

ISE 3230: Systems Modeling and Optimization for Analytics

Homework 3

(Due Friday 10/02/2020 at 11:59pm)

For problems that require coding, you should use the **CVXPY** package in Python and *Gurobi* as the solver. Please include your code, the relevant part of the output, and comment on it. Do not include your code and/or its output as an appendix to your homework. If you are familiar with LaTeX or a suitable markdown language for equations and willing to prepare your homework in Jupyter Notebook or Lab, you are welcome to do so.

Problem 1. (10 pts) Transform the following linear program into the standard form

$$\begin{aligned} \text{Minimize } z &= 2x_1 - 3x_2 + 5x_3 + x_4 \\ \text{subject to} \\ &-x_1 + 3x_2 - x_3 + 2x_4 \leq -12 \\ &5x_1 + x_2 + 4x_3 - x_4 \geq 10 \\ &3x_1 - 2x_2 + x_3 - x_4 = -8 \\ &x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Problem 2. (10 pts) Solve the following linear program graphically. Solve the problem in Python and validate the solution you found graphically.

$$\begin{aligned} \text{Maximize } z &= x_1 + 2x_2 \\ \text{subject to} \\ &-2x_1 + x_2 \leq 2 \\ &2x_1 + 5x_2 \geq 10 \\ &x_1 - 4x_2 \leq 2 \\ &x_1, x_2 \geq 0 \end{aligned}$$

Problem 3. Given the following system of linear equations,

$$x_1 + 3x_2 - x_3 + x_4 = 30$$

$$2x_1 + x_2 + 2x_3 + x_4 = 15$$

- (a) (10 pts) Find all basic solutions.
- (b) (10 pts) For each basic solution, specify the basic and nonbasic variables and basis matrix B .

Problem 4. Consider the following linear program.

$$\text{Maximize } z = 2x_1 + 2x_2$$

subject to

$$x_1 + 2x_2 \leq 20$$

$$-3x_1 + 4x_2 \leq 20$$

$$3x_1 + 2x_2 \leq 36$$

$$x_1, x_2 \geq 0$$

- (a) (10 pts) Sketch the feasible region.
- (b) (5 pts) Write the problem in the standard form.
- (c) (5 pts) Identify the defining variable for each hyperplane bounding the feasible region.
- (d) (10 pts) Specify the basic and nonbasic variables for each extreme point.
- (e) (5 pts) Graphically determine the optimal extreme point and specify the optimal basis matrix.

Problem 5. Consider the following linear program.

$$\text{Minimize } z = -x_1 - 2x_2$$

subject to

$$x_1 + x_2 \leq 16$$

$$-x_1 + x_2 \leq 5$$

$$x_1 \leq 12$$

$$-x_1 + 3x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

- (a) (10 pts) Solve this problem graphically.
- (b) (10 pts) Solve this problem in Python.