A novel Fermatean fuzzy BWM-VIKOR based multi-criteria decision-making approach for selecting health care waste treatment technology

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Abstract

Medical waste management (MWM) is a challenging issue for medical facility managers owing to its potential to prevent environmental and health risks. Many treatment technologies (TTs) have been used for MWM, and selecting appropriate TTs is a complex multi-criteria decision-making (MCDM) problem. In this paper, we propose a novel integrated MCDM method based on the best-worst method (BWM) and the VlseKriterijuska Optimizacija I Komoromisno Resenje (VIKOR) method under the Fermatean fuzzy environment to evaluate, rank, and select treatment technologies for medical waste management. First, novel distance measure and entropy measure for Fermatean fuzzy sets are developed, and their properties are examined. Second, different treatment technologies are evaluated by experts using Fermatean fuzzy sets. Third, the experts' weights are determined using a novel method based on the entropy measure, and the criteria weights are calculated by a novel hybrid criteria weight calculation method. Finally, the ranking of different treatment technologies is determined by the proposed method. To validate the proposed method, a real case study in Jinan, China is presented, where the proposed method is applied to determine the optimal health care waste (HCW) treatment technologies from five TTs with eight criteria. The results are compared to other MCDM methods, which shows the effectiveness and reliability of the proposed method. Moreover, sensitivity analysis is also carried out to show the robustness of the proposed method. From the results, it can be found that the proposed method could provide reliable and robust results for obtaining the optimal health care waste treatment technology.

Keywords: Medical waste management, Fermatean fuzzy set, VIKOR, Distance measure, Multi-criteria decision-making, Best-worst method

1. Introduction

For hospital and medical facility managers, how to properly treat and dispose heal care waste (HCW) is one of the most challenging issues, as contaminated and even infectious waste could exist [1]. Over the past decades, the amount of HCW has exponentially increased due to the rapid growth and expansion of medical facilities, especially in developing countries [2]. With the increasing volume of HCW, how to properly manage HCW has become a critical issue directly connected to the environment and public health [3]. Due to the existence of potentially hazardous and infectious waste in HCW, there often exist toxic chemical substances, infectious pathogens, heavy metals, and even radioactive substances. Inappropriate handling and decomposing of these wastes could lead to potential health hazards and environmental pollution, as many materials involved in HCW are potentially harmful to human health and the public environment. Hence, medical waste management (MWM), which is to manage and treat HCW, has received extensive attention in recent years, especially in many developing countries as they often face increasing demand for HCW management [4, 5].

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The core of MWM is the treatment technology, that is, how to treat and decompose the HCW. Generally, hospitals and medical facilities often have their own supplier for the treatment and disposal of medical waste, and there are often several suppliers available. For hospital and medical facility managers, they need to determine the most appropriate treatment solution for their waste through evaluations, where various factors of the treatment technology should be considered, such as loading capacity, waste type, environmental emissions, technical reliability, health and safety concerns, waste mass and volume reduction, and others [6]. Hence, the HCW treatment selection problem can be regarded as a complex multi-criteria decision-making (MCDM) problem, and it is possible to adopt proper MCDM methods to determine the appropriate treatment technology for MWM [7–9].

When adopting MCDM methods for treatment technology selection, the decisions are often obtained based on the judgments of experts. With that, uncertainty has become a common issue as the lack of information has often led to partial and uncertain judgments of experts. In practice, decision-makers (DM) often use linguistic terms to express their subjective judgments, which makes it increasingly hard to precisely model such information using crisp numbers, instead, uncertain methods such as fuzzy sets and evidence theory have been used in many cases [10–13].

The intuitionistic fuzzy sets (IFSs), proposed by Atanassov [14] on the basis of fuzzy set theory, is one of the widely used methods to handle uncertain information [15–17]. By using the membership degree and non-membership degree whose sum is no more than 1 to model human knowledge under uncertainty, the IFS has been successfully applied to various decision-making problems [18–20]. However, the constraint that the sum of the membership degree and non-membership degree should be no more than 1 could lead to some limitations. For instance, if the decision-maker assigns the membership degree with a value of 0.7 and the non-membership degree with a value of 0.5, that is, the sum of the membership degree and non-membership degree is $1.2 > 1$, which cannot be modeled by IFS. To this end, Yager [21] presented the idea of Pythagorean fuzzy sets (PFSs), where the square sum of the membership degree and non-membership degree should be no more than 1. Compared with IFS, PFS could provide more flexibility for the experts to model their uncertain judgments [22]. Recently, Senapati and Yager [23] introduced the concept of Fermatean fuzzy sets (FFSs), where the sum of the cubes of membership degree and non-membership degree is limited to 1, that is, $\mu^3 + \nu^3 \leq 1$. This means that FFS is more powerful than FS, IFS, and PFS as they all are contained in the space of FFS, and FFS could provide more flexibility for experts to express their judgments under a relatively high degree of uncertainty. On the other hand, compared with other uncertainty measures such as evidence theory and rough sets, with the inclusion of both the membership degree and the non-membership degree, the FFS could more effectively capture the uncertainty and subjectivity of human knowledge.

However, to the best of our knowledge, there have been very few studies that utilize Fermatean fuzzy sets for medical waste management. Moreover, current distance measures and entropy measures for FFSs are not well-suited for all situations, and there is hardly any research on the extension of the VIKOR method under the Fermatean fuzzy environment. The main motivations for this study can be summarized as follows:

(1) The current distance measures for FFSs often have some limitations when dealing with conflicting information, in addition, there lacks of a reliable method to measure the uncertainty of the FFS. To address these limitations, novel distance measure and entropy measure for FFSs should be studied.

(2) The MWM treatment technology selection problem is often a group decision-making problem, where several experts are involved in the decision-making process, each exhibits different importance. Therefore, there requires an effective expert weight calculation method to determine the weights of different experts.

(3) When selecting treatment technology for MWM, there often exist several criteria that should be taken into account, and how to accurately model the importance of these criteria remains a challenging issue. Hence, it is necessary to develop a reliable weighing method to properly calculate the criteria weights for MWM.

(4) In view of uncertain information in MWM, more flexibility is always preferred when dealing with human judgments under uncertainty. Compared with IFS and PFS, the FFS could provide more flexibility by expanding the domain of membership and non-membership degrees. Therefore, it is important to extend MCDM methods under the Fermatean fuzzy environment for MWM treatment technology selection.

This research gap has motivated us to extend the treatment technology selection problem in MWM under the Fermatean fuzzy environment. In this study, a novel distance measure and a novel entropy measure are introduced for Fermatean fuzzy sets, and the VIKOR method is extended with Fermatean fuzzy sets using the novel distance measure. Moreover, an integrated MCDM approach combining the BWM, the entropy-based objective weight calculation method, and the FFVIKOR method is introduced to select the optimal MWM treatment technology. This study has the following novelties:

(1) A novel distance measure and a novel entropy measure for FFS are defined, and the properties of these measures are proved. The proposed measures could more accurately measure the differences among FFSs and determine the degree of uncertainty of FFS.

(2) A novel objective expert weight calculation method is introduced, where the weights of the experts are determined based on the certainty degree of their judgments using the novel entropy measure.

(3) A hybrid criteria weight calculation method based on the novel distance measure and the BWM is proposed. In this method, the subjective weights are determined using the BWM, whereas the objective weights are determined based on the overall support degree of each criterion using the novel distance measure.

(4) The VIKOR method is extended with Fermatean fuzzy sets, and an integrated MCDM method called the FF-BWM-VIKOR method is proposed on the basis of that. The proposed method is applied to treatment technology selection for MWM, and the results are validated through comparative analysis and sensitivity analysis.

The remainder of the paper is organized as follows: Section 2 introduces some related works in this field. Section 3 reviews several basic concepts of FFSs. Section 4 introduces the novel distance measure and entropy measure for FFSs and describes the framework of the proposed FF-BWM-VIKOR method. A case study of medical waste management is presented in Section 5 to show the effectiveness of the proposed method. Section 6 provides further analysis and discussions of the experimental results. Finally, Section 8 concludes this paper.

2. Literature review

2.1. HCW technology selection

In the past decades, numerous researchers have focused on HCW treatment technologies. Voudrias [6] detailed the five most widely used technologies for infectious medical waste treatment, and selected the suitable one using the analytic hierarchy process (AHP) based on the assessments of environmental, economic, technical, and social criteria. Ho [24] evaluated different infectious medical waste disposal suppliers, and used fuzzy AHP to determine the objective weights of the critical evaluation criteria for selecting infectious medical waste disposal suppliers. Hsu et al. [25] conducted a study regarding the selection of infectious medical waste disposal firms, and it proposed an AHP-based method for selecting medical waste disposal firms based on the results of interviews with experts, thus reducing the effects of subjective information. Jangre et al. [26] studied on the identification and prioritization of factors that influence the business practices from MWM, and 18 factors are identified using the best-worst method (BWM). Aung et al. [27] conducted a study on the evaluation of medical waste management practices, and used AHP to assess the medical waste management system of 8 hospitals in Myanmar. Ozkan [28] analyzed the medical waste management practices in Turkey, and proposed two medical waste treatment option selection methods using the AHP and ELECTRE. Yazdani et al. [29] integrated the BWM and interval rough numbers for selecting the optimal medical waste disposal location, where a new interval rough number Dombi-Bonferroni mean operator is used.

Xiao [30] introduced a novel MCDM method for assessing health-care waste treatment technologies based on D numbers, where D numbers is used to model the judgments of experts. Ghoushchi et al. [31] proposed a novel MCDM method for selecting the optimal landfill for medical waste using spherical fuzzy step-wise weight assessment ratio analysis (SFSWARA) and spherical fuzzy weighted aggregated sum product assessment (SFWASPAS) methods under the spherical fuzzy environment. Liu et al. [32] proposed a Pythagorean fuzzy combined compromise solution (PF-CoCoSo) method to evaluate and rank different treatment technology alternatives, where the criteria weights are determined using a novel criteria weight calculation approach. Narayanamoorthy et al. [33] proposed a novel MCDM method combining the hesitant fuzzy subjective and objective weight integrated approach (HF-SOWIA) and the hesitant fuzzy multi-objective optimization on the basis of simple ratio analysis (HF-MOOSRA) for selecting the optimal alternative for medical waste management. Mishra et al. [34] conducted MWM treatment selection using the distance from average solution framework (EDAS) based on parametric divergence measures under the intuitionistic fuzzy environment. Manupati et al. [35] studied 9 HCW disposal techniques, and proposed a novel evaluation and selection framework based on socio-technical and triple bottom line perspectives, where the fuzzy VIKOR method is adopted to evaluate and rank these alternatives. Krishankumar et al. [36] investigated the selection of HCW treatment technology, and proposed a novel evaluation method based on distance from average solution approach with generalized orthopair fuzzy information (GOFI), where criteria weights are computed using entropy measures. Rani et al. [37] proposed an integrated MCDM method for evaluating and selecting the optimal HCW treatment technology based on Pythagorean fuzzy stepwise weight assessment ratio analysis (PF-SWARA) and additive ratio assessment (PF-ARAS) methods, where the attribute weights are determined using the PF-SWARA method and different alternatives are ranked based on PF-ARAS. Chen et al. [38] presented a novel decision-making method based on Z numbers and the TODIM method, and applied the proposed method to assess and select optimal treatment technology for MWM. However, these studies are limited to intuitionistic fuzzy sets and Pythagorean fuzzy sets, and they are unable to provide reliable results with Fermatean fuzzy information.

2.2. *Fermatean fuzzy sets*

FFS has received increasing attention in recent years. For instance, Senapati and Yager [23] introduced several functions for FFS, including the score function and accuracy function, and defined several operators for calculating with FFS. Senapati and Yager [39] presented the Fermatean fuzzy weighted averaging (FFWA) operator and Fermatean fuzzy weighted geometric (FFWG) operator for FFS, and applied these operations in MCDM problems. Simic et al. [40] combined the method based on the removal effects of criteria (MEREC) and combined compromise solution (CoCoSo) method under the Fermatean fuzzy environment, and applied the proposed MCDM method to adopt urban transport planning to the pandemic. Aydemir and Yilmaz Gunduz [41] used Dombi operations to develop Fermatean fuzzy aggregation operators, and extended the TOPSIS method with Fermatean fuzzy sets for MCDM problems. Yang et al. [42] presented a novel distance measure for FFSs, and proposed Fermatean fuzzy TOPSIS method on the basis of that for evaluating green low-carbon ports. Mishra et al. [43] studied the third-party reverse logistics providers selection problem, and proposed a hybrid method based on CRITIC and EDAS methods with Fermatean fuzzy sets. Saha et al. [44] proposed a double normalized MARCOS approach with Fermatean fuzzy information to evaluate and select warehouse locations for automotive industry.

2.3. *MCDM methods*

For MCDM problems, one important issue is to determine the weights of different decision criteria. As most MCDM problems often require several criteria to enable more comprehensive and reliable results, it is important to determine the importance of these criteria such that more balanced and accurate results could be obtained. There are mainly two kinds of approaches for criteria weight calculation: subjective methods and objective methods. Subjective methods mainly reflect the subjective opinion and judgments of the decision-maker (DM), where the weights of the criteria are determined based on the information provided by the experts. Due to its advantages of reflecting the actual opinion of experts, subjective methods have been widely adopted in criteria weight calculation. The most commonly used subjective methods include AHP method [45], DEMATEL method [46] and BWM [47], all of which are based on the pairwise comparison. It has been proven that for MCDM problems with n criteria, the AHP method would need to perform $n(n-1)/2$ pairwise comparisons, the DEMATEL method would require $n(n-1)$ pairwise comparisons, whereas BWM only requires $2n-3$ pairwise comparisons. In addition, it has also shown by Liang et al. [48] that BWM has higher consistency than AHP, and that has led to the increasing attention and application of the BWM. On the other hand, objective methods focus on the information contained in the decision matrix while ignoring the opinions of DMs. Generally, objective methods often rely on mathematical models, such as entropy measure [49], method based on the removal effects of criteria (MEREC), and multiple objective programming. As both the subjective methods and objective methods have certain advantages owing to their unique characteristics, there have been studies combining these two methods, and several hybrid criteria weight calculation approaches have been developed. For instance, Goodridge et al. [50] proposed a novel attribute weight calculation approach by combining the AHP method and sensitive simple additive weighting (SSAW). Liu et al. [51] integrated the AHP method with entropy measure to determine the weights of different factors for evaluating the feasibility of hydraulic fracturing in hydrate-bearing sediments. Yu et al. [52] introduced a new integrated weighing method based on stepwise weighted assessment ratio analysis II (SWARA II) and MEREC, and applied the proposed method to selecting appropriate offshore wind turbines. However, there have been few studies on the hybrid criteria weighting method under the Fermatean fuzzy environment.

On the other hand, many MCDM methods have been developed to handle the uncertainty in real-world MCDM problems in the literature [53]. Opricovic [54] proposed a novel MCDM method, called the VIKOR (Vlsekriterijumska Optimizacija I KOmpromisno Resenje) method, which is proposed to solve discrete MCDM problems with noncommensurable and conflicting criteria by determining compromise solutions for rank and select considering conflicting criteria. This method could reach a comprised solution that is the closest to the ideal solution. Devi [55] later extended the VIKOR method under the intuitionistic fuzzy environment, and applied the intuitionistic fuzzy VIKOR (IF-VIKOR) method to select robots for material handling task. Zeng et al. [56] extended the intuitionistic fuzzy VIKOR method with a novel score function, and combined it with the TOPSIS method to rank alternatives. Gul et al. [57] extended the VIKOR method with Pythagorean fuzzy sets, and proposed a Pythagorean fuzzy numbers-based-VIKOR (PFVIKOR) approach for safety risk assessment of underground mine. Liang et al. [58] introduced an integrated MCDM approach by combining the VIKOR method and the TODIM method under the Pythagorean fuzzy environment, and applied the proposed approach to evaluate Internet banking website quality. Zhou and Chen [59] introduced a generalized distance measure for PFSs, and extended the PFVIKOR method using the novel distance measure. Bakioglu and Atahan [60] combined the VIKOR method with AHP under the Pythagorean fuzzy environment, and proposed a novel hybrid MCDM method for prioritizing risks involved with self-driving vehicles. Gul et al. [61] combined VIKOR and BWM, and utilized the proposed method for the prioritization of control measures in risk assessment. Due to the lack of information, subjective human opinion, and complexity of the situation, uncertainty has become an inevitable feature in the process of HCW treatment, which has driven numerous studies on MWM using MCDM methods under different fuzzy environments.

3. Preliminaries

Proposed by Senapati and Yager [23], Fermatean fuzzy set (FFS) is a novel way to deal with uncertain information with more flexibility and efficiency. Similar to IFS and PFS, the FFS mainly includes two components, namely, membership degree and non-membership degree. Several basic concepts of FFS are detailed as follows:

Definition 1. Let X be the fixed universal set, a Fermatean fuzzy set F on X is defined as a function that applied to X, which can be represented as:

$$F = \{< x, \mu_F(x), \nu_F(x) > | x \in X\} \tag{1}$$

where $\mu_F : X \to [0, 1]$ is the membership degree, $\nu_F : X \to [0, 1]$ is the non-membership degree, and $\mu_F(x)^3 + \nu_F(x)^3 \leq 1$. For each $x \in X$, the indeterminacy degree is defined as:

$$\pi_F(x) = \sqrt[3]{1 - \mu_F(x)^3 - \nu_F(x)^3} \tag{2}$$

Additionally, $F = (\mu, \nu)$ is called a Fermatean fuzzy number (FFN), such that $\mu, \nu \in [0, 1]$ and $0 \leq \mu^3 + \nu^3 \leq 1$.

Remark. Compared with IFS and PFS, FFS limits the sum of the cubes of membership degree and non-membership degree to be no more than 1, thus having a larger domain (as shown in Fig 1). Hence, FFS could be more effective in dealing with uncertainty than IFS and PFS.

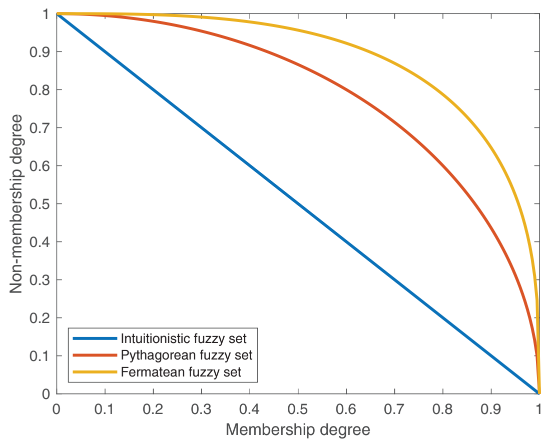

Figure 1: The relationship between IFS, PFS and FFS.

Definition 2. Let $F = (\mu, \nu)$ be a FFN, the score function of F is defined as:

$$S(F) = \mu^3 - \nu^3 \tag{3}$$

where $S(F) \in [-1, 1]$.

Definition 3. Let $F = (\mu, nu)$ be a FFN, the accuracy function of F is defined as:

$$A(F) = \mu^3 + \nu^3 \tag{4}$$

where $A(F) \in [0, 1]$.

For any two FFNs $F_1 = (\mu_1, \nu_1)$ and $F_2 = (\mu_2, \nu_2)$, there is:

(1) if $S(F_1) < S(F_2)$, then $S_1 \prec S_2$

(2) if $S(F_1) = S(F_2)$, then

  (a) if $A(F_1) < A(F_2)$, then $S_1 \prec S_2$

  (b) if $A(F_1) = A(F_2)$, then $S_1 \simeq S_2$

Definition 4. Let $F = (\mu, \nu)$, $F_1 = (\mu_1, \nu_1)$ and $F_2 = (\mu_2, \nu_2)$ be three FFNs and $\lambda > 0$ be a real number, then the basic operation laws on FFNs are defined as:

(1) $F^c = (\mu, \nu)$.

(2) $F_1 \oplus F_2 = \left(\left(\mu_1^3 + \mu_2^3 - \mu_1^3\mu_2^3\right)^{\frac{1}{3}}, \nu_1\nu_2\right)$.

(3) $F_1 \otimes F_2 = \left(\mu_1\mu_2, \left(\nu_1^3 + \nu_2^3 - \nu_1^3\nu_2^3\right)^{\frac{1}{3}}\right)$.

(4) $F^k = \left(\mu^k, \left(1 - (1 - \nu^3)^k\right)^{\frac{1}{3}}\right)$.

(5) $kF = \left(\left(1 - (1 - \mu^3)^k\right)^{\frac{1}{3}}, \nu^k\right)$.

Definition 5. Let A, B, and C be three FFSs on X, the distance measure is a mapping $D : FFS(X) \times FFS(X) \rightarrow [0, 1]$ with the following properties:

(D1) $0 \leq D(A, B) \leq 1$.

(D2) $D(A, B) = D(B, A)$.

(D3) $D(A, B) = 0$ iff $A = B$.

(D4) $D(A, A^c) = 1$ iff A is a crisp set.

(D5) If $A \subseteq B \subseteq C$, then $D(A, B) \leq D(A, C)$ and $D(B, C) \leq D(A, C)$.

Definition 6. Let A and B be two FFSs on X, the entropy measure is a mapping $E : FFS(X) \rightarrow [0, 1]$, satisfying the following properties:

(E1) $0 \leq E(A) \leq 1$.

(E2) $E(A) = 0$ iff A is a crisp set.

(E3) $E(A) = 1$ iff $\mu_A(x) = \nu_A(x)$.

(E4) $E(A) = E(A^c)$.

(E5) $E(A) \leq E(B)$ if $\mu_A(x) \leq \mu_B(x) \leq \nu_B(x) \leq \nu_A(x)$ or $\nu_A(x) \leq \nu_B(x) \leq \mu_B(x) \leq \mu_A(x)$.

Definition 7. Let $F_i = (\mu_i, \nu_i)(i = 1, 2, \ldots, n)$ be a set of FFNs and $w = (w_1, w_2, \ldots, w_n)$ be the corresponding weight vector with $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$. Then the Feratean fuzzy weighted averaging (FFWA) operator is a mapping $FFWA : F^n \to F$, such that

$$FFWA(F_1, F_2, \ldots, F_n) = (\sum_{i=1}^{n} w_i\mu_i, \sum_{i=1}^{n} w_i, \nu_i) \tag{5}$$

4. Methodology

4.1. New distance measure for FFSs

Definition 8. Let A and B be two FFSs on X, a novel distance measure for FFSs is defined as follows:

$$D_n(A, B) = 1 - \frac{1}{|X|} \sum_{x \in X} \frac{(\mu_A(x)^3 \wedge \mu_B(x)^3) + ((1 - \nu_A(x)^3) \wedge (1 - \nu_B(x)^3))}{(\mu_A(x)^3 \vee \mu_B(x)^3) + ((1 - \nu_A(x)^3) \vee (1 - \nu_B(x)^3))} \tag{6}$$

Theorem 1. The mapping $D_n(A, B)$ given by Eq. (6), is a distance measure for FFSs.

Proof. The proofs for (D1)-(D5) are given as follows:

(D1): It is obvious.

(D2): It is obvious.

(D3): Let $A = B$, hence $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$. Thus, according to (6), there is:

$$D_n(A, B) = 0.$$

On the other hand, let $D_n(A, B) = 0$, then, from Eq. (6), there is:

$$\begin{aligned}&(\mu_A(x)^3 \wedge \mu_B(x)^3) + ((1 - \nu_A(x)^3) \wedge (1 - \nu_B(x)^3))\\ =&(\mu_A(x)^3 \vee \mu_B(x)^3) + ((1 - \nu_A(x)^3) \vee (1 - \nu_B(x)^3))\end{aligned}$$

for all $x \in X$.

Since $\forall x \in X$,

$$(\mu_A(x)^3 \wedge \mu_B(x)^3) \leq (\mu_A(x)^3 \vee \mu_B(x)^3)$$

and

$$((1 - \nu_A(x)^3) \wedge (1 - \nu_B(x)^3)) \leq \mu_B(x)^3) + ((1 - \nu_A(x)^3) \vee (1 - \nu_B(x)^3)).$$

Hence,

$$D_n(A, B) = 0$$

when $\mu_A(x) = \mu_B(x)$ and $(1 - \nu_A(x)) = (1 - \nu_B(x))$.

That is,

$$A = B.$$

That concludes the proof.

(D4): It is obvious.

(D5): Let A, B, and C be three FFSs, and $A \subseteq B \subseteq C$, that is,

$$\mu_A(x) \leq \mu_B(x) \leq \mu_C(x)$$

and

$$\nu_A(x) \geq \nu_B(x) \geq \nu_C(x),$$

for all $x \in X$..

Clearly, there is

$$(1 - \nu_A(x)^3) \leq (1 - \nu_B(x)^3) \leq (1 - \nu_C(x)^3).$$

Then, according to Eq. (6), there is

$$\begin{aligned} D_n(A,B) &= 1 - \frac{1}{|X|}\sum_{x\in X}\frac{(\mu_A(x)^3 \wedge \mu_B(x)^3) + ((1-\nu_A(x)^3) \wedge (1-\nu_B(x)^3))}{(\mu_A(x)^3 \vee \mu_B(x)^3) + ((1-\nu_A(x)^3) \vee (1-\nu_B(x)^3))} \\ &= 1 - \frac{1}{|X|}\sum_{x\in X}\frac{\mu_A(x)^3 + (1-\nu_A(x)^3)}{\mu_B(x)^3 + (1-\nu_B(x)^3)} \end{aligned}$$

Similarly, there is

$$\begin{aligned} D_n(A,C) &= 1 - \frac{1}{|X|}\sum_{x\in X}\frac{(\mu_A(x)^3 \wedge \mu_C(x)^3) + ((1-\nu_A(x)^3) \wedge (1-\nu_C(x)^3))}{(\mu_A(x)^3 \vee \mu_C(x)^3) + ((1-\nu_A(x)^3) \vee (1-\nu_C(x)^3))} \\ &= 1 - \frac{1}{|X|}\sum_{x\in X}\frac{\mu_A(x)^3 + (1-\nu_A(x)^3)}{\mu_C(x)^3 + (1-\nu_C(x)^3)} \end{aligned}$$

Because

$$\frac{1}{\mu_B(x)^3 + (1-\nu_B(x)^3)} \geq \frac{1}{\mu_C(x)^3 + (1-\nu_C(x)^3)},$$

hence,

$$D_n(A,B) \leq D_n(A,C).$$

Similarly,

$$D_n(B,C) \leq D_n(A,C).$$

That concludes the proof. □

4.2. New entropy measure for FFSs

Definition 9. Let A be a FFS on X, a novel entropy measure for FFS is defined as follows:

$$E_n(A) = \frac{\sum_{x\in X}\mu_A(x)^3 \wedge \nu_A(x)^3}{\sum_{x\in X}\mu_A(x)^3 \vee \nu_A(x)^3} \tag{7}$$

Theorem 2. The mapping $E_n(A)$ given by Eq. (7) is an entropy measure for FFS.

Proof. The proofs for (E1)-(E5) are given as follows:

(E1): It is obvious.

(E2): Let A be a crisp set, i.e.,

$$\mu_A(x) = 0$$

or

$$\nu_A(x) = 0.$$

Then, from Eq. (7), there is

$$E(A) = 0.$$

On the other hand, let $E(A) = 0$, then according to Eq. (7), there is

$$\mu_A(x)^3 \wedge \nu_A(x)^3 = 0$$

for all $x \in X$.

Therefore, we have

$$\mu_A(x) = 0$$

or

$$\nu_A(x) = 0,$$

that is, A is a crisp set.

That concludes the proof.

(E3): Let $\mu_A(x) = \nu_A(x)$, then from Eq. (7), there is

$$E(A) = 1.$$

On the other hand, let $E(A) = 1$, then, from Eq. (7), there is

$$(\mu_A(x)^3 \wedge \nu_A(x)^3) = (\mu_A(x)^3 \vee \nu_A(x)^3)$$

for all $x \in X$.

Obviously,

$$\mu_A(x) = \nu_A(x).$$

That concludes the proof.

(E4): It is obvious.

(E5): Let A, B be two FFSs, and

$$\mu_A(x) \leq \mu_B(x) \leq \nu_B(x) \leq \nu_A(x).$$

Clearly, there is

$$\frac{\mu_A(x)^3}{\nu_A(x)^3} \leq \frac{\mu_B(x)^3}{\nu_B(x)^3}$$
$$\Leftrightarrow \frac{\mu_A(x)^3 \wedge \nu_A(x)^3}{\mu_A(x)^3 \vee \nu_A(x)^3} \leq \frac{\mu_B(x)^3 \wedge \nu_B(x)^3}{\mu_B(x)^3 \vee \nu_B(x)^3}.$$

Hence, for all $x \in X$, there is

$$E(A) \leq E(B).$$

Similarly, if

$$\nu_A(x) \leq \nu_B(x) \leq \mu_B(x) \leq \mu_A(x),$$

then

$$E(A) \leq E(B).$$

That concludes the proof. □

4.3. *FF-BWM-VIKOR method for MWM*

In this section, an integrated FF-BWM-VIKOR method is developed for medical waste management, which includes three stages: expert weights calculation, criteria weights calculation and treatment technology selection. The detailed process of the proposed method is described as follows.

4.3.1. *Stage I: Expert weight calculation*

Step 1: Determine the MWM problem

For the MWM problem, consider a set of m alternatives $T = \{T_1, T_2, \ldots, T_m\}$ with a set of n criteria $C = \{C_1, C_2, \ldots, C_n\}$. Let there be t decision-makers (DMs) $E = \{E_1, E_2, \ldots, E_t\}$ to provide evaluations, and the evaluation of the ith alternative T_i over the jth criterion C_j by the kth DM is denoted by a^k_{ij}, which is in the form of FFN. For the kth DM, its evaluations can be summarized as a decision matrix:

$$D_k = \begin{bmatrix} a^k_{11} & a^k_{12} & \cdots & a^k_{1n} \\ a^k_{21} & a^k_{22} & \cdots & a^k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a^k_{m1} & a^k_{m2} & \cdots & a^k_{mn} \end{bmatrix} \tag{8}$$

Step 2: Compute the DM weights

For the MWM problem, as multiple DMs are involved, how to determine the weights of different DMs is an important issue. In this paper, we propose an entropy-based DM weight calculation method using the proposed entropy measure, which is described as follows:

Step 2.1: Calculate the certainty degree of different DMs

The relative importance of different DMs can be characterized by their certainty degree on their evaluations, which can be calculated based on the entropy measure. For the kth DM, let its evaluation of the ith alternative over the jth criteria be $a_{ij}^k = (\mu_{ij,k}, \nu_{ij,k})$, the corresponding entropy is calculated using Eq. (7) as:

$$E(a_{ij}^k) = \frac{\mu_{ij,k}^3 \wedge \nu_{ij,k}^3}{\mu_{ij,k}^3 \vee \nu_{ij,k}^3} \tag{9}$$

Then, the certainty degree of the kth DM is computed by:

$$cer(E_k) = \sum_{i=1}^{m} \sum_{j=1}^{n} E(a_{ij}^k) \tag{10}$$

Step 2.2: Compute the weights of DMs

By normalizing the certainty degree, the weights of the DMs are given as:

$$\omega_k = \frac{cer(E_k)}{\sum_{k=1}^{t} cer(E_k)} \tag{11}$$

Clearly, $0 \leq \omega_k \leq 1$ and $\sum_{k=1}^{t} \omega_k = 1$.

Step 3: Aggregate the evaluations of different DMs

In order to construct the aggregated decision matrix and obtain more comprehensive results, the Fermatean fuzzy weighted averaging (FFWA) operator is adopted, and the aggregated Fermatean fuzzy decision matrix is obtained as:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \tag{12}$$

where a_{ij} is the aggregated FFN, and is computed using Eq. (5) as:

$$a_{ij} = FFWA(a_{ij}^1, a_{ij}^2, \ldots, a_{ij}^t) \tag{13}$$

4.3.2. Criteria weights calculation

Step 4: Compute the criteria weights

In order to determine the weight of each criterion, a novel hybrid weight calculation method combining the BWM and the proposed distance measure is introduced, which is described as follows:

Step 4.1: Calculate the support degree between different criteria

For any two criteria C_p and C_q, calculate the distance between the aggregated evaluations of these criteria of the same alternative as:

$$D(a_{ip}, a_{iq}) = 1 - \frac{(\mu_{ip}^3 \wedge \mu_{iq}^3) + ((1 - \nu_{ip}^3) \wedge (1 - \nu_{iq}^3))}{(\mu_{ip}^3 \vee \mu_{iq}^3) + ((1 - \nu_{ip}^3) \vee (1 - \nu_{iq}^3))} \tag{14}$$

where $a_{ip} = (\mu_{ip}, \nu_{ip})$ denotes the aggregated evaluation of the ith alternative over the pth criterion, and $a_{iq} = (\mu_{iq}, \nu_{iq})$ denotes the aggregated evaluation of the ith alternative over the qth criterion.

Then, construct the similarity matrix representing the similarity among different criteria as:

$$SMM = \begin{bmatrix} S_{1,1} & S_{1,2} & \cdots & S_{1,n} \\ S_{2,1} & S_{2,2} & \cdots & S_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{n,1} & S_{n,2} & \cdots & S_{n,n} \end{bmatrix} \tag{15}$$

where $S_{p,q} = 1 - D(a_{ip}, a_{iq})$ is the support degree between the evaluations on C_p and C_q.

Step 4.2: Determine the overall support degree of each criterion

Based on the similarity matrix, the overall support degree of the jth criterion is calculated as:

$$Sup(a_{ij}) = \sum_{p=1, j \neq p}^{n} S_{j,p} \tag{16}$$

Step 4.3: Calculate the importance degree of each criterion

For each criterion, the importance degree δ_j is calculated based on the overall support degree as:

$$\delta_j = \frac{1}{m(n-1)} \sum_{i=1}^{m} \sum_{j=1}^{n} Sup(a_{ij}) \tag{17}$$

Step 4.4: Calculate the objective weight of each criterion

The objective weight of each criterion is calculated by normalizing the importance degree as:

$$\gamma_j = \frac{\delta_j}{\sum_{j=1}^{n} \delta_j} \tag{18}$$

Step 4.5: Determine the best and the worst criteria

Based on the analysis of criteria, determine the best, i.e., most important, and the worst, i.e., least important criteria, denoted as C_B and C_W, respectively.

Step 4.6: Obtain the best-to-other vectors and others-to-worst vectors

For each DM, conduct pairwise comparison between the best criterion and other criteria, and construct the best-to-others vector as:

$$BO^k = (v_{B1}^k, v_{B2}^k, \ldots, v_{Bn}^k) \tag{19}$$

where v_{Bj}^k represents the preference of C_B over the jth criterion C_j by DM E_k, and $v_{BB}^k = 1$.

Similarly, the preference of each criterion over the worst criterion is determined, and the others-to-worst vector can be obtained as:

$$OW^k = (v_{1W}^k, v_{2W}^k, \ldots, v_{nW}^k) \tag{20}$$

where v_{jW}^k is the preference of C_j over C_W, and $v_{WW}^k = 1$.

Step 4.7: Calculate the individual criteria weights of each DM

For each DM, based on its best-to-others and others-to-worst vectors, the individual criteria weights can be calculated by solving the following optimization model:

$$\begin{aligned} &\min \max \left\{ \left| \frac{\epsilon_B^k}{\epsilon_j^k} - v_{Bj}^k \right|, \left| \frac{\epsilon_j^k}{\epsilon_W^k} \right| \right\} \\ &s.t. \begin{cases} \sum_{j=1}^{n} \epsilon_j^k = 1 \\ 0 \leq \epsilon_j^k \leq 1 \end{cases} \end{aligned} \tag{21}$$

The above model can be equivalently converted into the following optimization model:

$$\min \ \xi^k$$
$$s.t. \begin{cases} |\epsilon_B^k - v_{Bj}^k \epsilon_j^k| \leq \xi^k \\ |\epsilon_j^k - v_{jW}^k \epsilon_W^k| \leq \xi^k \\ \sum_{j=1}^{n} \epsilon_j^k = 1 \\ 0 \leq \epsilon_j^k \leq 1 \end{cases} \tag{22}$$

By solving Model (22), the optimal individual criteria $\epsilon^{k*} = (\epsilon_1^{k*}, \epsilon_2^{k*}, \ldots, \epsilon_n^{k*})$ of the kth DM E_k can be obtained.

Step 4.8: Determine the subjective weights of criteria

By combining the individual weights of different DMs, the subjective weights of the criteria can be computed as:

$$\eta_j = \sum_{k=1}^{t} \omega_k \epsilon_j^k \tag{23}$$

Step 4.9: Determine the weights of the criteria

The weights of the criteria are computed by combining the subjective weights and objective weights as:

$$w_j = \theta\gamma_j + (1-\theta)\eta_j \tag{24}$$

where γ_j represents the subjective weight of the jth criterion, η_j represents the objective weight of the jth criterion. $\theta(0 \leq \theta \leq 1)$ is the adjustment coefficient, which can be determined based on the actual situation. Clearly, $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$.

4.3.3. Treatment technology selection

Step 5: Construct the normalized decision matrix

Since both benefit criteria and cost criteria could be involved, the Fermatean fuzzy decision matrix A should be normalized as:

$$R = [\rho_{ij}]_{m \times n} \tag{25}$$

where

$$\rho_{ij} = \begin{cases} a_{ij} = (\mu_{ij}, \nu_{ij}) & \text{for benefit criterion} \\ (a_{ij})^c = (\nu_{ij}, \mu_{ij}) & \text{for cost criterion} \end{cases} \tag{26}$$

Step 6: Determine the best and the worst values of each criterion

For the jth criterion, determine the best values ϕ_j^+ and the worst value ϕ_j^- as:

$$\begin{aligned} \phi_j^+ &= \max_i \rho_{ij} \\ \phi_j^- &= \min_i \rho_{ij} \end{aligned} \tag{27}$$

Step 7: Calculate the specific group utility and the individual regret

Based on the novel distance measure, calculate the specific group utility S_i and the individual regret R_i of the ith alternative by calculating the distance between the best value and the ith alternative as:

$$S_i = \sum_{j=1}^{n} w_j \frac{D(\phi_j^+, \rho_{ij})}{D(\phi_j^+, \phi_j^-)} \tag{28}$$

$$R_i = \max_j w_j \frac{D(\phi_j^+, \rho_{ij})}{D(\phi_j^+, \phi_j^-)} \tag{29}$$

where w_j is the weight of the criterion C_j.

Step 8: Calculate the aggregating index of the alternatives

By using the specific group utility and the individual regret, calculate the aggregating index of each alternative as:

$$Q_i = \tau \frac{S_i - S^-}{S^+ - S^-} + (1-\tau)\frac{R_i - R^-}{R^+ - R^-} \tag{30}$$

where $S^+ = \max_i S_i$, $S^- = \min_i S_i$, $R^+ = \max_i R_i$, $R^- = \min_i R_i$. $\tau(0 \leq \tau \leq 1)$ is the weight of the specific group utility, and $(1-\tau)$ is the weight of the individual regret.

Step 9: Rank different alternatives according to S_i, R_i and Q_i

Rank the different alternatives according to the values of S_i, R_i and Q_i in ascending order, respectively. Correspondingly, three ranking orders $r_h(h = 1, 2, 3)$ could be obtained.

Step 10: Obtain the compromised solution

Based on the ranking orders, obtain the compromised solution where $A_{(1)}$ is ranked the best with minimum A if the following two conditions are satisfied:

Condition 1: Acceptable advantage

The difference between the first and the second alternatives satisfies:

$$Q(A_{(2)}) - Q(A_{(1)}) \geq \frac{1}{m-1} \tag{31}$$

Condition 2: Acceptable stability in decision-making

The best alternative $A_{(1)}$ must also be the best one according to the value of S or/and R. This compromise solution is stable in the decision-making process. When $\tau > 0.5$, it should be the strategy of maximum group utility, or "with veto" ($\tau < 0.5$), or "by consensus" ($\tau = 0.5$).

If one of these two conditions is not satisfied, a set of compromised solutions will be proposed, specifically,

(1) $A_{(1)}$ and $A_{(2)}$ are compromised solutions if Condition 2 is not satisfied.

(2) $A_{(1)}, A_{(2)}, \ldots, A_{(m)}$ are compromised solutions if Condition 1 is not satisfied, where the closeness of the alternative $A_{(m)}$ is determined by:

$$Q(A_{(m)}) - Q(A_{(1)}) < \frac{1}{m-1} \tag{32}$$

The process of the proposed FF-BWM-VIKOR method is shown in Fig 2.

5. Case study

In order to validate the proposed method, in this section, a practical case of infectious medical waste treatment technology selection in China is presented.

In Jinan, China, with the increasing development of hospitals and health care facilities, more and more medical waste is produced, most of which remains untreated. This has caused significant risks to public health and the environment as the spread of medical waste.

To this end, the proposed FF-BWM-VIKOR method is applied to assess and select appropriate medical waste treatment technology. Five different treatment technologies are considered, which are A_1: steam sterilization, A_2: incineration, A_3: chemical disinfection, A_4: microwave and A_5: landfill disposal. The details of the treatment technologies are listed as follows:

Steam sterilization (A_1): It is an easy-to-use and reliable way for medical waste treatment that could be monitored and validated. It mainly conducts heating in an autoclave using saturated steam under a certain pressure to attain the desired chamber temperature.

Incineration (A_2): It is the procedure of burning waste at high temperatures, normally at 900-1000 $°C$. It could offer the benefit of a rapid, easy disposal procedure, but there are emission concerns, even though specific states and areas actively promote incineration as the favored process of treatment.

Chemical disinfection (A_3): It mainly uses chemical agents for disinfection, particularly chlorine. It is most suitable for liquid wastes, even though it can also be applied to treat solid wastes.

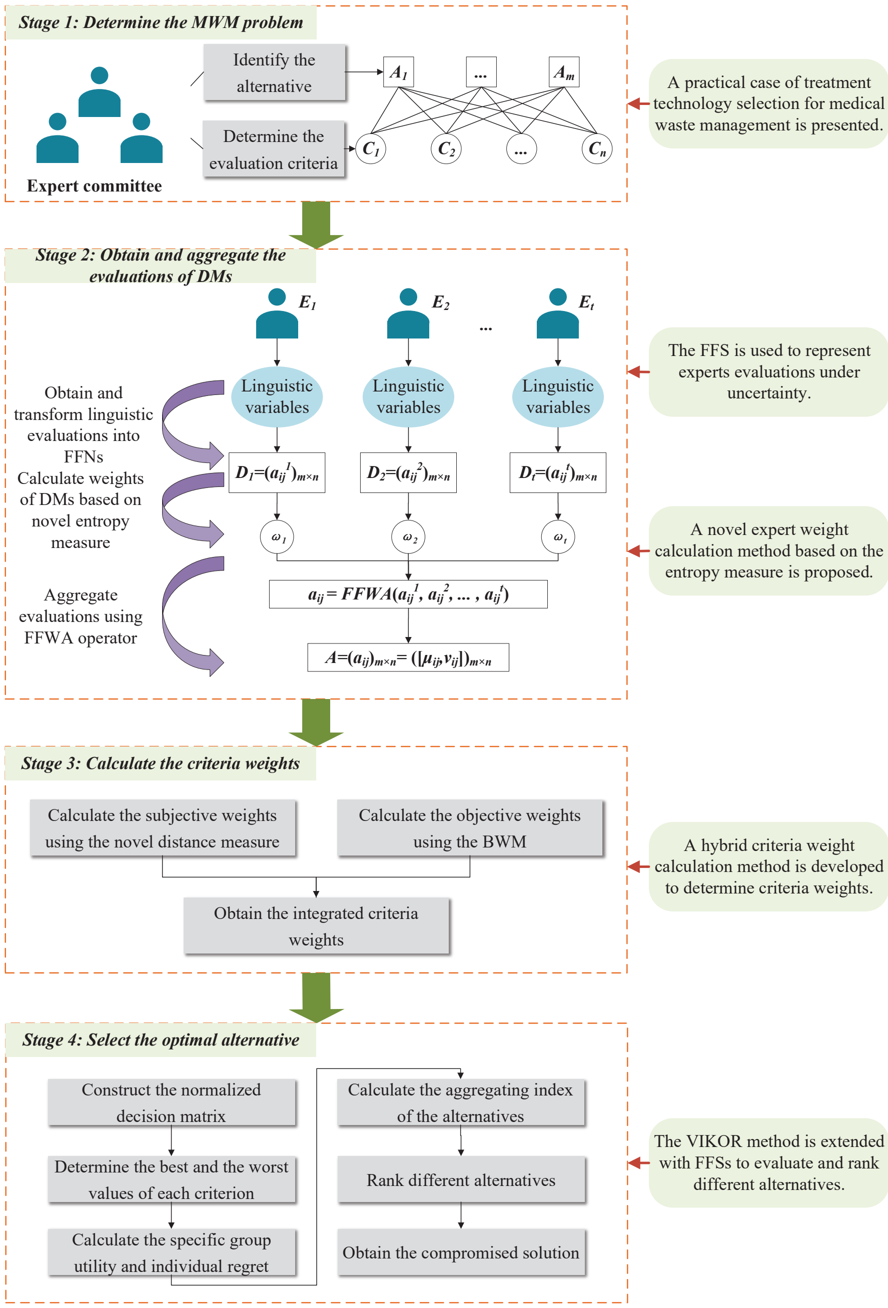

Figure 2: Process of the proposed method.

Microwave (A_4): It uses electromagnetic waves to disinfect waste, as the high frequency creates the molecules in the receiving body to vibrate quickly as they endeavor to support the varying electromagnetic field. Waste is initially shredded, mixed with water, and internally heated to neutralize all existing biological matter. The shredding process results in volume reduction and the energy usage is lower than that of an incinerator.

Landfill disposal (A_5): It is a general process for its low cost and simplicity. However, it could potentially threaten the well-being of the environment due to the infectious content.

These technologies are assessed in eight criteria, namely, "cost" (C_1), "waste residuals" (C_2), "release with health effects" (C_3), "energy consumption" (C_4), "reliability" (C_5), "volume reduction" (C_6), "treatment effectiveness" (C_7), and "public acceptance" (C_8). The framework of this case is shown in Fig 3.

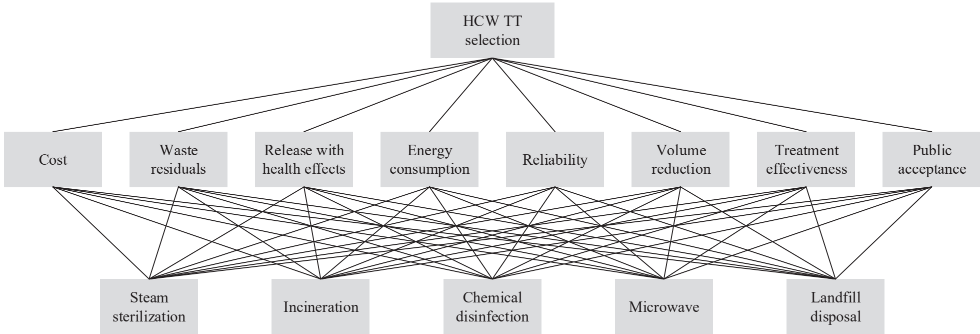

Figure 3: Framework of the case study.

To conduct the study, five experts (E_1, E_2, E_3, E_4, E_5) are invited to provide their evaluations, and an expert committee is constructed. In order to ensure the comprehensiveness and reliability of the evaluation, these experts are selected from different backgrounds, including university professor, environmental engineer, disposal expert, waste management engineer, and hospital manager. Two interviews are conducted with these experts, in the first interview, the proposed method was introduced to the experts, and questionnaires are given to the experts to obtain their judgments on criteria weights and all five alternatives. In the second interview, the questionnaires are collected, and the evaluations provided by the experts are further discussed to ensure the reliability of their judgments.

For each DM, in order to provide more flexibility for the judgments, both the pairwise comparisons among different criteria and the evaluations of different alternatives are expressed using linguistic variables. The linguistic variables of the criteria weight and alternative performance are listed in Table 1 and Table 2. Table 1 shows the pairwise comparisons in terms of linguistic variables. Table 2 presents the evaluations of the WMW alternatives in the form of linguistic variables and FFNs.

Table 1: Linguistic variables for pairwise comparison.

Linguistic variables	Values
Extremely more important (EM)	9
Very highly more important (VH)	8
Highly more important (H)	7
Slightly highly more important (SH)	6
Moderately highly more important (MH)	5
Moderately more important (MM)	4
Moderately lowly more important (ML)	3
Slightly more important (SM)	2
Equal important (E)	1

The linguistic evaluations of DMs on different medical waste treatment technologies are listed in Table 3, which are presented in the form of the nine-level linguistic variables listed in Table 2. According to Table 2, the linguistic evaluations are equivalently converted into FFNs, as listed in Table 4.

In order to determine the weights of the DMs, the certainty degrees of the DMs are calculated using the proposed entropy measure. For example, for the evaluation of E_1 on A_1 regarding C_2, i.e., $(0.95, 0.30)$, the entropy can be calculated as:

$$E_{12}^{1} = 0.0315$$

Similarly, by computing the entropy of each evaluation, the certainty degree of the DMs can be obtained as:

$$cer(E_1) = 33.9168, cer(E_2) = 29.2653, cer(E_3) = 29.5283, cer(E_4) = 31.1195, cer(E_5) = 29.4760$$

Table 2: Evaluations of alternatives in terms of linguistic variables.

Linguistic variables	Fermatean fuzzy numbers
Extremely high (EH)	(0.95,0.30)
Very high (VH)	(0.80,0.35)
High (H)	(0.75,0.40)
Moderately high (MH)	(0.60,0.45)
Moderate (M)	(0.50,0.55)
Moderately low (ML)	(0.45,0.60)
Low (L)	(0.40,0.75)
Very low (VL)	(0.35,0.80)
Extremely low (EL)	(0.30.0.95)

Table 3: Linguistic evaluations of DMs for MWM treatment technology alternatives.

DM	Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
	A_1	VH	EH	VL	EH	MH	EL	L	M
	A_2	EH	EH	VL	EH	EH	M	VH	VL
E_1	A_3	ML	EH	VH	EH	MH	EL	VH	EH
	A_4	H	H	H	ML	MH	VL	H	EL
	A_5	L	EL	EL	VH	H	L	EH	EL
	A_1	ML	ML	H	VH	VL	M	M	MH
	A_2	H	H	L	H	MH	VL	VL	M
E_2	A_3	EH	ML	MH	L	H	L	M	H
	A_4	EH	EH	M	VL	VL	L	VH	L
	A_5	VH	L	EH	ML	VL	L	MH	M
	A_1	ML	VH	MH	M	EH	L	H	H
	A_2	ML	MH	EL	EL	M	VH	EH	VL
E_3	A_3	MH	M	EL	ML	VL	VH	L	M
	A_4	VL	MH	L	MH	H	H	M	EL
	A_5	L	EH	VL	VH	M	EH	EL	ML
	A_1	EL	EH	EL	H	VH	VH	EL	ML
	A_2	L	VH	ML	EH	VL	L	VL	VL
E_4	A_3	VH	MH	M	VL	VH	MH	ML	M
	A_4	ML	EL	L	VL	VL	L	ML	EL
	A_5	EH	EH	M	M	ML	EH	ML	VL
	A_1	VH	ML	L	ML	EL	VL	EH	EH
	A_2	MH	EL	L	ML	VH	EL	EL	VL
E_5	A_3	MH	H	MH	M	M	L	H	VL
	A_4	H	VL	ML	MH	VH	EL	EH	H
	A_5	M	ML	M	L	M	M	VH	VH

Correspondingly, the weight of the DMs can be calculated using Eq. (11) as:

$$\omega_1 = 0.2212, \omega_2 = 0.1909, \omega_3 = 0.1926, \omega_4 = 0.2030, \omega_5 = 0.1923$$

In the next step, based on the decision matrices of different DMs and the weights of the DMs, the aggregated decision matrix is obtained by using the FFWA operator, and the aggregated decision Fermatean fuzzy decision matrix is listed in Table 5.

Next, in order to determine the weights of criteria, the support degrees between different criteria are computed. By using the novel distance measure, the distance between the aggregated evaluations of each two criteria of the same alternative could be calculated, and the similarity matrix could be constructed accordingly. For example, the similarity

Table 4: Fermatean decision matrices of the DMs.

DM	Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
	A_1	$(0.80, 0.35)$	$(0.95, 0.30)$	$(0.35, 0.80)$	$(0.95, 0.30)$	$(0.60, 0.45)$	$(0.30, 0.95)$	$(0.40, 0.75)$	$(0.50, 0.55)$
	A_2	$(0.95, 0.30)$	$(0.95, 0.30)$	$(0.35, 0.80)$	$(0.95, 0.30)$	$(0.95, 0.30)$	$(0.50, 0.55)$	$(0.80, 0.35)$	$(0.35, 0.80)$
E_1	A_3	$(0.45, 0.60)$	$(0.95, 0.30)$	$(0.80, 0.35)$	$(0.95, 0.30)$	$(0.60, 0.45)$	$(0.30, 0.95)$	$(0.80, 0.35)$	$(0.95, 0.30)$
	A_4	$(0.75, 0.40)$	$(0.75, 0.40)$	$(0.75, 0.40)$	$(0.45, 0.60)$	$(0.60, 0.45)$	$(0.35, 0.80)$	$(0.75, 0.40)$	$(0.30, 0.95)$
	A_5	$(0.40, 0.75)$	$(0.30, 0.95)$	$(0.30, 0.95)$	$(0.80, 0.35)$	$(0.75, 0.40)$	$(0.40, 0.75)$	$(0.95, 0.30)$	$(0.30, 0.95)$
	A_1	$(0.45, 0.60)$	$(0.45, 0.60)$	$(0.75, 0.40)$	$(0.80, 0.35)$	$(0.35, 0.80)$	$(0.50, 0.55)$	$(0.50, 0.55)$	$(0.60, 0.45)$
	A_2	$(0.75, 0.40)$	$(0.75, 0.40)$	$(0.40, 0.75)$	$(0.75, 0.40)$	$(0.60, 0.45)$	$(0.35, 0.80)$	$(0.35, 0.80)$	$(0.50, 0.55)$
E_2	A_3	$(0.95, 0.30)$	$(0.45, 0.60)$	$(0.60, 0.45)$	$(0.40, 0.75)$	$(0.75, 0.40)$	$(0.40, 0.75)$	$(0.50, 0.55)$	$(0.75, 0.40)$
	A_4	$(0.95, 0.30)$	$(0.95, 0.30)$	$(0.50, 0.55)$	$(0.35, 0.80)$	$(0.35, 0.80)$	$(0.40, 0.75)$	$(0.80, 0.35)$	$(0.40, 0.75)$
	A_5	$(0.80, 0.35)$	$(0.40, 0.75)$	$(0.95, 0.30)$	$(0.45, 0.60)$	$(0.35, 0.80)$	$(0.40, 0.75)$	$(0.60, 0.45)$	$(0.50, 0.55)$
	A_1	$(0.45, 0.60)$	$(0.80, 0.35)$	$(0.60, 0.45)$	$(0.50, 0.55)$	$(0.95, 0.30)$	$(0.40, 0.75)$	$(0.75, 0.40)$	$(0.75, 0.40)$
	A_2	$(0.45, 0.60)$	$(0.60, 0.45)$	$(0.30, 0.95)$	$(0.30, 0.95)$	$(0.50, 0.55)$	$(0.80, 0.35)$	$(0.95, 0.30)$	$(0.35, 0.80)$
E_3	A_3	$(0.60, 0.45)$	$(0.50, 0.55)$	$(0.30, 0.95)$	$(0.45, 0.60)$	$(0.35, 0.80)$	$(0.80, 0.35)$	$(0.40, 0.75)$	$(0.50, 0.55)$
	A_4	$(0.35, 0.80)$	$(0.60, 0.45)$	$(0.40, 0.75)$	$(0.60, 0.45)$	$(0.75, 0.40)$	$(0.75, 0.40)$	$(0.50, 0.55)$	$(0.30, 0.95)$
	A_5	$(0.40, 0.75)$	$(0.95, 0.30)$	$(0.35, 0.80)$	$(0.80, 0.35)$	$(0.50, 0.55)$	$(0.95, 0.30)$	$(0.30, 0.95)$	$(0.45, 0.60)$
	A_1	$(0.30, 0.95)$	$(0.95, 0.30)$	$(0.30, 0.95)$	$(0.75, 0.40)$	$(0.80, 0.35)$	$(0.80, 0.35)$	$(0.30, 0.95)$	$(0.45, 0.60)$
	A_2	$(0.40, 0.75)$	$(0.80, 0.35)$	$(0.45, 0.60)$	$(0.95, 0.30)$	$(0.35, 0.80)$	$(0.40, 0.75)$	$(0.35, 0.80)$	$(0.35, 0.80)$
E_4	A_3	$(0.80, 0.35)$	$(0.60, 0.45)$	$(0.50, 0.55)$	$(0.35, 0.80)$	$(0.80, 0.35)$	$(0.60, 0.45)$	$(0.45, 0.60)$	$(0.50, 0.55)$
	A_4	$(0.45, 0.60)$	$(0.30, 0.95)$	$(0.40, 0.75)$	$(0.35, 0.80)$	$(0.35, 0.80)$	$(0.40, 0.75)$	$(0.45, 0.60)$	$(0.30, 0.95)$
	A_5	$(0.95, 0.30)$	$(0.95, 0.30)$	$(0.50, 0.55)$	$(0.50, 0.55)$	$(0.45, 0.60)$	$(0.95, 0.30)$	$(0.45, 0.60)$	$(0.35, 0.80)$
	A_1	$(0.80, 0.35)$	$(0.45, 0.60)$	$(0.40, 0.75)$	$(0.45, 0.60)$	$(0.30, 0.95)$	$(0.35, 0.80)$	$(0.95, 0.30)$	$(0.95, 0.30)$
	A_2	$(0.60, 0.45)$	$(0.30, 0.95)$	$(0.40, 0.75)$	$(0.45, 0.60)$	$(0.80, 0.35)$	$(0.30, 0.95)$	$(0.30, 0.95)$	$(0.35, 0.80)$
E_5	A_3	$(0.60, 0.45)$	$(0.75, 0.40)$	$(0.60, 0.45)$	$(0.50, 0.55)$	$(0.50, 0.55)$	$(0.40, 0.75)$	$(0.75, 0.40)$	$(0.35, 0.80)$
	A_4	$(0.75, 0.40)$	$(0.35, 0.80)$	$(0.45, 0.60)$	$(0.60, 0.45)$	$(0.80, 0.35)$	$(0.30, 0.95)$	$(0.95, 0.30)$	$(0.75, 0.40)$
	A_5	$(0.50, 0.55)$	$(0.45, 0.60)$	$(0.50, 0.55)$	$(0.40, 0.75)$	$(0.50, 0.55)$	$(0.50, 0.55)$	$(0.80, 0.35)$	$(0.80, 0.35)$

Table 5: Aggregated Fermatean decision matrix.

Criterion	A_1	A_2	A_3	A_4	A_5
C_1	$(0.5643, 0.5677)$	$(0.6366, 0.4971)$	$(0.6742, 0.4343)$	$(0.6502, 0.4986)$	$(0.6072, 0.5438)$
C_2	$(0.7295, 0.4246)$	$(0.6890, 0.4831)$	$(0.6584, 0.4551)$	$(0.5910, 0.5791)$	$(0.6051, 0.5874)$
C_3	$(0.4740, 0.6771)$	$(0.3798, 0.7691)$	$(0.5662, 0.5445)$	$(0.5061, 0.6055)$	$(0.5128, 0.6389)$
C_4	$(0.6980, 0.4357)$	$(0.6905, 0.5020)$	$(0.5404, 0.5932)$	$(0.4683, 0.6210)$	$(0.5954, 0.5152)$
C_5	$(0.6026, 0.5638)$	$(0.6459, 0.4879)$	$(0.6019, 0.5068)$	$(0.5689, 0.5590)$	$(0.5165, 0.5747)$
C_6	$(0.4685, 0.6845)$	$(0.4704, 0.6767)$	$(0.4955, 0.6563)$	$(0.4371, 0.7321)$	$(0.6368, 0.5335)$
C_7	$(0.5720, 0.5985)$	$(0.5555, 0.6330)$	$(0.5850, 0.5256)$	$(0.6889, 0.4407)$	$(0.6277, 0.5243)$
C_8	$(0.6436, 0.4641)$	$(0.3786, 0.7523)$	$(0.6184, 0.5141)$	$(0.4056, 0.8061)$	$(0.4734, 0.6604)$

of the evaluations of A_1 is obtained as:

$$SMM_1 = \begin{bmatrix} 1.0000 & 0.4500 & 0.5381 & 0.5012 & 0.8629 & 0.5155 & 0.9123 & 0.6194 \\ 0.4500 & 1.0000 & 0.2422 & 0.8980 & 0.5215 & 0.2320 & 0.4351 & 0.7265 \\ 0.5381 & 0.2422 & 1.0000 & 0.2697 & 0.4643 & 0.9580 & 0.5566 & 0.3333 \\ 0.5012 & 0.8980 & 0.2697 & 1.0000 & 0.5808 & 0.2584 & 0.4846 & 0.8091 \\ 0.8629 & 0.5215 & 0.4643 & 0.5808 & 1.0000 & 0.4448 & 0.8343 & 0.7178 \\ 0.5155 & 0.2320 & 0.9580 & 0.2584 & 0.4448 & 1.0000 & 0.5332 & 0.3193 \\ 0.9123 & 0.4351 & 0.5566 & 0.4846 & 0.8343 & 0.5332 & 1.0000 & 0.5989 \\ 0.6194 & 0.7265 & 0.3333 & 0.8091 & 0.7178 & 0.3193 & 0.5989 & 1.0000 \end{bmatrix}$$

By using Eqs. (16)-(18), the overall support degree and the objective weight of each criterion are calculated, as listed in Table 6.

To determine the subjective weights, the best and the worst criteria are identified by the DMs, where C_7 is determined to be the best criterion and C_4 is determined to be the worst criterion. Moreover, the best-to-others and others-to-worst vectors are provided by the DMs, as shown in Table 8.

Based on the best-to-others and others-to-worst vectors, the subjective weights of the criteria are obtained by using

Table 6: Overall support degree and objective weight of each criterion.

Criterion	A_1	A_2	A_3	A_4	A_5	δ	γ
C_1	0.6285	0.5577	0.6173	0.4974	0.7579	0.6117	0.1324
C_2	0.5008	0.5321	0.6612	0.5734	0.7436	0.6022	0.1303
C_3	0.4803	0.3392	0.7269	0.5704	0.6310	0.5496	0.1189
C_4	0.5431	0.5346	0.6488	0.5420	0.7334	0.6004	0.1299
C_5	0.6324	0.5567	0.7505	0.5773	0.6795	0.6393	0.1383
C_6	0.4659	0.4165	0.4964	0.4456	0.7138	0.5076	0.1099
C_7	0.6221	0.4751	0.7513	0.4203	0.7261	0.5990	0.1296
C_8	0.5892	0.3465	0.7431	0.3498	0.5268	0.5111	0.1106

Table 7: Best-to-others and others-to-worst vectors.

Criterion	E_1		E_2		E_3		E_4		E_5	
	v_{Bj}	v_{jW}	v_{Bj}	v_{jW}	v_{Bj}	v_{jW}	v_{Bj}	v_{jW}	v_{Bj}	v_{jW}
C_1	5	4	6	3	4	5	5	3	4	4
C_2	6	3	6	4	5	4	6	4	5	5
C_3	4	6	5	4	4	5	3	6	4	5
C_4	8	1	9	1	8	1	8	1	9	1
C_5	3	6	4	6	3	7	4	5	4	4
C_6	7	2	8	3	7	3	6	2	8	2
C_7	1	8	1	9	1	8	1	9	1	8
C_8	4	5	4	6	5	4	4	5	4	6

Eqs. (22). For example, the optimization model for E_1 is constructed as:

$$
\min\ \xi^1
$$

$$
s.t. \begin{cases}
|\epsilon_7^1 - 5\epsilon_1^1| \le \xi^1,\ |\epsilon_7^1 - 6\epsilon_2^1| \le \xi^1,\ |\epsilon_7^1 - 4\epsilon_3^1| \le \xi^1,\ |\epsilon_7^1 - 8\epsilon_4^1| \le \xi^1 \\
|\epsilon_7^1 - 3\epsilon_5^1| \le \xi^1,\ |\epsilon_7^1 - 7\epsilon_6^1| \le \xi^1,\ |\epsilon_7^1 - 1\epsilon_7^1| \le \xi^1,\ |\epsilon_7^1 - 4\epsilon_8^1| \le \xi^1 \\
|\epsilon_1^1 - 4\epsilon_4^1| \le \xi^1,\ |\epsilon_2^1 - 3\epsilon_4^1| \le \xi^1,\ |\epsilon_3^1 - 6\epsilon_4^1| \le \xi^1,\ |\epsilon_4^1 - 1\epsilon_4^1| \le \xi^1 \\
|\epsilon_5^1 - 6\epsilon_4^1| \le \xi^1,\ |\epsilon_6^1 - 2\epsilon_4^1| \le \xi^1,\ |\epsilon_7^1 - 8\epsilon_4^1| \le \xi^1,\ |\epsilon_8^1 - 5\epsilon_4^1| \le \xi^1 \\
\sum_{j=1}^{n} \epsilon_j^1 = 1 \\
0 \le \epsilon_j^1 \le 1
\end{cases}
$$

By solving the optimization models constructed based on the best-to-others and others-to-worst vectors of the DMs, the corresponding weights determined by each DM and the subjective weights are obtained, as shown in Table 9.

Table 8: Subjective weight of each criterion.

Criterion	E_1	E_2	E_3	E_4	E_5	η
C_1	0.0902	0.0814	0.1109	0.0876	0.0990	0.0937
C_2	0.0752	0.0794	0.0887	0.0730	0.0966	0.0823
C_3	0.1127	0.0977	0.1108	0.1460	0.0992	0.1137
C_4	0.0338	0.0349	0.0336	0.0365	0.0362	0.0350
C_5	0.1502	0.1221	0.1478	0.1095	0.1012	0.1267
C_6	0.0644	0.0611	0.0633	0.0726	0.0604	0.0645
C_7	0.3607	0.4013	0.3561	0.3650	0.3865	0.3734
C_8	0.1127	0.1221	0.0887	0.1095	0.1208	0.1108

Further, based on Table 6 and Table 8, the criteria weights could be obtained by combining the objective weights and the subjective weights. In this case, the coefficient $\theta = 0.5$, and the criteria weights are obtained as:

$$w = (0.1130, 0.1063, 0.1163, 0.0825, 0.1325, 0.0872, 0.2515, 0.1107)$$

In order to determine the most appropriate treatment technology, the FF-VIKOR approach is adopted. It should be noted that in the studied case, criteria C_1, C_2, C_3, and C_4 are cost criteria, and the others are benefit criteria. Corresponding, the aggregated Fermatean fuzzy decision matrix in Table 5 should be normalized. By using Eq. (26), the normalized decision matrix is obtained, as shown in Table 9.

Table 9: Normalized decision matrix.

Criterion	A_1	A_2	A_3	A_4	A_5
C_1	(0.5677, 0.5643)	(0.4971, 0.6366)	(0.4343, 0.6742)	(0.4986, 0.6502)	(0.5438, 0.6072)
C_2	(0.4246, 0.7295)	(0.4831, 0.6890)	(0.4551, 0.6584)	(0.5791, 0.5910)	(0.5874, 0.6051)
C_3	(0.6771, 0.4740)	(0.7691, 0.3798)	(0.5445, 0.5662)	(0.6055, 0.5061)	(0.6389, 0.5128)
C_4	(0.4357, 0.6980)	(0.5020, 0.6905)	(0.5932, 0.5404)	(0.6210, 0.4683)	(0.5152, 0.5954)
C_5	(0.6026, 0.5638)	(0.6459, 0.4879)	(0.6019, 0.5068)	(0.5689, 0.5590)	(0.5165, 0.5747)
C_6	(0.4685, 0.6845)	(0.4704, 0.6767)	(0.4955, 0.6563)	(0.4371, 0.7321)	(0.6368, 0.5335)
C_7	(0.5720, 0.5985)	(0.5555, 0.6330)	(0.5850, 0.5256)	(0.6889, 0.4407)	(0.6277, 0.5243)
C_8	(0.6436, 0.4641)	(0.3786, 0.7523)	(0.6184, 0.5141)	(0.4056, 0.8061)	(0.4734, 0.6604)

Next, for each criterion, the best and the worst values are determined according to Eq. (27), as shown in Table 10.

Table 10: Best and worst values of each criterion.

Criterion	Best value	Worst value
C_1	(0.5677, 0.5643)	(0.4343, 0.6742)
C_2	(0.5791.0.5910)	(0.4246, 0.7295)
C_3	(0.7691, 0.3798)	(0.5445, 0.5662)
C_4	(0.6210, 0.4683)	(0.4357, 0.6980)
C_5	(0.6459, 0.4879)	(0.5165, 0.5747)
C_6	(0.6368, 0.5335)	(0.4371, 0.7321)
C_7	(0.6889, 0.4407)	(0.5555, 0.6330)
C_8	(0.6436, 0.4641)	(0.3786, 0.7523)

By using Eq. (28) and Eq. (29), the group utility and the individual regret of each alternative are calculated based on the novel distance measure, as listed in Table 11. Correspondingly, the aggregating index of each alternative is computed using Eq. (30), where $\tau = 0.5$, and the results are listed in Table 11.

Table 11: Overall compromised solution of the alternatives.

Alternative	Group utility		Individual regret		Aggregating index	
	Value	Ranking	Value	Ranking	Value	Ranking
A_1	0.6218	3	0.2238	2	0.8194	2
A_2	0.6548	1	0.2515	1	1.0000	1
A_3	0.6456	2	0.1744	3	0.7059	3
A_4	0.4559	5	0.1093	5	0.0000	5
A_5	0.5308	4	0.1325	4	0.2700	4

Hence, based on the group utility, individual regret and aggregating index, three ranking orders could be obtained, as listed in Table 11. It can be found that both Condition 1 and Condition 2 are satisfied. Therefore, the ranking order obtained based on the aggregating index is the acceptable solution, that is, A_2 is evaluated to be the best treatment technology for medical waste management, and the ranking order of the treatment technologies is $A_2 \succ A_1 \succ A_3 \succ A_5 \succ A_4$.

6. Results

According to the results of the case study, the proposed FF-BWM-VIKOR method could evaluate several medical waste treatment technologies and select the most appropriate treatment technology. From the results, it can be found that incineration is evaluated to be the best alternative for medical waste management in Jinan, which is intuitive to the opinions of experts. Moreover, this result is also consistent with the actual situation and previous studies. In Jinan, incineration is one of the most widely used and recommended technologies for medical waste treatment for its low cost, high capacity, and great effectiveness. It could also eliminate the threat that medical waste may pose completely. Steam sterilization and chemical disinfection are ranked second and third, respectively, which are also some of the widely used technologies in Jinan. In order to better illustrate the effectiveness and feasibility of the proposed method, we conduct further analysis.

6.1. Comparative analysis

In order to validate the proposed FF-BWM-VIKOR method, we further investigated the rankings obtained by several other methods, including Pythagorean fuzzy TOPSIS, Pythagorean fuzzy VIKOR, Fermatean fuzzy TOPSIS, Fermatean fuzzy EDAS, Fermatean fuzzy WASPAS, and Fermatean fuzzy MARCOS using the same case. Table 12 shows the results obtained by the proposed FF-BWM-VIKOR method and the comparative methods. From Table 12, it can be found that A_2, i.e., incineration, is evaluated to be the optimal treatment technology by all the methods except PF-TOPSIS, which shows the reliability of the results obtained by the proposed method. However, it is worth noting that the rankings of some low-ranked alternatives have changed, which is caused by the fact that different methods evaluate these alternatives differently. In general, the comparison results further validate the consistency and robustness of the proposed method as other methods have also selected incineration as the optimal treatment technology for medical waste management. The ranking orders of these methods are also illustrated in Fig 4.

Table 12: Comparative results.

Method	Ranking order
Proposed FF-BWM-VIKOR	$A_2 \succ A_1 \succ A_3 \succ A_5 \succ A_4$
Pythagorean fuzzy TOPSIS	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
Pythagorean fuzzy VIKOR	$A_2 \succ A_3 \succ A_1 \succ A_4 \succ A_5$
Fermatean fuzzy TOPSIS	$A_2 \succ A_3 \succ A_1 \succ A_5 \succ A_4$
Fermatean fuzzy EDAS	$A_2 \succ A_3 \succ A_5 \succ A_1 \succ A_4$
Fermatean fuzzy WASPAS	$A_2 \succ A_1 \succ A_3 \succ A_4 \succ A_5$
Fermatean fuzzy MARCOS	$A_2 \succ A_1 \succ A_3 \succ A_5 \succ A_4$

Since the ranking order obtained by the proposed FF-BWM-VIKOR method is not exactly the same as the results obtained by other methods, it is necessary to calculate the correlation between them. In this study, Spearman's rank correlation coefficients are calculated, as shown in Fig 5. The rank correlation coefficients between the proposed FF-BWM-VIKOR method and the comparative methods are obtained as $(0.55, 0.80, 0.90, 0.70, 0.90, 1.00)$, close to $+1$, indicating that the results of all methods are correlated with each other. Therefore, it can be found that the results obtained by the proposed method are highly consistent with other methods, which further validates the reliability and robustness of the proposed method. From the comparative analysis, the advantages of the proposed FF-BWM-VIKOR method can be obtained as follows:

(1) Both the proposed FF-BWM-VIKOR method and FF-TOPSIS, FF-EDAS, FF-WASPAS, and FF-MARCOS methods are developed based on FFSs, whereas the PF-TOPSIS and PF-VIKOR methods are developed based on PFSs. Compared with PFSs, the FFSs could provide more flexibility in reflecting uncertain information, thus enhancing the reliability of the proposed method.

(2) The proposed method adopts a novel entropy-based expert weight calculation, where the certainty degree of each DM is calculated by using the proposed entropy measure. This procedure ensures the DM weights are determined based on the reliability of their evaluations, hence increasing the reliability of the results without the requirement of prior information. There is no procedure among the compared methods that supports this kind of calculation.

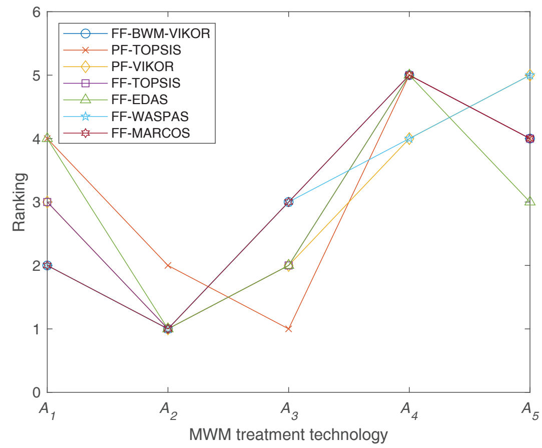

Figure 4: Comparative analysis results.

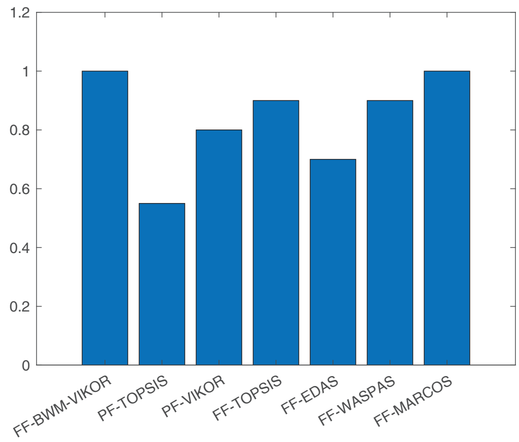

Figure 5: Spearman's rank correlation coefficients of different methods.

(3) The proposed method uses a hybrid criteria weight calculation method to more reliably and accurately determine the weights of the criteria. By using the novel distance measure, the subjective weights of criteria, which are derived based on the evaluations of the DMs, could be obtained. Moreover, by adopting the BWM, the objective weights of the criteria are calculated, which requires significantly fewer pairwise comparisons compared with AHP. Combining the subjective weights and the objective weights, the criteria weight calculation procedure could enable more balanced and reliable results.

(4) In the proposed FF-BWM-VIKOR method, by combining the results of the group utility, individual regret and aggregating index, the compromised solution could be obtained. In most cases, it can be ensured that the optimal solution not only has the best performance, but also has the lowest regret, thus enhancing the reliability and feasibility of the obtained results. From the comparison, it can be found that the results obtained by the proposed method are more flexible, reliable, and reasonable.

6.2. *Sensitivity analysis*

In order to better analyze the performance and behavior of the proposed method, sensitivity analysis is conducted in this section. Here, two sets of sensitivity analyses are conducted, corresponding to the two coefficients used in the proposed method.

Table 13: Criteria weights with different values of θ.

θ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.0	0.0937	0.0823	0.1137	0.0350	0.1267	0.0645	0.3734	0.1108
0.1	0.0975	0.0871	0.1142	0.0445	0.1279	0.0691	0.3490	0.1108
0.2	0.1014	0.0919	0.1147	0.0540	0.1290	0.0736	0.3247	0.1108
0.3	0.1053	0.0967	0.1152	0.0635	0.1302	0.0781	0.3003	0.1107
0.4	0.1092	0.1015	0.1158	0.0730	0.1314	0.0827	0.2759	0.1107
0.5	0.1130	0.1063	0.1163	0.0825	0.1325	0.0872	0.2515	0.1107
0.6	0.1169	0.1111	0.1168	0.0920	0.1337	0.0917	0.2271	0.1107
0.7	0.1208	0.1159	0.1173	0.1014	0.1349	0.0963	0.2028	0.1107
0.8	0.1246	0.1207	0.1179	0.1109	0.1360	0.1008	0.1784	0.1106
0.9	0.1285	0.1255	0.1184	0.1204	0.1372	0.1053	0.1540	0.1106
1.0	0.1324	0.1303	0.1189	0.1299	0.1383	0.1099	0.1296	0.1106

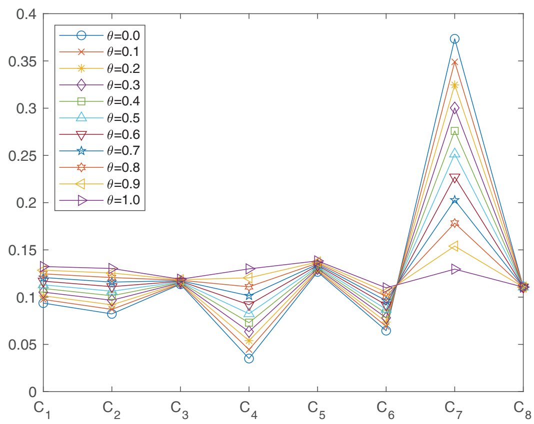

Figure 6: Changes in criteria weights with different values of θ.

Firstly, consider the coefficient θ used for determining the weights of the criteria. Eleven different values are considered for θ, and the results are shown in Table 13 and Fig 6. From Table 13 and Fig 6, the values of θ increase from 0.0 to 1.0, which indicates different preferences on the subjective weight and the objective weight. Correspondingly, the weights of the criteria have fluctuated, and the ranking of different criteria based on their weight has also changed. Moreover, the ranking of the alternatives is illustrated in Fig 7 and Fig 8. From Fig 7 and Fig 8, it can be found that with the changes of θ, though the value of Q changes, the ranking order remains the same for most cases, which further shows the consistency and robustness of the proposed method.

Secondly, consider the coefficient τ used for obtaining the aggregating index Q. Eleven different values are considered for τ, and the results are shown in Table 14 and Fig 9. From Table 14 and Fig 9, when the values of τ increase from 0.0 to 1.0, the results obtained by the proposed method are relatively stable. Specifically, A_2 is always ranked first, whereas A_4 is always the last one. From this sensitivity analysis, it can be concluded that the proposed FF-BWM-VIKOR method is consistent and robust, and it could provide stable and reliable results for MWM.

7. Discussion

In this study, the FF-BWM-VIKOR method is proposed to select the most appropriate treatment technology for MWM in Jinan China. The results obtained from experts' evaluations provide some insights into the importance of the evaluation criteria and appropriate treatment technologies for MWM. For the criteria weights, it is determined that treatment effectiveness (C_4) is the most important criterion, with the highest criterion weight (0.2515), which is in line with the opinions of experts, as the effectiveness of the treatment technology is the utmost concern when

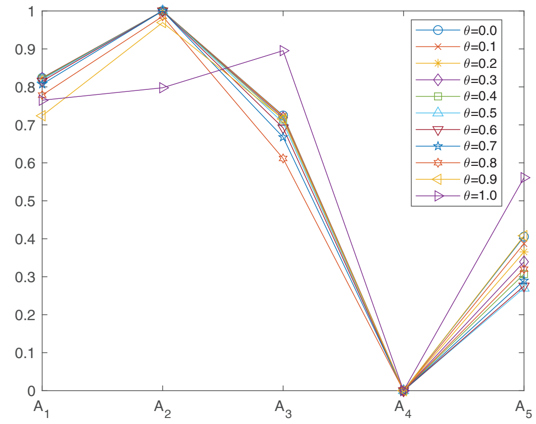

Figure 7: Changes in aggregating index with different values of θ.

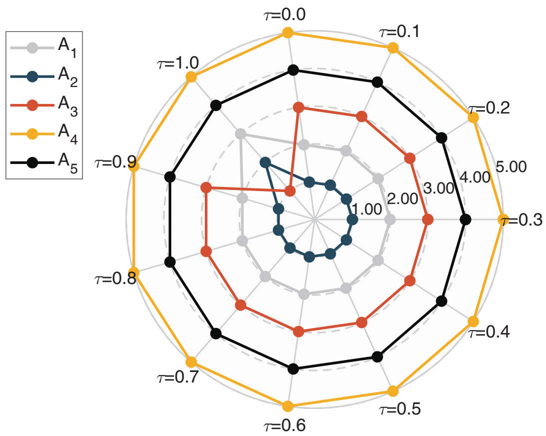

Figure 8: Changes in ranking order with different values of θ.

Table 14: Aggregating index with different values of τ.

τ	A_1	A_2	A_3	A_4	A_5
0.0	0.8048	1.0000	0.4582	0.0000	0.1636
0.1	0.8077	1.0000	0.5078	0.0000	0.1849
0.2	0.8107	1.0000	0.5573	0.0000	0.2061
0.3	0.8136	1.0000	0.6069	0.0000	0.2274
0.4	0.8165	1.0000	0.6564	0.0000	0.2487
0.5	0.8194	1.0000	0.7059	0.0000	0.2700
0.6	0.8223	1.0000	0.7555	0.0000	0.2913
0.7	0.8252	1.0000	0.8050	0.0000	0.3126
0.8	0.8281	1.0000	0.8545	0.0000	0.3339
0.9	0.8311	1.0000	0.9041	0.0000	0.3552
1.0	0.8340	1.0000	0.9536	0.0000	0.3765

selecting appropriate technologies. Reliability (C_5) is also ranked relatively high among the evaluation criteria, and

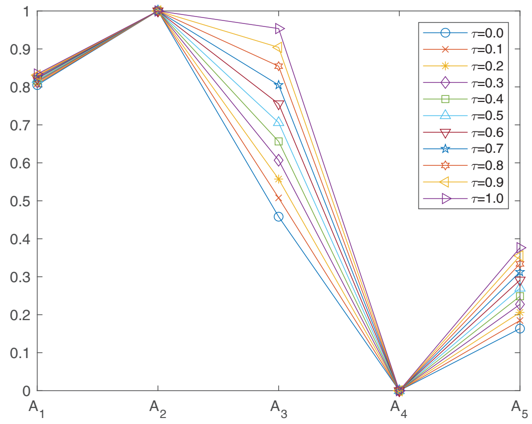

Figure 9: Changes in aggregating index with different values of τ.

it is fully in line with the actual situation. It is worth noting that cost (C_1) and release with health effects (C_3) have very close importance. The cost criterion, which is based on the economic aspect of the treatment technology, has gained attention as the lower cost would significantly increase the preferability of the treatment technology, and it is assigned with relatively high weight (0.1130). The release with health effects criterion focuses on the environmental aspect, and it is highly rated with a weight of 0.1163. This is in line with the actual situation since possible environmental impacts should always be avoided when conducting proper medical waste management. The last criterion is energy consumption (C_4) with a weight of 0.0825, showing that for medical waste management, the possible energy consumption should be considered, however, it can be minimized with good planning.

Moreover, the ranking of the treatment technologies can be explained as follows:

(1) The best treatment technology is incineration (A_2), which is fully in line with the actual situation. Incineration mainly burns waste at very high temperatures, thus offering a rapid, effective, and economical way for medical waste management. This result shows that for medical waste management, effective and efficient treatment technology is always preferred, though there are some emission concerns, it is still favored for its effectiveness. This is also supported by the actual application of incineration in Jinan.

(2) Steam sterilization (A_1) is ranked second. This is used to disinfect infectious health care waste, which could result in significant inactivation. As steam sterilization has shown to be effective in reducing waste volume while having low cost, it has been widely used in various applications, which supports the results.

(3) Chemical disinfection (A_3) is ranked third in this study, which is in line with the actual situation. Chemical disinfection mainly uses sodium hypochlorite as disinfectant, and it could accomplish relatively high inactivation. The decision of using chemical disinfection to treat medical waste has attracted much attention and has been adopted in several applications for its efficiency and low emission, which is supported by the ranking results.

(4) Compared with other technologies, landfill disposal (A_5) is less preferred. It often disposes medical waste in a sanitary landfill, sometimes after certain disinfection techniques. This is not always preferred as it could lead to contamination and it often requires a significant amount of land. Compared with other treatment technologies, landfill disposal is clearly less preferred, and the ranking results obtained by the proposed method are supported by the actual situation.

(5) Microwave (A_4) is the least favored treatment technology for medical waste management. It is essentially a steam-based technology as it is based on moist heat and steam generated by microwave energy. However, it has higher cost and higher energy consumption, while having lower effectiveness. In the proposed method, it is ranked last among different treatment technologies, which is in line with the actual situation.

In this study, the BWM and VIKOR are integrated under Fermatean fuzzy environment, and the proposed method is applied to the health care waste technology selection in Jinan, China. From the results of this study, the following managerial implications and characteristics can be obtained:

(1) The proposed method simultaneously considers subjective and objective information when determining the

DMs' weights and criteria weights, where the certainty of the DMs' evaluation is considered in measuring the DMs' weights, and the FF-BWM is adopted in measuring the criteria weights, thus ensuring the reliability of the obtained criteria weights. Moreover, by using the FF-VIKOR method, the proposed method could generate reliable and comprehensive results that satisfy the compromised conditions. Therefore, the results obtained by the proposed method are guaranteed to be optimal.

(2) The application of Fermatean fuzzy set can be regarded as a way to model imprecise or subjective information, especially when the DM is not able to provide crisp numbers with complete certainty. By adopting Fermatean fuzzy sets, the more reliable and accurate modeling of evaluation could be enabled with the help of the membership degree and the non-membership. Moreover, compared with IFS and PFS, the applied FFS is shown to be more effective with its higher flexibility.

(3) With the novel distance measure and entropy measure, the proposed method could provide better performance than other FF-based methods since it could more effectively capture the certainty of the DMs' evaluation using the novel entropy measure and more accurately measure the difference between FFs using the novel distance measure. The inclusion of the novel distance measure and entropy measure further enhances the reliability of the proposed method.

8. Conclusion

This study presents an integrated Fermatean fuzzy decision-making approach for selecting HCW treatment technology. The proposed approach introduces a hybrid framework combining the BWM and VIKOR under the Fermatean fuzzy environment, which is consisted of three stages: expert weights calculation, criteria weights calculation and treatment technology selection. For the first stage, experts are weighted based on their evaluation information using a novel entropy measure. In the second stage, a hybrid criteria weight calculation method considering both the subjective weight and objective weight based on a novel FFS distance measure and the BWM is presented to determine the weights of the evaluation criteria. In the third stage, the novel Fermatean fuzzy VIKOR method is used to evaluate and rank different treatment technologies. The reliability and effectiveness of the proposed method are validated through comparative analysis. Furthermore, two sensitivity analyses are conducted to show the consistency and robustness of the proposed method.

This study mainly makes the following contributions:

(1) A novel distance measure and a novel entropy measure are developed for Fermatean fuzzy sets, and the properties of these measures are proved. Compared with other distance measures, the novel distance measure could more effectively measure the difference among FFSs. In addition, the proposed entropy measure presents a novel way to measure the uncertainty of FFSs.

(2) A hybrid criteria weight calculation method considering both the subjective weight and objective weight is proposed. The novel distance measure is used to calculate the support degree and objective weight of each criterion based on the evaluations. The BWM is adopted to determine the subjective weights of the criteria while considering the weights of experts. As shown in sensitivity analysis, the criteria weight calculation method could provide reliable and stable results.

(3) An integrated decision-making approach that combines the BWM and VIKOR under the Fermatean fuzzy environment is proposed. The Fermatean fuzzy sets are adopted to represent the uncertain evaluations of experts, the proposed hybrid criteria weights calculation method is used to determine the weights of the criteria, and the novel Fermatean fuzzy VIKOR method is used to evaluate and rank different medical waste treatment technologies. As shown in the case study, the proposed method could provide reliable and robust results.

A case study of health care waste treatment technology selection in Jinan is presented to demonstrate the process and effectiveness of the proposed method. The findings show that incineration is the best treatment technology for medical waste management, which is obtained based on the evaluations of several criteria, including cost, reliability and treatment effectiveness. The results are in line with the actual situation, and could provide insights into the adoption and application of appropriate treatment technology for medical waste management.

There are some limitations to this study. Firstly, the case study is conducted based on a relatively small group of experts, which could limit the reliability and comprehensiveness of the results, and the inclusion of a large group of experts could be studied in the future. Secondly, the proposed method adopts the VIKOR method, for future research, other MCDM methods could be studied.

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