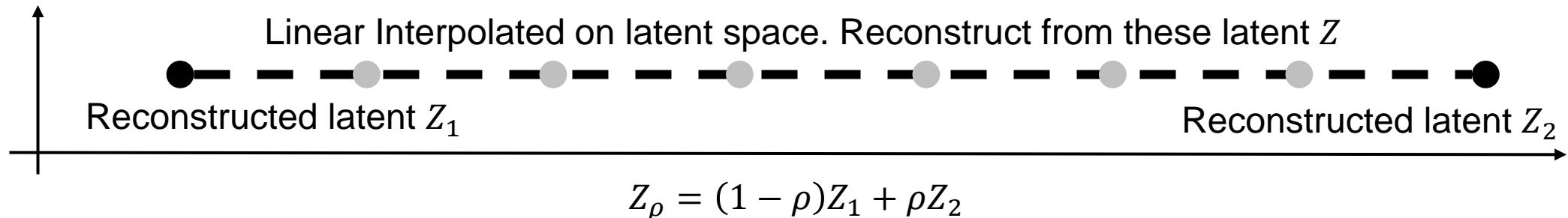
- Short-run MCMC procedure  $\xrightarrow{\text{regard as}}$   $K$ -layer generator model  $M_\theta(Z, \xi)$

$$Z = \arg \min_z L(Z) = \|X - M_\theta(Z)\|^2$$

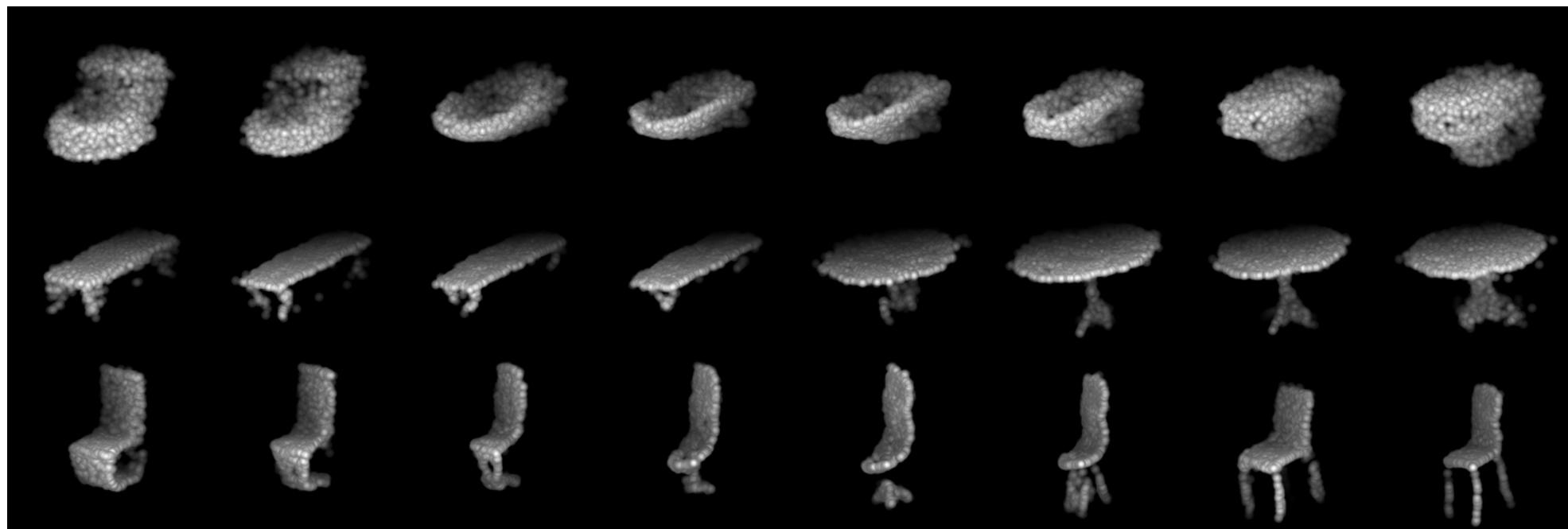


Lowest reconstruction loss in **ALL** category

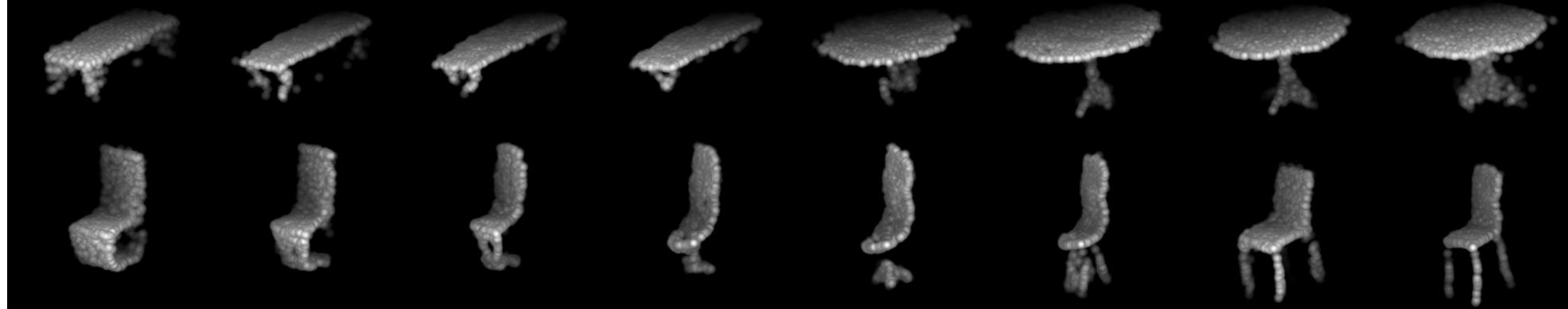
# Interpolation Results



Toilet



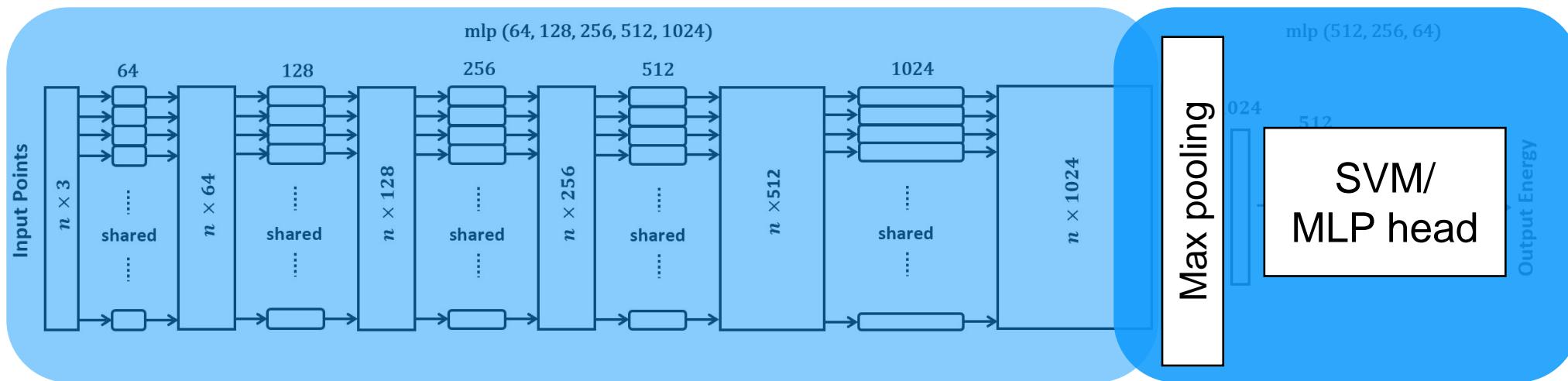
Table



Chair



# Representation Learning



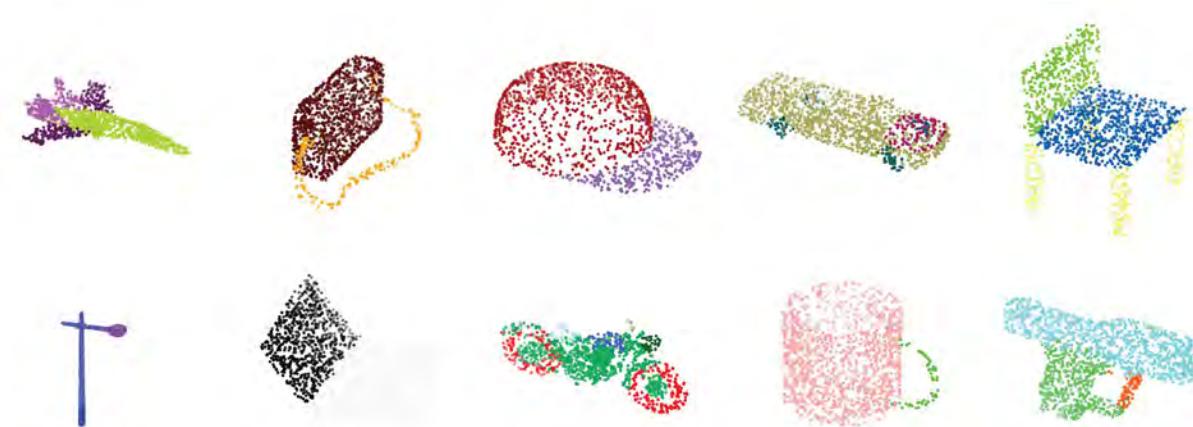
Unsupervised Learning  
**EBM Generative Feature Learning**

Supervised Learning  
**Downstream Task Learning**

# Representation Learning

Supervised Learning  
Downstream Task Learning

Method	Accuracy
SPH [18]	79.8%
LFD [4]	79.9%
PANORAMA-NN [33]	91.1%
VConv-DAE [34]	80.5%
3D-GAN [38]	91.0%
3D-WINN [16]	91.9%
3D-DescriptorNet [44]	92.4%
Primitive GAN [19]	92.2%
FoldingNet [51]	94.4%
I-GAN [1]	95.4%
PointFlow [50]	93.7%
Ours	93.7%



Classification

Segmentation

# 1. Learning

**Generative PointNet: Energy-Based Learning on Unordered Point Sets**

Current challenge?

Unordered point set is non-trivial to deal with;  
No good generative model for point cloud.

Why EBM helps?

No assisting network needed;  
Derived from PointNet

How to model and sample?

Short-run MCMC by Langevin Dynamic  
Regarded as k-layer generator

One more thing...

Representation learning  
on Classification and Segmentation

# 2. Representing

Energy-based Implicit Function  
for 3D shape representation

# 3. Controlling

Energy-based Continuous  
Inverse Optimal Control

# 1. Learning

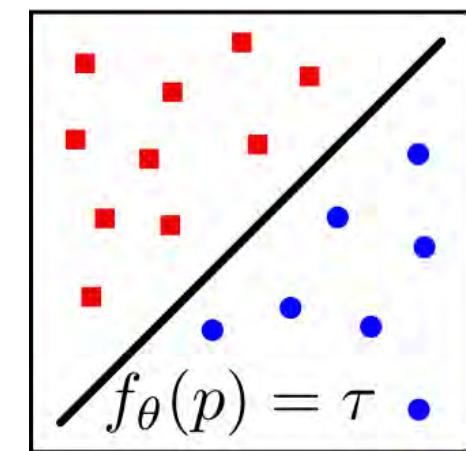
Generative PointNet: Energy-Based Learning on Unordered Point Sets

$$p_{\theta} = \frac{1}{Z_{\theta}} \exp f_{\theta}$$

on  
Energy-Based Model

# 2. Representing

Energy-based Implicit Function for 3D shape representation

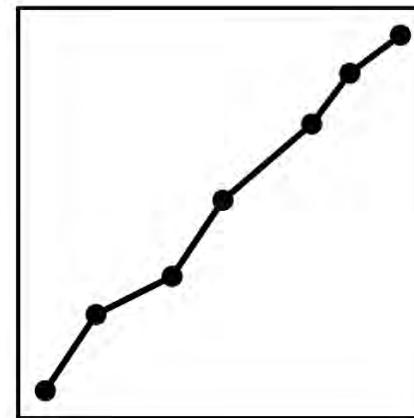
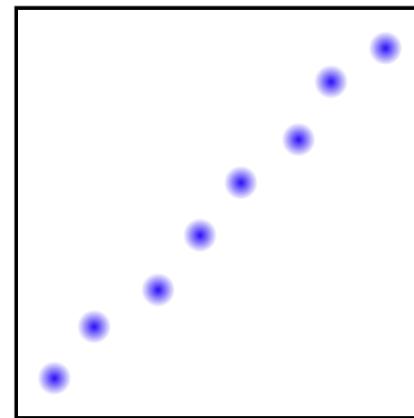
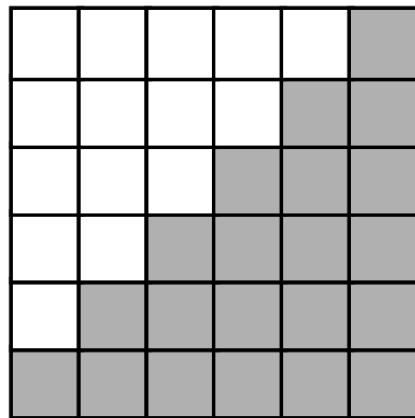


# 3. Controlling

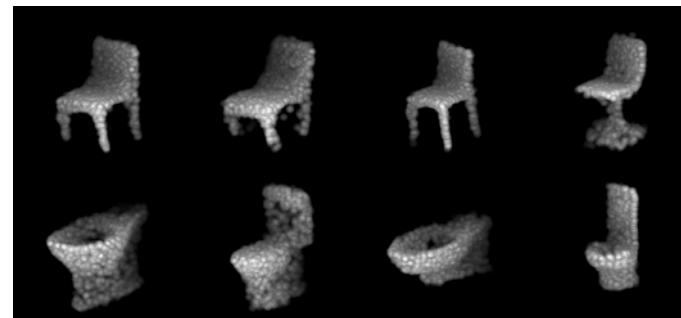
Energy-based Continuous Inverse Optimal Control

Implicit Representation

# Represent a 3D shape



Voxel

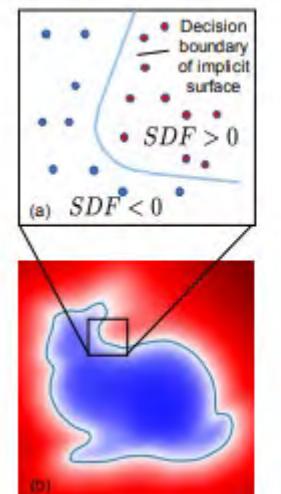
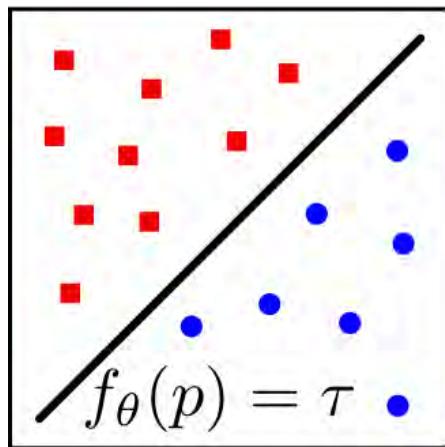


Point Cloud



Mesh

# Related Work

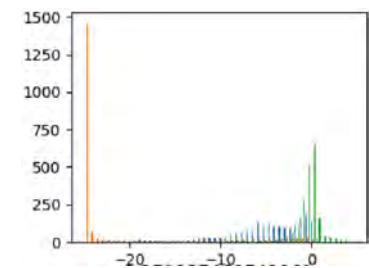
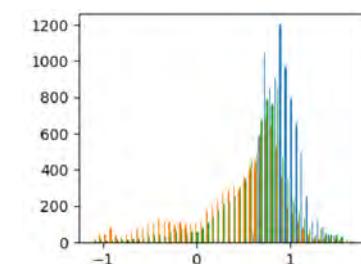
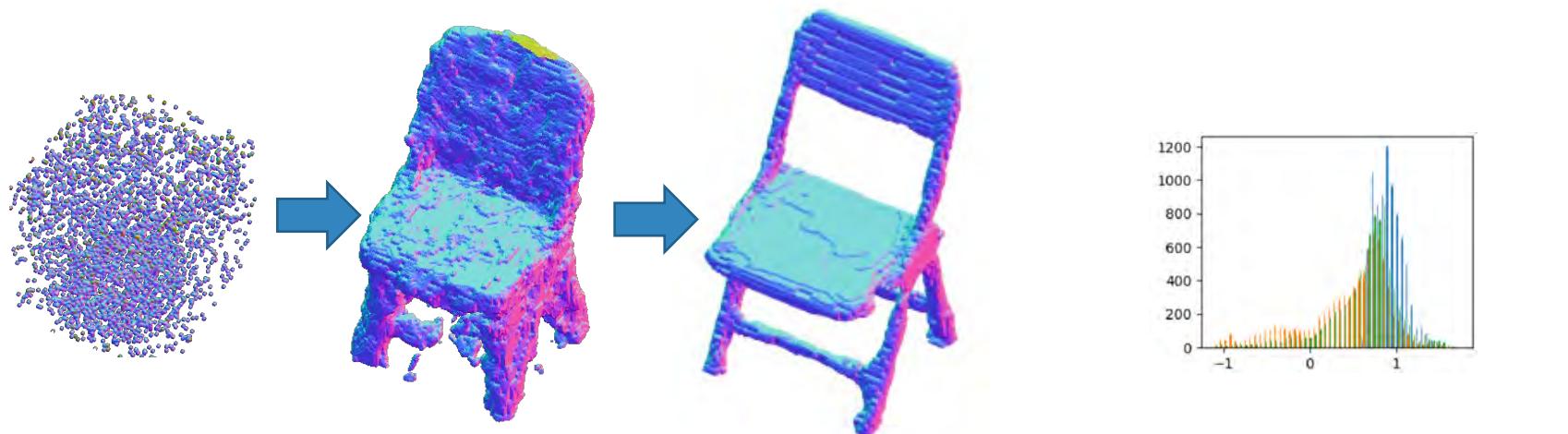


- **Occupancy Network (1 = on the surface; 0 = off the surface)**
  - Need sample negative points (point off the surface)
  - Train as a classifier.
- **Signed distance function (distance to the surface)**
  - Only work on watertight object.
  - Must have explicit definition of “in” and “out”
  - Need to calculate SDF over all training point.

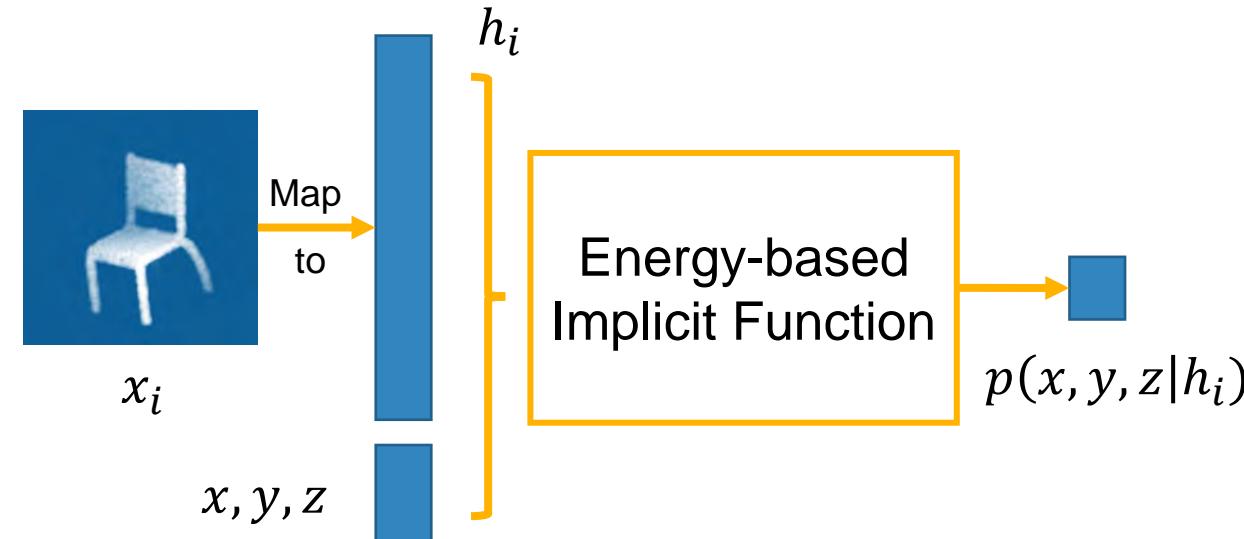
# Energy-Based Implicit Function

- Defined on point

$$p_{\theta}(x, y, z) = \frac{1}{Z(\theta)} \exp f_{\theta}(x, y, z)$$



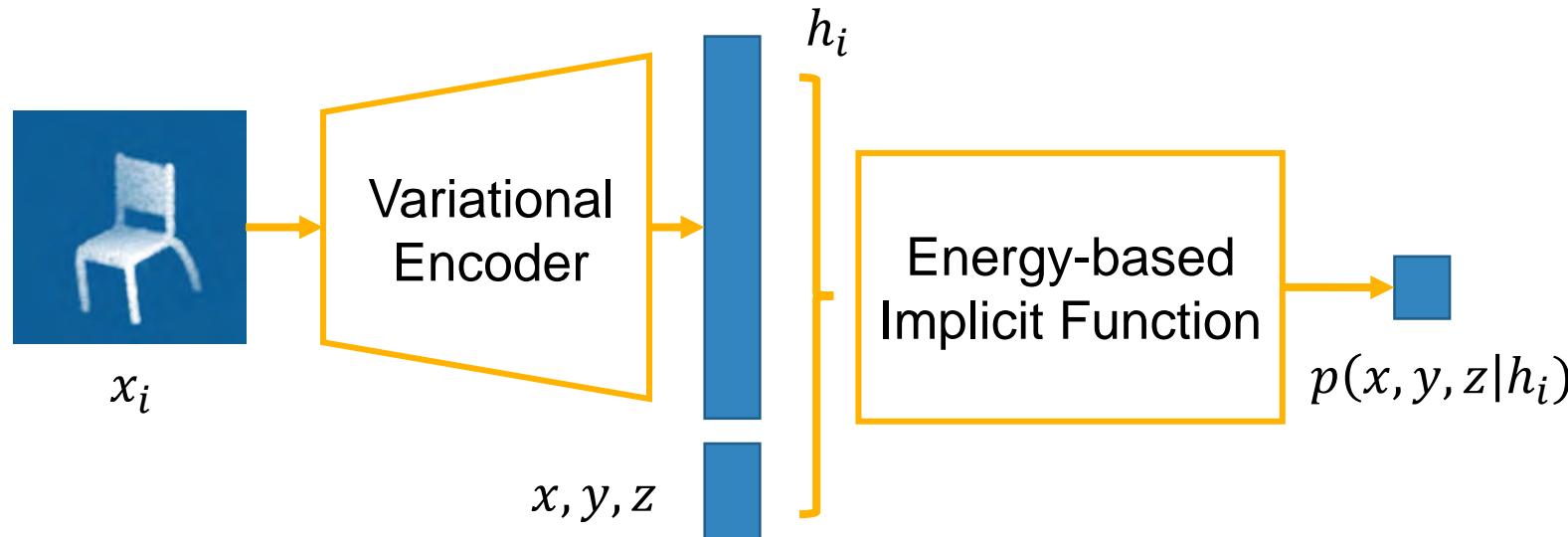
# Conditional EBIFF meets VAE



MLE Loss

Loss: 
$$L(\phi | X_i) = E_{q_\phi(z|x_i)} \log p_\theta(x|z)$$

# Conditional EBIFF meets VAE



Variational Loss:

$$L(\phi|X_i) = E_{q_{\phi}(Z|x_i)} \log p_{\theta}(x|z) - KL(q_{\phi(z)} \| p_{0(z)})$$

Reparametrized trick:

$$\sum_{\{x,y,z\} \in X_i} \log p_{\theta}(x, y, z | \mu_i + \epsilon \sigma_i) \quad \frac{1}{2} \sum_{j=1}^J (1 + 2 \log \sigma - \mu_j^2 - \sigma_j^2)$$

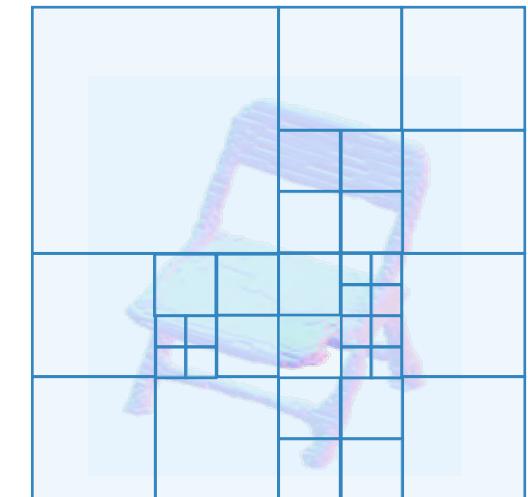
# Importance Sampling

Maximum Likelihood Estimation use gradient descent:

$$\frac{\partial}{\partial \theta} l(\theta) = E_{q_{data}} \left[ \frac{\partial}{\partial \theta} f_\theta(X) \right] - E_{p_\theta} \left[ \frac{\partial}{\partial \theta} f_\theta(X) \right]$$

Importance Sampling

$$E_p[h(x)] = \int h(x)p(x)dx = \int \frac{p(x)}{q(x)} h(x)q(x)dx = E_q \left[ \frac{p(x)}{q(x)} h(x) \right]$$

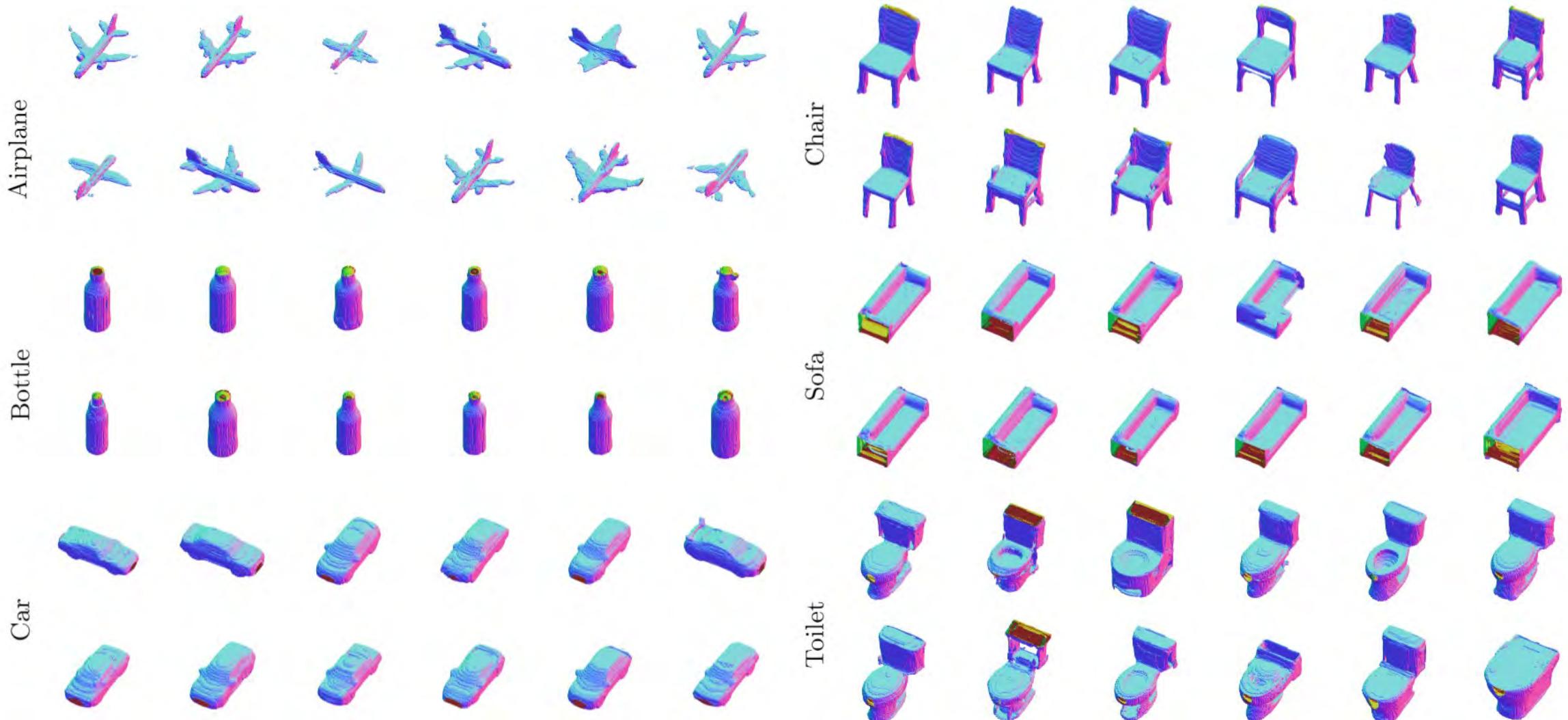


To make a better approximation:

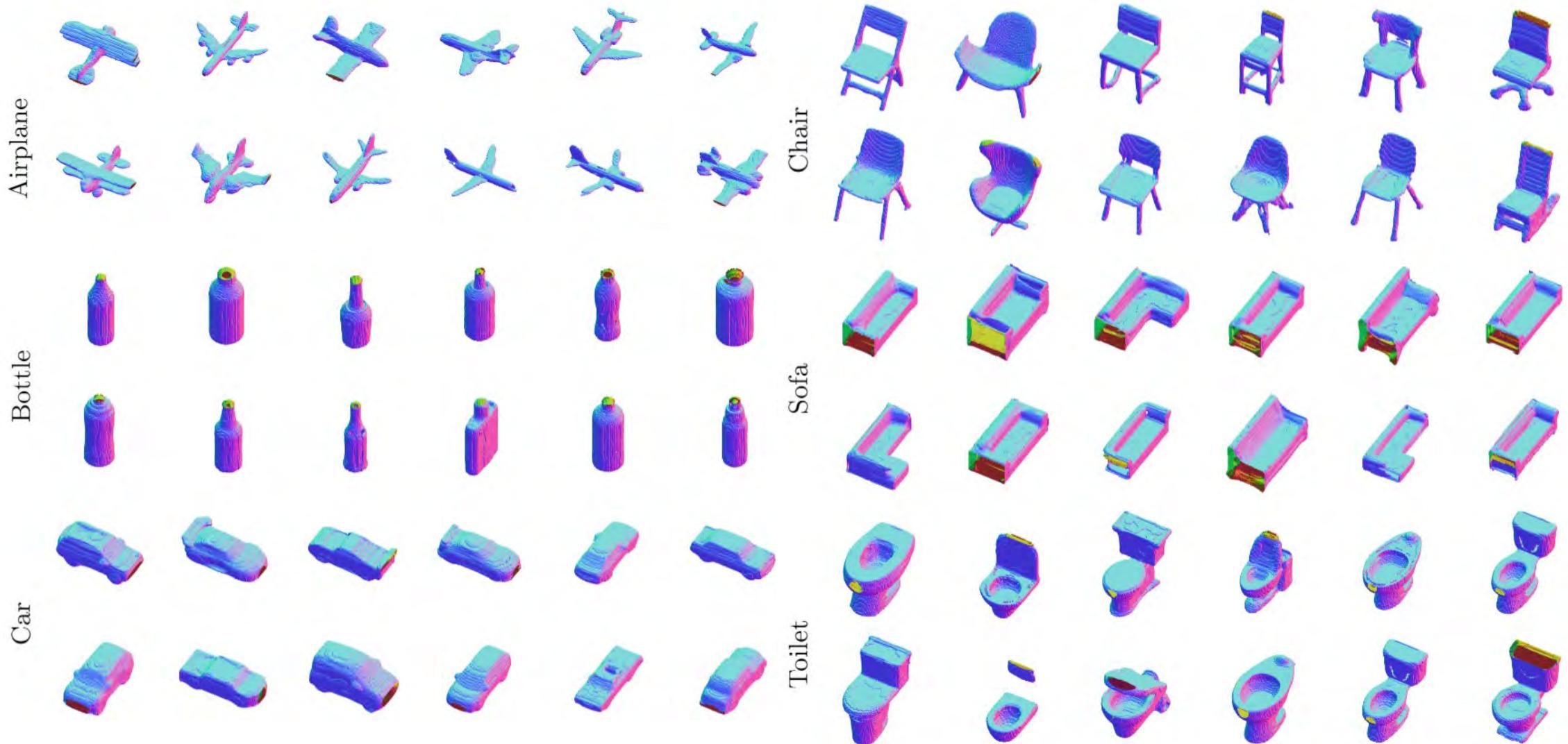
1. The reference should be easy to sample. --- use uniform distribution
2. The reference should be not too far away from the target distribution --- Piecewise Uniform

For point  $x$  in a specific cube  $G$ :  $q(x) = \frac{1}{Z_q} \exp f_\theta(\bar{x})$ , Where  $\bar{x}$  is the center point of the cube  $G$ .

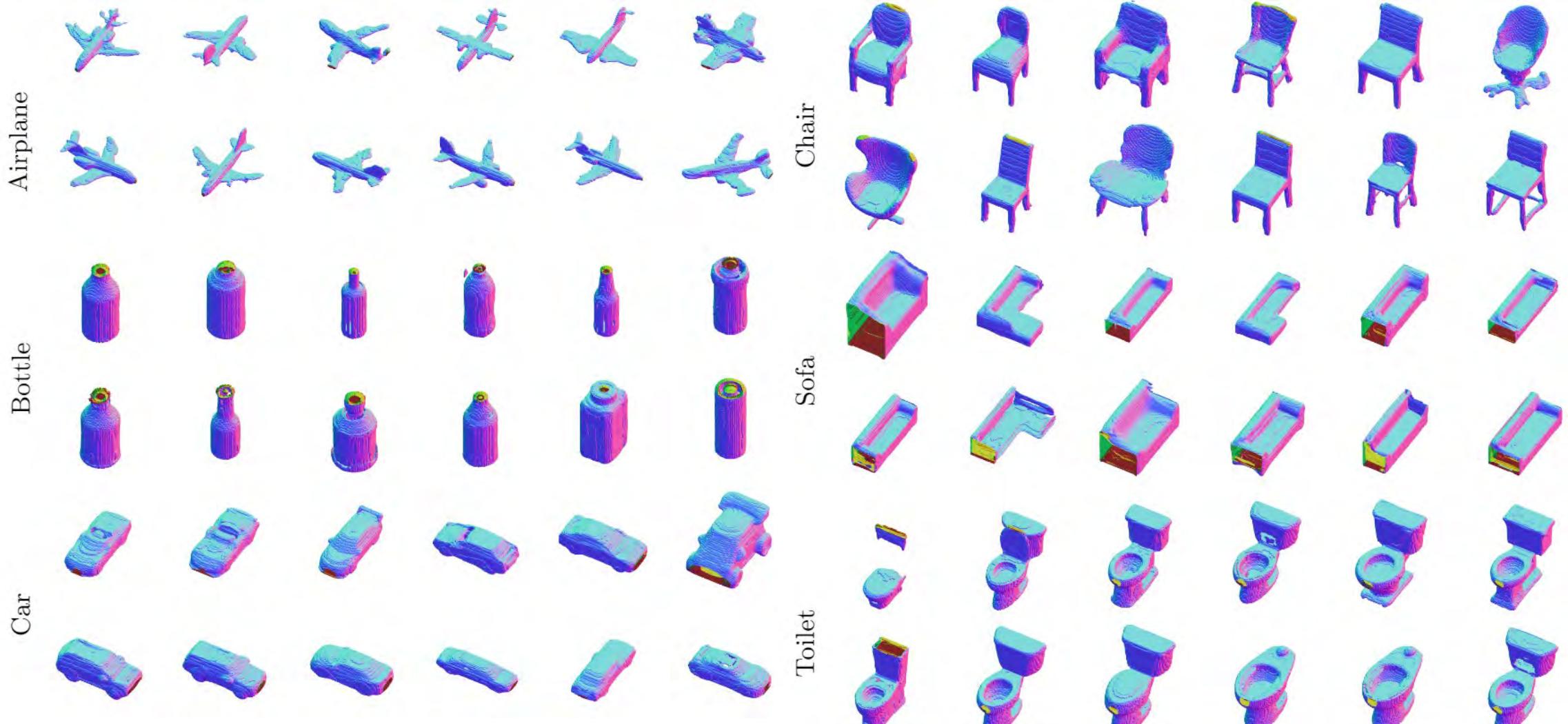
# Generation Results



# Reconstruction Results

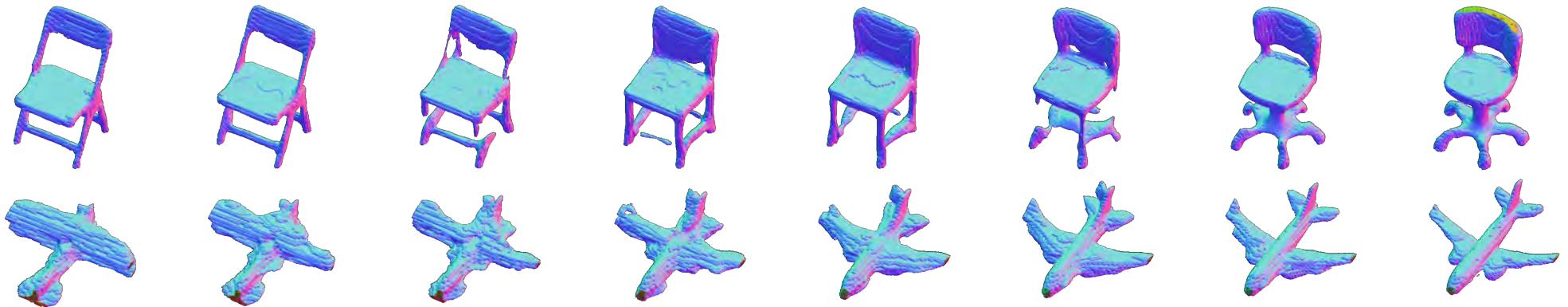


# Testing reconstruction

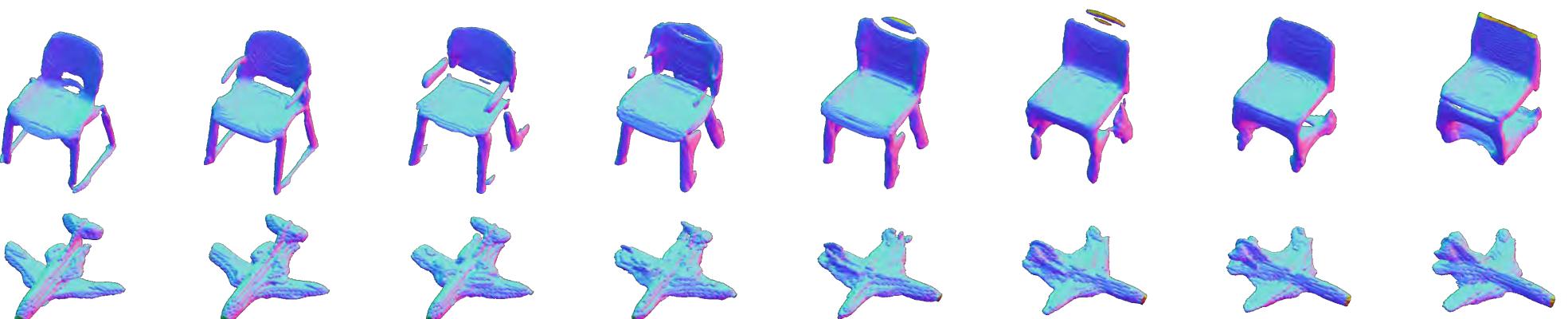


# Interpolation Results

Training



Testing



# 1. Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

## Current challenge?

Cannot deal with non-watertight object;  
Need human-defined function.

## Why EBM helps?

A natural representation ---  $p(x, y, z)$   
No need to sample negative points

## How to model and sample?

Importance Sampling  
Multi-grid / volume-adaptive piecewise uniform

## One more thing...

Cooperate with VAE  
Good generation results

# 2. Representing

Energy-based Implicit Function  
for 3D shape representation

# 3. Controlling

Energy-based Continuous  
Inverse Optimal Control

# 1. Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

$$p_{\theta} = \frac{1}{Z_{\theta}} \exp f_{\theta} \quad \text{on}$$

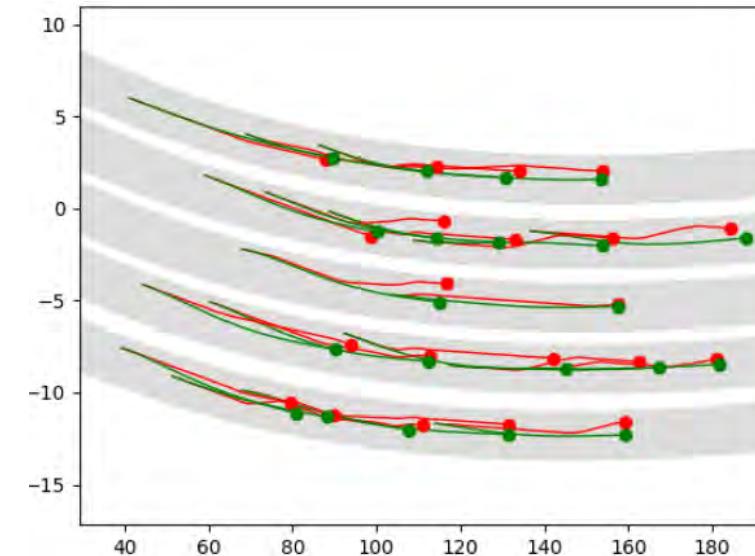
Energy-Based Model

# 2. Representing

Energy-based Implicit Function  
for 3D shape representation

# 3. Controlling

Energy-based Continuous  
Inverse Optimal Control



Inverse Optimal Control

# Continuous Inverse Optimal Control

## The MDP formulation for self-driving

State:  $x = \{\text{longitude}, \text{latitude}, \text{speed}, \text{heading angle}, \text{acceleration}, \text{steering angle}\};$

$x_t$ : State

change of  
acceleration, change  
of steering angle

$u_t$ : Control

Dynamic : The  
transition function.  
 $f$  is bicycle model.  
 $x_{t+1} = f(x_t, u_t)$

$f$ : Dynamic Function

# Continuous Inverse Optimal Control

The MDP formulation for self-driving

$x_t$ : State

$u_t$ : Control

$f$ : Dynamic Function

$C_\theta$ : Cost Function

$$U = \arg \min_U C_\theta(X, U)$$

# Continuous Inverse Optimal Control

The MDP formulation for self-driving

$x_t$ : State

$u_t$ : Control

$f$ : Dynamic Function

$C_\theta$ : Cost Function

Expert  
Demonstration:  
 $\{\tau_i\}$

Learn from

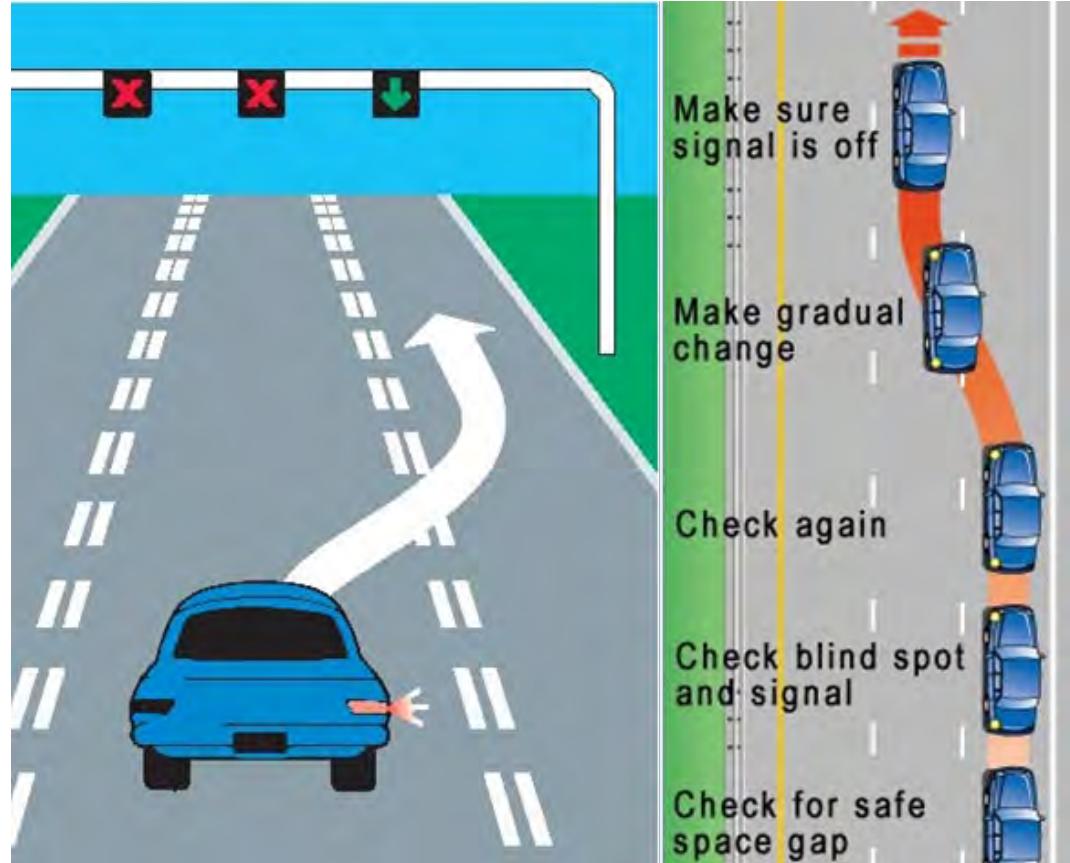
A sequence of state and control pair  $(X, U)$

$$\theta = \arg \min_{\theta} C_\theta(\tau_i)$$

# How human drive?

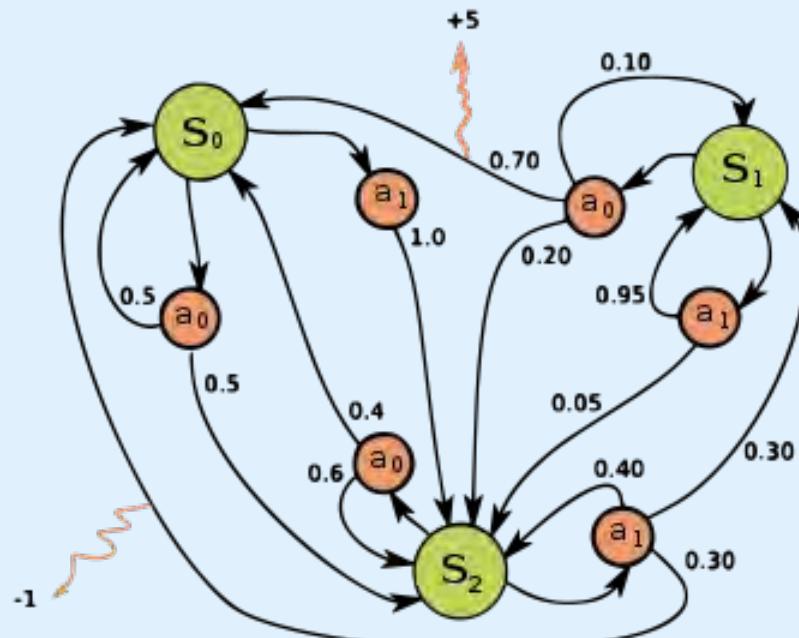


Instinct



Computation

# How an agent drives?



Fast thinking: Policy Method

Loss

Starting point

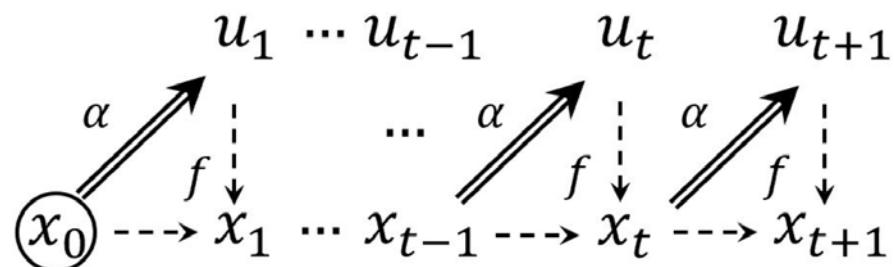


Slow thinking: Optimize Method

# Two type of model for optimal control

$$u = \arg \min_u Q_\theta(x, u)$$

$Q_\theta$ : The expected future cost

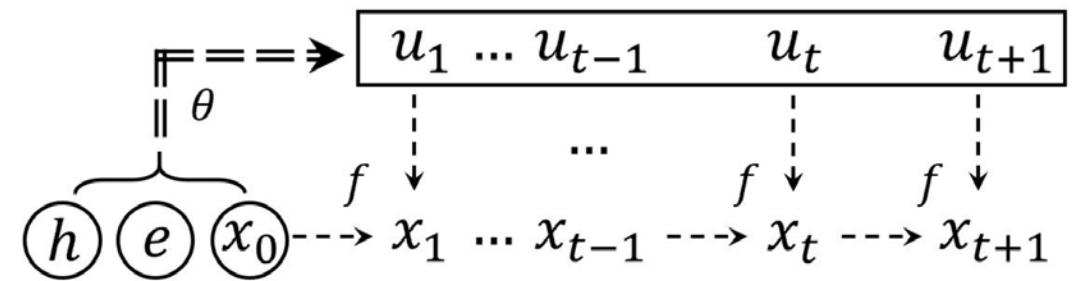


Fast thinking: Policy Method

$$U = \arg \min_U C_\theta(X, U)$$

$U$ : A sequence of control  $u$

$C_\theta$ : A sequence of cost  $c_\theta$



Slow thinking: Optimize Method

# Conditional EBM

Fast thinking: Policy Method

Slow thinking: Optimize Method

- Conditional Energy-based Model:

$$p_{\theta}(\tau|e, h) = p_{\theta}(\mathbf{u}|e, h) = \frac{1}{Z_{\theta}(e, h)} \exp[-C_{\theta}(\mathbf{x}, \mathbf{u}, e, h)]$$

Where  $Z_{\theta}(e, h)$  is the normalizing constant.  $e$  := environment;  $h$  := history.

- Previous work: Use Laplace approximation to approximate  $Z_{\theta}$

$$P(\mathbf{u}|\mathbf{x}_0) \approx e^{r(\mathbf{u})} \left[ \int e^{r(\mathbf{u}) + (\tilde{\mathbf{u}} - \mathbf{u})^T \mathbf{g} + \frac{1}{2} (\tilde{\mathbf{u}} - \mathbf{u})^T \mathbf{H} (\tilde{\mathbf{u}} - \mathbf{u})} d\tilde{\mathbf{u}} \right]^{-1} = \left[ \int e^{-\frac{1}{2} \mathbf{g}^T \mathbf{H}^{-1} \mathbf{g} + \frac{1}{2} (\mathbf{H}(\tilde{\mathbf{u}} - \mathbf{u}) + \mathbf{g})^T \mathbf{H}^{-1} (\mathbf{H}(\tilde{\mathbf{u}} - \mathbf{u}) + \mathbf{g})} d\tilde{\mathbf{u}} \right]^{-1} = e^{\frac{1}{2} \mathbf{g}^T \mathbf{H}^{-1} \mathbf{g}} |\mathbf{-H}|^{\frac{1}{2}} (2\pi)^{-\frac{d_{\mathbf{u}}}{2}},$$

# Conditional EBM

Fast thinking: Policy Method

Slow thinking: Optimize Method

$$p_{\theta}(\tau|e, h) = p_{\theta}(\mathbf{u}|e, h) = \frac{1}{Z_{\theta}(e, h)} \exp[-C_{\theta}(\mathbf{x}, \mathbf{u}, e, h)]$$

- Our method: Sampling-based Approach / Optimization-based Approach:

$$\frac{\partial}{\partial \theta} l(\theta) = \frac{1}{n} \sum_{i=1}^n \left[ \frac{\partial}{\partial \theta} C_{\theta}(\tilde{\mathbf{x}}_i, \tilde{\mathbf{u}}_i, e, h) - \frac{\partial}{\partial \theta} C_{\theta}(\mathbf{x}_i, \mathbf{u}_i, e, h) \right]$$

Sampled through Langevin dynamics

or

Predicted through optimization method

# Conditional EBM

Fast thinking: Policy Method

Slow thinking: Optimize Method

$$p_{\theta}(\tau|e, h) = p_{\theta}(\mathbf{u}|e, h) = \frac{1}{Z_{\theta}(e, h)} \exp[-C_{\theta}(\mathbf{x}, \mathbf{u}, e, h)]$$

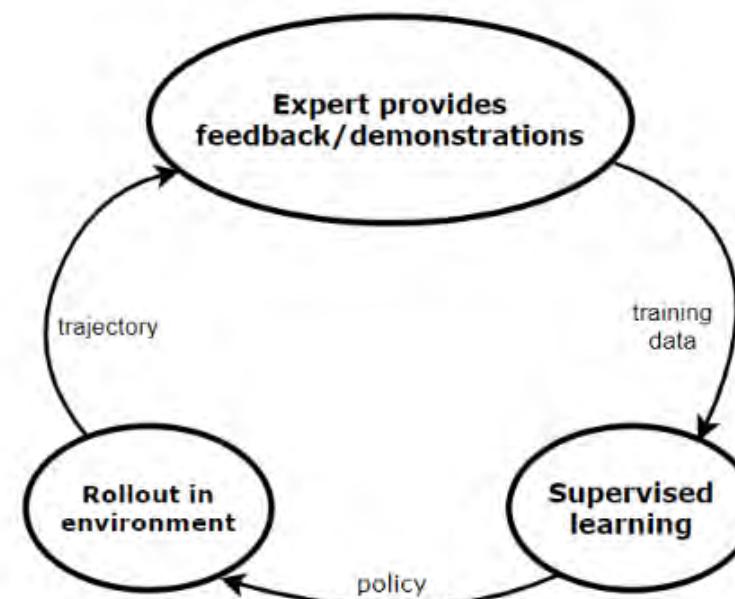
- Sample-based approach:
  - More exploration
  - Finding corner case
  - Better generalization
- Support more complex cost function:
  - 1D CNN
  - LSTM
  - MLP

# Two type of model for optimal control

Fast thinking: Policy Method

Slow thinking: Optimize Method

- Previous work: Imitate policy learnt from expert demonstration. Result used directly.



# Two type of model for optimal control

Fast thinking: Policy Method

Slow thinking: Optimize Method

- Our method: Imitate policy learn from the optimized result (slow thinking result)
- A generator is used as a fast initializer of the sampling (or the optimization):

$$q_\alpha(u, \xi | e, h) = q_\alpha(u | \xi, e, h)p(\xi)$$

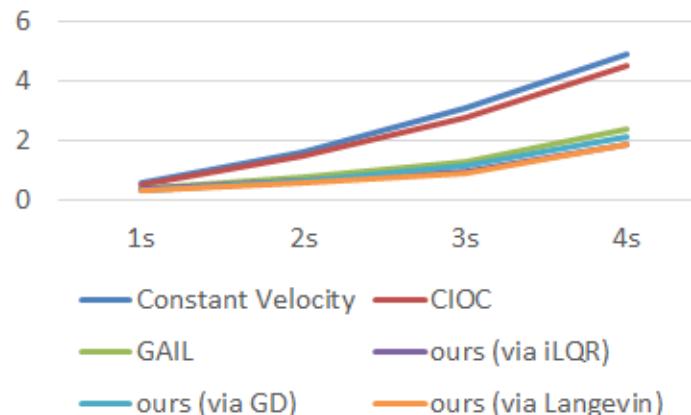
- Loss for encoder:

$$L_g(\alpha) = \frac{1}{n} \sum_{i=1}^n \| \tilde{u}_i - q_\alpha(\xi, e, h) \|^2$$

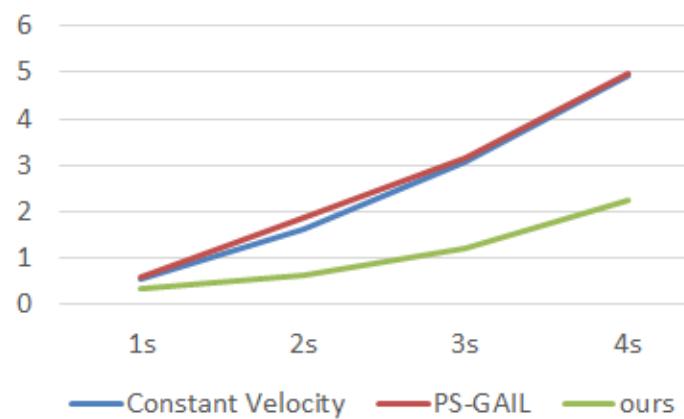
- Result is used as the initialization of the sampling / optimization

# EBOC: Predicted trajectories

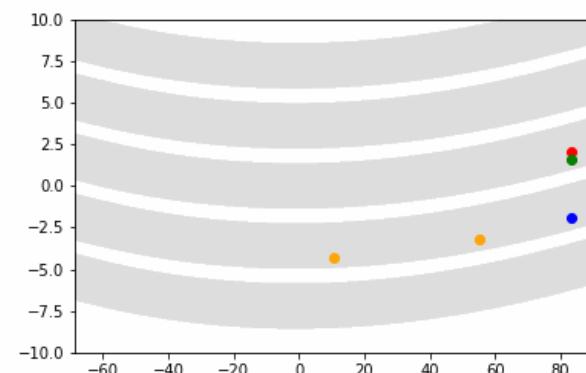
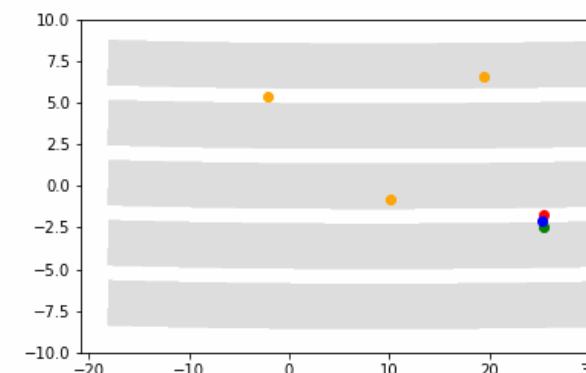
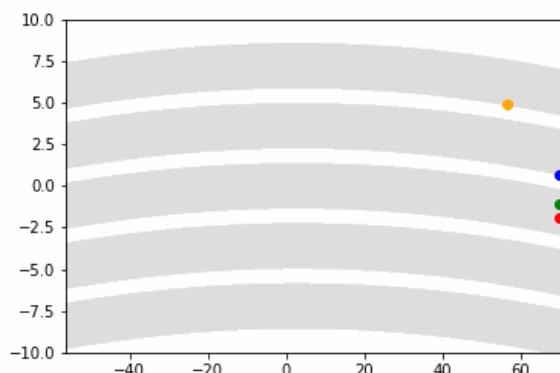
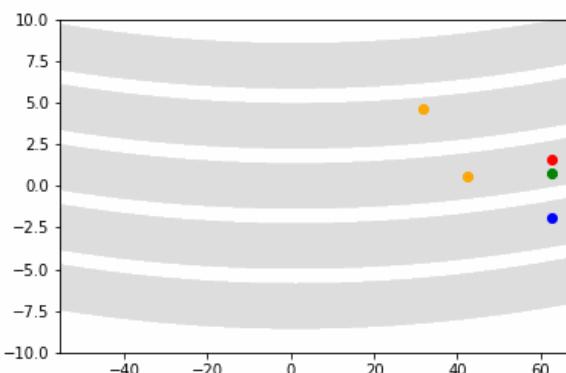
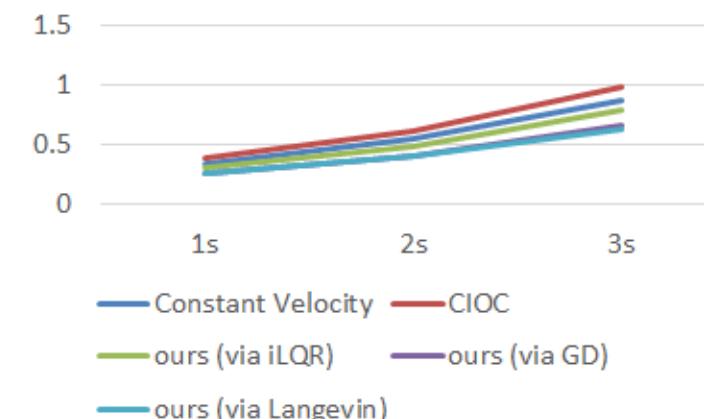
NGSIM single-agent



NGSIM multi-agent



ISEE single-agent



■ Ground Truth;

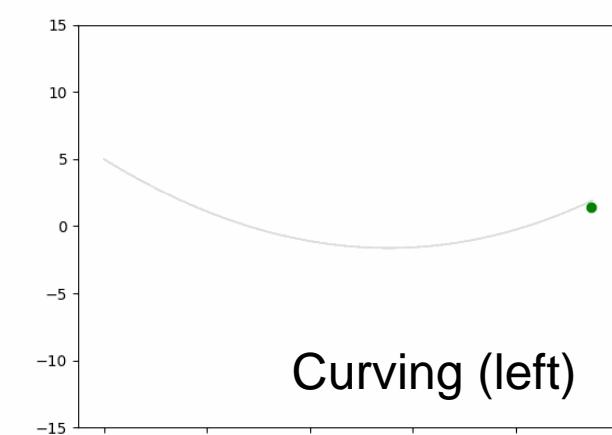
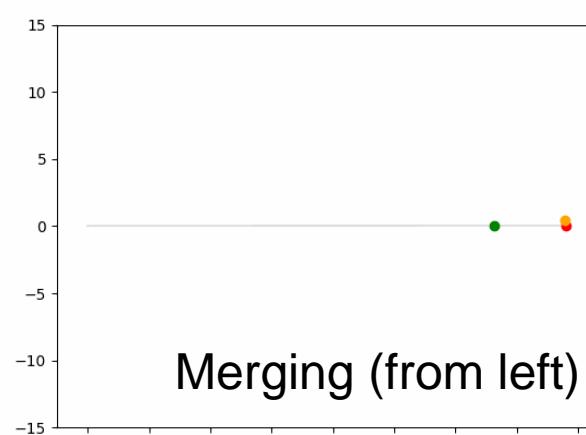
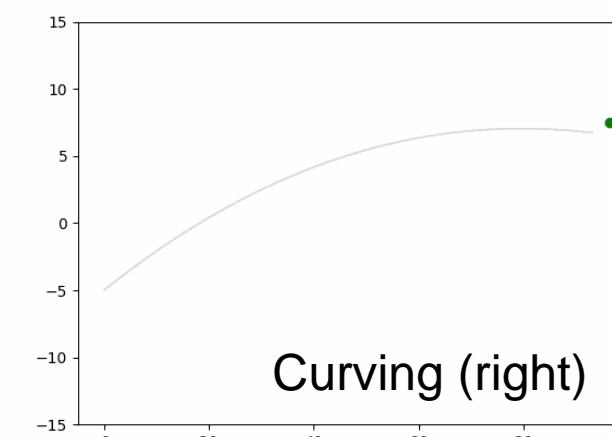
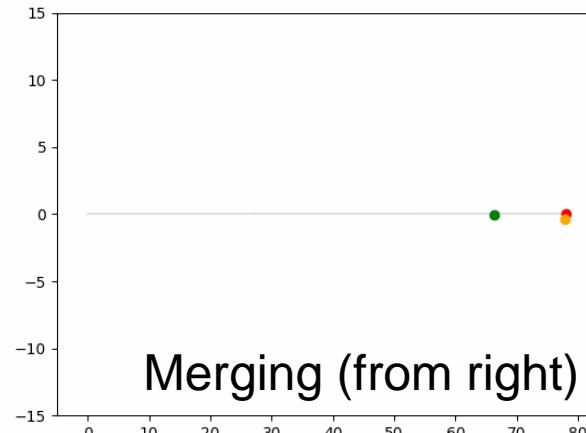
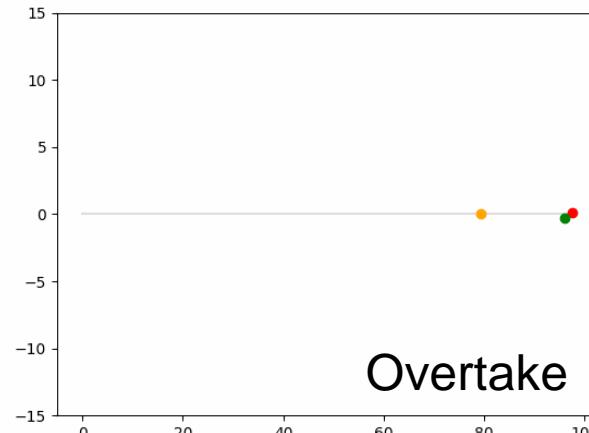
■ Ours;

■ GAIL;

■ Other Vehicle;

■ Lane.

# EBIOC: Toy examples

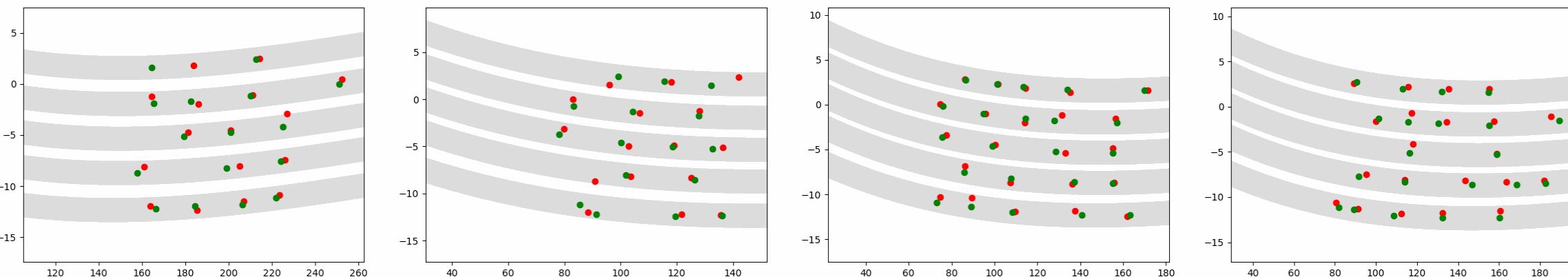


■ Green: predicted trajectories; ■ Orange: other vehicles; ■ Red: Rollout if keep constant control.

# EBIOC: Multi-agent Prediction Result

- Meta-cost: sum of each cost = Product of probability = A joint probability distribution
- Assumption: Fully cooperate + Share information

$$p_{\theta}(\mathbf{U}|\mathbf{e}, \mathbf{h}) = \prod_{k=1}^K p_{\theta}(\mathbf{u}^k | e, h^k) = \frac{1}{Z_{\theta}(\mathbf{e}, \mathbf{h})} \exp \left[ - \sum_{k=1}^K c_{\theta}(\mathbf{x}^k, \mathbf{u}^k, e, h^k) \right]$$



Multi-agent prediction on NGSIM US101 dataset (■ Lane; ■ Ground Truth; ■ Ours)

# 1. Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

Current challenge?

Trade off between exploration and exploitation  
Trade off between efficiency and accuracy

Why EBM helps?

Sample-based: Better exploration  
Combine policy method and optimization

How to model and sample?

Sample-base: Langevin Dynamic  
Optimized-based: iLQR

One more thing...

Multi-agent setting  
Joint EBM cooperative learning

# 2. Representing

Energy-based Implicit Function  
for 3D shape representation

# 3. Controlling

Energy-based Continuous  
Inverse Optimal Control

# Publications

---

- 2022.03 Jianwen Xie, Yaxuan Zhu, **Yifei Xu**, Dingcheng Li, Ping Li " Generative Learning with Latent Space Flow-based Prior Model" *In review*
- 2022.02 **Yifei Xu**<sup>†</sup>, Jingqiao Zhang<sup>†</sup>, Ru He<sup>†</sup>, Liangzhu Ge<sup>†</sup>, Chao Yang, Cheng Yang, Ying Nian Wu " SAS: Self-Augmented Strategy for Language Model Pre-training" *In Proc. 36th AAAI Conference on Artificial Intelligence (AAAI) 2022*
- 2021.06 **Yifei Xu**<sup>†</sup>, Jianwen Xie<sup>†</sup>, Zilong Zheng, Song-Chun Zhu, Ying Nian Wu " Generative PointNet : Deep Energy-Based Learning on Point Sets for 3D Generation and Reconstruction" *IEEE Conference on Computer Vision and Pattern Recognition (CVPR) 2021*.
- 2020.02 **Yifei Xu**, Jianwen Xie, Tianyang Zhao, Chris Baker, Yibiao Zhao, Ying Nian Wu " Energe-based Continous Inverse Optimal Control" *Journal version submitted to TNNLS; NeurIPS workshop on Machine Learning for Autonomous Driving, 2020*
- 2018.11 Tianyang Zhao, **Yifei Xu**, Mathew Monfort, Wongun Choi, Chris Baker, Yibiao Zhao, Yizhou Wang, Ying Nian Wu " Convolutional Spatial Fusion for Multi-Agent Trajectory Prediction" *IEEE Conference on Computer Vision and Pattern Recognition (CVPR) 2019*.
- 2017.03 Jianwen Xie, **Yifei Xu**, Erik Nijkamp, Ying Nian Wu, Song-Chun Zhu "Generative Hierarchical Structure Learning of Sparse FRAME Models" *IEEE Conference on Computer Vision and Pattern Recognition (CVPR) 2017*.

# Reference

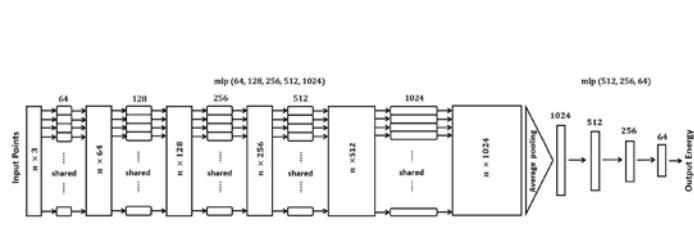
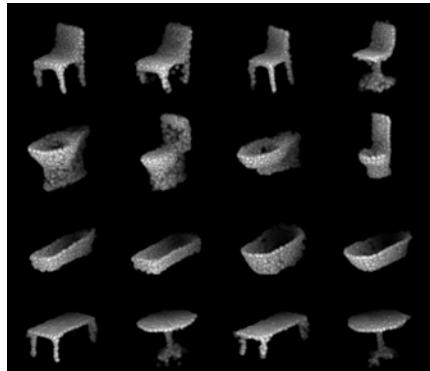
- [1] E. Todorov, "Optimal control theory," *Bayesian brain: probabilistic approaches to neural coding*, pp. 269–298, 2006.
- [2] W. Li and E. Todorov, "Iterative linear quadratic regulator design for nonlinear biological movement systems," in *ICINCO (1)*, 2004, pp. 222–229.
- [3] A. Bemporad, M. Morari, V. Dua, and E. N. Pistikopoulos, "The explicit linear quadratic regulator for constrained systems," *Automatica*, vol. 38, no. 1, pp. 3–20, 2002.
- [4] J. Xie, Y. Lu, S.-C. Zhu, and Y. Wu, "A theory of generative convnet," in *International Conference on Machine Learning*, 2016, pp. 2635–2644.
- [5] R. M. Neal *et al.*, "Mcmc using hamiltonian dynamics," *Handbook of markov chain monte carlo*, vol. 2, no. 11, p. 2, 2011.
- [6] P. J. Bickel and K. A. Doksum, *Mathematical statistics: basic ideas and selected topics, volumes I-II package*. CRC Press, 2015.
- [7] G. E. Hinton, "Training products of experts by minimizing contrastive divergence," *Neural Computation*, vol. 14, no. 8, pp. 1771–1800, 2002.
- [8] A. Hyvärinen, "Estimation of non-normalized statistical models by score matching," *Journal of Machine Learning Research*, vol. 6, pp. 695–709, 2005.
- [9] J. Xie, Y. Lu, R. Gao, S. Zhu, and Y. Wu, "Cooperative learning of energy-based model and latent variable model via mcmc teaching." *AAAI*, 2018.
- [10] J. Xie, Y. Lu, R. Gao, S.-C. Zhu, and Y. N. Wu, "Cooperative training of descriptor and generator networks," *IEEE transactions on pattern analysis and machine intelligence*, vol. 42, no. 1, pp. 27–45, 2018.
- [11] J. Xie, Z. Zheng, X. Fang, S.-C. Zhu, and Y. N. Wu, "Cooperative training of fast thinking initializer and slow thinking solver for multi-modal conditional learning," *arXiv preprint arXiv:1902.02812*, 2019.
- [12] B. D. Ziebart, A. L. Maas, J. A. Bagnell, and A. K. Dey, "Maximum entropy inverse reinforcement learning," in *AAAI*, vol. 8. Chicago, IL, USA, 2008, pp. 1433–1438.
- [13] S. C. Zhu, Y. Wu, and D. Mumford, "Filters, random fields and maximum entropy (frame): Towards a unified theory for texture modeling," *International Journal of Computer Vision*, vol. 27, no. 2, pp. 107–126, 1998.
- [14] M. Richardson and P. Domingos, "Markov logic networks," *Machine learning*, vol. 62, no. 1-2, pp. 107–136, 2006.
- [15] M. Wulfmeier, P. Ondruska, and I. Posner, "Maximum entropy deep inverse reinforcement learning," *arXiv preprint arXiv:1507.04888*, 2015.
- [16] J. Xie, S.-C. Zhu, and Y. N. Wu, "Learning energy-based spatial-temporal generative convnets for dynamic patterns," *IEEE transactions on pattern analysis and machine intelligence*, 2019.
- [17] J. Xie, W. Hu, S.-C. Zhu, and Y. N. Wu, "Learning sparse frame models for natural image patterns," *International Journal of Computer Vision*, vol. 114, no. 2-3, pp. 91–112, 2015.
- [27] J. Ho and S. Ermon, "Generative adversarial imitation learning," in *Advances in Neural Information Processing Systems*, 2016, pp. 4565–4573.
- [28] Y. Li, J. Song, and S. Ermon, "Infogail: Interpretable imitation learning from visual demonstrations," in *Advances in Neural Information Processing Systems*, 2017, pp. 3812–3822.
- [29] M. Monfort, A. Liu, and B. D. Ziebart, "Intent prediction and trajectory forecasting via predictive inverse linear-quadratic regulation," in *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence*, ser. AAAI'15. AAAI Press, 2015, pp. 3672–3678. [Online]. Available: <http://dl.acm.org/citation.cfm?id=2888116.2888226>
- [30] S. Levine and V. Koltun, "Continuous inverse optimal control with locally optimal examples," *arXiv preprint arXiv:1206.4617*, 2012.
- [31] A. Alahi, K. Goel, V. Ramamathan, A. Robicquet, L. Fei-Fei, and S. Savarese, "Social lstm: Human trajectory prediction in crowded spaces," in *Proceedings of the IEEE International Conference on Computer Vision and Pattern Recognition*, 2016.
- [32] A. Gupta, J. Johnson, L. Fei-Fei, S. Savarese, and A. Alahi, "Social gan: Socially acceptable trajectories with generative adversarial networks," in *Proceedings of the IEEE International Conference on Computer Vision and Pattern Recognition*, 2018.
- [33] A. Vemula, K. Muelling, and J. Oh, "Social attention: Modeling attention in human crowds," in *Proceedings of the International Conference on Robotics and Automation (ICRA) 2018*, May 2018.
- [34] N. Lee, W. Choi, P. Vernaza, C. B. Choy, P. H. Torr, and M. Chandraker, "Desire: Distant future prediction in dynamic scenes with interacting agents," in *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 2017, pp. 336–345.
- [35] N. Deo, A. Rangesh, and M. M. Trivedi, "How would surround vehicles move? a unified framework for maneuver classification and motion prediction," *IEEE Transactions on Intelligent Vehicles*, vol. 3, no. 2, pp. 129–140, 2018.
- [36] T. Zhao, Y. Xu, M. Monfort, W. Choi, C. Baker, Y. Zhao, Y. Wang, and Y. N. Wu, "Multi-agent tensor fusion for contextual trajectory prediction," in *The IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, June 2019.
- [37] J. Xie, Z. Zheng, X. Fang, S. Zhu, and Y. N. Wu, "Learning cycle-consistent cooperative networks via alternating MCMC teaching for unsupervised cross-domain translation," in *Thirty-Fifth AAAI Conference on Artificial Intelligence (AAAI)*, 2021, pp. 10430–10440.
- [38] J. Xie, Z. Zheng, and P. Li, "Learning energy-based model with variational auto-encoder as amortized sampler," in *Thirty-Fifth AAAI Conference on Artificial Intelligence (AAAI)*, 2021, pp. 10441–10451.
- [39] D. P. Kingma and M. Welling, "Auto-encoding variational bayes," in *International Conference on Learning Representations (ICLR)*, 2014.
- [40] H. Robbins and S. Monro, "A stochastic approximation method," *The annals of mathematical statistics*, pp. 400–407, 1951.
- [41] T. M. Cover and J. A. Thomas, *Elements of information theory, Second Edition*. Wiley, 2006.
- [42] W. K. Hastings, "Monte carlo sampling methods using markov chains and their applications," 1970.
- [43] T. Chen, E. B. Fox, and C. Guestrin, "Stochastic gradient hamiltonian monte carlo," in *International Conference on Machine Learning (ICML)*, vol. 32, 2014, pp. 1683–1691.
- [44] E. Nijkamp, M. Hill, T. Han, S. Zhu, and Y. N. Wu, "On the anatomy of mcmc-based maximum likelihood learning of energy-based models," in *AAAI Conference on Artificial Intelligence*, 2020, pp. 5272–5280.
- [45] J. Xie, R. Gao, Z. Zheng, S.-C. Zhu, and Y. N. Wu, "Learning dynamic generator model by alternating back-propagation through time," in *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 33, 2019, pp. 5498–5507.
- [46] P. Polack, F. Altché, B. d'Andréa Novel, and A. de La Fortelle, "The kinematic bicycle model: A consistent model for planning feasible trajectories for autonomous vehicles?" in *Intelligent Vehicles Symposium (IV), 2017 IEEE*. IEEE, 2017, pp. 812–818.
- [47] D. P. Kingma and J. Ba, "Adam: A method for stochastic optimization," *arXiv preprint arXiv:1412.6980*, 2014.
- [48] J. Colyar and J. Hakkias, "Us highway dataset," vol. Federal Highway Administration (FHWA), Tech. Rep. FHWA-HRT-07-030, 2007.
- [49] K. He, X. Zhang, S. Ren, and J. Sun, "Delving deep into rectifiers: Surpassing human-level performance on imagenet classification," in *IEEE International Conference on Computer Vision (ICCV)*, 2015, pp. 1026–1034.
- [50] A. Kuefeler, J. Morton, T. Wheeler, and M. Kochenderfer, "Imitating driver behavior with generative adversarial networks," in *Intelligent Vehicles Symposium (IV), 2017 IEEE*. IEEE, 2017, pp. 204–211.
- [51] J. Schulman, S. Levine, P. Abbeel, M. Jordan, and P. Moritz, "Trust region policy optimization," in *International conference on machine learning*, 2015, pp. 1889–1897.
- [52] J. Schulman, F. Wolski, P. Dhariwal, A. Radford, and O. Klimov, "Proximal policy optimization algorithms," *arXiv preprint arXiv:1707.06347*, 2017.
- [53] R. P. Bhattacharyya, D. J. Phillips, B. Wulfe, J. Morton, A. Kuefeler, and M. J. Kochenderfer, "Multi-agent imitation learning for driving simulation," in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2018, pp. 1534–1539.
- [54] R. P. Bhattacharyya, D. J. Phillips, C. Liu, J. K. Gupta, K. Driggs-Campbell, and M. J. Kochenderfer, "Simulating emergent properties of human driving behavior using multi-agent reward augmented imitation learning," in *Proceedings of the International Conference on Robotics and Automation (ICRA)*, May 2019.

# Thanks

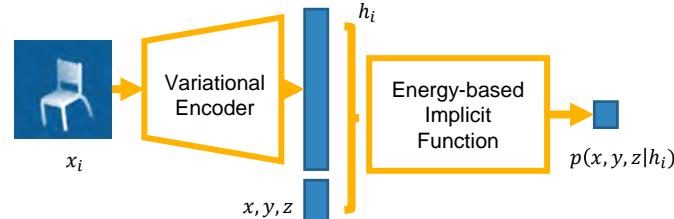
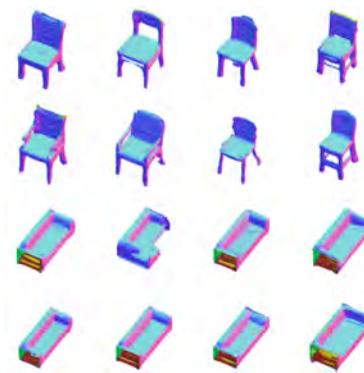


# Q&A

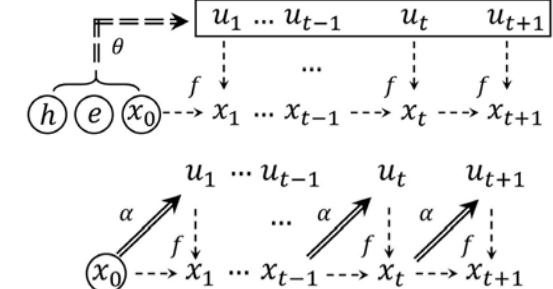
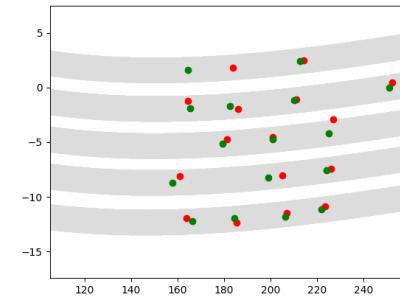
$$\text{Energy-based Model: } p(X) = \frac{1}{Z} \exp f(X)$$



Point Cloud



Implicit Function



Control