

Problem #1

$$\begin{aligned} \text{(a)} \quad & -u_1(1-y) \leq f_1(x) \\ & -u_2 y \leq f_2(x) \\ & y \in \{0, 1\} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & f_1(x) \leq u_1 x (1-y) \\ & -u_2 y \leq f_2(x) \\ & y \in \{0, 1\} \end{aligned}$$

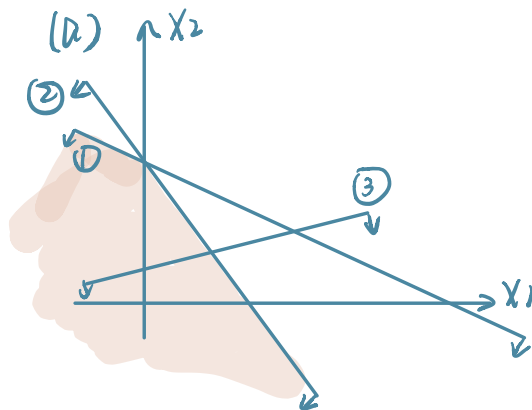
$$\begin{aligned} \text{(c)} \quad & (u_1 + u_2)(1-y) \geq -f_1(x) \quad -u_1 y \leq f_1(x) \\ & (u_1 + u_2)y \geq -f_2(x) \quad -u_2(1-y) \leq f_2(x) \\ & y \in \{0, 1\} \end{aligned}$$

Problem # 2

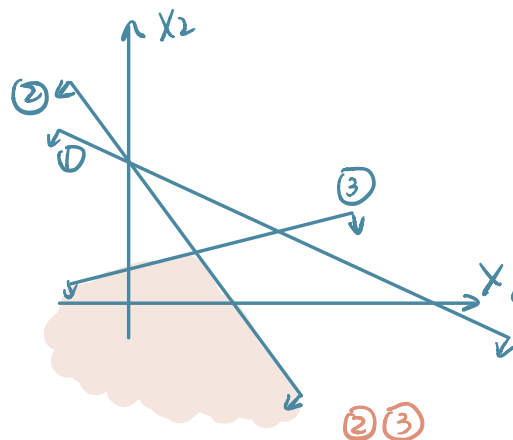
$$x_1 + 2x_2 \leq 12 \quad (1)$$

$$3x_1 + 2x_2 \leq 12 \quad (2)$$

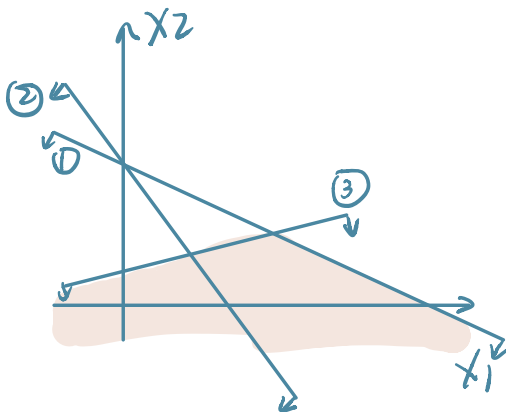
$$-x_1 + 3x_2 \leq 3 \quad (3)$$



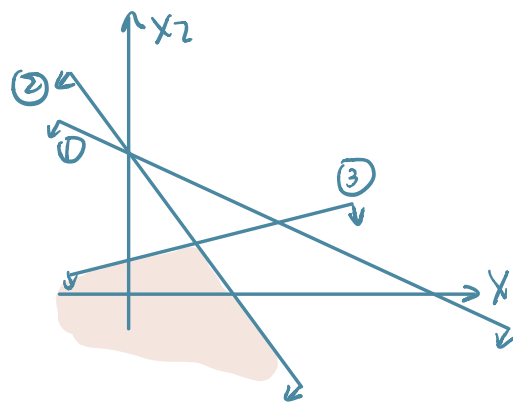
(1)(2)



(2)(3)



(1)(3)



(1)(2)(3)

(b)

$$x_1 + 2x_2 \leq 12 + M(1 - y_1)$$

$$3x_1 + 2x_2 \leq 12 + M(1 - y_2)$$

$$-x_1 + 3x_2 \leq 3 + M(1 - y_3)$$

$$y_1 + y_2 + y_3 \geq 2$$

y_1, y_2, y_3 are binary numbers.

$M = \text{constant}$

Problem #3.

mixed integer programming
maximize $z = f_1(x_1) + f_2(x_2)$

s.t.

$$x_1 + 3x_2 \leq 12 + M(1 - y_1)$$
$$2x_1 + x_2 \leq 16 + M(1 - y_2)$$
$$x_1 + x_2 \leq 9 + M(1 - y_3)$$

$$\max z = f_1(x_1) + f_2(x_2)$$

$$y_1 + y_2 + y_3 \geq 2$$

$$x_1 = x_{11} + x_{12}$$

$$x_2 = x_{21} + x_{22}$$

$$f_1(x_1) = 10 + 2x_{11} + x_{12}$$

$$f_2(x_2) = 8 + x_{21} + 3x_{22}$$

$$5z_1 \leq x_{11} \leq 5$$

$$0 \leq x_{12} \leq 8$$

$$0 \leq x_{12} \leq M$$

$$2z_2 \leq x_{21} \leq 2$$

$$0 \leq x_{22} \leq 8$$

$$0 \leq x_{22} \leq M$$

$$x_1, x_2 \geq 0$$

y_1, y_2, y_3, z_1, z_2 are binary #s.

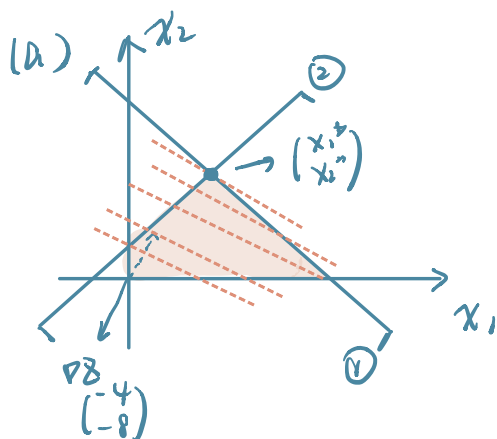
Problem #4.

$$\text{minimize } z = -4x_1 - 8x_2$$

$$\text{s.t. } 2x_1 + 2x_2 \leq 19$$

$$-2x_1 + 2x_2 \leq 3$$

$$x_1, x_2 \in \mathbb{I}^+$$



$$x_1^* = 4$$

$$x_2^* = 5$$

$$z^* = -4 \times 4 - 8 \times 5 = -16 - 40 = -56$$

(b) cvxpy

```

3
4
5 @author: fionafei
6 """
7
8
9 import cvxpy as cp
10
11
12
13 x = cp.Variable(2, nonneg = True) # vector variable
14
15
16 #obj_func=170*x[0]+220*x[1]+250*x[2]+55*x[3]
17 obj_func_neg=-4*x[0]-8*x[1]
18
19 constraints = []
20 constraints.append(x[0]>=0)
21 constraints.append(x[1]>=0)
22 constraints.append(2*x[0]+2*x[1]<=19)
23 constraints.append(-2*x[0]+2*x[1]<=3)
24
25
26
27 #problem = cp.Problem(cp.Maximize(obj_func), constraints)
28 problem = cp.Problem(cp.Minimize(obj_func_neg), constraints)
29
30 #problem.solve(solver=cp.CVXOPT, verbose = True)
31 #problem.solve(verbose = True)
32 problem.solve(solver=cp.GUROBI, verbose = True)
33
34 print("obj_func =")
35 #print(obj_func.value)
36 print(obj_func_neg.value)
37 print("x =")
38 print(x.value)

```

```

Value: 1 Min: 0 Max: 1 Default: 1
Changed value of parameter QCPDual to 1
Prev: 0 Min: 0 Max: 1 Default: 0
Gurobi Optimizer version 9.0.3 build v9.0.3rc0 (mac64)
Optimize a model with 6 rows, 2 columns and 8 nonzeros
Model fingerprint: 0xe66ed663
Coefficient statistics:
  Matrix range     [1e+00, 2e+00]
  Objective range  [4e+00, 8e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [3e+00, 2e+01]
Presolve removed 6 rows and 2 columns
Presolve time: 0.00s
Presolve: All rows and columns removed
Iteration   Objective    Primal Inf.    Dual Inf.      Time
   0        -6.0000000e+01   0.000000e+00   0.000000e+00      0s

Solved in 0 iterations and 0.01 seconds
Optimal objective -6.000000000e+01
obj_func =
-60.0
x =
[4.  5.]

```

Problem #5

7 full, 7 half, 7 empty.

↓

7. same.

$$\text{Total wine} = 7 + \frac{7}{2} = \frac{14+7}{2} = \frac{21}{2}.$$

Amount for each person:

$$\frac{21}{2} \cdot \frac{1}{3} = \underline{\underline{\frac{7}{2}}}$$

x_{1i} = # of full bottles to person 1, 2, 3

x_{2i} = # ... half-full ...

x_{3i} = # ... empty ...

$i = 1, 2, 3.$

person 1

$$x_{11} + x_{12} + x_{13} = 7$$

$$x_{21} + x_{22} + x_{23} = 7$$

$$x_{31} + x_{32} + x_{33} = 7$$

$$x_{11} + x_{21} + x_{31} = 7$$

$$x_{12} + x_{22} + x_{32} = 7$$

$$x_{13} + x_{23} + x_{33} = 7$$

$$x_{11} + \frac{x_{12}}{2} = \frac{7}{2}$$

$$x_{21} + \frac{x_{22}}{2} = \frac{7}{2}$$

$$x_{31} + \frac{x_{32}}{2} = \frac{7}{2}$$

$$x_{ij} \in \mathbb{I}^+, \quad i=1, 2, 3, \quad j=1, 2, 3$$

objective fun:

$$\min z = \sum_{i,j=1}^3 0 \cdot x_{ij}$$

```

8 import cvxpy as cp
9
10
11
12
13 x = cp.Variable((3,3), nonneg = True) # vector variable
14
15
16 #obj_func=170*x[0]+220*x[1]+250*x[2]+55*x[3]
17 obj_func_neg=0*x[0,0]+0*x[0,1]+0*x[0,2]+0*x[1,0]+0*x[1,1]+0*x[1,2]+0*x[2,0]+0*x[2,1]+0*x[2,2]
18
19 constraints = []
20 constraints.append(x[0,0]>=0)
21 constraints.append(x[0,1]>=0)
22 constraints.append(x[0,2]>=0)
23 constraints.append(x[1,0]>=0)
24 constraints.append(x[1,1]>=0)
25 constraints.append(x[1,2]>=0)
26 constraints.append(x[2,0]>=0)
27 constraints.append(x[2,1]>=0)
28 constraints.append(x[2,2]>=0)
29
30 constraints.append(x[0,0]+x[1,0]+x[2,0]==7)
31 constraints.append(x[0,1]+x[1,1]+x[2,1]==7)
32 constraints.append(x[0,2]+x[1,2]+x[2,2]==7)
33
34 constraints.append(x[0,0]+x[0,1]+x[0,2]==7)
35 constraints.append(x[1,0]+x[1,1]+x[1,2]==7)
36 constraints.append(x[2,0]+x[2,1]+x[2,2]==7)
37
38 constraints.append(x[0,0]+x[0,1]/2==3.5)
39 constraints.append(x[1,0]+x[1,1]/2==3.5)
40 constraints.append(x[2,0]+x[2,1]/2==3.5)
41
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43
44
45 #problem = cp.Problem(cp.Maximize(obj_func), constraints)
46 problem = cp.Problem(cp.Minimize(obj_func_neg), constraints)
47
48 #problem.solve(solver=cp.CVXOPT, verbose = True)
49 #problem.solve(verbose = True)
50 problem.solve(solver=cp.GUROBI, verbose = True)
51
52 print("obj_func =")
53 #print(obj_func.value)
54 print(obj_func_neg.value)
55 print("x =")
56 print(x.value)
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```

Prev: 0 Min: 0 Max: 1 Default: 0
Gurobi Optimizer version 9.0.3 build v9.0.3rc0 (mac64)
Optimize a model with 27 rows, 9 columns and 42 nonzeros
Model fingerprint: 0xb402285e
Coefficient statistics:
Matrix range [5e-01, 1e+00]
Objective range [0e+00, 0e+00]
Bounds range [0e+00, 0e+00]
RHS range [4e+00, 7e+00]
Presolve removed 27 rows and 9 columns
Presolve time: 0.01s
Presolve: All rows and columns removed
Iteration Objective Primal Inf. Dual Inf. Time
0 0.0000000e+00 0.000000e+00 0.000000e+00 0s
Solved in 0 iterations and 0.01 seconds
Optimal objective 0.00000000e+00
obj_func =
0.0
x =
[[0. 7. 0.]
[3.5 0. 3.5]
[3.5 0. 3.5]]

Problem #6.

z_i = if choose company i , $i = A, B, C$

x_i = # of minutes for company i , $i = A, B, C$

y_i = amount of money paid for company i , $i = A, B, C$

$$\text{min } z = y_A + y_B + y_C$$

$$\text{s.t. } x_A = x_{A1} + x_{A2}$$

$$x_C = x_{C1} + x_{C2}$$

$$x_A + x_B + x_C = W \geq 3000$$

$$0 \leq x_A \leq z_A M$$

$$0 \leq x_B \leq z_B M$$

$$0 \leq x_C \leq z_C M$$

$$1000 z_{1A} \leq x_{A1} \leq 1000$$

$$0 \leq x_{A2} \leq z_{A,M}$$

$$1000 z_{1C} \leq x_{C1} \leq 1000$$

$$0 \leq x_{C2} \leq z_{1C} M$$

$$y_A = 10 z_A + 0.05 x_{A1} + 0.04 x_{A2}$$

$$y_B = 20 z_B + 0.04 x_B$$

$$y_C = 25 z_C + 0.05 x_{C1} + 0.035 x_{C2}$$

$$z_A + z_B + z_C = 1$$

$z_A, z_B, z_C, z_{1A}, z_{1C}$ are binary numbers


```

1  #!/usr/bin/env python3
2  # -*- coding: utf-8 -*-
3  """
4  Created on Sat Nov 14 13:10:17 2020
5
6  @author: fionafei
7  """
8
9  import cvxpy as cp
10
11  x = cp.Variable((5,1), integer = True) # vector variable
12  z = cp.Variable((3,1), boolean = True)
13  w = cp.Variable((2,1), boolean = True)
14
15
16  #obj_func=170*x[0]+220*x[1]+250*x[2]+55*x[3]
17  obj_func_neg=10*z[0,0]+0.05*x[0,0]+0.04*x[1,0]+20*z[1,0]+0.04*x[2,0]+25*z[2,0]
18  +0.05*x[3,0]+0.035*x[4,0]
19
20  constraints = []
21  constraints.append(x[0,0]+x[1,0]+x[2,0]+x[3,0]+x[4,0]>=3000)
22  constraints.append(x>=0)
23  constraints.append(x[0,0]+x[1,0]<=z[0,0]*3000)
24  constraints.append(x[2,0]<=z[1,0]*3000)
25  constraints.append(x[3,0]+x[4,0]<=z[2,0]*3000)
26  constraints.append(x[0,0]<=1000)
27  constraints.append(x[0,0]>=1000+w[0,0])
28  constraints.append(x[1,0]<=3000+w[0,0])
29  constraints.append(x[3,0]<=1000)
30  constraints.append(x[3,0]>=1000+w[1,0])
31  constraints.append(x[4,0]<=3000+w[1,0])
32  constraints.append(z[0,0]+z[1,0]+z[2,0]==1)
33  |
34
35  #problem = cp.Problem(cp.Maximize(obj_func), constraints)
36  problem = cp.Problem(cp.Minimize(obj_func_neg), constraints)
37
38  #problem.solve(solver=cp.CVXOPT,verbose = True)
39  #problem.solve(verbose = True)
40  problem.solve(solver=cp.GUROBI,verbose = True)
41
42  print("obj_func =")
43  #print(obj_func.value)
44  print(obj_func_neg.value)
45  print("x =")
46  print(x.value)
47

```

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Matrix range      [1e+00, 3e+03]
Objective range   [4e-02, 2e+01]
Bounds range      [1e+00, 1e+00]
RHS range         [1e+00, 3e+03]
Presolve removed 16 rows and 10 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds
Thread count was 1 (of 4 available processors)

Solution count 1: 140

Optimal solution found (tolerance 1.00e-04)
Best objective 1.400000000000e+02, best bound 1.400000000000e+02, gap 0.0000%
obj_func =
140.0
x =
[[1000.]
 [2000.]
 [  0.]
 [  0.]
 [  0.]]

```