

Problem # 1

Standard form:

$$\min_{x_1, x_2, x_3, x_4, x_5, x_6} z = 2x_1 - 3x_2 + 5x_3 + x_4$$

$$\text{subject to } -x_1 + 3x_2 - x_3 + 2x_4 + x_5 = -12$$

$$5x_1 + x_2 + 4x_3 - x_4 - x_5 = 10$$

$$3x_1 - 2x_2 + x_3 - x_4 = -8$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Problem 2

$$\max z = x_1 + 2x_2$$

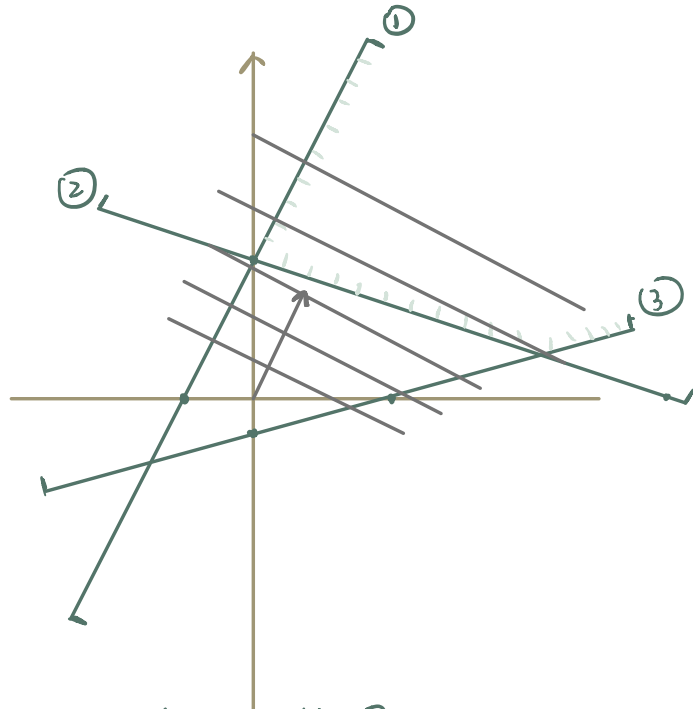
$$\text{s.t. } -2x_1 + x_2 \leq 2 \quad (1)$$

$$2x_1 + 5x_2 \geq 10 \quad (2)$$

$$x_1 - 4x_2 \leq 2 \quad (3)$$

$$x_1, x_2 \geq 0$$

$$\nabla z = \begin{pmatrix} dz/dx_1 \\ dz/dx_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$



The contour lines are parallel with (1).

There's no max solution.

```

3      """
4      Created on Fri Oct  2 16:20:06 2020
5
6      @author: fionafei
7      """
8
9      import cvxpy as cp
10
11
12
13      x = cp.Variable(2, nonneg = True) # vector variable
14
15
16      obj_func = x[0] + 2*x[1]
17      #obj_func_neg = -2*x[0] - 3*x[1]
18
19      constraints = []
20      constraints.append(-2*x[0] + x[1] <= 2)
21      constraints.append(2*x[0] + 5*x[1] >= 10)
22      constraints.append(x[0] - 4*x[1] <= 2)
23
24
25      problem = cp.Problem(cp.Maximize(obj_func), constraints)
26      #problem = cp.Problem(cp.Minimize(obj_func_neg), constraints)
27
28      #problem.solve(solver=cp.CVXOPT, verbose = True)
29      #problem.solve(verbose = True)
30      problem.solve(solver=cp.GUROBI, verbose = True)
31
32      print("obj_func =")
33      print(obj_func.value)
34      #print(obj_func_neg.value)
35      print("x =")
36      print(x.value)
37

```

```

Using license file /Users/fionafei/gurobi.lic
Academic license - for non-commercial use only
Parameter OutputFlag unchanged
Value: 1 Min: 0 Max: 1 Default: 1
Changed value of parameter QCPDual to 1
Prev: 0 Min: 0 Max: 1 Default: 0
Gurobi Optimizer version 9.0.3 build v9.0.3rc0 (mac64)
Optimize a model with 5 rows, 2 columns and 8 nonzeros
Model fingerprint: 0xfb429990
Coefficient statistics:
  Matrix range      [1e+00, 5e+00]
  Objective range   [1e+00, 2e+00]
  Bounds range      [0e+00, 0e+00]
  RHS range         [2e+00, 1e+01]
Presolve removed 3 rows and 1 columns
Presolve time: 0.01s

Solved in 0 iterations and 0.01 seconds
Infeasible or unbounded model
obj_func =
None
x =
None

```

Problem 3

$$x_1 + 3x_2 - x_3 + x_4 = 30$$

$$2x_1 + x_2 + 2x_3 + x_4 = 15$$

(a). basic solution.

$$A = \begin{pmatrix} 1 & 3 & -1 & 1 \\ 2 & 1 & 2 & 1 \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \text{ and } b = \begin{pmatrix} 30 \\ 15 \end{pmatrix}$$

the solution of this system is,

① x_1, x_2

$$\overset{B}{\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{5} & -\frac{1}{5} \end{pmatrix} \begin{pmatrix} 30 \\ 15 \end{pmatrix} = \begin{bmatrix} 3 \\ 9 \end{bmatrix}$$

$$\begin{cases} x_1 + 3x_2 = 30 \\ 2x_1 + x_2 = 15 \end{cases} \Rightarrow \begin{cases} x_1 = 3 \\ x_2 = 9 \end{cases}$$

It's solution is

$$(x_1, x_2) = 3, 9$$

$$(x_3, x_4) = 0, 0$$

$$\text{Basic: } \overset{B}{\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}}$$

Non-basic variables:

$$x_3, x_4$$

$$\text{basic var: } x_1, x_2$$

② x_1, x_3

$$\begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{4} \end{pmatrix} \cdot \begin{pmatrix} 30 \\ 15 \end{pmatrix} = \begin{pmatrix} \frac{75}{4} \\ -\frac{45}{4} \end{pmatrix}$$

$$\begin{cases} x_1 - x_3 = 30 \\ 2x_1 + 2x_3 = 15 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{75}{4} \\ x_3 = -\frac{45}{4} \end{cases}$$

It's solution is

$$(x_1, x_2, x_3, x_4) = \left(\frac{75}{4}, 0, -\frac{45}{4}, 0\right)$$

$$B = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$$

$$\text{N-B V: } x_2, x_4$$

$$\text{BV: } x_1, x_3$$

③ x_1, x_4

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 30 \\ 15 \end{bmatrix}$$

$$\begin{cases} x_1 + x_4 = 30 & x_1 = -15 \\ 2x_1 + x_4 = 15 & x_4 = 45 \end{cases}$$

It's solution is:

$$(x_1, x_2, x_3, x_4) = (-15, 0, 0, 45)$$

$$B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

NBV: x_2, x_3

BV: x_1, x_4

④ x_2, x_3

$$\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 15 \end{bmatrix}$$

$$\begin{cases} 3x_2 - x_3 = 30 & x_2 = \frac{25}{7} \\ x_2 + 2x_3 = 15 & x_3 = \frac{15}{7} \end{cases}$$

It's solution is:

$$(x_1, x_2, x_3, x_4) = (0, \frac{25}{7}, \frac{15}{7}, 0)$$

$$B = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

NBV: x_1, x_4

BV: x_2, x_3

⑤ x_2, x_4

$$\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 30 \\ 15 \end{bmatrix}$$

$$\begin{cases} 3x_2 + x_4 = 30 & x_2 = \frac{15}{2} \\ x_2 + x_4 = 15 & x_4 = \frac{15}{2} \end{cases}$$

It's solution is:

$$(x_1, x_2, x_3, x_4) = (0, \frac{15}{2}, 0, \frac{15}{2})$$

$$B = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

N-BV: x_1, x_3

BV: x_2, x_4

⑥ x_3, x_4

$$\begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 30 \\ 15 \end{bmatrix}$$

$$\begin{cases} -x_3 + x_4 = 30 & x_3 = -5 \\ 2x_3 + x_4 = 15 & x_4 = 25 \end{cases}$$

It's solution is:

$$(x_1, x_2, x_3, x_4) = (0, 0, -5, 25)$$

$$B = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$$

N-BV: x_1, x_2

BV: x_3, x_4

Problem #4

maximize $z = 2x_1 + 2x_2$

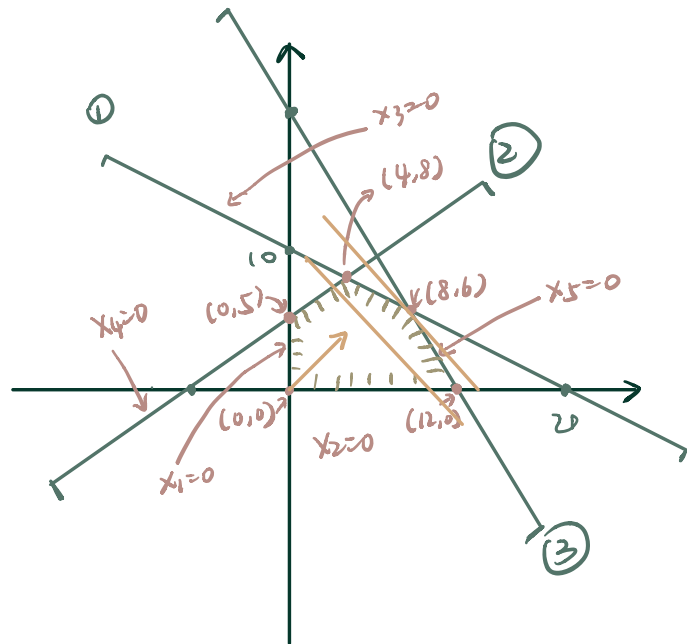
x_1, x_2, x_3, x_4, x_5

s.t. $x_1 + 2x_2 + x_3 = 20$

$-3x_1 + 4x_2 + x_4 = 20$

$3x_1 + 2x_2 + x_5 = 36$

$x_1, x_2, x_3, x_4, x_5 \geq 0$



(d)

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ -3 & 4 & 0 & 1 & 0 \\ 3 & 2 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \\ 36 \end{bmatrix}$$

point	Non-basic variable = 0	basic variable	Matrix B
(0,0)	x_1, x_2	x_3, x_4, x_5	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(12,0)	x_2, x_5	x_1, x_3, x_4	$\begin{bmatrix} 1 & 1 & 0 \\ -3 & 0 & 1 \\ 3 & 0 & 0 \end{bmatrix}$
(8,6)	x_3, x_5	x_1, x_2, x_4	$\begin{bmatrix} 1 & 2 & 0 \\ -3 & 4 & 1 \\ 3 & 2 & 0 \end{bmatrix}$
(0,5)	x_1, x_4	x_2, x_3, x_5	$\begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix}$
(4,8)	x_3, x_4	x_1, x_2, x_5	$\begin{bmatrix} 1 & 2 & 0 \\ -3 & 4 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

(e) The optimal extreme point is at (8,6).

The optimal basic matrix is $\begin{bmatrix} 1 & 2 & 0 \\ -3 & 4 & 1 \\ 3 & 2 & 0 \end{bmatrix}$

Problem #5

$$\text{minimize } z = -x_1 - 2x_2$$

s.t.

$$x_1 + x_2 \leq 16 \quad (1)$$

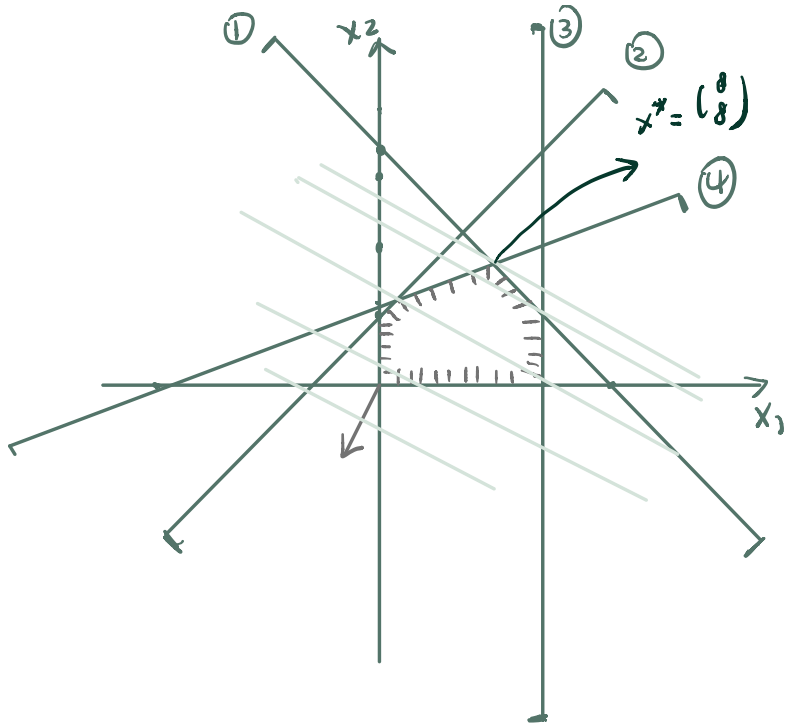
$$-x_1 + x_2 \leq 5 \quad (2)$$

$$x_1 \leq 12 \quad (3)$$

$$-x_1 + 3x_2 \leq 16 \quad (4)$$

$$x_1, x_2 \geq 0$$

$$\nabla z = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$



$$\begin{cases} x_1 + x_2 = 16 \\ -x_1 + 3x_2 \leq 16 \end{cases} \quad \begin{cases} x_1^* = 8 \\ x_2^* = 8 \end{cases}$$

$$z^* = -1 \cdot 8 + (-2) \cdot 8 = -8 - 16 = -24$$

```

1  #!/usr/bin/env python3
2  #- coding: utf-8 -*-
3  """
4  Created on Wed Sep 30 00:35:28 2020
5
6  @author: fionafei
7  """
8
9  import cvxpy as cp
10
11
12  x = cp.Variable(2, nonneg = True) # vector variable
13
14
15  obj_func_neg = -1*x[0] - 2*x[1]
16  #obj_func_neg = -2*x[0] - 3*x[1]
17
18  constraints = []
19  constraints.append(x[0]+x[1]<=16)
20  constraints.append(-1*x[0]+x[1]<=5)
21  constraints.append(x[0]<=12)
22  constraints.append(-1*x[0]+3*x[1]<=16)
23
24
25
26  #problem = cp.Problem(cp.Maximize(obj_func), constraints)
27  problem = cp.Problem(cp.Minimize(obj_func_neg), constraints)
28
29  #problem.solve(solver=cp.CVXOPT, verbose = True)
30  #problem.solve(verbose = True)
31  problem.solve(solver=cp.GUROBI, verbose = True)
32
33  print("obj_func =")
34  print(obj_func_neg.value)
35  #print(obj_func_neg.value)
36  print("x =")
37  print(x.value)
38

```

```

Value: 1 Min: 0 Max: 1 Default: 1
Changed value of parameter QCPDual to 1
Prev: 0 Min: 0 Max: 1 Default: 0
Gurobi Optimizer version 9.0.3 build v9.0.3rc0 (mac64)
Optimize a model with 6 rows, 2 columns and 9 nonzeros
Model fingerprint: 0x22d73c7e
Coefficient statistics:
  Matrix range    [1e+00, 3e+00]
  Objective range [1e+00, 2e+00]
  Bounds range    [0e+00, 0e+00]
  RHS range       [5e+00, 2e+01]
Presolve removed 3 rows and 0 columns
Presolve time: 0.00s
Presolved: 3 rows, 2 columns, 6 nonzeros

Iteration   Objective    Primal Inf.    Dual Inf.     Time
   0        -3.2000000e+01  1.007817e+01  0.000000e+00   0s
   1        -2.4000000e+01  0.000000e+00  0.000000e+00   0s

Solved in 1 iterations and 0.01 seconds
Optimal objective -2.400000000e+01
obj_func =
-24.0
x =
[ 8.  8.]

```