AE 410 Assignment -2

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ASSIGNMENT-2

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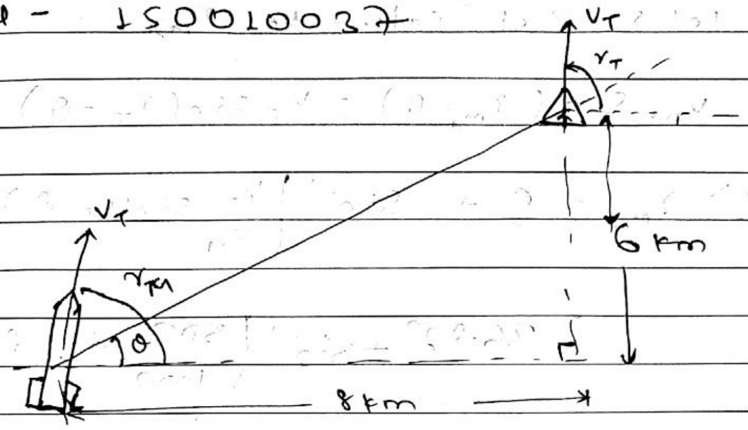
Q.1.



from above figure, we have $\tan\theta = \frac{6}{8}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

Also according to question

$V_T = 500\ ms^{-1}$; $\gamma_T = 0^\circ$

$V_M = 1000\ ms^{-1}$; $\gamma_M = 60^\circ$

(a.) LOS rate calculation, $\dot{\theta}$

$$\dot{\theta} = \frac{V_T \sin(\gamma_T - \theta) - V_M \sin(\gamma_M - \theta)}{r}$$

$$= \frac{500 \times \sin(0^\circ - 36.87^\circ) - 1000 \times \sin(60^\circ - 36.87^\circ)}{\sqrt{8000^2 + 6000^2}}$$

$$= -0.0693 \text{ rad/s} \simeq -0.07 \text{ rad/s}$$

Closing speed, $V_c = -V_r$

$$= -[V_T \cos(\gamma_T - \theta) - V_M \cos(\gamma_M - \theta)]$$

$$= -[500 \times \cos(0^\circ - 36.87^\circ) - 1000 \times \cos(60^\circ - 36.87^\circ)]$$

$$= 519.62\ ms^{-1}$$

Missile's heading error calculation
let's first calculate γ_{Md}

$$V_M \sin(\gamma_{Md} - \theta) = V_T \sin(\gamma_T - \theta)$$

$$\Rightarrow \gamma_{Md} = \theta + \sin^{-1}\left[\frac{V_T}{V_M} \sin(\gamma_T - \theta)\right]$$

$$= 36.87^\circ + \sin^{-1}\left(\frac{500}{1000} \sin(0^\circ - 36.87^\circ)\right)$$

$$= 19.41^\circ$$

Heading error, $HE = \gamma_M - \gamma_{Md} = 60^\circ - 19.41^\circ$
$= 40.59^\circ$

(b) If target moves towards missile at flight path angle of 150° with reference,

$$\gamma_T = 150^\circ$$

$$\dot{\theta} = \frac{V_T \sin(\gamma_T - \theta) - V_M \sin(\gamma_M - \theta)}{r} = 6.31 \times 10^{-3} \text{ rad s}^{-1}$$

$$\dot{\theta} \simeq 0.0063 \text{ rad s}^{-1}$$

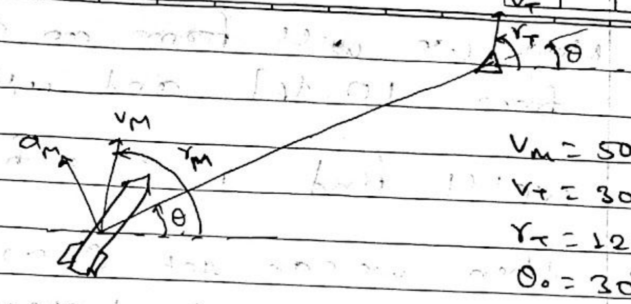
Since LOS rate is non zero, hence not on collision triangle but rate is small and nearly zero.

$$V_{r_0} = V_T \cos(\gamma_T - \theta) - V_M \cos(\gamma_M - \theta) = -1116.03 \text{ ms}^{-1}$$

$$V_{\theta_0} = V_T \sin(\gamma_T - \theta) - V_M \sin(\gamma_M - \theta) = 66.99 \text{ ms}^{-1}$$

$$r_{miss} = r_0 \sqrt{\frac{V_{\theta_0}^2}{V_{r_0}^2 + V_{\theta_0}^2}} = 599.17 \text{ m}$$

Q.2



$V_M = 500\ m/s$
$V_T = 300\ m/s$
$\gamma_T = 120^\circ$
$\theta_0 = 30^\circ$
$r_0 = 10\ km$

for pure persuit guidance, $\gamma_M = \theta$
since the missile always chases the target by directly moving towards it.

Now, we have the relation

$$t = \frac{r_0[V_T\cos(\gamma_T-\theta_0)+V_M] - r[V_T\cos(\gamma_T-\theta)+V_M]}{-V_T^2+V_M^2} \quad \text{—(I)}$$

At ~~the~~ collision, $t = t_f$ & $r = 0$

$$\Rightarrow t_f = \frac{r_0[V_T\cos(\gamma_T-\theta_0)+V_M]}{-V_T^2+V_M^2} = 31.25\ \text{sec.} \quad \text{—(II)}$$

we have relation between r and θ as

$$r = K\frac{\{\sin(\gamma_T-\theta)\}^{\nu-1}}{[1+\cos(\gamma_T-\theta)]^{\nu}} \quad \text{—(III)}$$

$\nu = \frac{V_T}{V_M} = 0.6$

$K = \frac{r_0[1+\cos(\gamma_T-\theta_0)]^{\nu}}{[\sin(\gamma_T-\theta_0)]^{\nu-1}} = 10\,000$

from (I) & (II) we have

$$\boxed{\theta = \cancel{\gamma_T} - \cos^{-1}\left[\frac{(t_f-t)(V_T^2-V_M^2)V_M}{\cancel{\gamma} V_T} - \cancel{\frac{V_M}{V_T}}\right]} \quad \text{—(IV)}$$

$$\theta = \gamma_T - \cos^{-1}\left[\frac{(t_f-t)(V_T^2+V_M^2)}{\gamma V_T} - \frac{V_M}{V_T}\right] \quad \text{—(IV)}$$

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Now we will form an array of time
~~from [0, t_f] and using (III) & (IV)~~
we'll find γ as a function of time

then we can get θ as function of time
then we can get trajectories of missile
~~of~~ and target
And Guidance command can be found
by

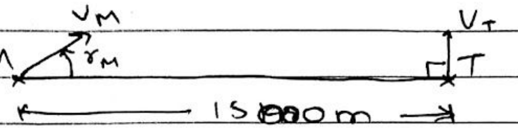
$$a_M = \frac{V_M V_T \sin^2(\gamma_T - \theta)}{K\left\{\tan\left(\frac{\gamma_T - \theta}{2}\right)\right\}^{\nu}}$$

but since no solver of MATLAB (fsolve, fzero, isqnonlin) was able to give solution to implicit equation correctly.
so I tried another way.

used $\dot{\gamma}, \dot{\theta}, \dot{\gamma}_M, a_M$ expressions and integrated for a small time step to get next step value by $\gamma(k+1) = \gamma(k) + \text{rdot} * dt$

Plots:-

Q.3.



$r_0 = 15000$ m ; $V_M = 500\ ms^{-1}$; $V_T = 300$ m/s

$\theta_0 = 0°$; $\gamma_T = 90°$

$$V_r = V_T \cos(\gamma_T - \theta) - V_M \cos(\gamma_M - \theta)$$

$$= V_T \cos(90) - V_M \cos(\gamma_M - \theta)$$

$$V_\theta = V_T \sin(\gamma_T - \theta) - V_M \sin(\gamma_M - \theta) = 0$$

$$\Rightarrow 300 \times \sin(90°) - 500 \sin(\gamma_M) = 0$$

$$\Rightarrow \gamma_M = \sin^{-1}\left(\frac{3}{5}\right) = 36.87°$$

Q.3.(a) γ_M, launch angle $= 36.87°$.

Q.3.(b) HE $= \pm 20°$

In PPN, $\dot{\gamma}_M = N\dot{\theta} \Rightarrow a_M = N V_M \dot{\theta}$

for TPN $\dot{\gamma}_M = \dfrac{a_M \cos(\gamma_M - \theta)}{V_M}$; $a_M = -N' V_r \dot{\theta}$

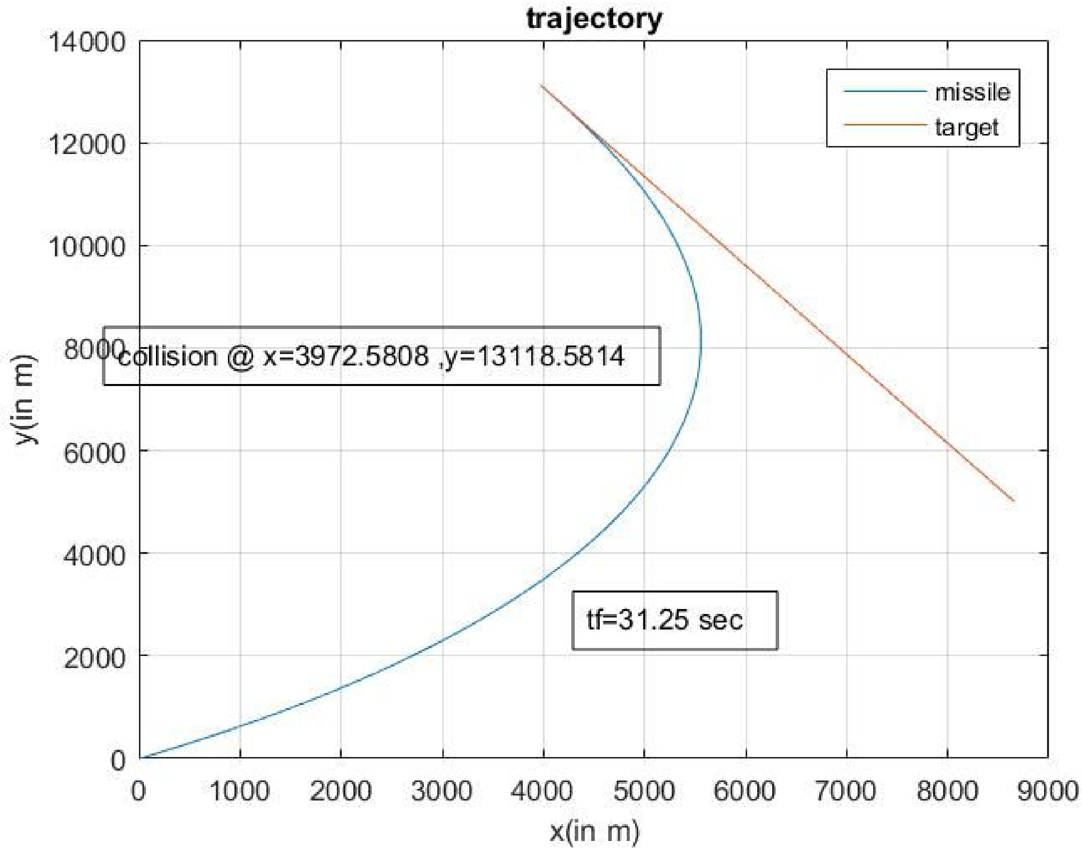
$$\dot{V}_M = a_M \sin(\gamma_M - \theta)$$

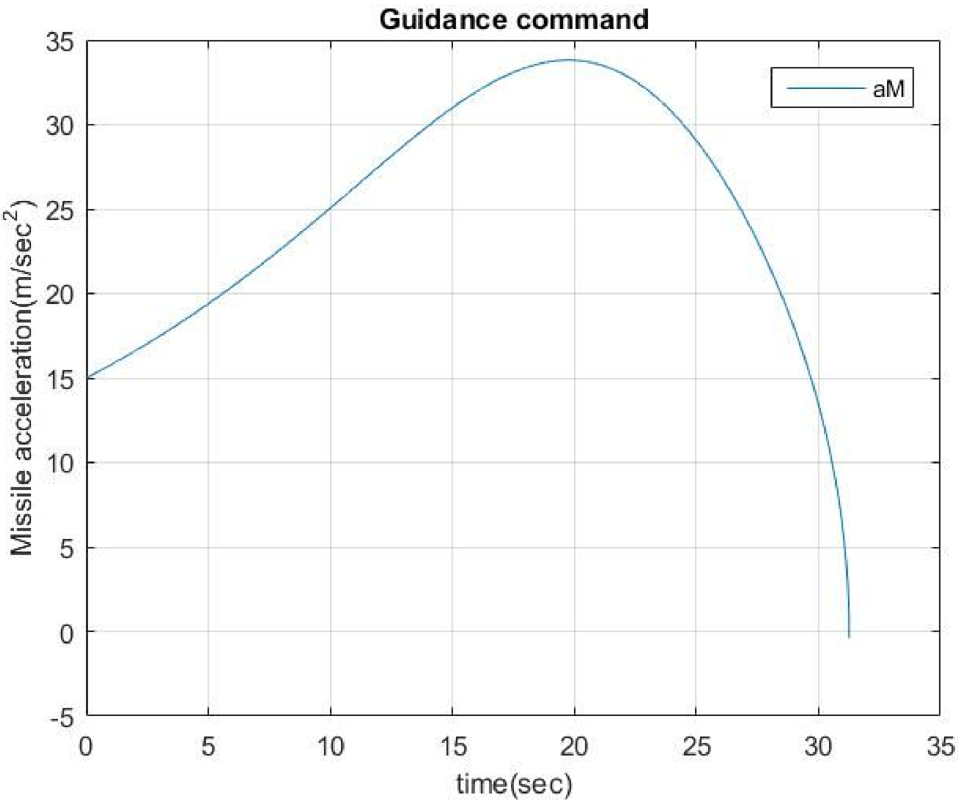
Plots:-

Plots for Question 2 and 3

Q.2.(a)

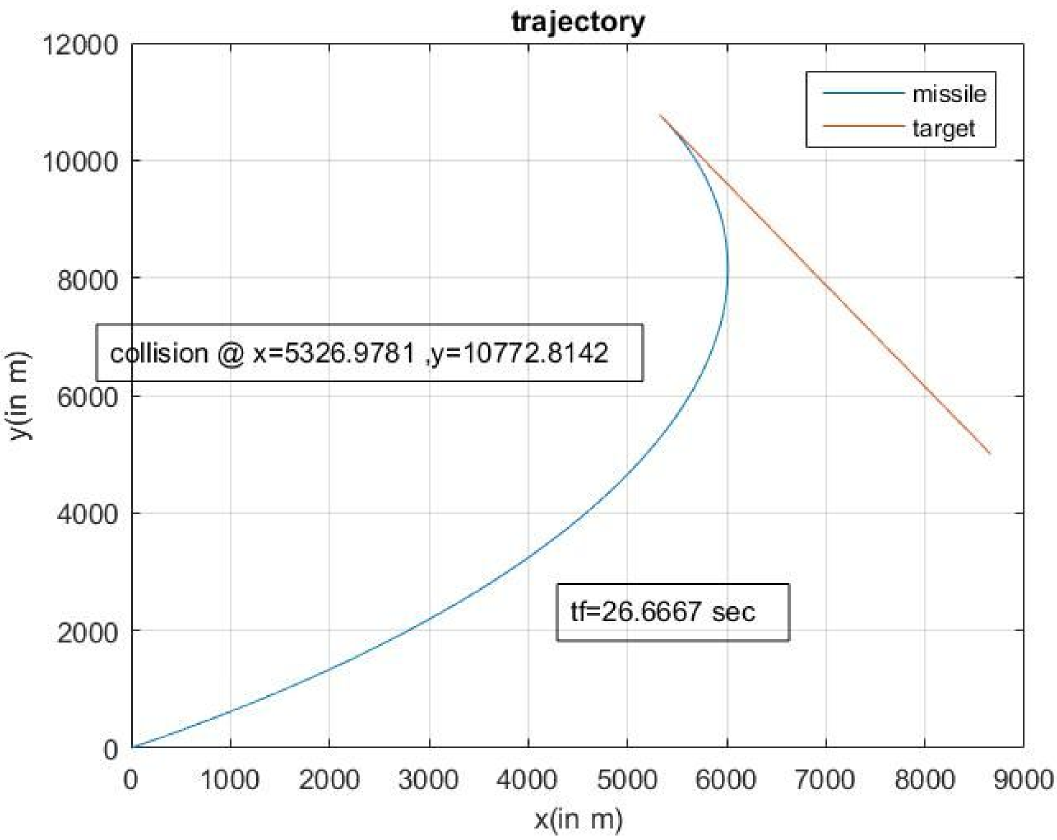
Trajectory and guidance command plot when missile follows pure pursuit guidance.

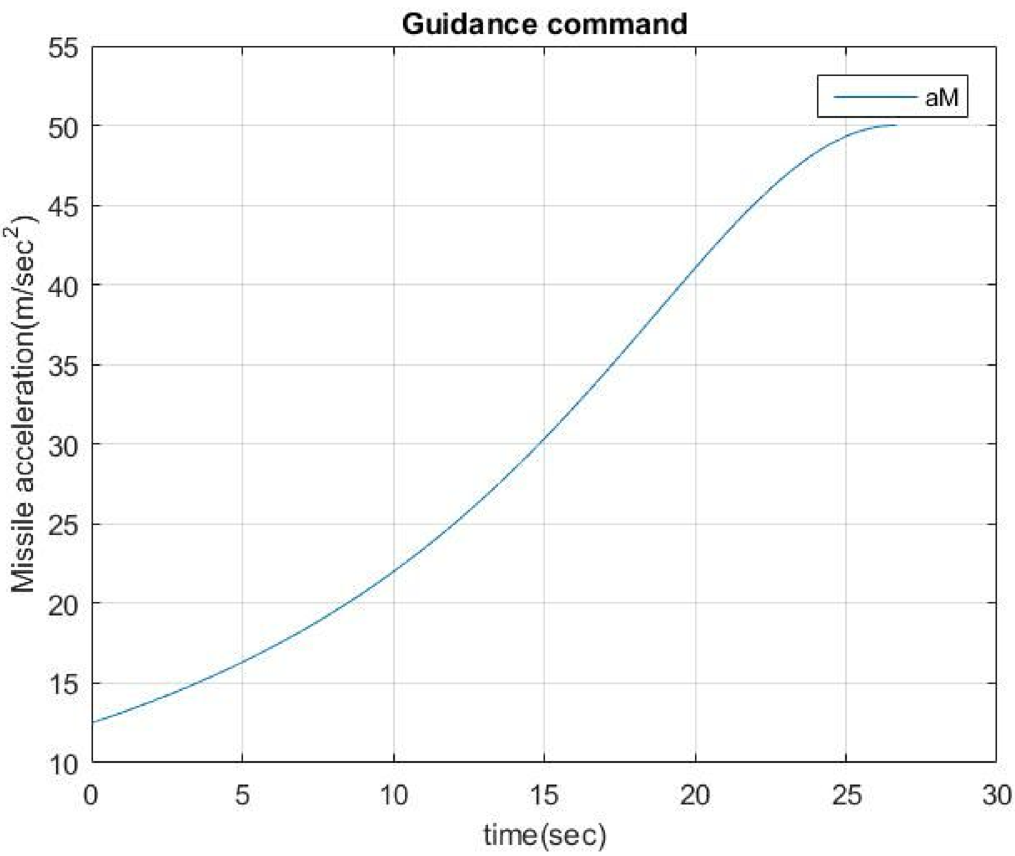




Q.2.(b)

When velocity of target, VT = 250 m/s





Here, result agrees with the theory. The acceleration requirement towards the interception depends upon $\nu = \frac{V_M}{V_T}$

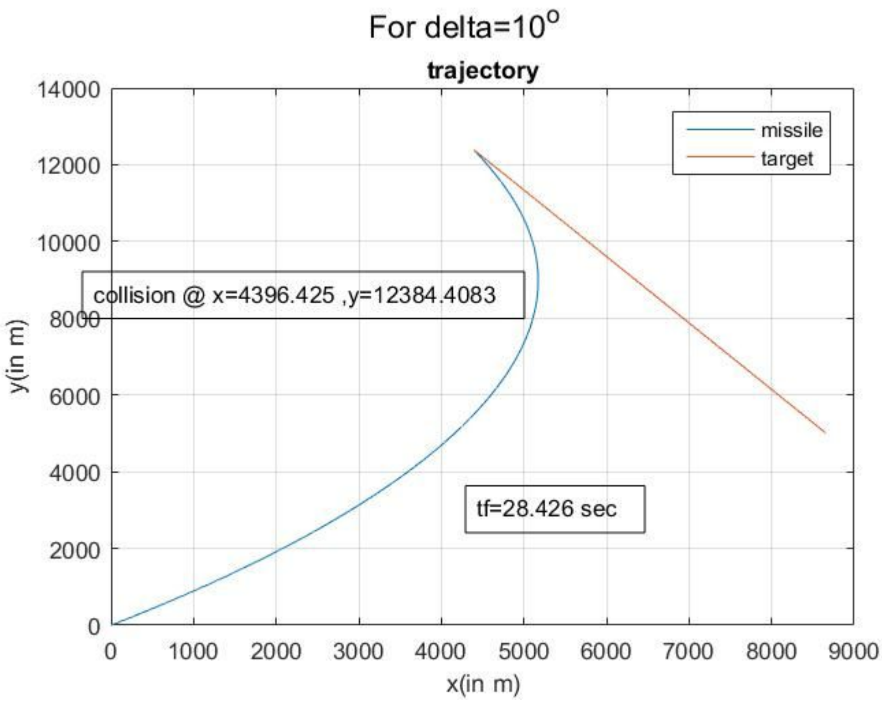
For $1 < \nu < 2$ a_M is zero

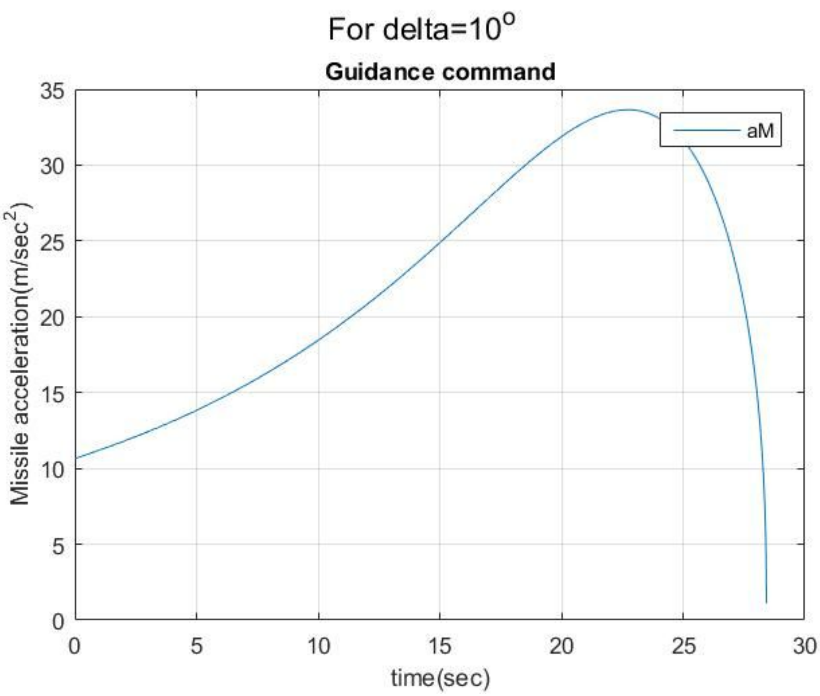
For $\nu = 2$ a_M is finite which is the case when V_T =250

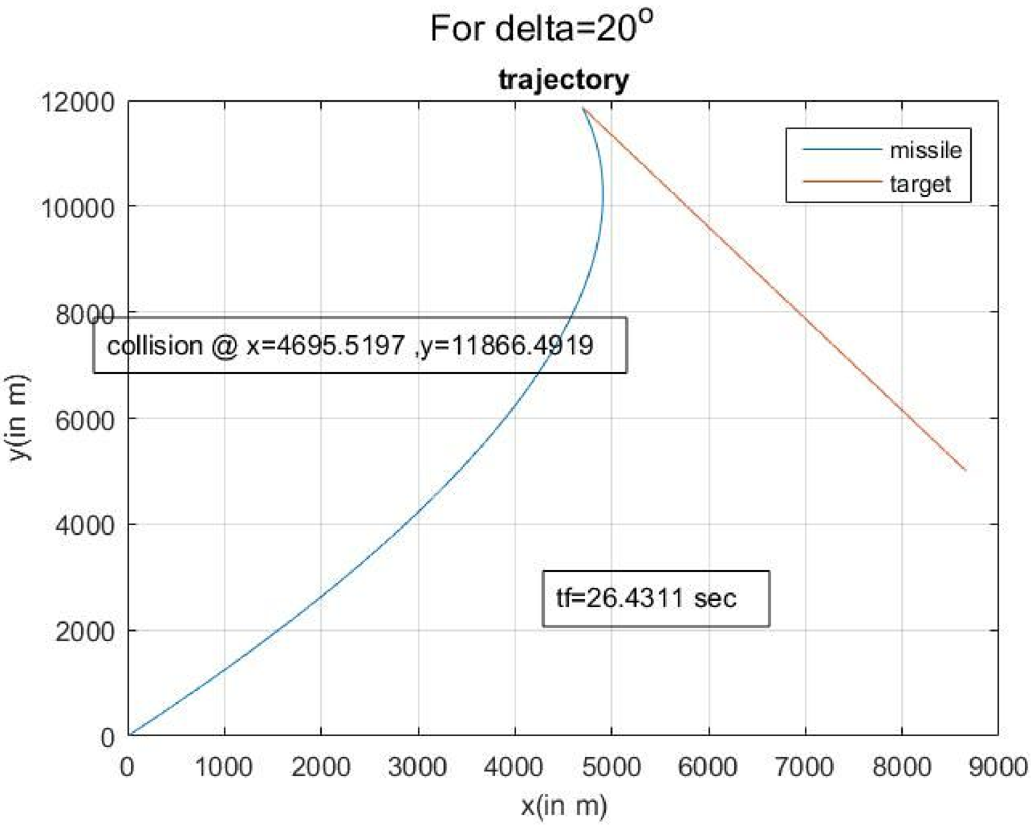
For $\nu > 2$ a_M is infinite

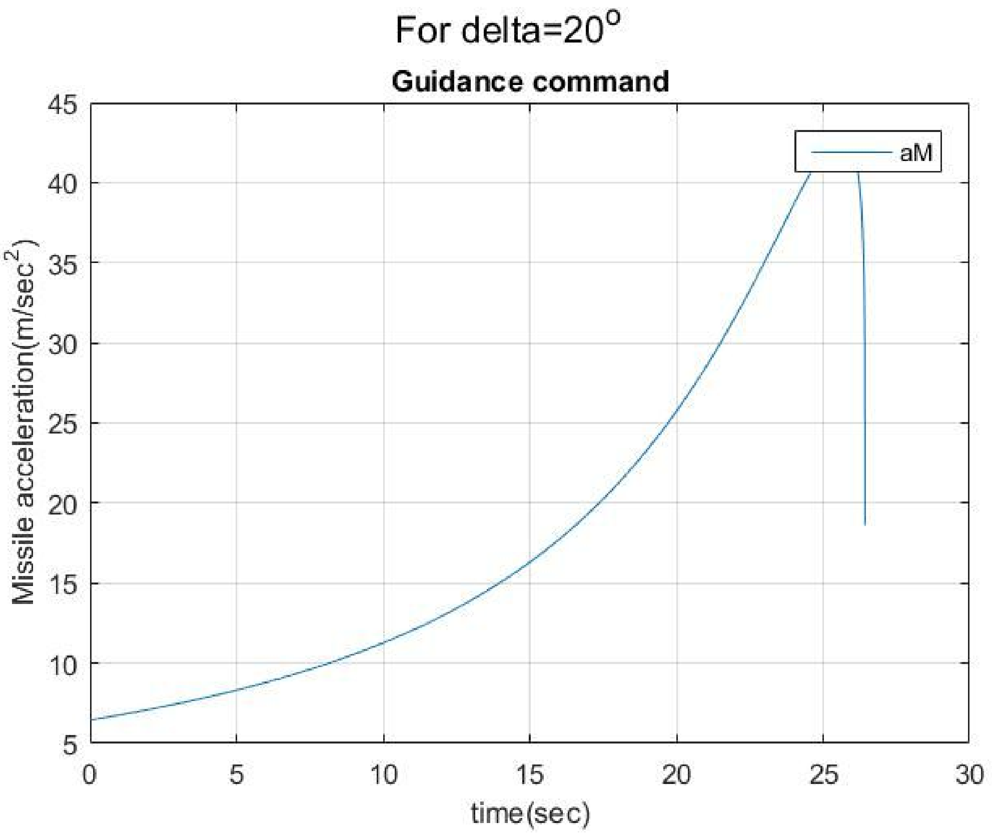
Q.2(c)

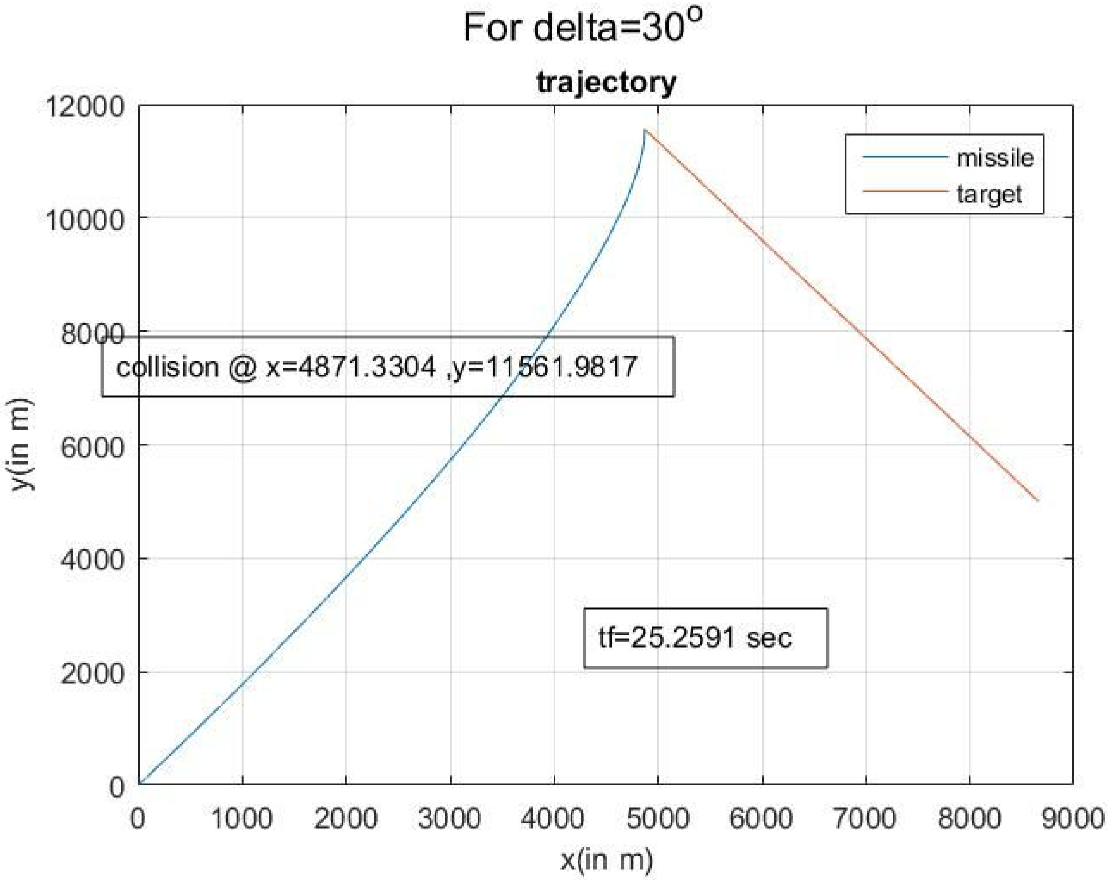
Plots of trajectories and guidance commands for deviated pursuit guidance for various deviation angles.

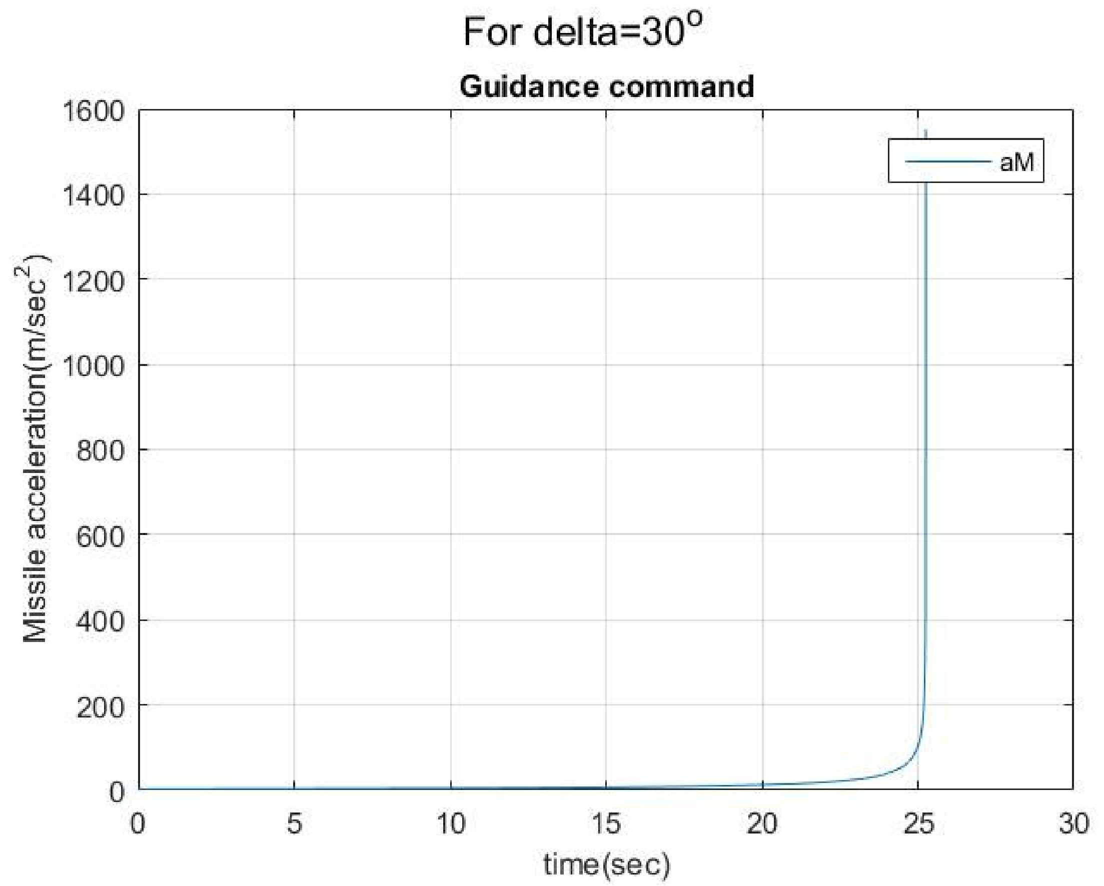





For delta=20^{o}
trajectory
missile
target
collision @ x=4695.5197 ,y=11866.4919
tf=26.4311 sec
y(in m)
x(in m)
12000
10000
8000
6000
4000
2000
0
1000
3000
5000
7000
9000


For delta=20^{o}
Guidance command
aM
Missile acceleration(m/sec^{2})
time(sec)
45
40
35
30
25
20
15
10
5
0
30

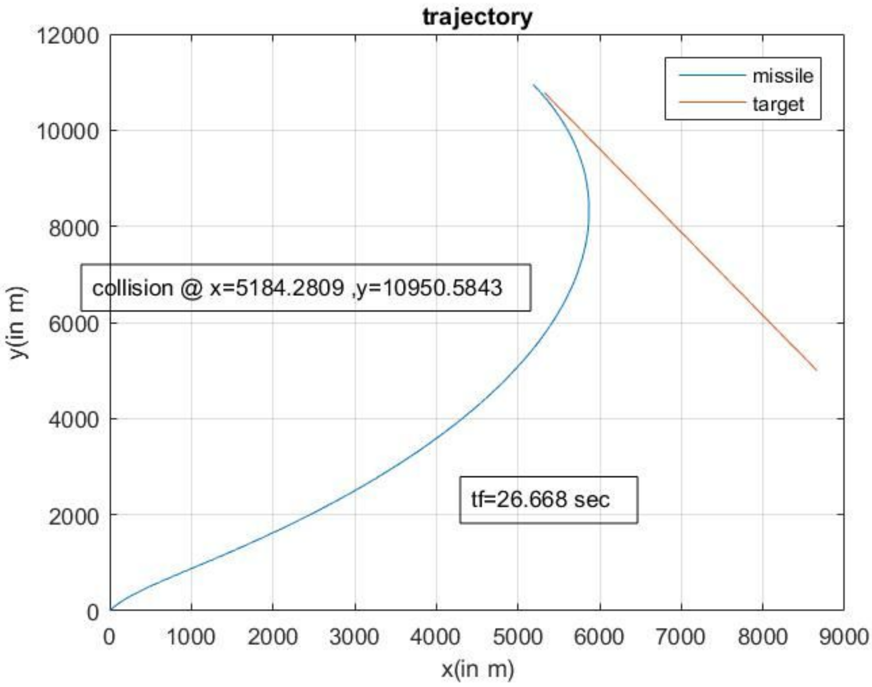

For delta=30^o
trajectory
missile
target
collision @ x=4871.3304 ,y=11561.9817
tf=25.2591 sec
y(in m)
x(in m)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
0
2000
4000
6000
8000
10000
12000

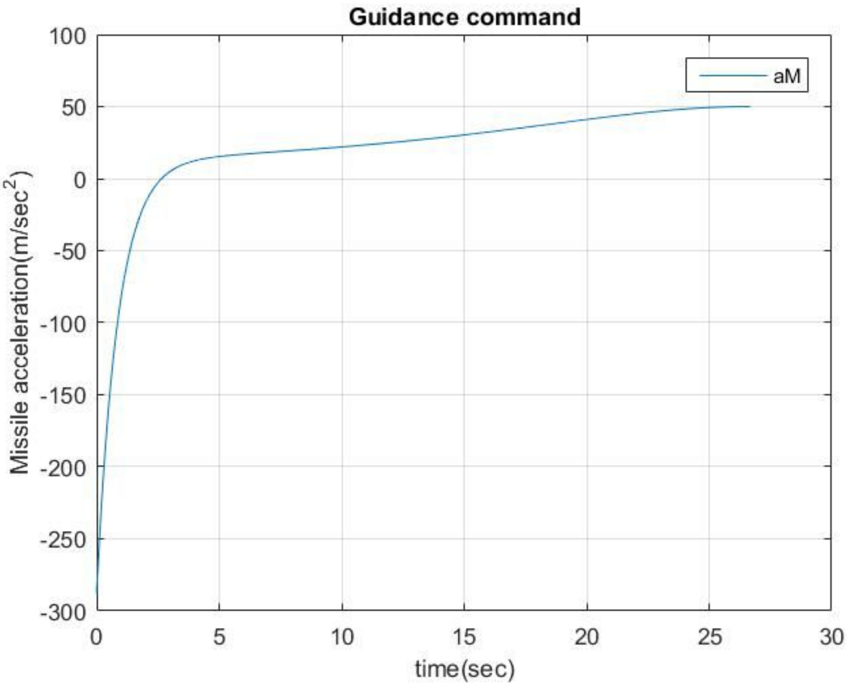

For delta=30^o
Guidance command
aM
Missile acceleration(m/sec^2)
time(sec)
0
5
10
15
20
25
30
0
200
400
600
800
1000
1200
1400
1600

From the plots for different deviation angle it’s seen that the guidance command first increases then decreases with increasing deviation angle but for further increase in delta causes the required guidance command to be very high (a very steep increase).
Also with increasing delta the collision time is decreasing slightly, the x coordinate of collision is increasing and y coordinate of collision is decreasing.

Q.2.(d)

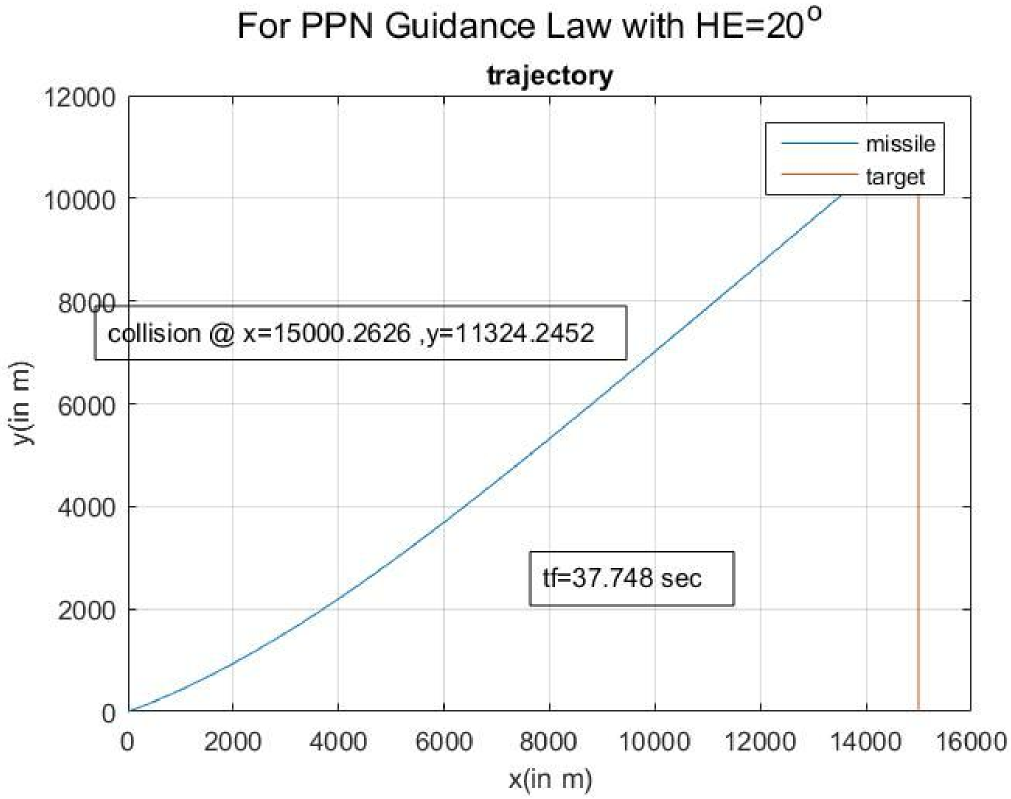
The missile has an initial deviation of 30° anticlockwise in its velocity direction. We can implement the pure pursuit law for this as $a_M = V_M\dot{\theta} - K(\gamma_M - \theta)$, Here I used K=10

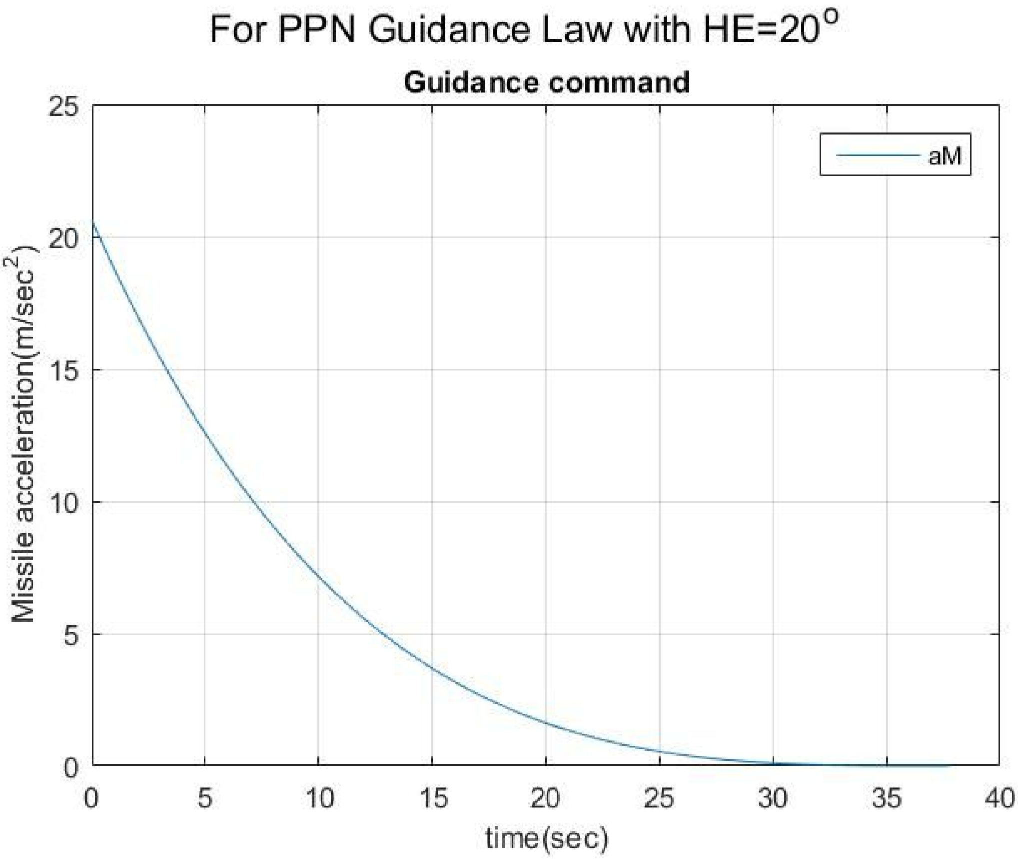




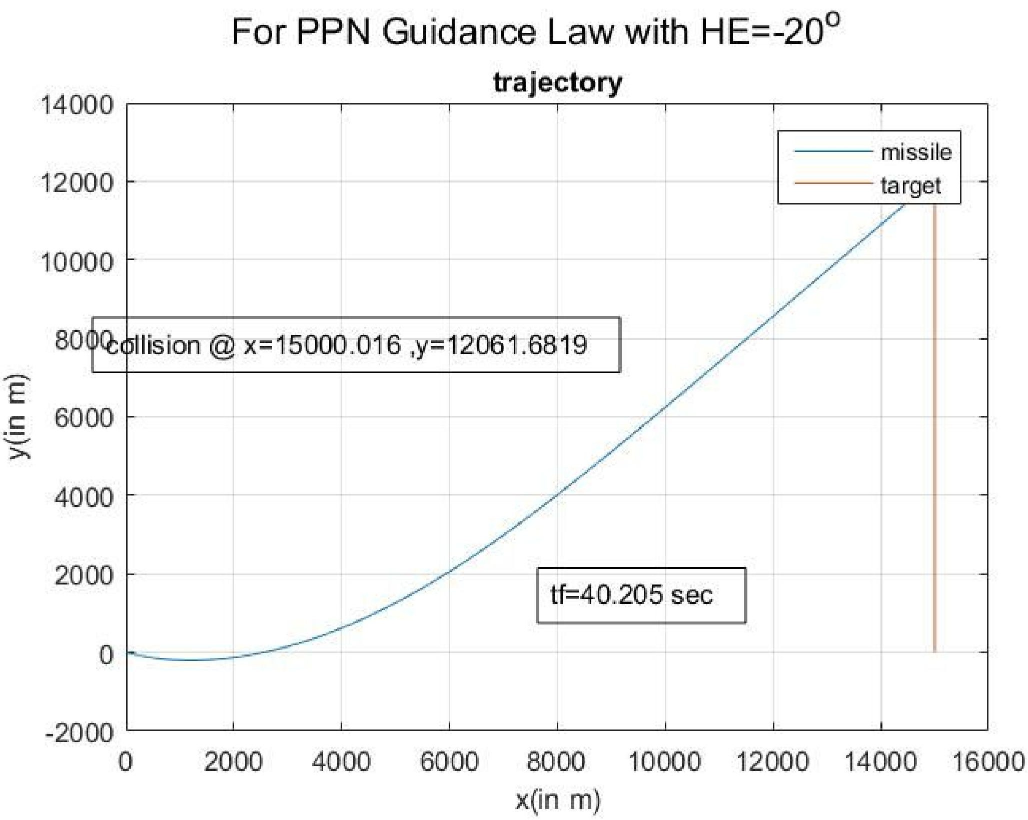
Q.3.(b).

Plots for PPN with Heading error = +20° and N = 5

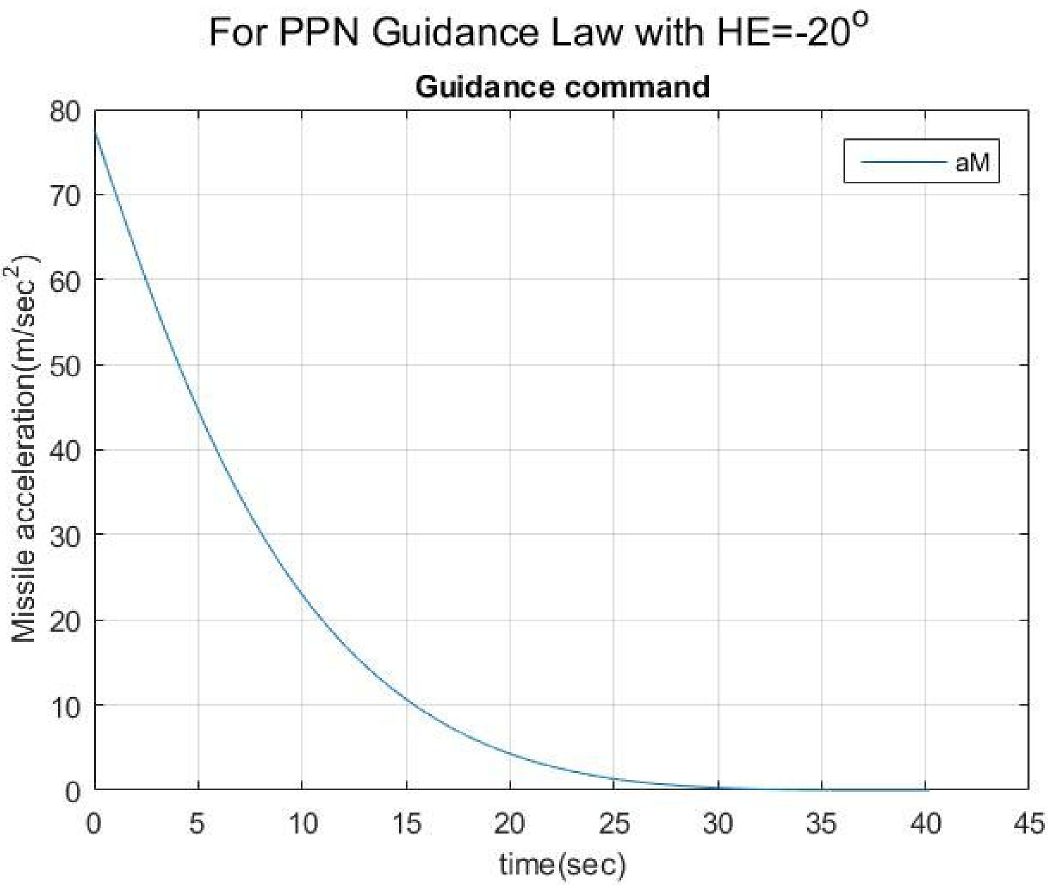




For HE = -20°

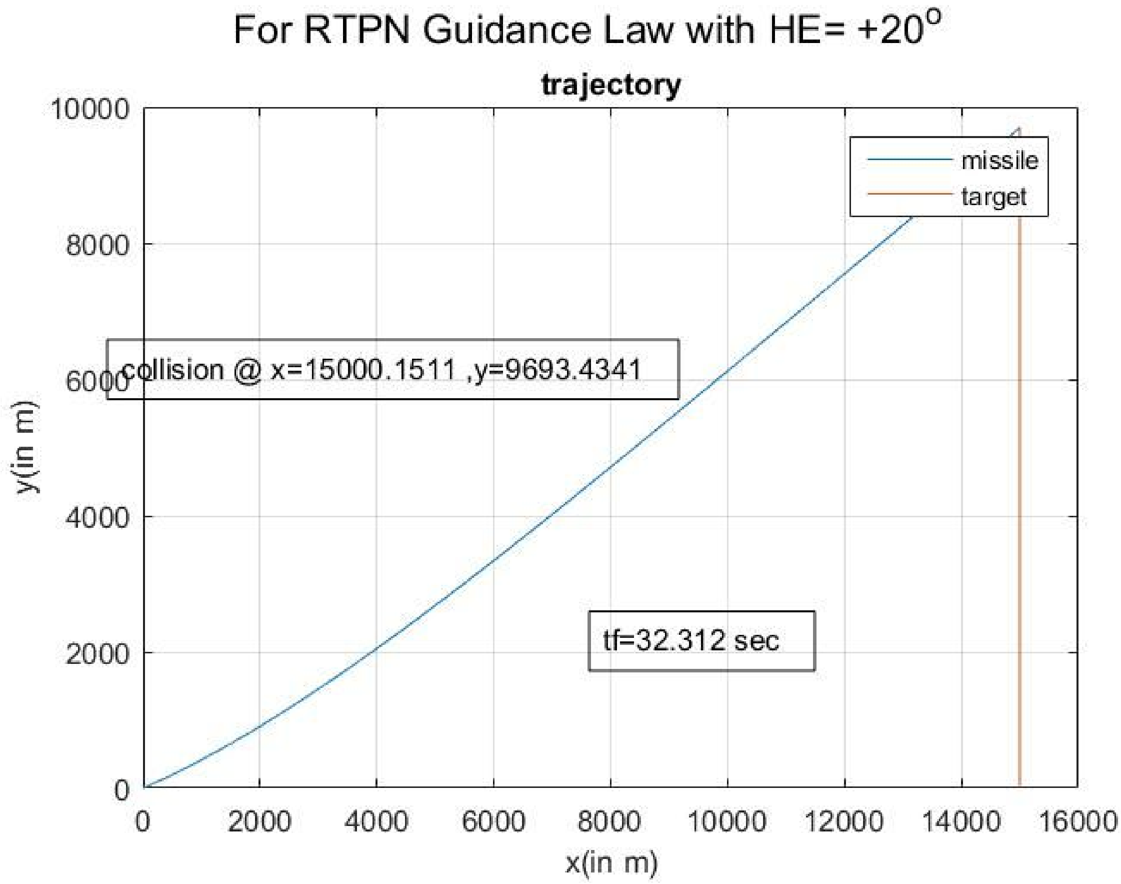


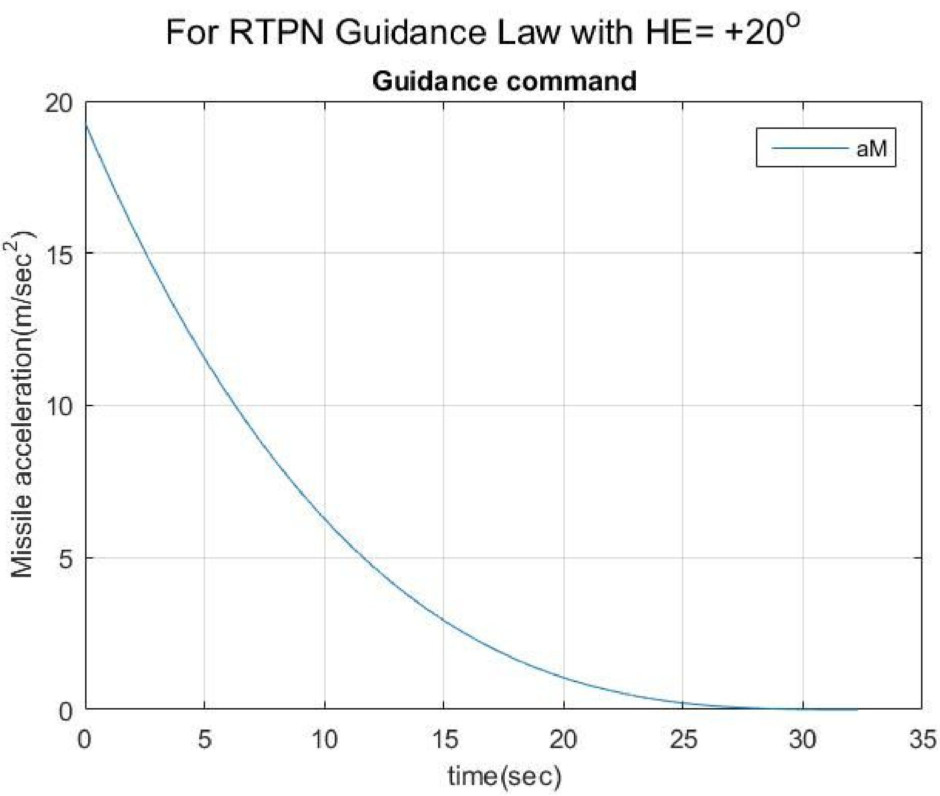
For PPN Guidance Law with HE=-20°
trajectory
missile
target
collision @ x=15000.016 ,y=12061.6819
tf=40.205 sec
y(in m)
x(in m)
14000
12000
10000
8000
6000
4000
2000
0
-2000
0
2000
4000
6000
8000
10000
12000
14000
16000



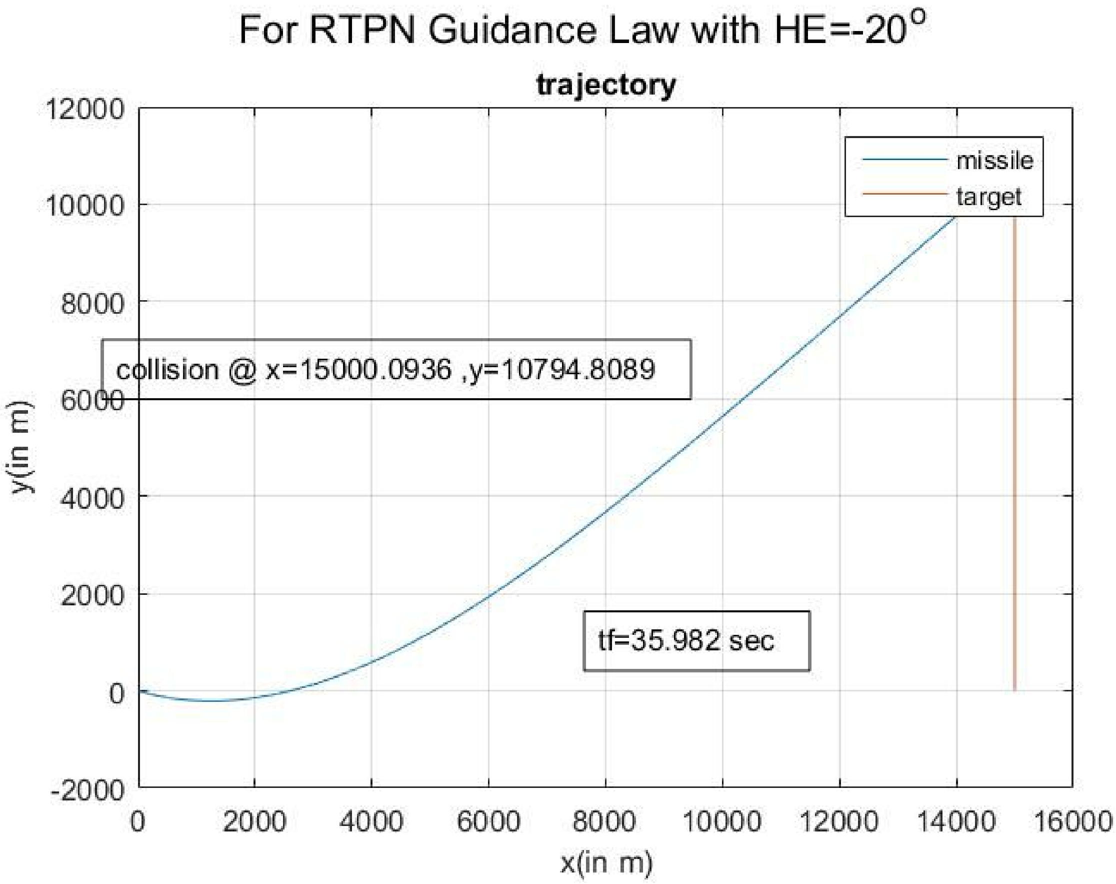
For PPN Guidance Law with HE=-20°
Guidance command
aM
Missile acceleration(m/sec^2)
time(sec)
80
70
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35
40
45

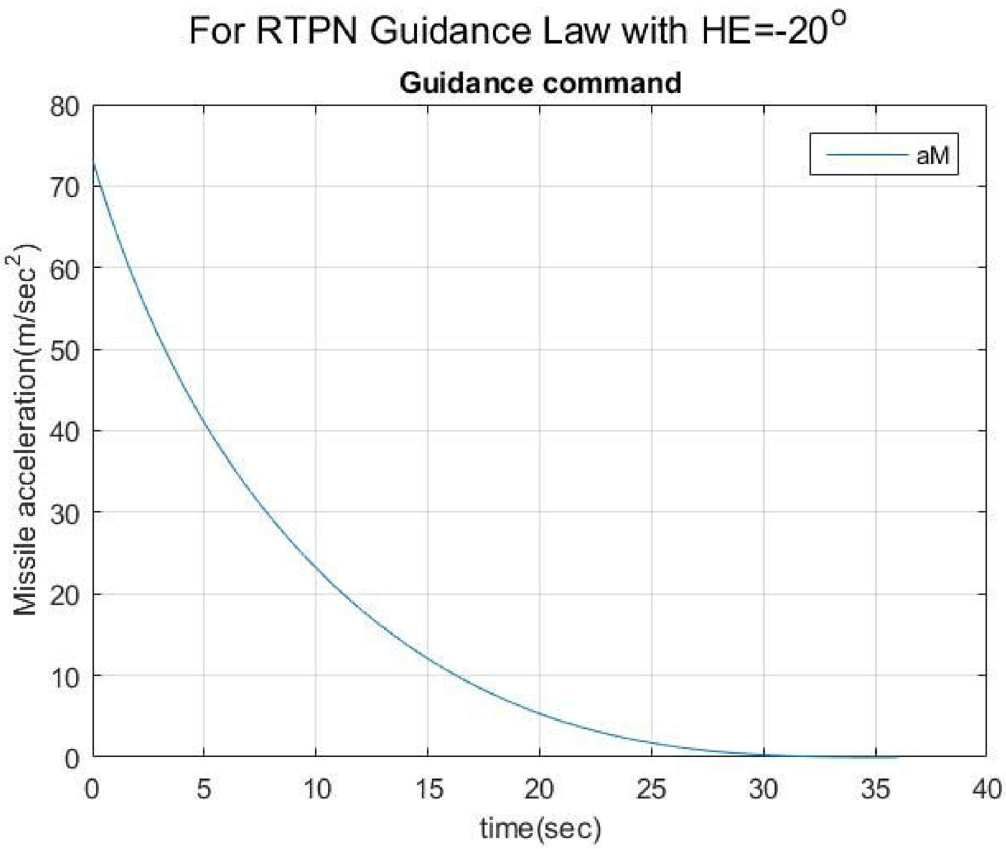
Plots for RTPN with HE =+20° and N=5





Plots for RTPN with HE =-20°

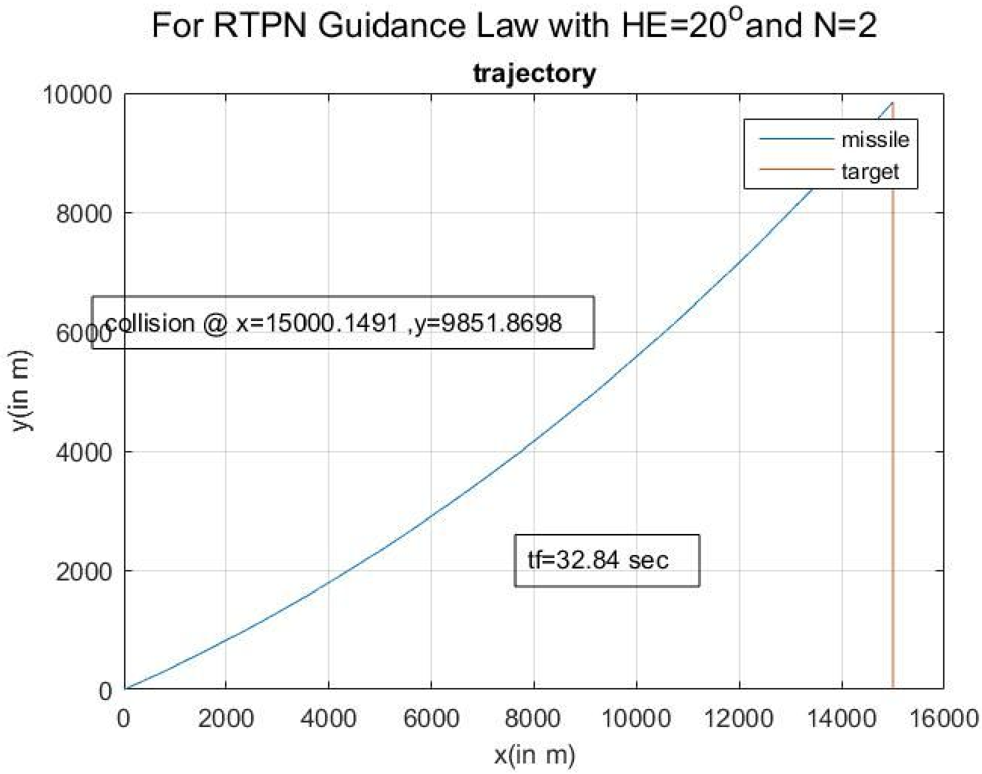


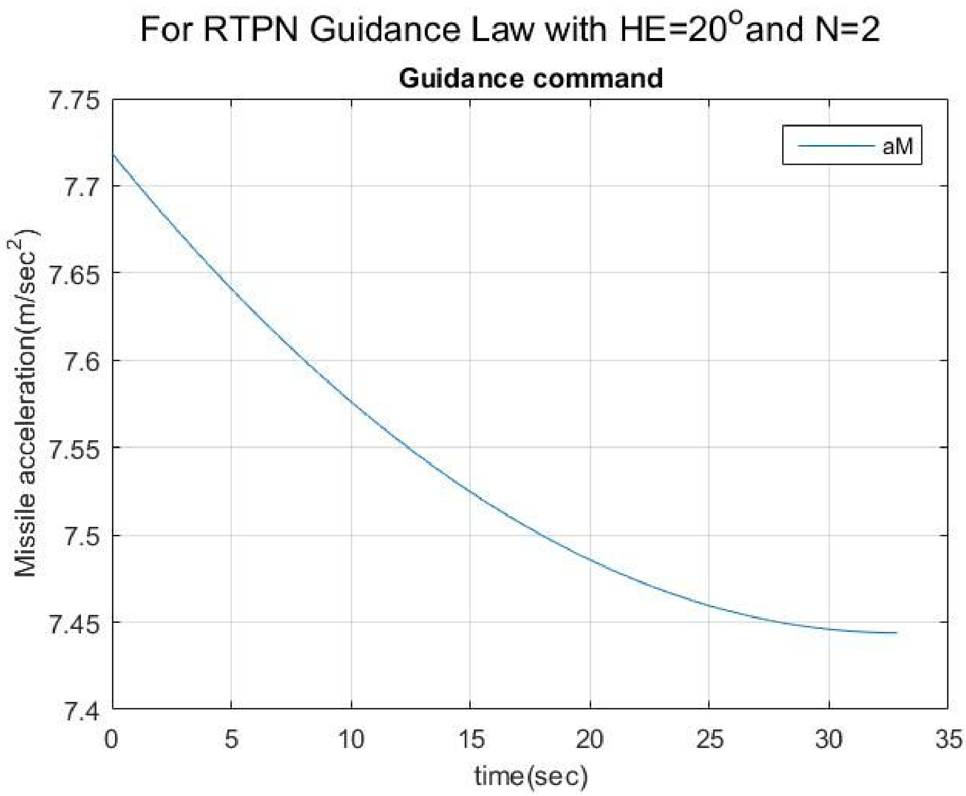


Q.3.(c).

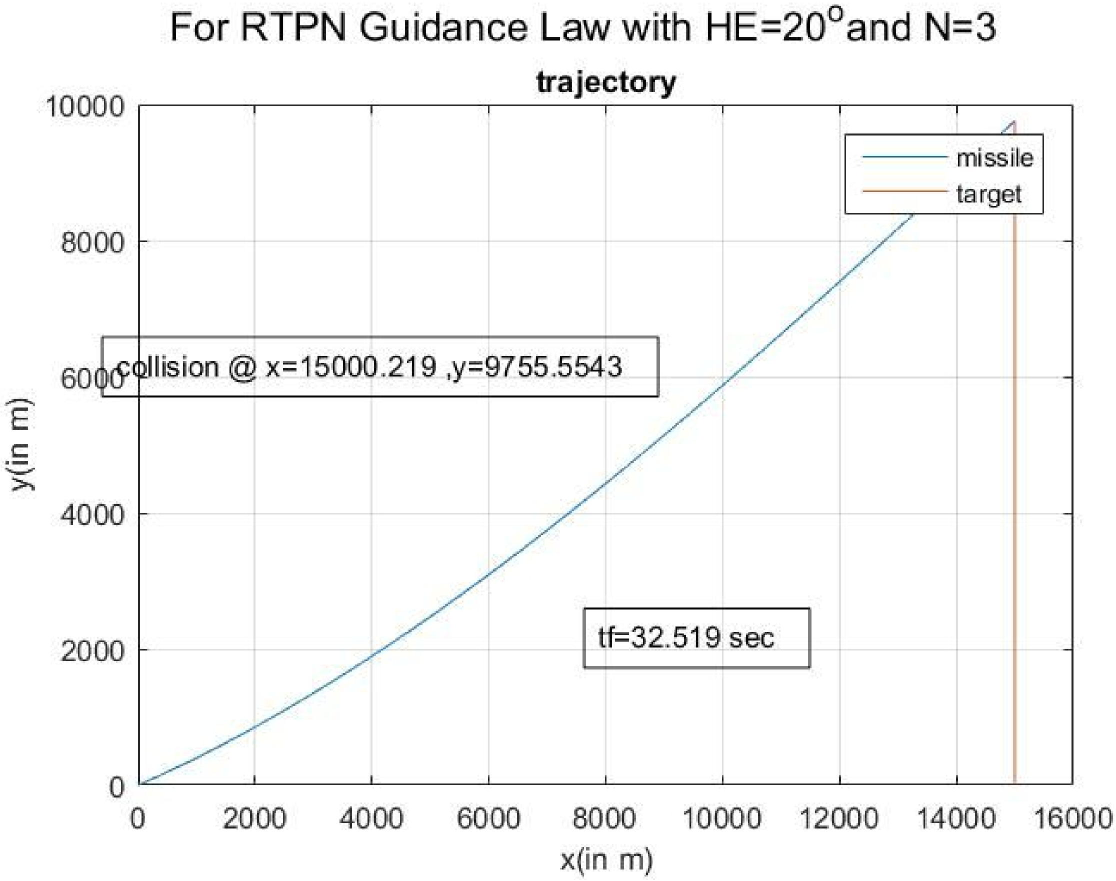
Note plots for N=5 have already been done in (b) part

Plots for RTPN with HE=20^o and N=2

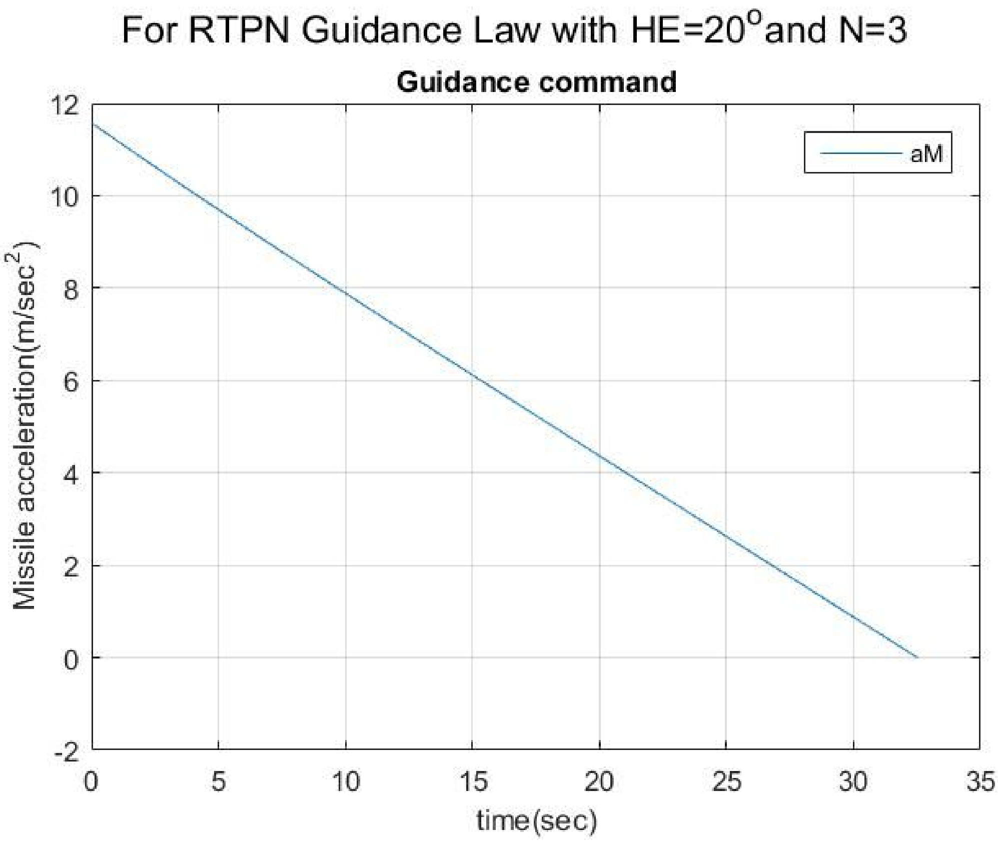




For N= 3

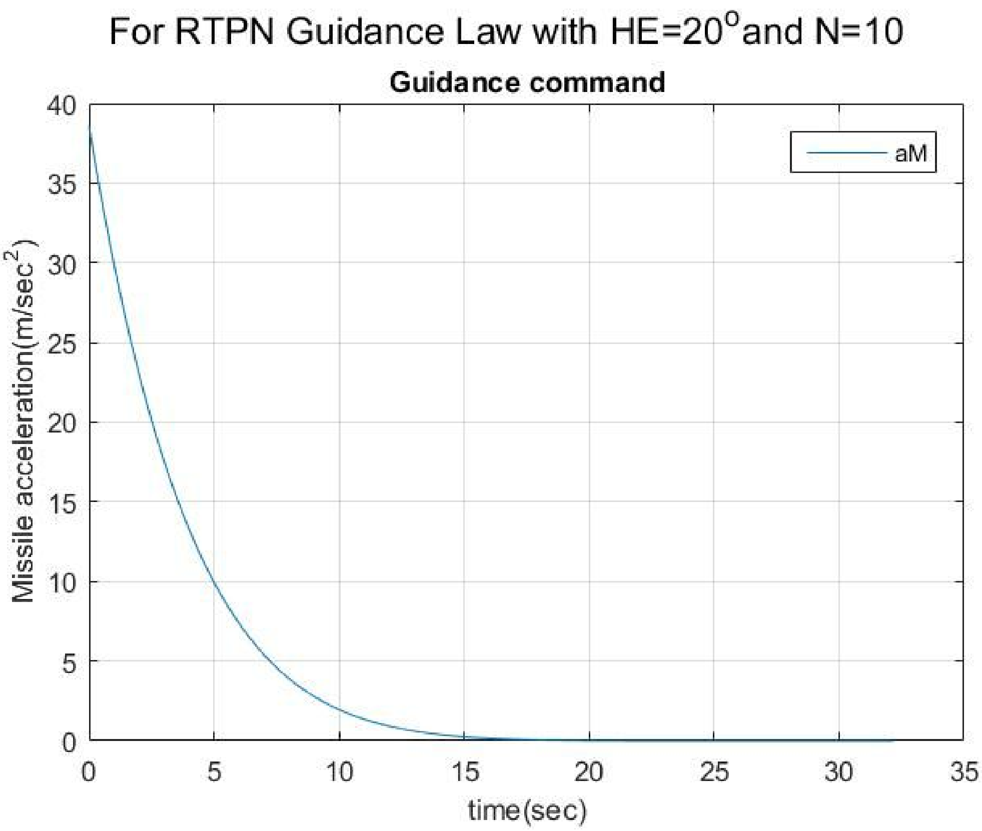


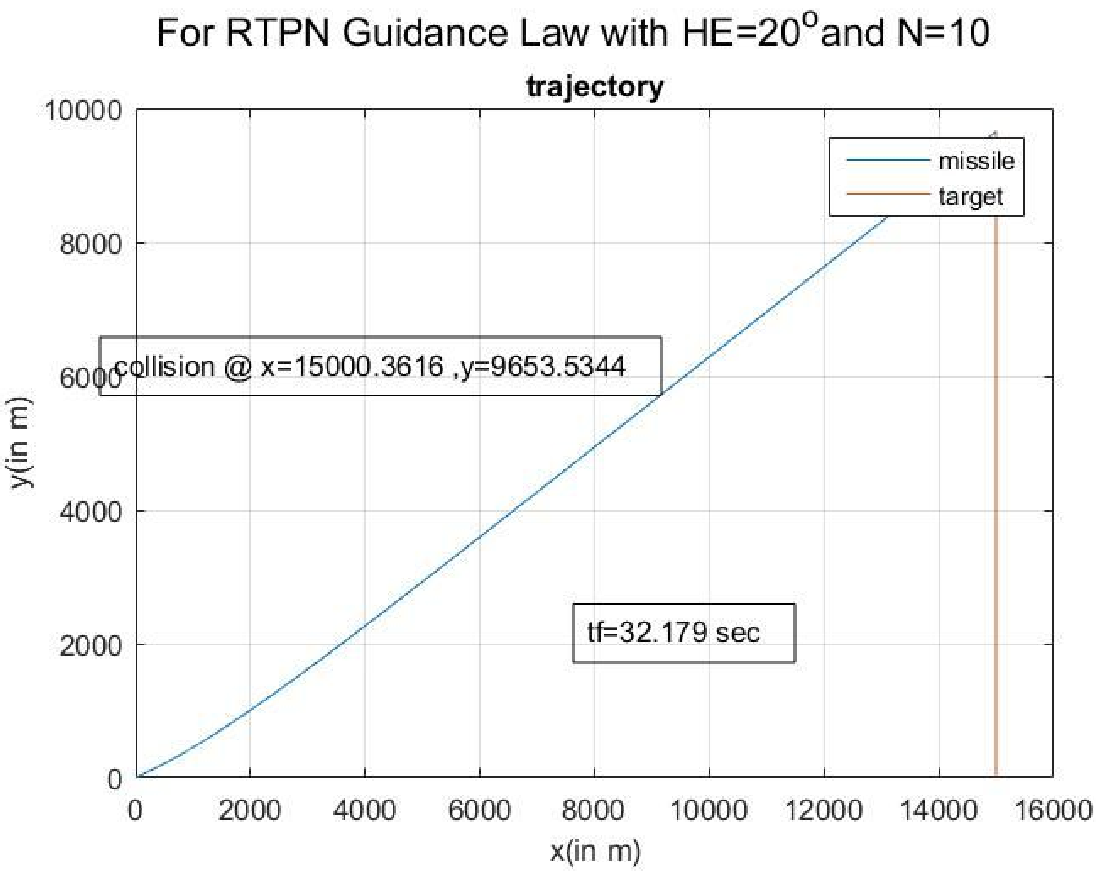
For RTPN Guidance Law with HE=20^{o}and N=3
trajectory
missile
target
collision @ x=15000.219 ,y=9755.5543
tf=32.519 sec
y(in m)
x(in m)



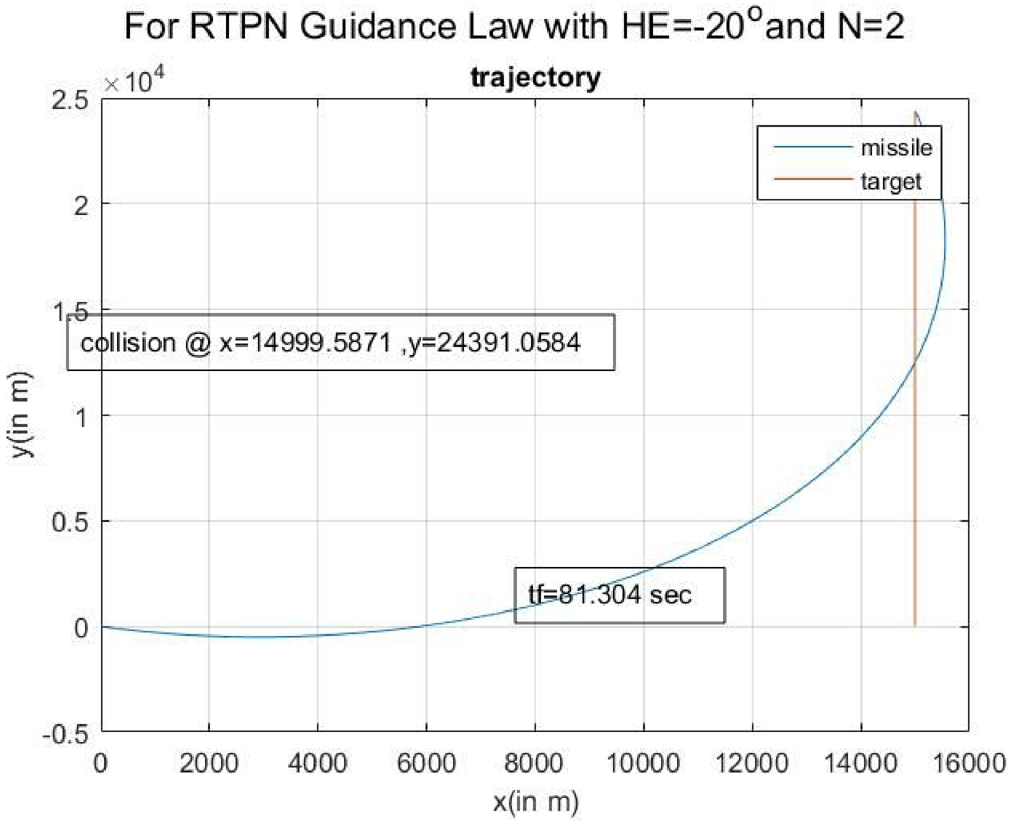
For RTPN Guidance Law with HE=20^{o}and N=3
Guidance command
aM
Missile acceleration(m/sec^2)
time(sec)

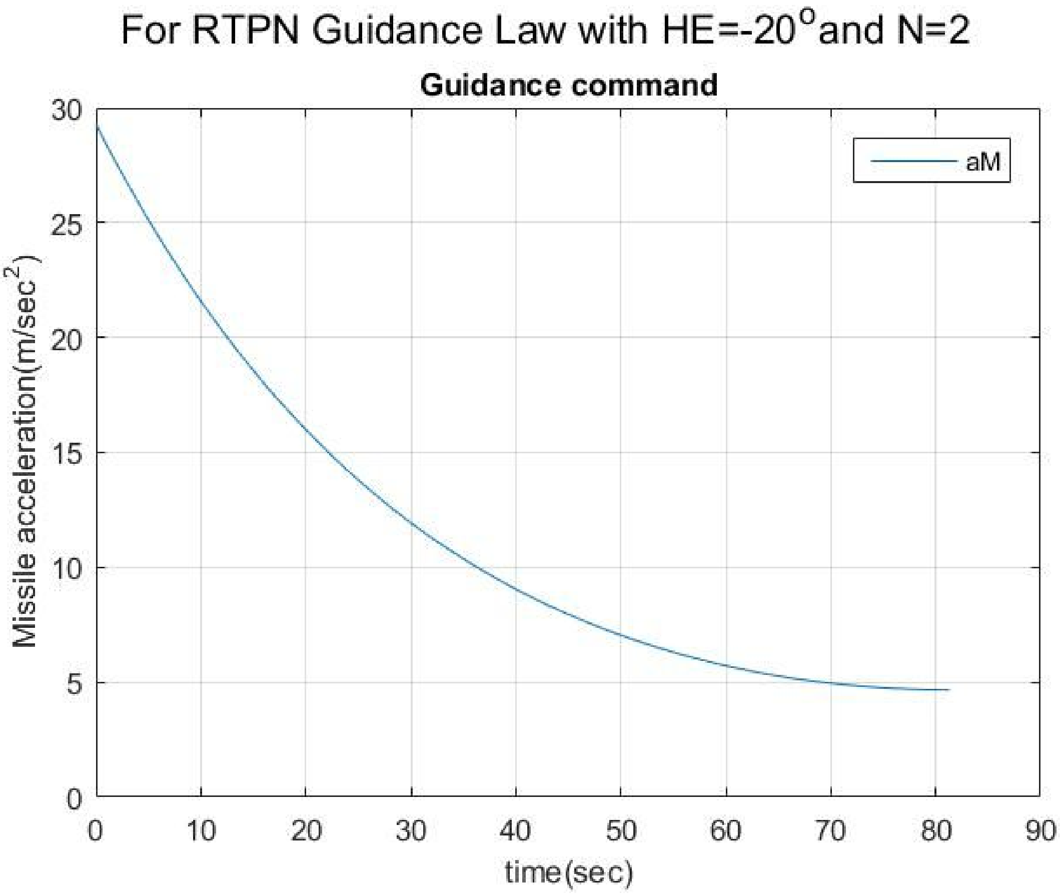
For N=10

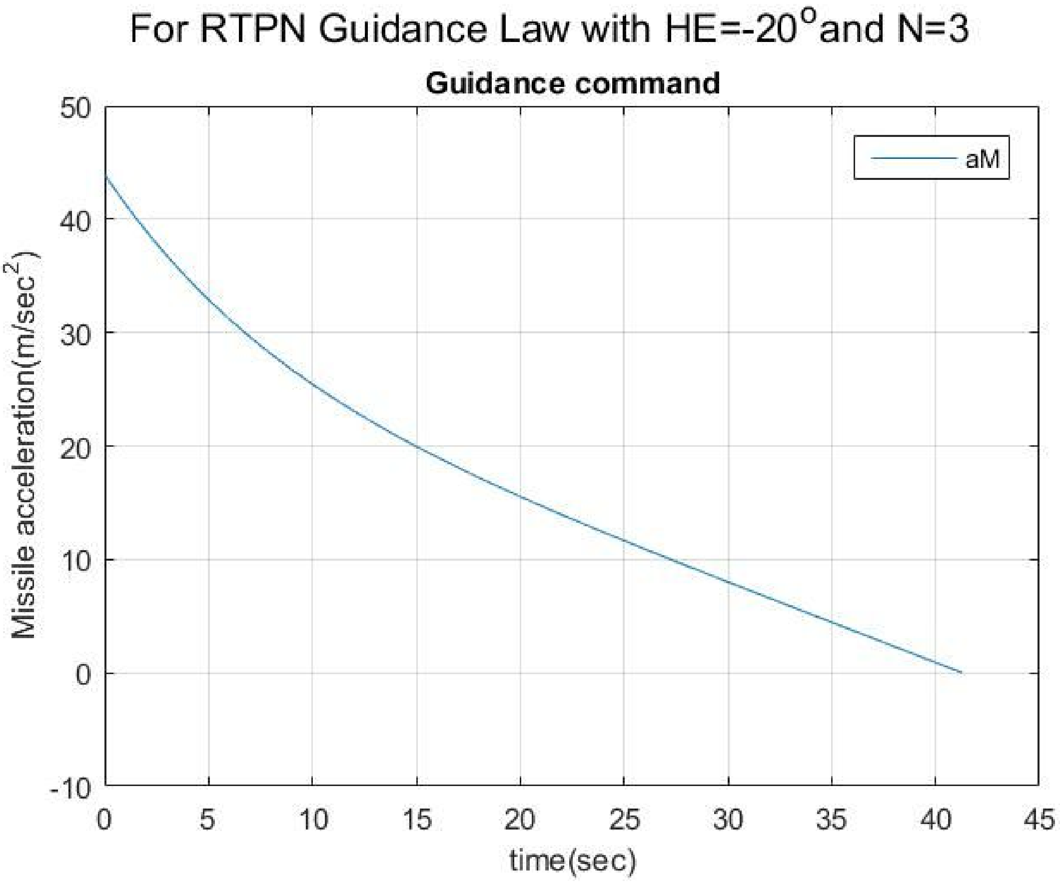

For RTPN Guidance Law with HE=20°and N=10
Guidance command
aM
Missile acceleration(m/sec²)
time(sec)
0
5
10
15
20
25
30
35
40

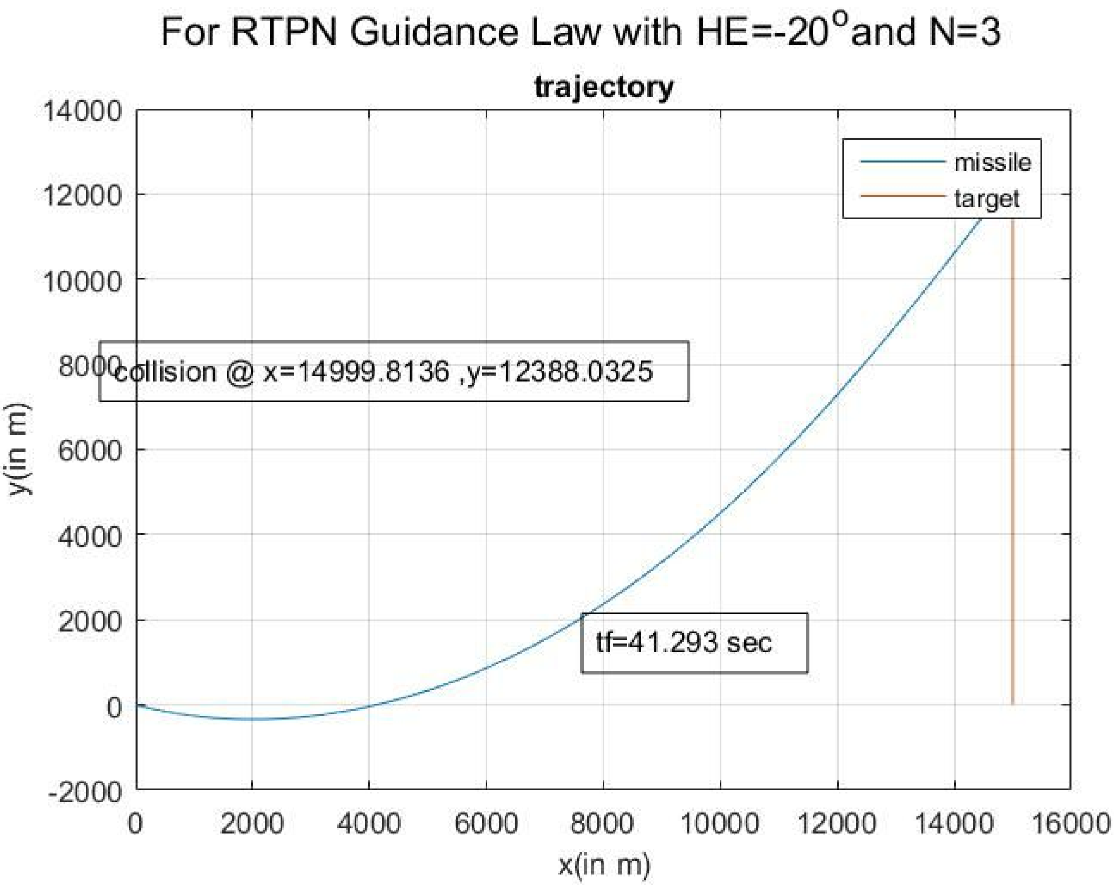

For RTPN Guidance Law with HE=20°and N=10
trajectory
missile
target
collision @ x=15000.3616 ,y=9653.5344
tf=32.179 sec
y(in m)
x(in m)
0
2000
4000
6000
8000
10000
12000
14000
16000

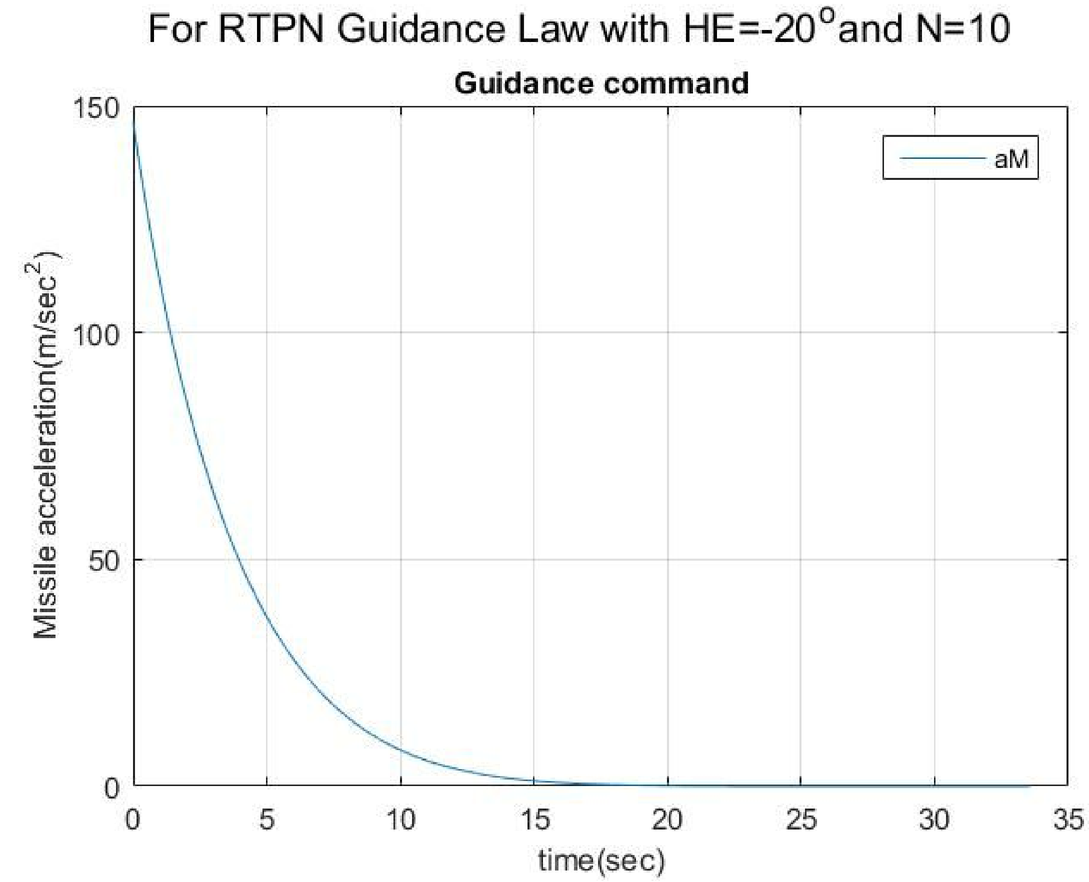
Plots for RTPN with HE = -20

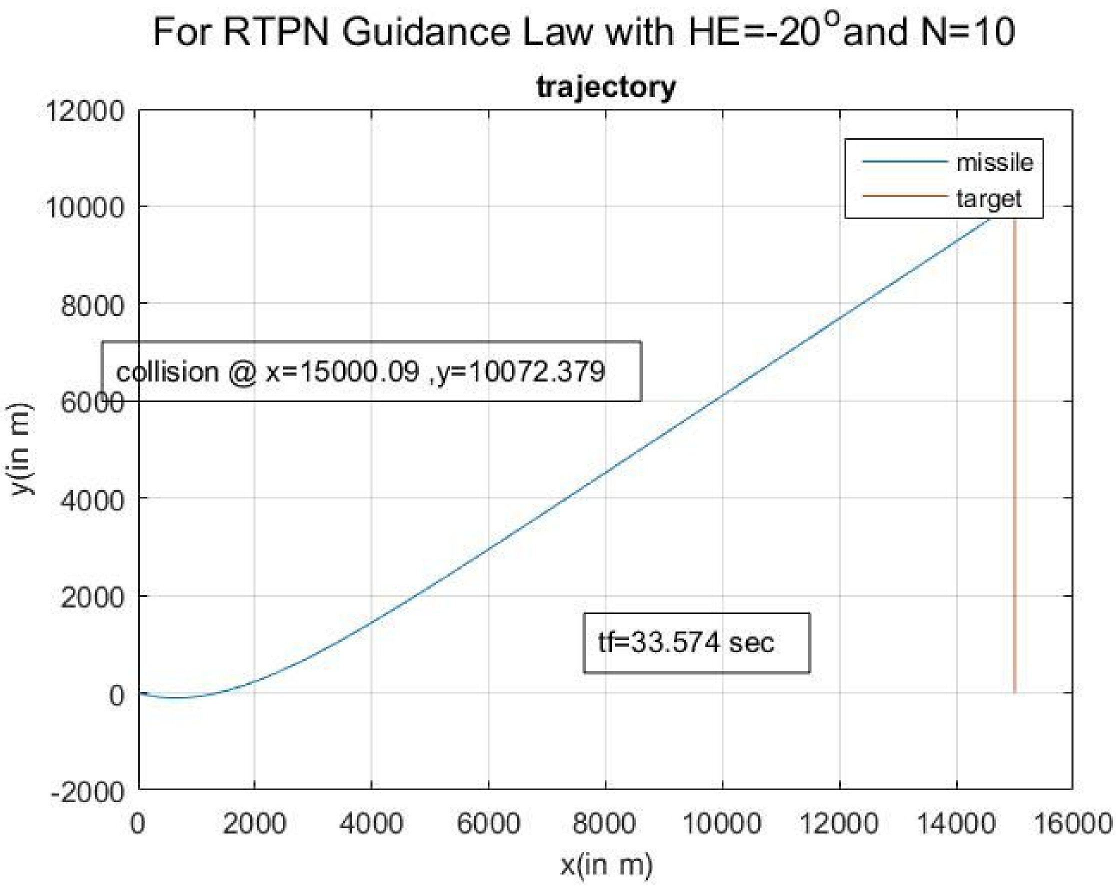




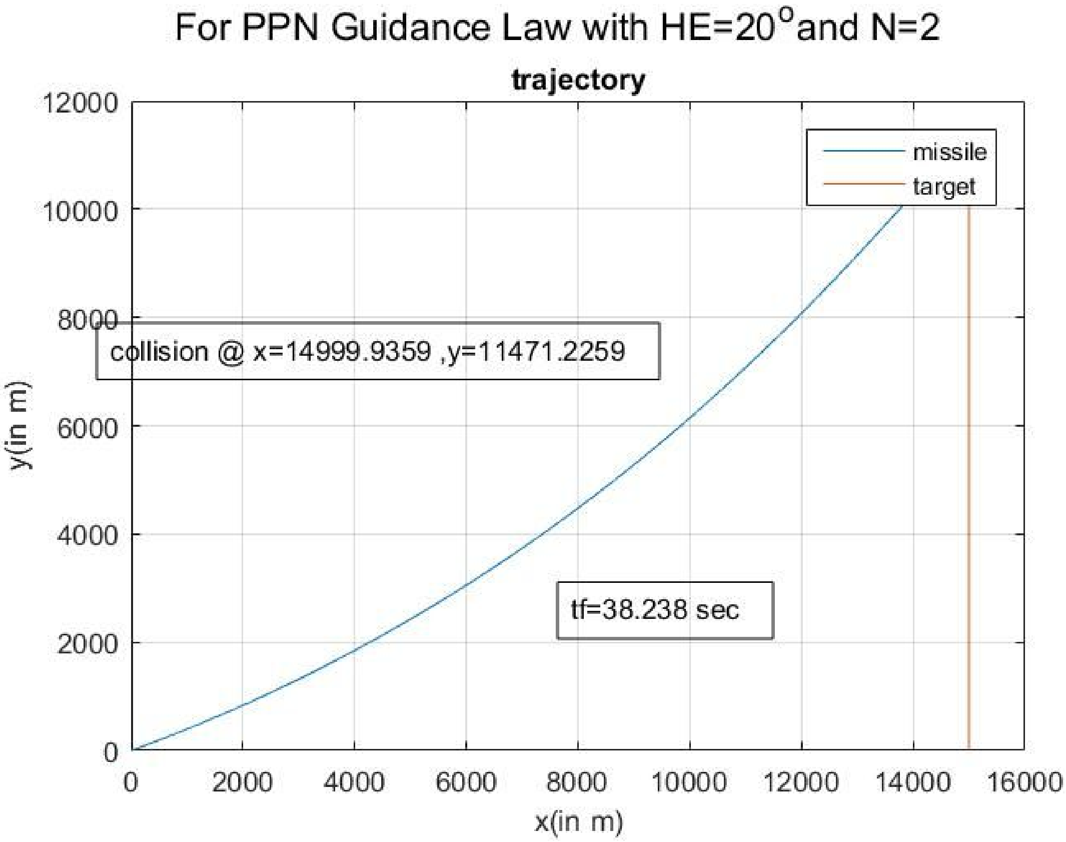

For RTPN Guidance Law with HE=-20°and N=3
Guidance command
aM
Missile acceleration(m/sec²)
time(sec)
50
40
30
20
10
0
-10
0
5
10
15
20
25
30
35
40
45

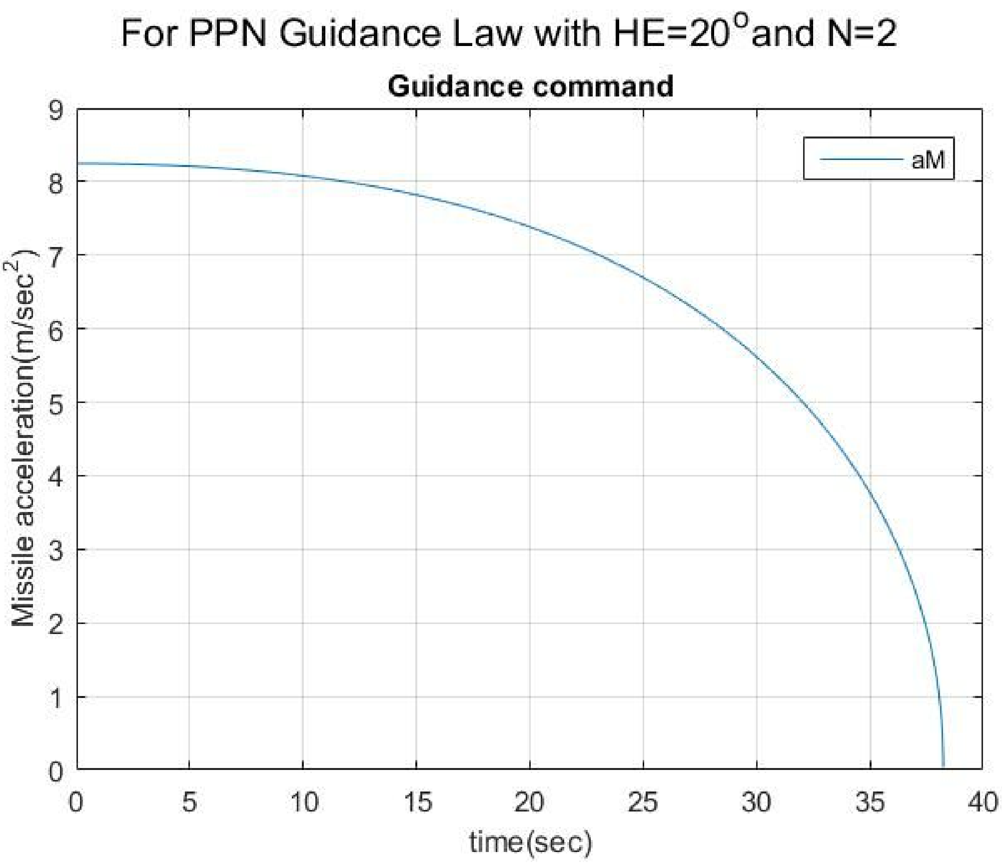

For RTPN Guidance Law with HE=-20°and N=3
trajectory
missile
target
collision @ x=14999.8136 ,y=12388.0325
tf=41.293 sec
y(in m)
x(in m)
14000
12000
10000
8000
6000
4000
2000
0
-2000
0
2000
4000
6000
8000
10000
12000
14000
16000

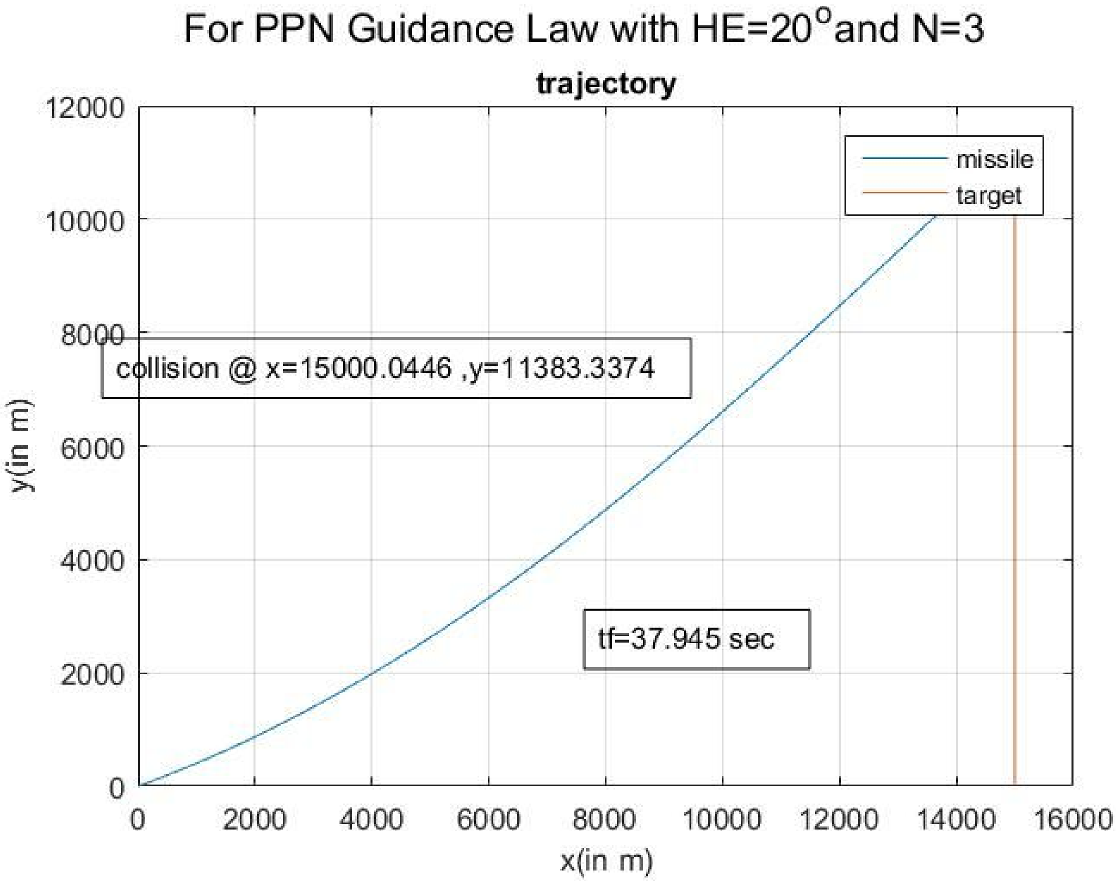

For RTPN Guidance Law with HE=-20^{o}and N=10
Guidance command
aM
Missile acceleration(m/sec^2)
time(sec)
150
100
50
0
0
5
10
15
20
25
30
35

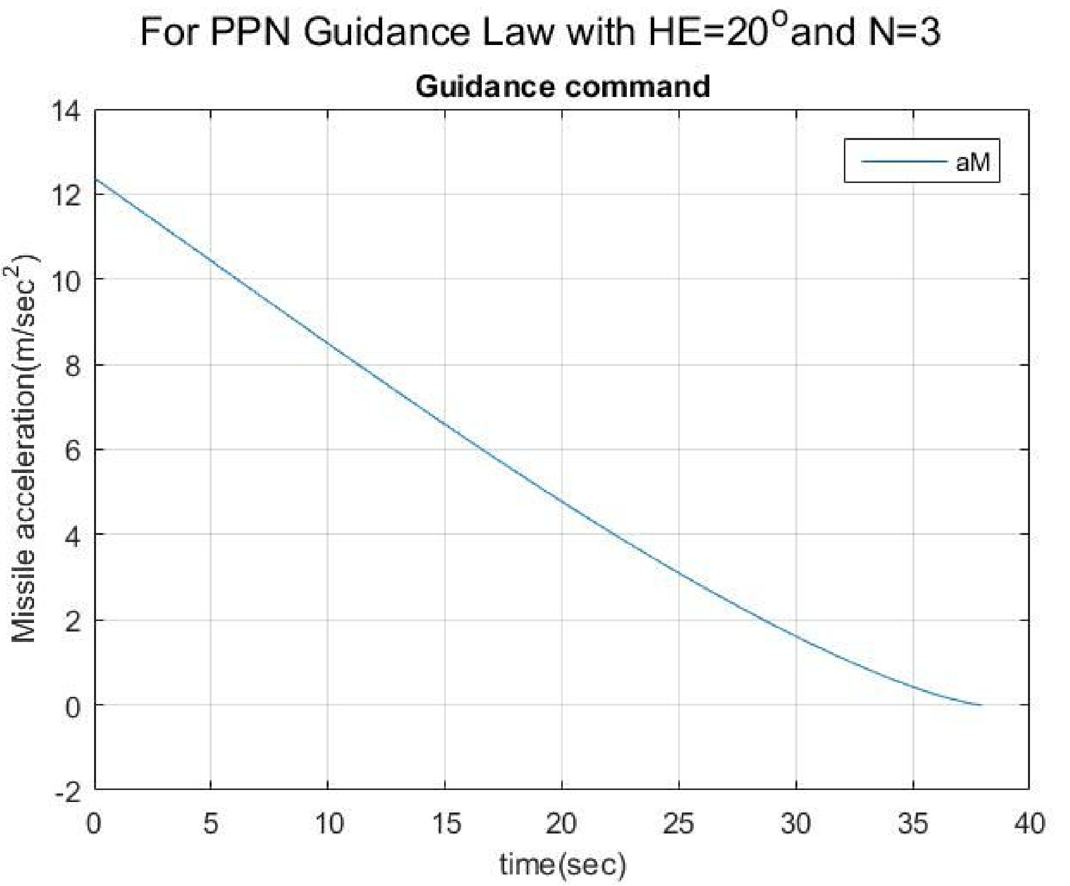

For RTPN Guidance Law with HE=-20^{o}and N=10
trajectory
missile
target
collision @ x=15000.09 ,y=10072.379
tf=33.574 sec
y(in m)
x(in m)
12000
10000
8000
6000
4000
2000
0
-2000
0
2000
4000
6000
8000
10000
12000
14000
16000

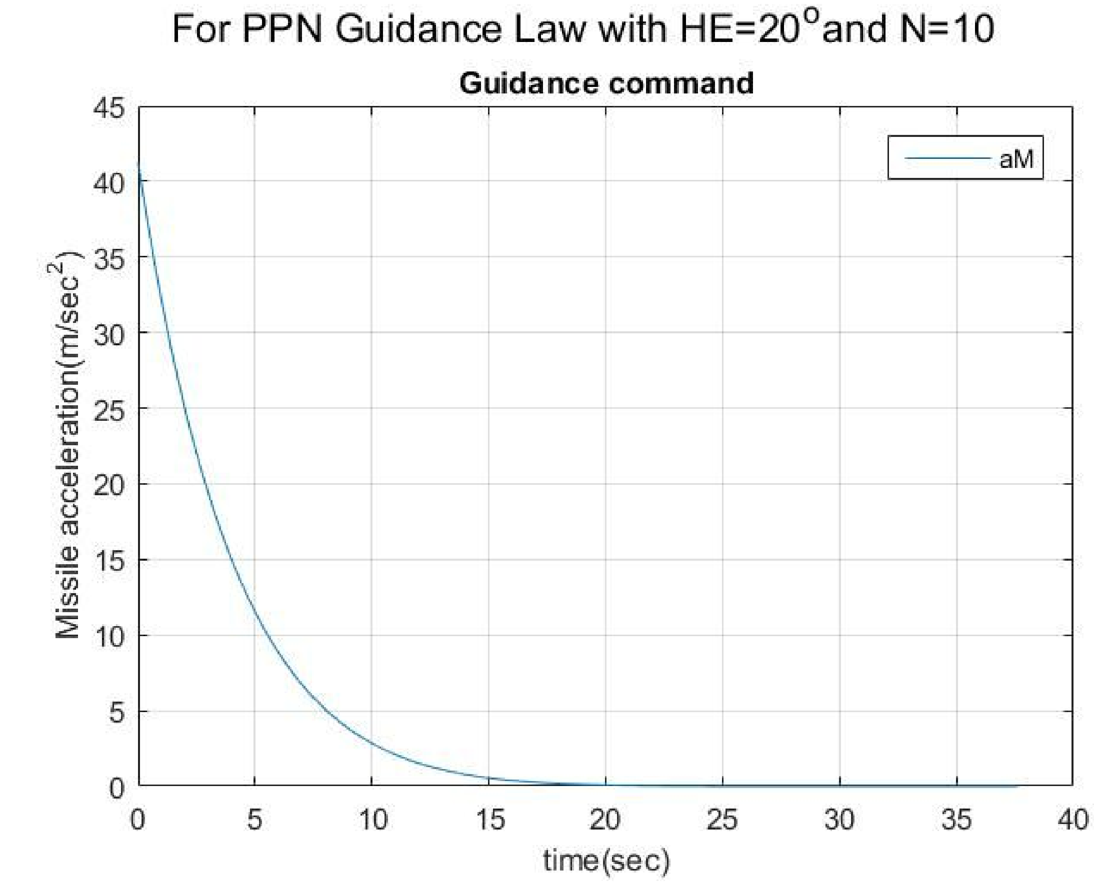
Plots for PPN with HE =20° ; Plots for N=5 have already been generated in part(b)

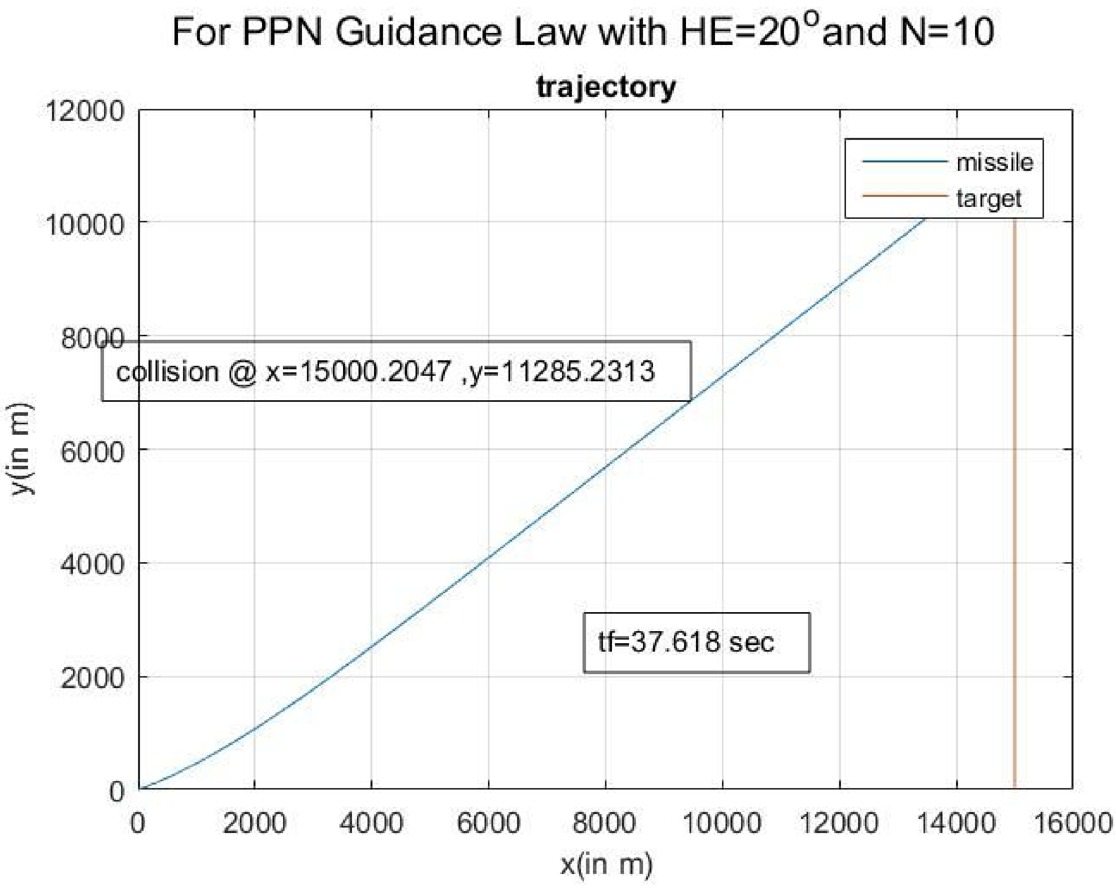




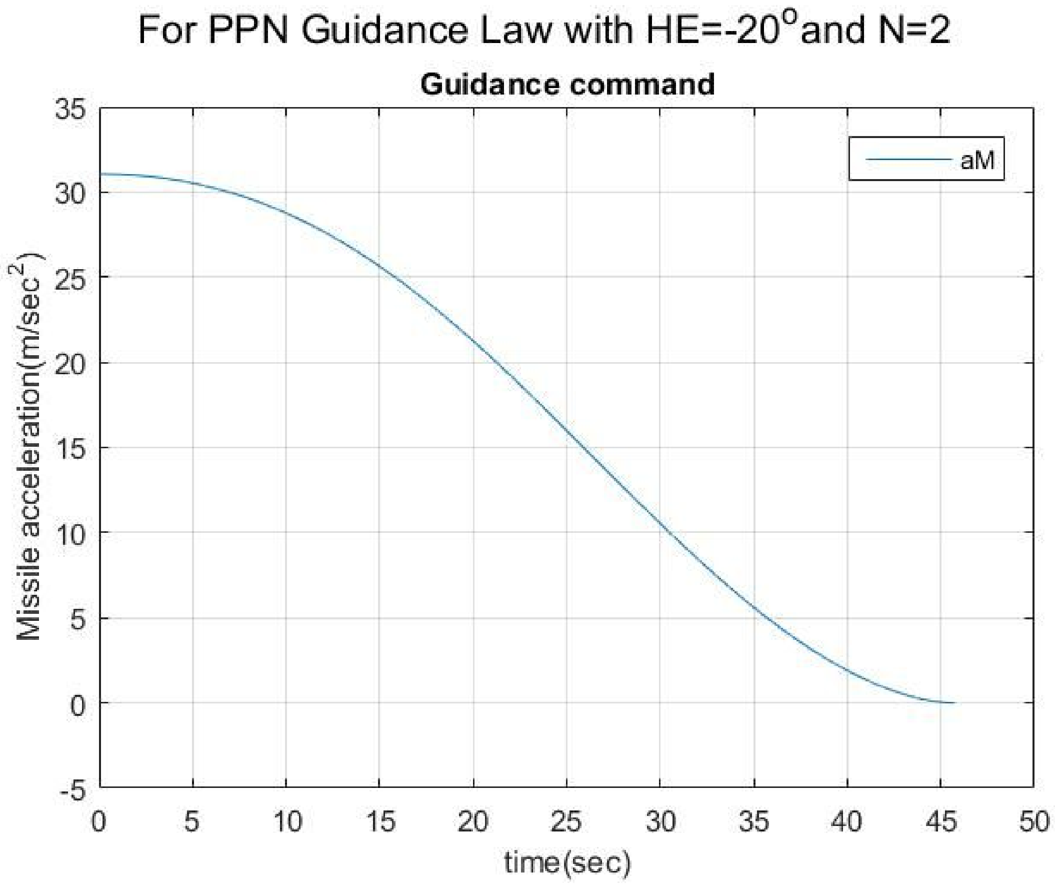

For PPN Guidance Law with HE=20°and N=3
trajectory
missile
target
collision @ x=15000.0446 ,y=11383.3374
tf=37.945 sec
y(in m)
x(in m)
12000
10000
8000
6000
4000
2000
0
0
2000
4000
6000
8000
10000
12000
14000
16000

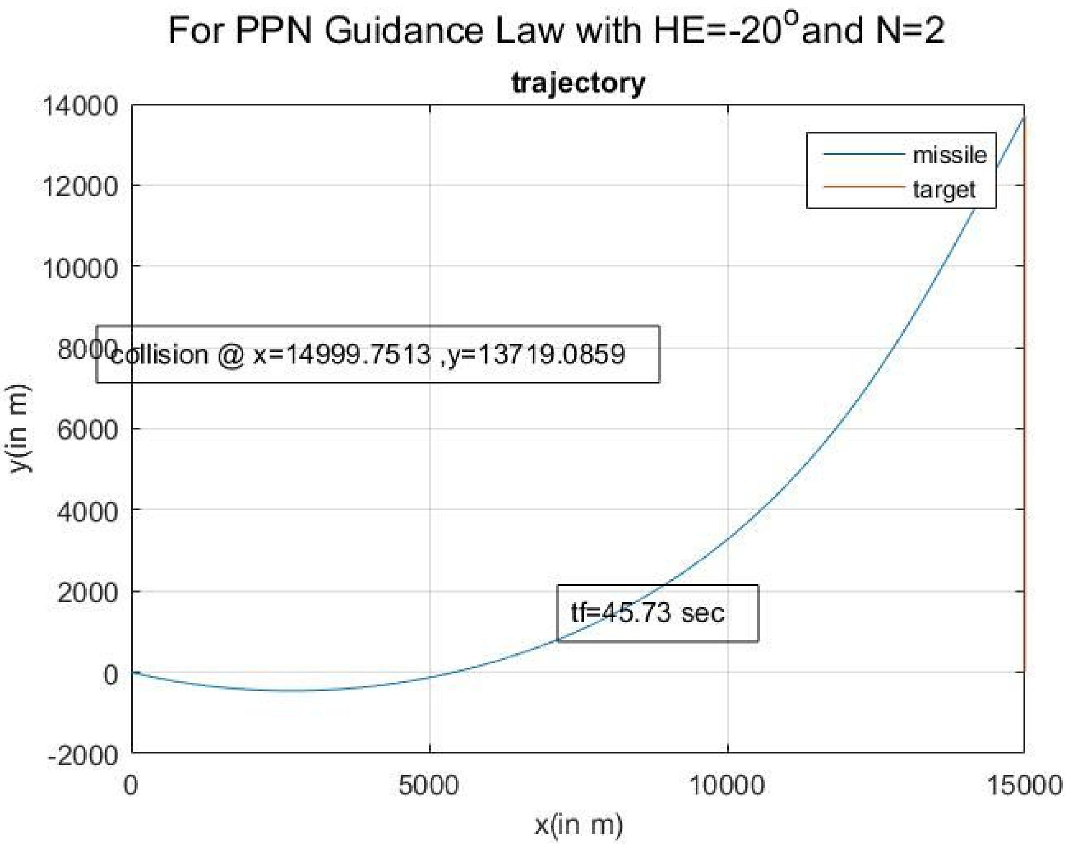

For PPN Guidance Law with HE=20°and N=3
Guidance command
aM
Missile acceleration(m/sec²)
time(sec)
14
12
10
8
6
4
2
0
-2
0
5
10
15
20
25
30
35
40

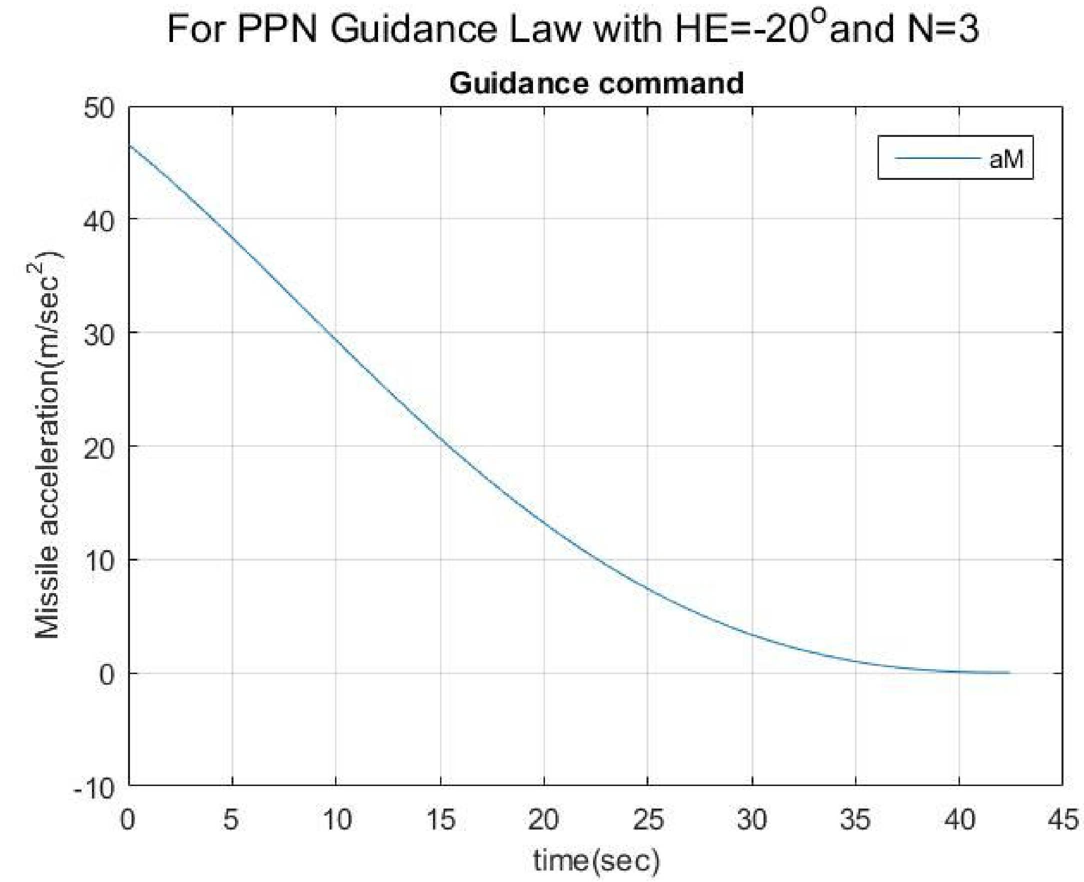

For PPN Guidance Law with HE=20°and N=10
Guidance command
aM
Missile acceleration(m/sec²)
time(sec)
0
5
10
15
20
25
30
35
40
45

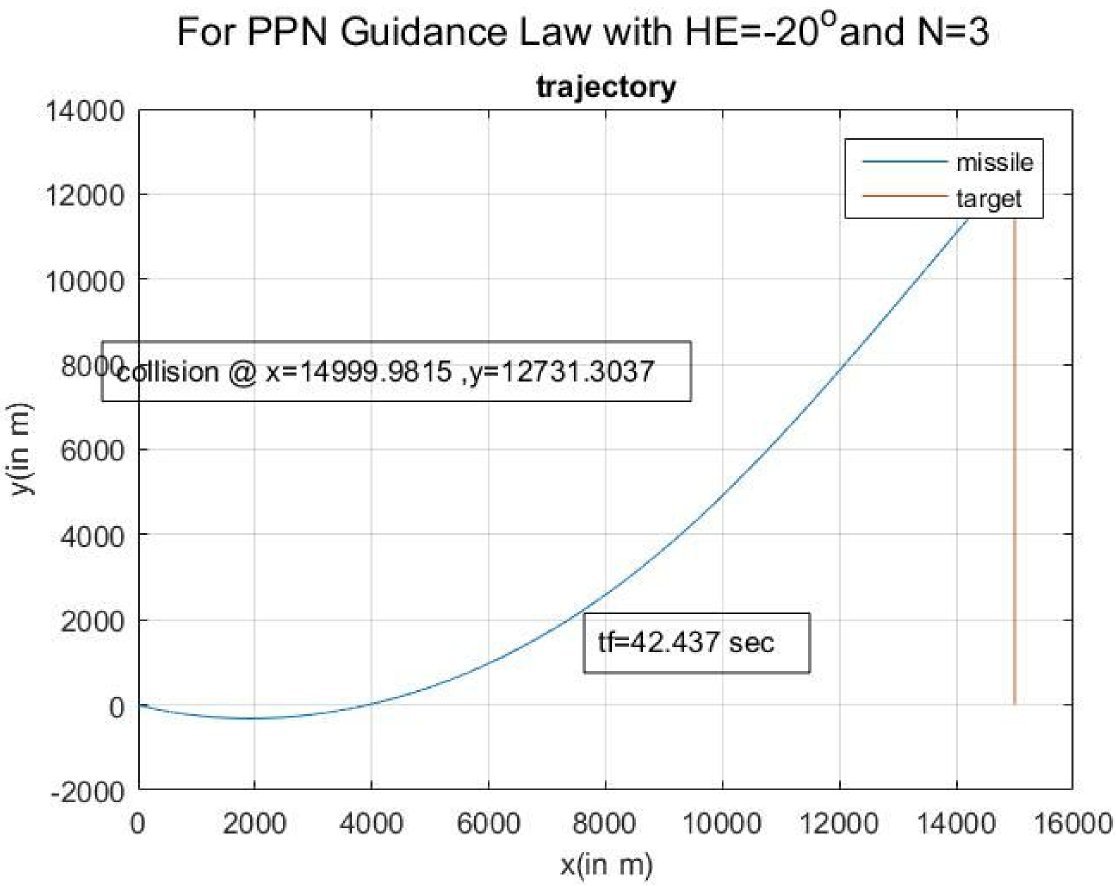

For PPN Guidance Law with HE=20°and N=10
trajectory
missile
target
collision @ x=15000.2047 ,y=11285.2313
tf=37.618 sec
y(in m)
x(in m)
0
2000
4000
6000
8000
10000
12000
14000
16000

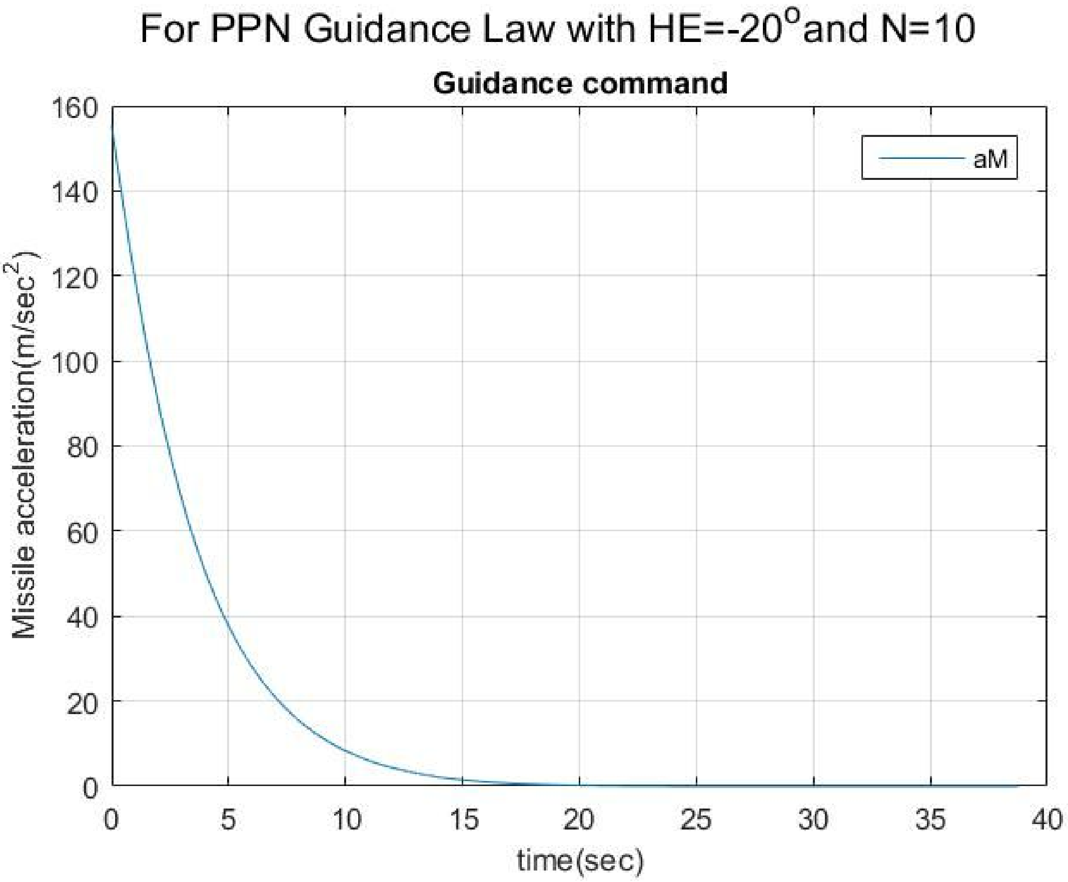
Plots for PPN with HE = -20°

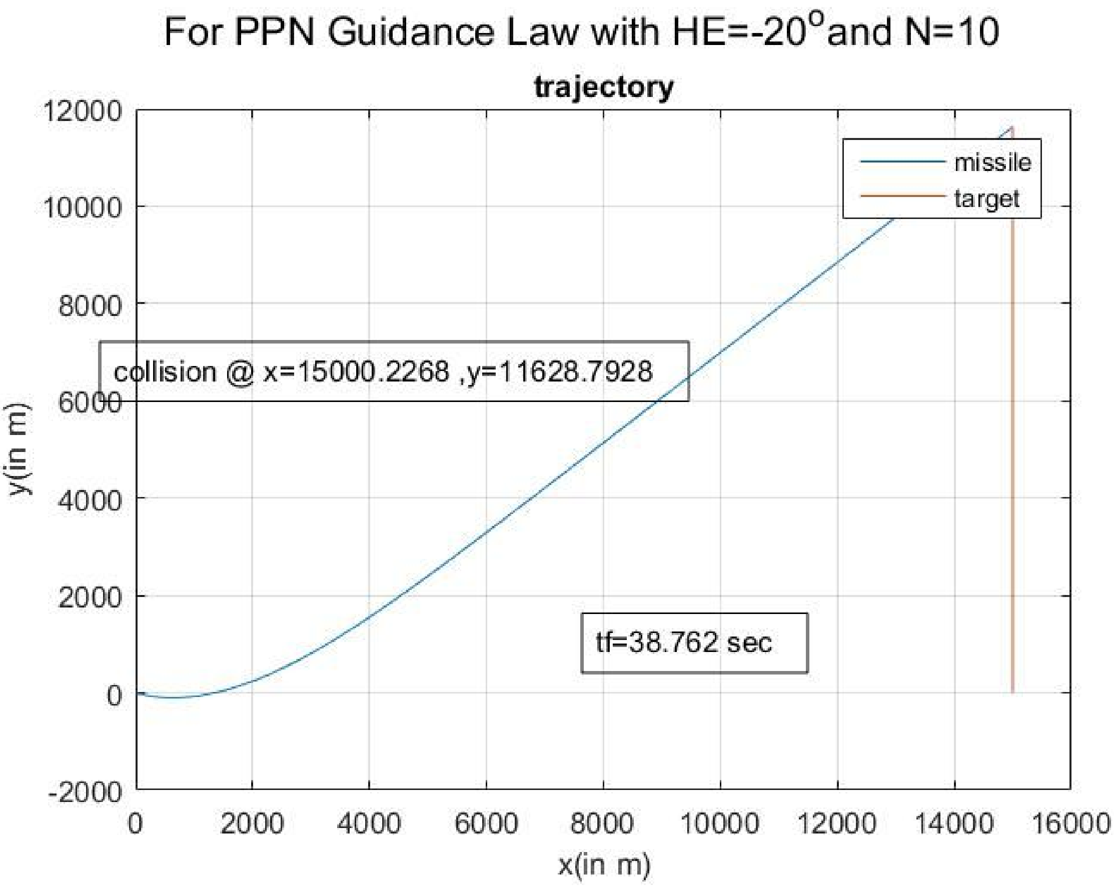





For PPN Guidance Law with HE=-20^{o}and N=3
Guidance command
aM
Missile acceleration(m/sec^2)
time(sec)
50
40
30
20
10
0
-10
0
5
10
15
20
25
30
35
40
45


For PPN Guidance Law with HE=-20^{o}and N=3
trajectory
missile
target
collision @ x=14999.9815 ,y=12731.3037
tf=42.437 sec
y(in m)
x(in m)
14000
12000
10000
8000
6000
4000
2000
0
-2000
0
2000
4000
6000
8000
10000
12000
14000
16000


For PPN Guidance Law with HE=-20°and N=10
Guidance command
aM
Missile acceleration(m/sec²)
time(sec)
0
5
10
15
20
25
30
35
40
160
140
120
100
80
60
40
20
0


For PPN Guidance Law with HE=-20°and N=10
trajectory
missile
target
collision @ x=15000.2268 ,y=11628.7928
tf=38.762 sec
y(in m)
x(in m)
12000
10000
8000
6000
4000
2000
0
-2000
0
2000
4000
6000
8000
10000
12000
14000
16000

For RTPN with HE =20: As the value of N increases the guidance command requirement decreases with time . Also with increase in N the acceleration requirement decreases more rapidly. Collision time is almost same for all cases. But for HE = -20 the collision time varies significantly. Guidance command follows the same pattern.
For PPN with HE = -20 : In this case also as the acceleration requirement decreases but the collision time does not differ much. For HE =20 the nature of curve of the guidance command changes its nature from concave to convex.

Note: The MATLAB Code used to generate the code is in separate file for each part of the problem. The same code can be run for multiple times by changing variables like N or deviation angle and plots will get generated for each case.