ISE 3230: Systems Modeling and Optimization for Analytics

Homework 5

(Due Friday 11/13/2020 at 11:59pm)

Instructions

(1) For problems that require coding, your should use the CVXPY package in Python and *Gurobi* as the solver. Please include your code, the relevant part of the output, and comment on it. Do not include your code and/or its output as an appendix to your homework.

(2) If you are familiar with LaTex or a suitable markdown language for equations and willing to prepare your homework in Jupyter Notebook or Lab, you are welcome to do so.

Problem 1. Suppose that $-u_1 \leq f_1(\mathbf{x}) \leq u_1$ and $-u_2 \leq f_2(\mathbf{x}) \leq u_2$, where u_1 and u_2 are constants. Show how to use integer programming techniques to model the following.

(a) (5 pts) Either $f_1(\mathbf{x}) \geq 0$ or $f_2(\mathbf{x}) \geq 0$.

(b) (5 pts) If $f_1(\mathbf{x}) \geq 0$, then $f_2(\mathbf{x}) \geq 0$.

(c) (5 pts) Either $f_1(\mathbf{x}) \geq 0$ or $f_2(\mathbf{x}) \geq 0$, but both are not nonnegative.

Problem 2. Consider the following constraint set. At least two of the following must hold:

$$\begin{aligned} x_1 + 2x_2 &\leq 12 \\ 3x_1 + 2x_2 &\leq 12 \\ -x_1 + 3x_2 &\leq 3 \end{aligned}$$

(a) (5 pts) Sketch the feasible regions.

(b) (10 pts) Use integer programming techniques to write an equivalent Mixed Integer Linear Program (MILP).

Problem 3. (20 pts) Formulate the following problem as a mixed integer programming problem.

$$
\begin{aligned}
&\text{Maximize } z = f_1(x_1) + f_2(x_2) \\
&\quad \text{subject to} \\
&\qquad \text{at least two of the following hold:} \\
&\qquad x_1 + 3x_2 \leq 12 \\
&\qquad 2x_1 + x_2 \leq 16 \\
&\qquad x_1 + x_2 \leq 9 \\
&\qquad x_1, x_2 \geq 0 \\
&\qquad \text{where} \\
&\qquad f_1(x_1) = \begin{cases} 10 + 2x_1, & \text{if } 0 \leq x_1 \leq 5 \\ 15 + x_1, & \text{if } x_1 \geq 5 \end{cases} \\
&\qquad f_2(x_2) = \begin{cases} 8 + x_2, & \text{if } 0 \leq x_2 \leq 2 \\ 4 + 3x_2, & \text{if } x_2 \geq 2 \end{cases}
\end{aligned}
$$

Problem 4. Consider the following problem:

$$
\begin{aligned}
&\text{Minimize } z = -4x_1 - 8x_2 \\
&\text{subject to} \\
&\qquad 2x_1 + 2x_2 \leq 19 \\
&\qquad -2x_1 + 2x_2 \leq 3 \\
&\qquad x_1, x_2 \in \mathbb{I}^+
\end{aligned}
$$

(a) (5 pts) Solve this problem graphically.
(b) (10 pts) Verify your answer by solving the problem in CVXPY.

Problem 5. (15 pts) Suppose that you have 7 full wine bottles, 7 half-full, and 7 empty. You would like to divide the 21 bottles among three individuals so that each will receive exactly 7. Additionally, each individual must receive the same quantity of wine. Express the problem as MILP constraints and find a solution in CVXPY. (*Hint*: Use a dummy objective function in which all the objective coefficients are zeros.)

Problem 6. (20 pts) A household uses at least 3000 minutes of long-distance telephone calls monthly and can choose to use the services of any of three companies: A, B and C. Company A charges a fixed monthly fee of \$10 and 5 cents per minute for the first 1000 minutes and 4 cents per minute for all additional minutes. Company B's monthly fee is \$20 with a flat 4 cents per minute. Company C's monthly charge is \$25 with 5 cents per minute for the first 1000 minutes and 3.5 cents per minute beyond that limit. Which company should be selected to minimize the total monthly charge? Formulate the problem and solve it in CVXPY.