Problem #0

algebra of simplex

Minimize $z = -x_1 - 2x_2$

s.t. $x_1 + x_2 \leq 16$

$-x_1 + x_2 \leq 5$

$x_1 \leq 12$

$-x_1 + 3x_2 \leq 16$

$x_1, x_2 \geq 0$

select x_B such that the corresponding B is easily invertible.

$m = 4$

$$x_B = \begin{pmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} \quad B = [a_3 | a_4 | a_5 | a_6] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 4}$$

$B^{-1} = I_3 \quad x_N = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Basic form if the set $x_N = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$z = -x_1 - 2x_2$

$x_3 = 16 - x_1 - x_2$

$x_4 = 5 + x_1 - x_2$

$x_5 = 12 - x_1$

$x_6 = 16 + x_1 - 3x_2$

$z = 0$

$$x_B = \begin{pmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 16 \\ 5 \\ 12 \\ 16 \end{pmatrix}$$

→ this BFS correspond to EP $p = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\frac{\partial z}{\partial x_1} = -1 < 0 \quad \frac{\partial z}{\partial x_2} = -2 < 0$$

↓

solution is not optimal.

select x_2 as the entering variable

$x_B = \beta - \sum x_j \alpha_j \rightarrow j=2$ (index of the entry value)

$x_B = \beta - x_2 \underline{\alpha}_2$

$$\begin{pmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \underbrace{\begin{pmatrix} 16 \\ 5 \\ 12 \\ 16 \end{pmatrix}}_{\beta} - x_2 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 3 \end{pmatrix}$$

$x_3 = 16 - x_2(-2) \geq 0 \longrightarrow x_2 \uparrow$ up to $+\infty$

$x_4 = 5 - x_2(1) \geq 0 \longrightarrow x_2 \uparrow$ up to 5

$x_5 = 12 - x_2(1) \geq 0 \longrightarrow x_2 \uparrow$ up to 12

$x_6 = 16 - x_2(1) \geq 0 \longrightarrow x_2 \uparrow$ up to 16

So x_2 can increase upto

$\min\{+\infty, 5, 12, 16\} = 5$

$\min\{\frac{5}{1}, \frac{12}{1}, \frac{16}{1}\} = 5$

As we increase x_2 upto 5, x_4 will become zero.

Hence, x_4 is the departing variable.

New basic form:

$x_B = \begin{pmatrix} x_3 \\ x_2 \\ x_5 \\ x_6 \end{pmatrix} \qquad x_N = \begin{pmatrix} x_1 \\ x_4 \end{pmatrix}$

use $x_2 = 5 - x_4 + x_1$ in all the equations in the previous basic form to establish new one.

$z = -x_1 - 2(5 - x_4 + x_1) = -3x_1 - 10 + 2x_4$

$x_3 = 16 - x_1 - (5 - x_4 + x_1) = 16 - 2x_1 - 5 + x_4 = -2x_1 + x_4 + 11$

$x_2 = 5 - x_4 + x_1$

$x_5 = 12 - x_1$

$x_6 = 16 + x_1 - 3(5 - x_4 + x_1) = 16 - 2x_1 - 15 + 3x_4 = -2x_1 + 3x_4 + 1$

$\frac{\partial z}{\partial x_1} = -3 < 0 \qquad \frac{\partial z}{\partial x_4} = 2 > 0$

Solution not optimal. choose x_1.

$$\begin{pmatrix} x_3 \\ x_2 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 11 \\ 5 \\ 12 \\ 1 \end{pmatrix} - x_1 \begin{pmatrix} 2 \\ -1 \\ 1 \\ 2 \end{pmatrix} \qquad \min\left\{\frac{11}{2}, \frac{12}{1}, \underset{\substack{\uparrow \\ x_6}}{\frac{1}{2}}\right\}$$

$$x_B = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_5 \end{pmatrix} \qquad x_N = \begin{pmatrix} x_4 \\ x_6 \end{pmatrix}$$

NBF. $x_6 = -2x_1 + 3x_4 + 1$

$x_1 = -\frac{x_6 - 3x_4 - 1}{2} = -\frac{1}{2}x_6 + \frac{3}{2}x_4 + \frac{1}{2}$

$z = -3(-\frac{1}{2}x_6 + \frac{3}{2}x_4 + \frac{1}{2}) - 10 + 2x_4 = \frac{3}{2}x_6 - \frac{5}{2}x_4 - \frac{23}{2}$

$x_3 = -2(-\frac{1}{2}x_6 + \frac{3}{2}x_4 + \frac{1}{2}) + x_4 + 11 = x_6 - 3x_4 - 1 + x_4 + 11 = x_6 - 2x_4 + 10$

$x_2 = 5 - x_4 - \frac{1}{2}x_6 + \frac{3}{2}x_4 + \frac{1}{2} = -\frac{1}{2}x_6 + \frac{1}{2}x_4 + \frac{11}{2}$

$x_5 = 12 - (-\frac{1}{2}x_6 + \frac{3}{2}x_4 + \frac{1}{2}) = \frac{1}{2}x_6 - \frac{3}{2}x_4 + \frac{23}{2}$

$x_1 = -\frac{1}{2}x_6 + \frac{3}{2}x_4 + \frac{1}{2}$

$$\frac{\partial z}{\partial x_4} = -\frac{5}{2} < 0 \qquad \frac{\partial z}{\partial x_6} = \frac{3}{2} > 0$$

not optimal. Choose x_4.

$$\begin{pmatrix} x_3 \\ x_2 \\ x_5 \\ x_1 \end{pmatrix} = \begin{pmatrix} 10 \\ \frac{11}{2} \\ \frac{23}{2} \\ \frac{1}{2} \end{pmatrix} - x_4 \begin{pmatrix} 2 \\ -\frac{1}{2} \\ \frac{3}{2} \\ -\frac{3}{2} \end{pmatrix} \qquad \min\left\{\frac{10}{2}, \frac{23}{2}\cdot\frac{2}{3} = \frac{23}{3}\right\}$$

x_3 (departing)

$$x_B = \begin{pmatrix} x_5 \\ x_2 \\ x_4 \\ x_1 \end{pmatrix} \qquad x_N = \begin{pmatrix} x_3 \\ x_6 \end{pmatrix} \qquad x_3 = x_6 - 2x_4 + 10$$

$$x_4 = \frac{x_3 - 10 - x_6}{-2} = -\frac{1}{2}x_3 + \frac{1}{2}x_6 + 5$$

$$z = \frac{3}{2}x_6 - \frac{5}{2}\left(-\frac{1}{2}x_3 + \frac{1}{2}x_6 + 5\right) - \frac{23}{2} = \frac{5}{4}x_3 + \frac{1}{4}x_6 - 24$$

$$x_4 = -\frac{1}{2}x_3 + \frac{1}{2}x_6 + 5$$

$$x_2 = -\frac{1}{2}x_6 + \frac{1}{2}\left(-\frac{1}{2}x_3 + \frac{1}{2}x_6 + 5\right) + \frac{11}{2} = \left(-\frac{1}{2} + \frac{1}{4}\right)x_6 - \frac{1}{4}x_3 + \frac{5}{2} + \frac{11}{2} = -\frac{1}{4}x_3 - \frac{1}{4}x_6 + 8$$

$$x_5 = \frac{1}{2}x_6 - \frac{3}{2}\left(-\frac{1}{2}x_3 + \frac{1}{2}x_6 + 5\right) + \frac{23}{2} = \left(\frac{1}{2} - \frac{3}{4}\right)x_6 + \frac{3}{4}x_3 - \frac{15}{2} + \frac{23}{2} = \frac{3}{4}x_3 - \frac{1}{4}x_6 + 4$$

$$x_1 = -\frac{1}{2}x_6 + \frac{3}{2}\left(-\frac{1}{2}x_3 + \frac{1}{2}x_6 + 5\right) + \frac{1}{2} = \left(-\frac{1}{2} + \frac{3}{4}\right)x_6 - \frac{3}{4}x_3 + \frac{15}{2} + \frac{1}{2} = -\frac{3}{4}x_3 + \frac{1}{4}x_6 + 8$$

optimal assessment

$$\frac{\partial z}{\partial x_6} = \frac{1}{4} > 0 \qquad \frac{\partial z}{\partial x_3} = \frac{5}{4} > 0$$

The solution is optimal

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ 0 \\ 5 \\ 4 \\ 0 \end{bmatrix} \qquad z^* = -8 - 2(8) = -24$$

Problem #1 Python

Minimize $z = -16x_1 - 15x_2$

s.t.

$40x_1 + 31x_2 \le 124$

$-x_1 + x_2 \le 1$

$x_1 \le 3$

$x_1, x_2 \ge 0$

$z + 16x_1 + 15x_2 = 0$

$40x_1 + 31x_2 + S_1 = 124$

$-x_1 + x_2 + S_2 = 1$

$x_1 + S_3 = 3$

(a) Simplex Algorithm Tableau Form.

C_N $\quad X_N$ $\quad C_B$ $\quad X_B$

	x_1	x_2	x_3	x_4	x_5	z	RHS
z	16	15	0	0	0	1	0
x_3	40	31	1	0	0	0	124
x_4	-1	1	0	1	0	0	1
x_5	$\boxed{1}$	0	0	0	1	0	3

a_1 $\quad$ pivot element $\quad$ B^{-1} $\quad$ $B^{-1}b$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_B B^{-1} a_1 - C_1 = 16 > 0$

$C_B B^{-1} a_2 - C_2 = 15 > 0$

$\therefore$ solution is not optimal.

x_1: entering variable

$$\min \left\{ \frac{124}{40}, \frac{3}{1} \right\}$$

$\uparrow$ x_5 departing.

C_B X_N C_N

	x_1	x_2	x_3	x_4	x_J	z	RHS
z	0	15	0	0	-16	1	-48
x_3	0	31	1	0	-40	0	4
x_4	0	1	0	1	1	0	4
x_1	1	0	0	0	1	0	3

X_B pivot element α_J B^{-1} $B^{-1}b$

$$B = \begin{bmatrix} 1 & 0 & 40 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_B B^{-1} \alpha_J - C_J = -16 < 0$

$C_B B^{-1} \alpha_2 - C_2 = 15 > 0$

not optimal solution

x_2: entering variable.

$$\min\left\{\frac{4}{31}, \frac{4}{1}\right\}$$

↑ x_3 departing

X_N C_B C_N

	x_1	x_2	x_3	x_4	x_J	z	RHS
z	0	0	$-\frac{15}{31}$	0	$-\frac{104}{31}$	1	$-\frac{780}{31}$
x_2	0	1	$\frac{1}{31}$	0	$-\frac{40}{31}$	0	$\frac{4}{31}$
x_4	0	0	$-\frac{1}{31}$	1	$\frac{71}{31}$	0	$\frac{120}{31}$
x_1	1	0	0	0	1	0	3

X_B B^{-1} ↑ $B^{-1}b$

$$B=\begin{bmatrix} 30 & 0 & 40 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_B B^{-1} a_5 - C_5 = \frac{104}{31} > 0$$

$$C_B B^{-1} a_3 - C_3 = -\frac{15}{31} < 0$$

solution not optimal

x_5 : entering var.

$$\min\left\{\frac{120}{31}\cdot\frac{31}{71} = \frac{120}{71},\ \frac{3}{1}\right\}$$

$\uparrow$ x_4 departing

C_X $\quad$ C_N $\qquad$ $x_N = x_3, x_5$

z	0	0	$-\frac{15}{31}$	0	$-\frac{104}{31}$	1	$-\frac{1548}{31}$
x_2	0	1	$\frac{1}{31}$	0	$-\frac{40}{31}$	0	$\frac{4}{31}$
x_4	0	0	$-\frac{1}{31}$	1	$\boxed{\frac{71}{31}}$	0	$\frac{120}{31}$
x_1	1	0	0	0	1	0	3

X_B $\qquad$ B^{-1} $\qquad$ $B^{-1}b$

$$B=\begin{bmatrix} 31 & 0 & 40 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

C_B [-15 0 -16]

C_N [0 0]

$< 0 \therefore$ optimal solution

z	0	0	$-\frac{31}{71}$	$-\frac{104}{71}$	0	1	-55.6	$x_N = [x_1\ x_2]$
x_2	0	1	$-\frac{1}{71}$	$\frac{40}{71}$	0	0	$\frac{104}{71}$	
x_5	0	0	$-\frac{1}{71}$	$\frac{31}{71}$	1	0	$\frac{120}{71}$	
x_1	1	0	$\frac{1}{71}$	$-\frac{31}{71}$	0	0	$\frac{13}{71}$	

X_B $\qquad$ B^{-1} $\qquad$ $B^{-1}b$

$$x_B^* = \begin{pmatrix} x_2 \\ x_5 \\ x_1 \end{pmatrix} = \begin{pmatrix} \frac{164}{71} \\ \frac{120}{71} \\ \frac{93}{71} \end{pmatrix} \qquad x_N^* = \begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$z^* = -55.6$$

$$B = \begin{bmatrix} 31 & 0 & 40 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \qquad C_B\,[-15 \quad 0 \quad -16]$$

$$C_N\,[0 \quad 0]$$

```python
# -*- coding: utf-8 -*-
"""
Created on Fri Oct  2 16:20:06 2020

@author: fionafei
"""

import cvxpy as cp


x = cp.Variable(2, nonneg  = True) # vector variable


#obj_func=x[0]+2*x[1]
obj_func_neg=-16*x[0]-15*x[1]

constraints = []
constraints.append(40*x[0]+31*x[1]<=124)
constraints.append(-1*x[0]+x[1]<=1)
constraints.append(x[0]<=3)


#problem = cp.Problem(cp.Maximize(obj_func), constraints)
problem = cp.Problem(cp.Minimize(obj_func_neg), constraints)

#problem.solve(solver=cp.CVXOPT,verbose = True)
#problem.solve(verbose = True)
problem.solve(solver=cp.GUROBI,verbose = True)

print("obj_func =")
#print(obj_func.value)
print(obj_func_neg.value)
print("x =")
print(x.value)
```

```
   Value: 1  Min: 0  Max: 1  Default: 1
Changed value of parameter QCPDual to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.0.3 build v9.0.3rc0 (mac64)
Optimize a model with 5 rows, 2 columns and 7 nonzeros
Model fingerprint: 0xc456305f
Coefficient statistics:
  Matrix range     [1e+00, 4e+01]
  Objective range  [2e+01, 2e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+00, 1e+02]
Presolve removed 3 rows and 0 columns
Presolve time: 0.01s
Presolved: 2 rows, 2 columns, 4 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0   -6.0000000e+01   6.000000e+00   0.000000e+00      0s
       1   -5.5605634e+01   0.000000e+00   0.000000e+00      0s

Solved in 1 iterations and 0.01 seconds
Optimal objective -5.560563380e+01
obj_func =
-55.605633802816904
x =
[1.30985915 2.30985915]
```

Problem #2

artificial objective function
min $z' = x_5$

Minimize $z = -2x_1 - 5x_2$
s.t. $3x_1 + 2x_2 \geq 6$
$2x_1 + x_2 \leq 2$
$x_1, x_2 \geq 0$

$3x_1 + 2x_2 - x_3 + x_5 = 6$
$2x_1 + x_2 + x_4 = 2$
$x_1, x_2, x_3, x_4, x_5 \geq 0$

	z'	x_1	x_2	x_3	x_4	x_5	RHS
z'	1	0	0	0	0	-1	0
x_5	0	3	2	-1	0	1	6
x_4	0	2	1	0	1	0	2

$C_B B^{-1} a_1 - C_1 = 2 > 0$
$C_B B^{-1} a_2 - C_2 = 5 > 0$

not optimal $\rightarrow$ x_2 entering var

min$\{3, 2\}$
x_4 departing

	z	x_1	x_2	x_3	x_4	x_5	RHS
z	1	3	2	-1	0	0	6
x_5	0	3	2	-1	0	1	6
x_4	0	$\boxed{2}$	1	0	1	0	2

pivot element

							RHS
z	1	0	$\frac{1}{2}$	-1	$-\frac{3}{2}$	0	3
x_5	0	0	$\frac{1}{2}$	-1	$-\frac{3}{2}$	1	3
x_1	0	1	$\boxed{\frac{1}{2}}$	0	$\frac{1}{2}$	0	1

pivot

x_2 entering var
x_1 departing

							RHS
z'	1	-1	0	-1	-2	0	2
x_5	0	-1	0	-1	-2	1	2
x_2	0	2	1	0	1	0	2

$C_B B^{-1} a_1 - C_1 = -1 < 0$

$C_B B^{-1} a_3 - C_3 = -1 < 0$

$C_B B^{-1} a_4 - C_4 = -2 < 0$

This problem is not feasible because x_5 has positive value.

Artificial obj fun $\neq 0$.

$C_B B^{-1} a_1 - C_1 = -8 < 0$

$C_B B^{-1} a_4 - C_4 = -5 < 0$

$\therefore$ solution is optimal but not feasible.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -2 \\ 0 \end{bmatrix} = z^* = -10$$

< 0

$\therefore$ non-feasible basic solution

original problem not feasible.

Problem #3 python

Minimize $z = -2x_1 - 2x_2 - 4x_3$

s.t. $2x_1 + x_2 + x_3 \le 2$

$3x_1 + 4x_2 + 2x_3 \ge 8$

$x_1, x_2, x_3 \ge 0$

artificial obj $z' = x_6$

$2x_1 + x_2 + x_3 + x_4 = 2$

$3x_1 + 4x_2 + 2x_3 - x_5 + x_6 = 8$

$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$

	z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	1	0	0	0	0	0	-1	0
x_4	0	2	1	1	1	0	0	2
x_6	0	3	4	2	0	-1	1	8

phase I

	z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	1	3	4	2	0	-1	0	-8
x_3	0	2	1	1	1	0	0	2
x_5	0	3	$\boxed{4}$	2	0	-1	1	8

$\min\{2, 2\}$

$\uparrow$ x_6 departing

	z							RHS
z	1	0	0	0	0	0	-1	0
x_4	0	$\frac{5}{4}$	0	$\frac{1}{2}$	1	$\frac{1}{4}$	$-\frac{1}{4}$	0
x_2	0	$\frac{3}{4}$	1	$\frac{1}{2}$	0	$-\frac{1}{4}$	$\frac{1}{4}$	2

Drop x_6.

	z						RHS
z	1	$\frac{1}{2}$	0	3	0	$\frac{1}{2}$	-4
x_4	0	$\frac{5}{4}$	0	$\frac{1}{2}$	1	$\frac{1}{4}$	0
x_2	0	$\frac{3}{4}$	1	$\frac{1}{2}$	0	$-\frac{1}{4}$	2

phase II

pivot

$<0 \quad \therefore$ optimal solution

$$
\begin{array}{c|ccccccc}
 & z & & & & & & \text{RHS} \\
z & 1 & -\frac{28}{4} & 0 & 0 & -6 & -1 & -4 \\
x_3 & 0 & \frac{10}{4} & 0 & 1 & 2 & \frac{1}{2} & 0 \\
x_2 & 0 & -\frac{1}{2} & 1 & 0 & -1 & -\frac{1}{2} & 2
\end{array}
$$

$$
x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad z^* = -4
$$

$$
x_B^* = \begin{pmatrix} x_2^* \\ x_3^* \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}
$$

$$
x_N^* = 0
$$

```
# -*- coding: utf-8 -*-
"""
Created on Fri Oct  2 16:20:06 2020

@author: fionafei
"""

import cvxpy as cp



x = cp.Variable(3, nonneg  = True) # vector variable


#obj_func=x[0]+2*x[1]
obj_func_neg=-2*x[0]-2*x[1]-4*x[2]

constraints = []
constraints.append(2*x[0]+x[1]+x[2]<=2)
constraints.append(3*x[0]+4*x[1]+2*x[2]>=8)
#constraints.append(x[0]<=3)


#problem = cp.Problem(cp.Maximize(obj_func), constraints)
problem = cp.Problem(cp.Minimize(obj_func_neg), constraints)

#problem.solve(solver=cp.CVXOPT,verbose = True)
#problem.solve(verbose = True)
problem.solve(solver=cp.GUROBI,verbose = True)

print("obj_func =")
#print(obj_func.value)
print(obj_func_neg.value)
print("x =")
print(x.value)
```

```
Parameter OutputFlag unchanged
   Value: 1  Min: 0  Max: 1  Default: 1
Changed value of parameter QCPDual to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.0.3 build v9.0.3rc0 (mac64)
Optimize a model with 5 rows, 3 columns and 9 nonzeros
Model fingerprint: 0x08e74972
Coefficient statistics:
  Matrix range     [1e+00, 4e+00]
  Objective range  [2e+00, 4e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [2e+00, 8e+00]
Presolve removed 5 rows and 3 columns
Presolve time: 0.01s
Presolve: All rows and columns removed
Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0   -4.0000000e+00   0.000000e+00   0.000000e+00      0s

Solved in 0 iterations and 0.01 seconds
Optimal objective -4.000000000e+00
obj_func =
-4.0
x =
[0. 2. 0.]
```

Problem #4

minimize $-7x_1 - 3x_2 - 2x_3$

s.t. $4x_1 + x_2 + x_3 \le 18$

$3x_1 + 2x_2 + x_3 \le 14$

$x_1, x_2, x_3 \ge 0$

	z	x_1	x_2	x_3	x_4	x_5	RHS
z	1	0	0	0	-1	-1	-32
x_1	0	1	-1	0	1	-1	4
x_3	0	0	5	1	-3	4	2

$$B^{-1}b = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 18 \\ 14 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$z = C_B B^{-1} \cdot b = [-7 \ \ -2]\begin{bmatrix} 4 \\ 2 \end{bmatrix} = -32$$

$$\alpha_1 = B^{-1}a_1 = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C_B B^{-1} a_2 - C_2 = 7 - 10 + 3 = 0$$

x_1, x_3 basic $\rightarrow$
$$\begin{array}{cc} x_1 & x_3 \end{array}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\alpha_2 = B^{-1}a_2 = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$z^* = -32 \quad C_B B^{-1} a_2 - C_2 = 0$

Alternative optional solution

letting x_2 be entering variable might lead to another alternative optimal solution.

Problem#5

minimize $-2x_1+4x_2$. slack: x_3, x_4.

	z	x_1	x_2	x_3	x_4	RHS
z	1	b	-1	f	g	-8
x_3	0	c	0	1	$\frac{1}{5}$	4
x_1	0	d	e	0	2	a

a)

x_1, x_3 are in basic

$\Rightarrow f, b=0$

$$C_B X_b = [0, -2]\begin{bmatrix}4\\a\end{bmatrix} = -8 = z$$

$\Rightarrow a=4$

$$C_B \alpha_4 - C_4 = [0, 2]\begin{bmatrix}\frac{1}{5}\\2\end{bmatrix} - 0$$

$\Rightarrow g=-4$

$$C_b\alpha_2 - C_2 = -1 = [0\ -2]\begin{bmatrix}0\\e\end{bmatrix} - 4 = -2e-4 = -1$$

$-2e=3 \Rightarrow e=-\frac{3}{2}$

x_1 is a basic variable

$\Rightarrow c=0, d=1$

b)

$B^{-1} = \begin{bmatrix}1 & \frac{1}{5}\\0 & 2\end{bmatrix}$ (use matrix calculator)

c) $\frac{\partial x_3}{\partial x_2} = 0$ $\frac{\partial z}{\partial x_4} = 4$

c) Yes the tableau is optimal because both $\frac{\partial z}{\partial x_2}$, $\frac{\partial z}{\partial x_4}$ are positive.

Problem #6

Minimize $z = -x_1 + 2x_2 - 3x_3$

s.t. $x_1 - x_2 + x_3 \le 8$

$x_1 - x_2 - x_3 \le 12$

$x_1 \geqslant -2$

$x_2 \le 0$

x_3 unrestricted.

$x_1' = x_1 + 2 \rightarrow x_1 = x_1' - 2$

$x_2' = -x_2 \rightarrow x_2 = -x_2'$

$x_3 = x_3^+ - x_3^-$

min $z = -(x_1' - 2) + 2(-x_2') - 3(x_3^+ - x_3^-)$

s.t. $(x_1' - 2) - (-x_2') + (x_3^+ - x_3^-) \le 8$

$(x_1' - 2) - (-x_2') - (x_3^+ - x_3^-) \le 12$

min

$z = -x_1' + 2 - 2x_2' - 3x_3^+ + 3x_3^-$

s.t. $x_1' - 2 + x_2' + x_3^+ - x_3^- \le 8$

$x_1' - 2 + x_2' - x_3^+ + x_3^- \le 12$

$z' = z - 2$

min $z' = -x_1' - 2x_2' - 3x_3^+ + 3x_3^-$

s.t. $x_1' + x_2' + x_3^+ - x_3^- \le 10$

$x_1' + x_2' - x_3^+ + x_3^- \le 14$

Standard form:

$$\min\ z' = -x_1' - 2x_2' - 3x_3^+ + 3x_3^-$$

$$x_1' + x_2' + x_3^+ - x_3^- + x_4 = 10$$

$$x_1' + x_2' - x_3^+ + x_3^- + x_5 = 14$$

$$x_1',\ x_2',\ x_3^+,\ x_3^-,\ x_4,\ x_5 \geqslant 0$$

	z'	x_1'	x_2'	x_3^+	x_3^-	x_4	x_5	RHS	
z'	1	1	2	3	-3	0	0	0	x_3^+ entering
x_4	0	1	1	$\boxed{1}$	-1	1	0	10	x_4 departing
x_5	0	1	1	-1	1	0	1	14	

pivot

$< 0 \rightarrow$ optimal solution

z'	1	-2	-1	0	0	-3	0	-30
x_3^+	0	1	1	1	-1	1	0	10
x_5	0	2	2	0	0	1	1	24

$z = z' + 2 = -30 + 2 = -28 \quad \therefore\ z'^* = -30$

$x_3^* = 10 \qquad x_3^+ = 10$

$\qquad\qquad\quad x_3^- = 0$

$x_1^* = x_1' - 2 = 0 - 2 = -2$

$x_2^* = -x_2' = 0$

$x_4^* = 0\ ,\ x_5^* = 24$

$$x = \begin{bmatrix} x_1' \\ x_2' \\ x_3^+ \\ x_3^- \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \\ 0 \\ 0 \\ 24 \end{bmatrix} = z'^* = -30$$