Screen Shot 2020-09-30 at 12.39.48 AMScreen Shot 2020-09-30 at 12.39.48 AMScreen Shot 2020-09-30 at 12.39.48 AM

Problem #1

Standard form:

$$\min_{x_1, x_2, x_3, x_4, x_5, x_6} \quad z = 2x_1 - 3x_2 + 5x_3 + x_4$$

subject to

$$-x_1 + 3x_2 - x_3 + 2x_4 + x_5 = -12$$

$$x_1 - 3x_2 + x_3 - 2x_4 - x_5 = 12$$

$$5x_1 + x_2 + 4x_3 - x_4 - x_5 = 10$$

$$-3x_1 + 2x_2 - x_3 + x_4 = 8$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Problem 2

$$\max z = x_1 + 2x_2$$

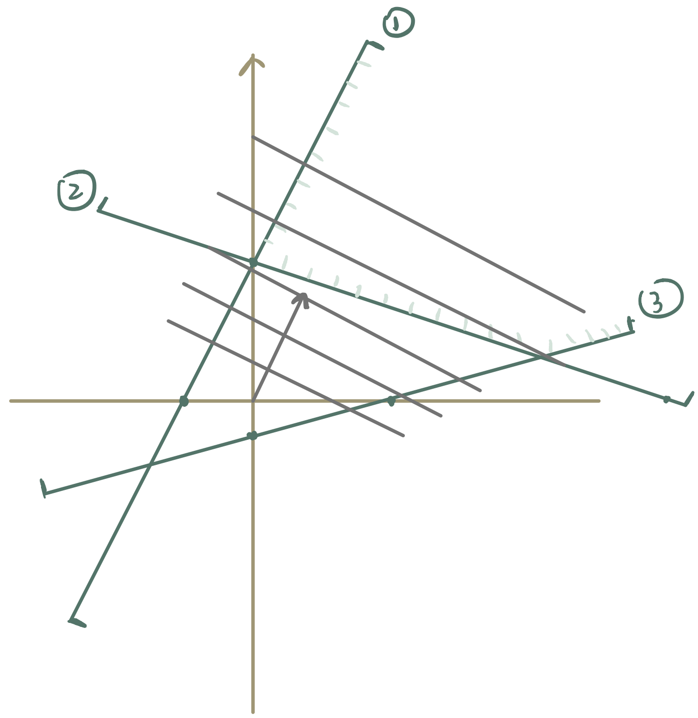
$$\text{s.t.} \quad -2x_1 + x_2 \leq 2 \quad (1)$$

$$2x_1 + 5x_2 \geq 10 \quad (2)$$

$$x_1 - 4x_2 \leq 2 \quad (3)$$

$$x_1, x_2 \geq 0$$

$$\nabla z = \begin{pmatrix} dz/dx_1 \\ dz/dx_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$



It's unbounded.

There's no max solution.

```
"""
Created on Fri Oct  2 16:20:06 2020

@author: fionafei
"""

import cvxpy as cp



x = cp.Variable(2, nonneg  = True) # vector variable


obj_func=x[0]+2*x[1]
#obj_func_neg=-2*x[0]-3*x[1]

constraints = []
constraints.append(-2*x[0]+x[1]<=2)
constraints.append(2*x[0]+5*x[1]>=10)
constraints.append(x[0]-4*x[1]<=2)


problem = cp.Problem(cp.Maximize(obj_func), constraints)
#problem = cp.Problem(cp.Minimize(obj_func_neg), constraints)

#problem.solve(solver=cp.CVXOPT,verbose = True)
#problem.solve(verbose = True)
problem.solve(solver=cp.GUROBI,verbose = True)

print("obj_func =")
print(obj_func.value)
#print(obj_func_neg.value)
print("x =")
print(x.value)
```

```
Using license file /Users/fionafei/gurobi.lic
Academic license - for non-commercial use only
Parameter OutputFlag unchanged
   Value: 1  Min: 0  Max: 1  Default: 1
Changed value of parameter QCPDual to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.0.3 build v9.0.3rc0 (mac64)
Optimize a model with 5 rows, 2 columns and 8 nonzeros
Model fingerprint: 0xfb429990
Coefficient statistics:
  Matrix range     [1e+00, 5e+00]
  Objective range  [1e+00, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [2e+00, 1e+01]
Presolve removed 3 rows and 1 columns
Presolve time: 0.01s

Solved in 0 iterations and 0.01 seconds
Infeasible or unbounded model
obj_func =
None
x =
None
```

Problem 3

$$x_1 + 3x_2 - x_3 + x_4 = 30$$
$$2x_1 + x_2 + 2x_3 + x_4 = 15$$

(a). basic solution.

$$A = \begin{pmatrix} 1 & 3 & -1 & 1 \\ 2 & 1 & 2 & 1 \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \text{ and } \quad b = \begin{pmatrix} 30 \\ 15 \end{pmatrix}$$

the solution of this system is,

① x_1, x_2

$$\overset{B}{\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{pmatrix} -\frac{1}{5} & \frac{3}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{pmatrix} \begin{pmatrix} 30 \\ 15 \end{pmatrix} = \begin{bmatrix} 3 \\ 9 \end{bmatrix}$$

$$\begin{cases} x_1 + 3x_2 = 30 \\ 2x_1 + x_2 = 15 \end{cases} \Rightarrow \begin{array}{l} x_1 = 3 \\ x_2 = 9 \end{array}$$

It's solution is

$(x_1, x_2) = 3, 9$

$(x_3, x_4) = 0, 0$

Basic B: $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$

non-basic variables:
x_3, x_4

basic var: x_1, x_2

② x_1, x_3

$$\begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{4} \end{pmatrix} \cdot \begin{pmatrix} 30 \\ 15 \end{pmatrix} = \begin{pmatrix} \frac{75}{4} \\ -\frac{45}{4} \end{pmatrix}$$

$$\begin{cases} x_1 - x_3 = 30 \\ 2x_1 + 2x_3 = 15 \end{cases} \Rightarrow \begin{array}{l} x_1 = \frac{75}{4} \\ x_3 = -\frac{45}{4} \end{array}$$

It's solution is

$(x_1, x_2, x_3, x_4) = (\frac{75}{4}, 0, -\frac{45}{4}, 0)$

$B = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$

N-B V: x_2, x_4

BV: x_1, x_3

③ x_1, x_4

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 30 \\ 15 \end{bmatrix}$$

$$\begin{cases} x_1 + x_4 = 30 \\ 2x_1 + x_4 = 15 \end{cases} \quad \begin{matrix} x_1 = -15 \\ x_4 = 45 \end{matrix}$$

It's solution is:

$(x_1, x_2, x_3, x_4) = (-15, 0, 0, 45)$

$B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

NBV: x_2, x_3

BV: x_1, x_4

④ x_2, x_3

$$\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 15 \end{bmatrix}$$

$$\begin{cases} 3x_2 - x_3 = 30 \\ x_2 + 2x_3 = 15 \end{cases} \quad \begin{matrix} x_2 = \frac{75}{7} \\ x_3 = \frac{15}{7} \end{matrix}$$

It's solution is:

$(x_1, x_2, x_3, x_4) = (0, \frac{75}{7}, \frac{15}{7}, 0)$

$B = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

NBV: x_1, x_4

BV: x_2, x_3

⑤ x_2, x_4

$$\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 30 \\ 15 \end{bmatrix}$$

$$\begin{cases} 3x_2 + x_4 = 30 \\ x_2 + x_4 = 15 \end{cases} \quad \begin{matrix} x_2 = \frac{15}{2} \\ x_4 = \frac{15}{2} \end{matrix}$$

It's solution is:

$(x_1, x_2, x_3, x_4) = (0, \frac{15}{2}, 0, \frac{15}{2})$

$B = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$

N-BV: x_1, x_3

BV: x_2, x_4

⑥ x_3, x_4

$$\begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 30 \\ 15 \end{bmatrix}$$

$$\begin{cases} -x_3 + x_4 = 30 \\ 2x_3 + x_4 = 15 \end{cases} \quad \begin{matrix} x_3 = -5 \\ x_4 = 25 \end{matrix}$$

It's solution is:

$(x_1, x_2, x_3, x_4) = (0, 0, -5, 25)$

$B = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$

N-BV: x_1, x_2

BV: x_3, x_4

Problem #4

maximize $z = 2x_1 + 2x_2$

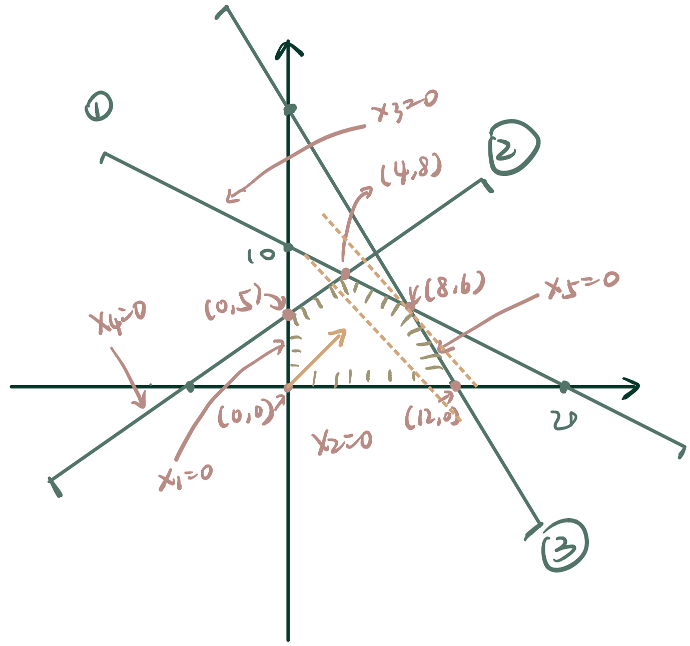
x_1, x_2, x_3, x_4, x_5

s.t. $x_1 + 2x_2 + x_3 = 20$

$-3x_1 + 4x_2 + x_4 = 20$

$3x_1 + 2x_2 + x_5 = 36$

$x_1, x_2, x_3, x_4, x_5 \geq 0$



(d)

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ -3 & 4 & 0 & 1 & 0 \\ 3 & 2 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \\ 36 \end{bmatrix}$$

point	Non-basic variable = 0	basic variable	Matrix B
(0,0)	x_1, x_2	x_3, x_4, x_5	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(12,0)	x_2, x_5	x_1, x_3, x_4	$\begin{bmatrix} 1 & 1 & 0 \\ -3 & 0 & 1 \\ 3 & 0 & 0 \end{bmatrix}$
(8,6)	x_3, x_5	x_1, x_2, x_4	$\begin{bmatrix} 1 & 2 & 0 \\ -3 & 4 & 1 \\ 3 & 2 & 0 \end{bmatrix}$
(0,5)	x_1, x_4	x_2, x_3, x_5	$\begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix}$
(4,8)	x_3, x_4	x_1, x_2, x_5	$\begin{bmatrix} 1 & 2 & 0 \\ -3 & 4 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

(e) The optimal extreme point is at (8,6).

The optimal basic matrix is $\begin{bmatrix} 1 & 2 & 0 \\ -3 & 4 & 1 \\ 3 & 2 & 0 \end{bmatrix}$

Problem #5

minimize $z = -x_1 - 2x_2$

s.t.

$x_1 + x_2 \le 16$ ①

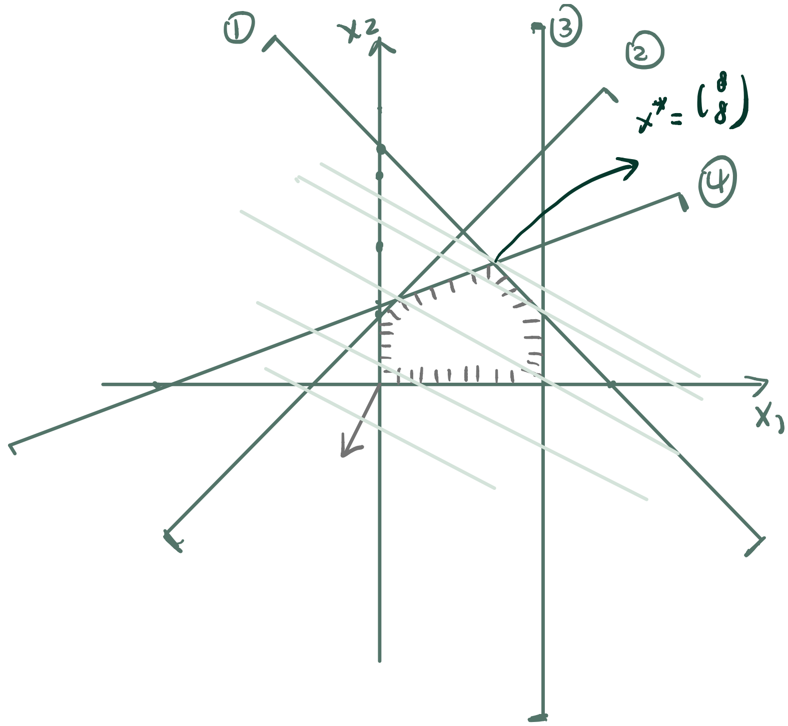
$-x_1 + x_2 \le 5$ ②

$x_1 \le 12$ ③

$-x_1 + 3x_2 \le 16$ ④

$x_1, x_2 \ge 0$

$\nabla z = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$



$$\begin{cases} x_1 + x_2 = 16 \\ -x_1 + 3x_2 \le 16 \end{cases} \qquad \begin{cases} x_1^* = 8 \\ x_2^* = 8 \end{cases}$$

$$z^* = -1 \cdot 8 + (-2) \cdot 8 = -8 - 16$$
$$= -24$$

```python
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
Created on Wed Sep 30 00:35:28 2020

@author: fionafei
"""

import cvxpy as cp


x = cp.Variable(2, nonneg  = True) # vector variable


obj_func_neg= -1*x[0] - 2*x[1]
#obj_func_neg=-2*x[0]-3*x[1]

constraints = []
constraints.append(x[0]+x[1]<=16)
constraints.append(-1*x[0]+x[1]<=5)
constraints.append(x[0]<=12)
constraints.append(-1*x[0]+3*x[1]<=16)



#problem = cp.Problem(cp.Maximize(obj_func), constraints)
problem = cp.Problem(cp.Minimize(obj_func_neg), constraints)

#problem.solve(solver=cp.CVXOPT,verbose = True)
#problem.solve(verbose = True)
problem.solve(solver=cp.GUROBI,verbose = True)

print("obj_func =")
print(obj_func_neg.value)
#print(obj_func_neg.value)
print("x =")
print(x.value)
```

```
   Value: 1  Min: 0  Max: 1  Default: 1
Changed value of parameter QCPDual to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.0.3 build v9.0.3rc0 (mac64)
Optimize a model with 6 rows, 2 columns and 9 nonzeros
Model fingerprint: 0x22d73c7e
Coefficient statistics:
  Matrix range     [1e+00, 3e+00]
  Objective range  [1e+00, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [5e+00, 2e+01]
Presolve removed 3 rows and 0 columns
Presolve time: 0.00s
Presolved: 3 rows, 2 columns, 6 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0   -3.2000000e+01   1.007817e+01   0.000000e+00      0s
       1   -2.4000000e+01   0.000000e+00   0.000000e+00      0s

Solved in 1 iterations and 0.01 seconds
Optimal objective -2.400000000e+01
obj_func =
-24.0
x =
[8. 8.]
```