An intuitionistic fuzzy weighted influence non-linear gauge system for equipment evaluation under system-of-systems warfare environment

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Abstract

System-of-systems (SoS) warfare has become the main form of modern warfare, where various equipment interact and interconnect to achieve operational goals. The complexity of equipment SoS poses a challenge for evaluating equipment under such warfare environments, as it requires considering multiple elements such as tasks and capabilities to ensure reliable evaluations. This paper proposes a novel equipment evaluation approach for SoS warfare environments based on the intuitionistic fuzzy weighted influence non-linear gauge system (IF-WINGS). The approach employs intuitionistic fuzzy sets (IFS) to represent uncertain information related to tasks, capabilities, and equipment. By extending the WINGS method with IFS, the proposed approach uncovers the interdependencies among the elements within the equipment SoS using the causal relation graph, considering both the strength and influence intensity of these elements. Additionally, the proposed approach evaluates and ranks different alternatives using relative closeness, which takes into account the distance of each alternative from the best and worst solutions, and this enables a more reliable and effective evaluation of alternatives. A novel equipment evaluation approach that considers the interrelationships among various elements within the equipment SoS is introduced based on IF-WINGS. To demonstrate the effectiveness of the proposed method, a case study is presented, and the results are compared to other methods, with Spearman's rank correlation coefficients of the comparative methods obtained as $(0.96, 0.96, 1.00, 1.00)$, respectively. The results show that the proposed method could provide reliable and robust results for equipment evaluation in SoS warfare

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environments. By introducing the IF-WINGS method into equipment evaluation under SoS environment, this study provides a novel and effective way that considers the complex relationships among different elements in SoS warfare, enabling reliable and reasonable evaluation. In conclusion, the IF-WINGS method proves to be a valuable tool for assessing equipment performance in complex and interconnected warfare scenarios.

Keywords: Equipment evaluation, Weighted influence non-linear gauge system, Multi-criteria decision-making, Intuitionistic fuzzy set, System-of-systems warfare

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1. Introduction

The rapid advancement of information and control technologies has transformed the nature of warfare, shifting from traditional platform-centric approaches to network-centric, system of systems (SoS) warfare (Zhao et al., 2015; Sun & Fang, 2020; Moffat, 2010). In SoS warfare, various equipment can achieve various capabilities through interrelationship, enabling them to fulfill operational tasks and achieve operational goals (Sun et al., 2022; Chen et al., 2023; Ding et al., 2018). Understanding how different equipment operates within the context of SoS warfare and assessing their impact on the overall outcome is crucial for determining the equipment's role and identifying the most pivotal components within the SoS warfare framework. Therefore, equipment evaluation holds significant importance in the analysis and development of SoS warfare strategies.

Traditionally, equipment evaluation involves two main approaches: simulation-based and data-based methods. In the simulation-based approach, military operations are modeled and simulated using techniques such as agent-based methods and dynamic models, and the evaluation of equipment is then based on the results of these simulations (Ding et al., 2017; Wu et al., 2019; Yajie et al., 2019). For instance, Yun et al. (2020) employed an agent-based modeling and simulation approach to assess the impact of high energy laser weapons (HELW) on the mission effectiveness of unmanned combat aerial vehicles (UCAVs), where a 4-level design framework is introduced based on a system-of-systems (SoS) oriented design. Similarly, Jia et al. (2019) proposed an operational effectiveness evaluation method for swarming UAVs combat systems using a system dynamics (SD) model. They utilized the rate-variable in-trees modeling method to build nine in-trees models for the subsystems of the swarming unmanned aerial vehicles (UAVs) combat system. However, the simulation-based approach is often limited to specific scenarios, which hinders its broader applicability. On the other hand, the data-based approach treats equipment evaluation as a multi-criteria decision-making (MCDM) problem, considering several criteria to evaluate the equipment's performance. Various MCDM methods have been adopted for equipment evaluation, including the analytic hierarchy process (AHP) (Dagdeviren et al., 2009; Zhang et al., 2005), evidential reasoning (Tianle et al., 2022; Luo et al., 2015), and rule-based systems (Gao et al., 2020). For instance, Bi et al. (2021) proposed an integrated approach using the interval-valued evidential reasoning algorithm, AHP, and the two-grade interval ranking method for evaluating the effectiveness of weapon systems under interval uncertainty. Additionally, Han et al. (2022) extended the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method under a linguistic hesitant Pythagorean fuzzy environment to evaluate the weapon contribution rate. While the data-based approach has proven effective for equipment evaluation in platform-centric warfare, its application in SoS warfare has been limited thus far.

In evaluating equipment within the context of SoS warfare, several challenges and shortcomings need to be addressed:

(1) In contrast to conventional equipment applications, SoS warfare involves achieving operational goals through interactions and interconnections among various equipment components. Given the complex nature of SoS warfare, equipment evaluation must consider their impact not only on the overall operational goal but also on the tasks and capabilities required for successful operations. Therefore, it is important to develop a robust method that can adequately capture the interrelationships among the operational goal, tasks, capabilities, and equipment within the SoS framework.

(2) The complexity of SoS warfare often leads to encountering uncertain and incomplete information, which may not be easily quantified or expressed in precise terms. Moreover, acquiring precise information for equipment evaluation can be challenging, as it often relies on expert judgments to make informed assessments. However, expert judgments can also be influenced by the uncertainty and incompleteness inherent in the SoS, resulting in subjective and linguistic information. Hence, there is a pressing need to establish a reliable mechanism to represent expert judgments effectively during equipment evaluation, considering the uncertainties and subjectivity inherent in the SoS context.

Decision-making problems have long attracted the attention of numerous researchers, and there have been many studies with emphasis on decision-making, including both MCDM methods (Shi et al., 2023; Prakash Garg et al., 2023) and artificial intelligence-based methods (Sunnetci et al., 2023; Sunnetci & Alkan, 2023, 2022). Numerous MCDM methods have been developed to address decision-making scenarios that involve interrelationships among various criteria. Some prominent methods include the analytic network process (ANP) (Kumar et al., 2021; Ghosh et al., 2021; Wu et al., 2020; Luo et al., 2020), decision-making trial and evaluation laboratory (DEMATEL) method (Yazdi et al., 2020; Du & Li, 2021; Braga et al., 2021; Li et al., 2020; Chen et al., 2021), and weighted influence non-linear gauge system (WINGS) (Tavana et al., 2023). Among these methods, WINGS, a novel structural model derived from DEMATEL, has garnered significant attention in recent years. It extends the capabilities of DEMATEL by incorporating both the strength and influence intensity of factors, making it effective for evaluating alternatives in scenarios where interrelations between factors cannot be overlooked (Michnik, 2013), and it has been successfully applied to various decision-making problems (Tavana et al., 2022b; Kaviani et al., 2020; Tavana et al., 2022a; Zhang & Xu, 2023). For instance, Tavana et al. (2021) introduced the fuzzy WINGS to assess advanced technology projects at the Kennedy Space Center, where intertwined criteria and their causal relations in complex problems were modeled using WINGS. Wang et al. (2021) enhanced the WINGS model with a radial basis function neural network and applied it to analyze and evaluate green financial supportive factors. Additionally, Govindan et al. (2023) utilized WINGS to assess and rank barriers to implementing blockchain technology-based platforms in the healthcare sector from a balanced scorecard perspective. However, a significant challenge that remains unresolved is effectively representing uncertain judgments of experts within the WINGS framework. As expert judgments are inherently uncertain and subjective, capturing and incorporating such information properly becomes crucial. Addressing this issue requires further research and development to enhance the applicability and reliability of the WINGS method in decision-making contexts involving uncertain and subjective expert inputs.

On the other hand, handling uncertain information has long been a focal point in MCDM. The concept of fuzzy sets, introduced by Zadeh (1965), provides a way to model uncertainty using membership functions. Over the years, fuzzy sets have garnered extensive attention for their ability to represent and model information under uncertainty, enabling decision-makers to express their judgments more flexibly. With the progress of decades, several extensions of fuzzy sets have emerged, and intuitionistic fuzzy sets (IFS) have been one of the most significant developments. Proposed by Atanassov (1986), IFS introduces a non-membership function to represent non-determinacy arising from decision-makers hesitation, thereby providing greater flexibility to decision-makers. In comparison to traditional fuzzy sets, IFS has proven to be more effective as it employs both membership and non-membership functions to represent the uncertain judgments of experts. As a result, IFS has found applications in various problem domains, including supplier selection (Rahimi et al., 2021; Singh et al., 2023; Jin & Garg, 2023; Zhang et al., 2022), alternative evaluation (Buran & Erçek, 2022; Chen, 2023; Mishra et al., 2020; Tumsekcali et al., 2021), and others (Garg & Rani, 2022; Wang et al., 2023; Dymova et al., 2022; Yu et al., 2022; Tao et al., 2021; Alcantud et al., 2020). For example, Xiao (2019) proposed a novel distance measure between IFS based on the Jensen-Shannon divergence, presenting an algorithm for pattern classification. Ecer (2022) introduced a new intuitionistic fuzzy extension of the MAIRCA framework, called intuitionistic fuzzy MAIRCA (IF-MAIRCA), to assess coronavirus vaccines based on multiple evaluation criteria, where IFS is used to represent the assessment information under uncertainty. In another study, Gao et al. (2021) developed a fuzzy MCDM framework that integrates IFS, score function, linear weighting method, prospect theory, and ANP to select sites for a large-scale rooftop photovoltaic project. Furthermore, Alkan & Kahraman (2022) introduced intuitionistic fuzzy multi-distance based evaluation for aggregated dynamic decision analysis (IF-DEVADA) for waste disposal location selection, utilizing IFS to provide a more accurate representation of data and better handle uncertainties. Due to its advantages in representing uncertain information, IFS holds great potential for application in equipment evaluation problems, where uncertainty often arises and requires effective handling.

However, to the best of our knowledge, there have not been studies that utilize WINGS method for equipment evaluation. Moreover, current extensions of the WINGS method are not well-suited for uncertain environment, and there is hardly any research on the extension of the WINGS method under intuitionistic fuzzy environment, which prompts this study. The main motivations for this study can be summarized as follows:

(1) Equipment evaluation within the SoS environment benefits from considering elements such as tasks and capabilities, as they help determine the equipment's importance more effectively and accurately. The WINGS method's ability to incorporate both the strength and influence intensity of criteria is crucial for decision-making problems with complex interrelationships. Hence, adopting the WINGS method for equipment evaluation is essential in this context.

(2) Uncertain information is common in equipment evaluation, requiring flexibility and reliability when handling human judgments under uncertainty. The IFS offers an effective extension of fuzzy sets, allowing for the modeling of uncertain information with higher flexibility through both the membership and non-membership functions. Therefore, extending the WINGS method under intuitionistic fuzzy environment is of paramount importance to address uncertainty adequately.

(3) In the context of alternative ranking within the WINGS method, a quantitative and reliable mechanism is needed, as the current approach considers rankings

obtained from different scores, which may lead to inconsistencies. The TOPSIS method, which determines the preference of alternatives based on their distances to the best and worst solutions and uses relative closeness to rank them, has demonstrated effectiveness in many MCDM problems. Thus, extending the WINGS method with the concept of TOPSIS is necessary to develop a reliable approach for alternative ranking in equipment evaluation.

Motivated by the above-mentioned research gaps, this study presents a novel equipment evaluation approach under SoS environment based on the intuitionistic fuzzy sets and the WINGS method. In this study, the IFS is adopted to represent the uncertain evaluation information, and the WINGS method is extended with IFS to determine the importance of the equipment while considering the interrelationships among the tasks, capabilities and equipment of the SoS. Moreover, a novel ranking mechanism is introduced for the WINGS method to determine the ranking of the equipment. This study has the following novelties:

(1) The WINGS method is extended with intuitionistic fuzzy sets, where the IFS represents the strength and influence intensity of the criteria. Compared with conventional WINGS method, by employing IFS to handle uncertain information, the intuitionistic fuzzy WINGS (IF-WINGS) method offers a more effective means to model the importance of criteria under uncertainty.

(2) A novel ranking mechanism based on the TOPSIS method is introduced for the WINGS approach. By calculating the distance of each alternative to the best and worst solutions and using relative closeness, the extended approach can quantitatively and effectively determine the ranking of alternatives, which could enable more reliable and reasonable results in comparison with conventional WINGS method.

(3) A novel equipment evaluation method is proposed, considering the interrelationships among tasks, capabilities, and equipment within the SoS framework. Unlike previous studies that only focused on the evaluation criteria without considering the interrelationship among different elements of the equipment SoS, the proposed method adopts the IF-WINGS method to build the causal relation graph for the equipment SoS. By considering the interrelationship within the equipment SoS, this approach offers enhanced reliability and accuracy in determining the importance of equipment.

The rest of the paper is organized as follows: Section 2 provides a brief introduction to several basic concepts of intuitionistic fuzzy sets. Section 3 outlines the proposed IF-WINGS method, and two numerical examples are presented in Section 4 to illustrate its application. The novel equipment evaluation method is introduced in Section 5, and a practical case is demonstrated in Section 6. Further analysis and discussions on the results are presented in Section 7. Finally, Section 8 concludes the paper.

2. Preliminaries

Proposed by Atanassov (1986), the intuitionistic fuzzy set is an extension of the classical fuzzy set. Unlike the classical fuzzy set, which uses only the membership function to represent the degree of belongingness, the intuitionistic fuzzy set introduces an additional function, i.e., the non-membership function. These two functions are used simultaneously to express the uncertainty and hesitancy in decision-making, providing a more comprehensive representation of uncertain information.

Definition 1. (Atanassov, 1986) Let X be a fixed set, then an intuitionistic fuzzy set $\tilde{A}$ can be defined as:

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | x \in X\} \tag{1}$$

where $\mu_{\tilde{A}} : X \to [0,1]$ is the membership function, and $\nu_{\tilde{A}} : X \to [0,1]$ is the non-membership function, such that $0 \leq \mu_{\tilde{A}} + \nu_{\tilde{A}} \leq 1$, $x \in X$.

Furthermore, the hesitancy degree of x is defined as:

$$\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \mu_{\tilde{A}}(x) \tag{2}$$

which indicates the degree of uncertainty of the IFS. The bigger the hesitancy degree is, the more uncertainty there is, conversely, the smaller the hesitancy degree is, the more information the IFS could provide, and the more accurate the IFS is.

For simplicity, $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | x \in X\}$ is often represented using an intuitionistic fuzzy number (IFN) $\tilde{A} = (\mu_{\tilde{A}}, \nu_{\tilde{A}})$, where $\mu_{\tilde{A}} \in [0,1]$, $\nu_{\tilde{A}} \in [0,1]$ and $0 \leq \mu_{\tilde{A}} + \nu_{\tilde{A}} \leq 1$, and the hesitancy degree can be derived using Eq. (2).

Definition 2. (Atanassov, 1986) Let $\tilde{A} = (\mu_{\tilde{A}}, \nu_{\tilde{A}})$ be an IFN, the score function of $\tilde{A}$ is defined as:

$$S(\tilde{A}) = \mu_{\tilde{A}} - \nu_{\tilde{A}} \tag{3}$$

Definition 3. (Atanassov, 1986) Let $\tilde{A} = (\mu_{\tilde{A}}, \nu_{\tilde{A}})$ be an IFN, the accuracy function of $\tilde{A}$ is defined as:

$$H(\tilde{A}) = \mu_{\tilde{A}} + \nu_{\tilde{A}} \tag{4}$$

Definition 4. (Atanassov, 1986) Let $\tilde{A} = (\mu_{\tilde{A}}, \nu_{\tilde{A}})$ and $\tilde{B} = (\mu_{\tilde{B}}, \nu_{\tilde{B}})$ be two IFNs, the distance between $\tilde{A}$ and $\tilde{B}$ is calculated as:

$$d(\tilde{A}, \tilde{B}) = \frac{1}{2}\left(|\mu_{\tilde{A}} - \mu_{\tilde{B}}| + |\nu_{\tilde{A}} - \nu_{\tilde{B}}| + |\mu_{\tilde{B}} + \nu_{\tilde{B}} - \mu_{\tilde{A}} - \nu_{\tilde{A}}|\right) \tag{5}$$

Definition 5. (Zeng et al., 2019) Let $\tilde{A} = (\mu_{\tilde{A}}, \nu_{\tilde{A}})$ be an IFN, the modified score measure for $\tilde{A}$ is calculated as:

$$E(A) = \frac{1}{2}\left(\mu_{\tilde{A}} - \nu_{\tilde{A}} - \pi_{\tilde{A}} \times \frac{\log_2(1+\pi_{\tilde{A}})}{100} + 1\right) \tag{6}$$

Let $\tilde{A} = (\mu_{\tilde{A}}, \nu_{\tilde{A}})$ and $\tilde{B} = (\mu_{\tilde{B}}, \nu_{\tilde{B}})$ be two IFNs, and λ be a positive number, the operations of IFNs can be defined as:

(1) $\tilde{A} \oplus \tilde{B} = (\mu_{\tilde{A}} + \mu_{\tilde{B}} - \mu_{\tilde{A}}\mu_{\tilde{B}}, \nu_{\tilde{A}}\nu_{\tilde{B}})$.

(2) $\tilde{A} \otimes \tilde{B} = (\mu_{\tilde{A}}\mu_{\tilde{B}}, \nu_{\tilde{A}} + \nu_{\tilde{B}} - \nu_{\tilde{A}}\nu_{\tilde{B}})$.

(3) $\lambda\tilde{A} = \left(1 - (1-\mu_{\tilde{A}})^{\lambda}, \nu_{\tilde{A}}^{\lambda}\right)$.

(4) $\tilde{A}^{\lambda} = \left(\mu_{\tilde{A}}^{\lambda}, 1 - (1-\nu_{\tilde{A}})^{\lambda}\right)$.

Let $\tilde{A} = (\mu_{\tilde{A}}, \nu_{\tilde{A}})$ and $\tilde{B} = (\mu_{\tilde{B}}, \nu_{\tilde{B}})$ be two IFNs, there is

1. If $S(\tilde{A}) > S(\tilde{B})$, then $\tilde{A} > \tilde{B}$
2. If $S(\tilde{A}) < S(\tilde{B})$, then $\tilde{A} < \tilde{B}$
3. If $S(\tilde{A}) = S(\tilde{B})$, then:
   (a) If $H(\tilde{A}) > H(\tilde{B})$, then $\tilde{A} > \tilde{B}$
   (b) If $H(\tilde{A}) < H(\tilde{B})$, then $\tilde{A} < \tilde{B}$
   (c) If $H(\tilde{A}) = H(\tilde{B})$, then $\tilde{A} = \tilde{B}$

Definition 5. (Xu, 2007) Let $\tilde{A}_i = (\mu_{\tilde{A}_i}, \nu_{\tilde{A}_i}), (i = 1, 2, \ldots, n)$ be an IFN, where its weight is expressed as ω_i with $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^{n} \omega_i = 1$, then the inituitionistic fuzzy weighted average operator (IFWA), which is a mapping $IFWA : \Theta^n \to \Theta$, is defined as:

$$IFWA(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = \oplus_{i=1}^{n}(\omega_i \tilde{A}_i) = \left(1 - \prod_{i=1}^{n}(1-\mu_{\tilde{A}_i})^{\omega_i}, \prod_{i=1}^{n}\nu_{\tilde{A}_i}^{\omega_i}\right) \tag{7}$$

3. The IF-WINGS method

DEMATEL is a powerful tool for identifying and analyzing cause-effect relationships among various elements in MCDM problems. It is particularly valuable for determining the elements within complex interconnected systems by representing the interdependencies among elements with cause-effect relations. On the other hand, the WINGS method is a novel structural approach inspired by DEMATEL. While

DEMATEL employs “influence intensity” to capture the impact of one element on another, the WINGS method introduces a second measure, “strength of elements”, in addition to the influence intensity. Despite its strengths, the WINGS method has faced criticism due to its limitations in handling uncertain information. To address this issue, this study proposes a novel approach called the intuitionistic fuzzy WINGS method, designed to effectively handle uncertain, ambiguous, and subjective information within the WINGS framework. By incorporating IFSs, the method offers enhanced capabilities for dealing with uncertainty, making it a more reliable and robust tool for equipment evaluation and decision-making problems. The procedure of the proposed IF-WINGS method can be summarized as follows.

Step 1: The initial step involves determining the criteria, sub-criteria, and alternatives based on existing literature and expert knowledge.

Step 2: In this step, experts establish the interdependencies among the criteria using a causal relation graph. The graph represents the criteria as nodes, and the relationships between criteria are depicted with arrows. The direction of the arrow indicates the influence relation, encompassing cause and effect relationships. An illustrative example of the causal relation graph is shown in Fig. 1, where C_i and C_j represent criteria, and criterion C_i is influencing criterion C_j.

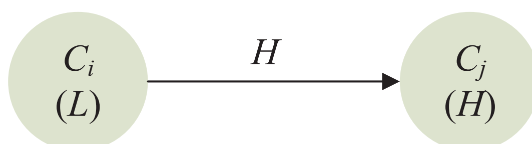

Figure 1: An illustrative example of the causal relation graph.

Step 3: The experts specify the strengths of the criteria and influence intensities using linguistic terms, as listed in Table 1. The strength values are denoted within the nodes, while the influence intensities are indicated on the connecting arrows. In Fig. 1, the strengths of criteria C_i and C_j are determined as low and high, respectively, while the influence intensity of criterion C_i on criterion C_j is designated as high.

Step 4: Using the criteria strengths and their influence intensities, the intuitionistic fuzzy direct strength-influence matrix $D = [d_{ij}]_{m\times n}$ is constructed as follows:

$$D = \begin{bmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & \cdots & (\mu_{1n}, \nu_{1n}) \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & \cdots & (\mu_{2n}, \nu_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{m1}, \nu_{m1}) & (\mu_{m2}, \nu_{m2}) & \cdots & (\mu_{mn}, \nu_{mn}) \end{bmatrix} \tag{8}$$

Table 1: Linguistic terms and the corresponding IFNs

Linguistic term	IFN
No influence (N)	(0.1,0.9)
Very low influence (VL)	(0.3,07)
Low influence (L)	(0.5,0.5)
High influence (H)	(0.7,0.3)
Very high influence (VH)	(0.9,0.1)

where d_{ii} denotes the strength of the ith criterion, and the element in the ith row and jth column represents the influence intensity of criterion C_i on criterion C_j.

Step 5: The expected intuitionistic fuzzy direct strength-influence matrix $\hat{D}$ is obtained by calculating the expected value of the IFNs as follows:

$$\hat{D} = [\hat{d}_{ij}]_{n \times n} \tag{9}$$

where $\hat{d}_{ij}$ is the expected value of the IFN d_{ij}, and it is calculated using Eq. (6).

Step 6: The normalized intuitionistic fuzzy direct strength-influence matrix $\tilde{D}$ is obtained by dividing each element of the intuitionistic fuzzy direct strength-influence matrix $\hat{D}$ by the sum of all elements, as follows:

$$\tilde{D} = [\tilde{d}_{ij}]_{n \times n} \tag{10}$$

with

$$\tilde{d}_{ij} = \frac{\hat{d}_{ij}}{s} \tag{11}$$

where s is determined by the sum of all elements in $\hat{D}$ as:

$$s = \sum_{i=1}^{n} \sum_{j=1}^{n} \hat{d}_{ij} \tag{12}$$

Step 7: Based on the normalized intuitionistic fuzzy direct strength-influence matrix $\tilde{D}$, the total intuitionistic fuzzy strength-influence matrix T is obtained as follows (Michnik, 2013):

$$T = \frac{\tilde{D}}{I - \tilde{D}} \tag{13}$$

where I is the $n \times n$ identity matrix.

Step 8: By considering the elements in the total intuitionistic fuzzy strength-influence matrix, the total impact score r_i and the total receptive score c_j for each criterion can be calculated as follows (Michnik, 2013):

$$r_i = \sum_{j=1}^{n} t_{ij} \tag{14}$$

$$c_j = \sum_{i=1}^{n} t_{ij} \tag{15}$$

Subsequently, the total engagement $(r_i + c_i)$ and the role $(r_i - c_i)$ of different criteria are determined.

Step 9: In this step, different alternatives are plotted based on their $(r_i + c_i)$ values and $(r_i - c_i)$ values, with the best and worst solutions serving as reference points. The plot utilizes the $(r_i + c_i)$ value as the horizontal dimension (x-axis) and the $(r_i - c_i)$ value as the vertical dimension (y-axis). The alternatives are then ranked according to their Euclidean distance from the best and worst solutions. This plot enables a visual representation of the alternatives' performance and facilitates decision-making based on their relative positions.

The process of the proposed IF-WINGS method is shown in Fig 2.

4. Numerical examples

In this section, the effectiveness and reliability of the proposed method are validated through two numerical examples originally presented by Michnik (2013).

4.1. Example 1

In this example, a problem with three criteria influencing each other is considered. The process of the proposed intuitionistic fuzzy WINGS method is described as follows.

Step 1: The problem under consideration involves three criteria denoted as C_1, C_2, and C_3, as per Michnik (2013).

Step 2: The interdependencies among the criteria are determined by analyzing the relationships among them, as shown in Fig 3.

Step 3: The criteria's strengths and their influence intensities are specified based on expert analysis, as shown in Fig 4. These values are used for the subsequent analysis.

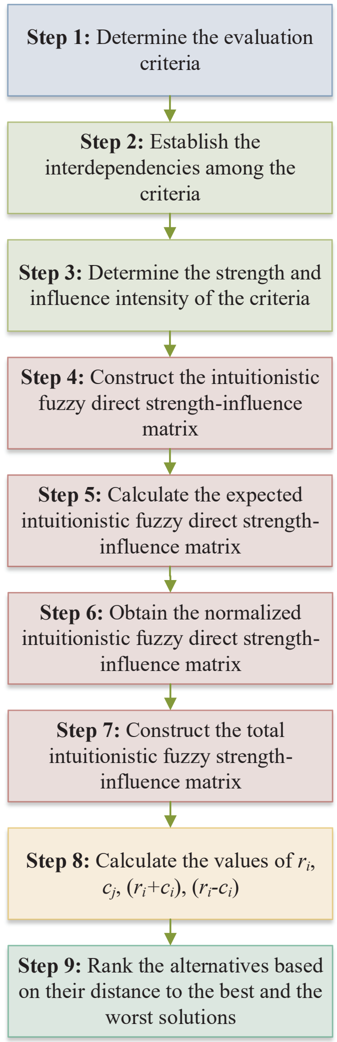

Figure 2: Process of the IF-WINGS method.

Step 4: Using the criteria's strengths and influence intensities, the intuitionistic fuzzy direct strength-influence matrix is constructed, which is expressed as follows:

$$D = \begin{bmatrix} (0.9, 0.1) & (0.3, 0.7) & (0.9, 0.1) \\ (0.7, 0.3) & (0.5, 0.5) & (0.5, 0.5) \\ (0.5, 0.5) & (0.7, 0.3) & (0.5, 0.5) \end{bmatrix}$$

Step 5: By using Eq. (6), the expected intuitionistic fuzzy direct strength-

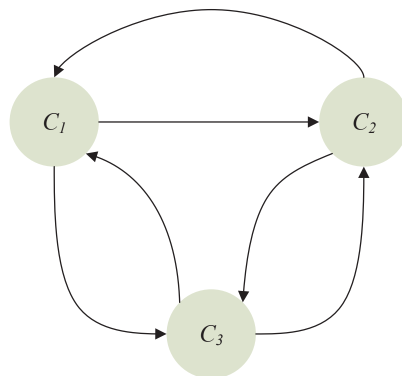

Figure 3: The interdependencies among the criteria of Example 1.

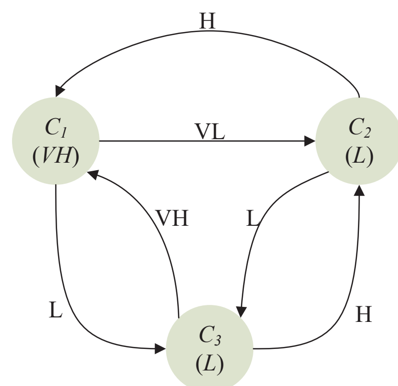

Figure 4: The causal relation graph of Example 1.

influence matrix can be obtained as:

$$\hat{D} = \begin{bmatrix} 0.9 & 0.3 & 0.9 \\ 0.7 & 0.5 & 0.5 \\ 0.5 & 0.7 & 0.5 \end{bmatrix}$$

Step 6: Based on the expected intuitionistic fuzzy direct strength-influence matrix, the normalized intuitionistic fuzzy direct strength-influence matrix is obtained, and is expressed by:

$$\tilde{D} = \begin{bmatrix} 0.1636 & 0.0545 & 0.1636 \\ 0.1273 & 0.0909 & 0.0909 \\ 0.0909 & 0.1273 & 0.0909 \end{bmatrix}$$

Step 7: Based on the normalized intuitionistic fuzzy direct strength-influence matrix, the total intuitionistic fuzzy strength-influence matrix is calculated as follows:

$$T = \begin{bmatrix} 0.2373 & 0.1069 & 0.2334 \\ 0.1882 & 0.1319 & 0.1471 \\ 0.1501 & 0.1692 & 0.1439 \end{bmatrix}$$

Step 8: Based on the total intuitionistic fuzzy strength-influence matrix, the total impact score r_i and the total receptive score c_j can be obtained by calculating the sum of rows and columns of the total intuitionistic fuzzy strength-influence matrix, and the results are obtained as:

$$r = \begin{bmatrix} 0.5776 \\ 0.4672 \\ 0.4632 \end{bmatrix}$$
$$c = \begin{bmatrix} 0.5756 & 0.4080 & 0.5244 \end{bmatrix}$$

Then, the total engagement $(r_i + c_i)$ and the role $(r_i - c_i)$ can be calculated as:

$$(r + c) = \begin{bmatrix} 1.1532 \\ 0.8751 \\ 0.9876 \end{bmatrix}$$
$$(r - c) = \begin{bmatrix} 0.0020 \\ 0.0592 \\ -0.0612 \end{bmatrix}$$

Step 9: By considering the total impact score r_i, the total receptive score c_j, the total engagement $(r_i + c_i)$ and the role $(r_i - c_i)$, the rankings of different criteria could be obtained as:

$$r_i : C_1 \succ C_2 \succ C_3$$
$$c_j : C_1 \succ C_3 \succ C_2$$
$$r_i + c_i : C_1 \succ C_3 \succ C_2$$
$$r_i - c_i : C_2 \succ C_1 \succ C_3$$

4.2. Example 2

In this example, originated from Michnik (2013), two alternatives A_1 and A_2 are ranked considering two intertwined criteria C_1 and C_2, where the criterion C_1 is influenced by criterion C_2.

Step 1: In this problem, two alternatives, namely, A_1 and A_2 and two criteria, namely, C_1 and C_2, are considered.

Step 2: Based on the analysis of the alternatives and criteria, the interdependencies among the alternatives and the criteria are determined, and the result is shown in Fig 5.

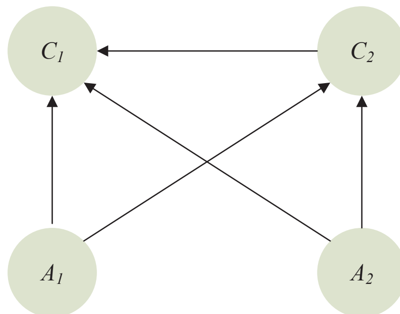

Figure 5: The interdependencies among the alternatives and the criteria of Example 2.

Step 3: Based on the interdependencies among the alternatives and the criteria, the strength and influence intensities of the alternatives and criteria are determined by the experts, as shown in Fig 6.

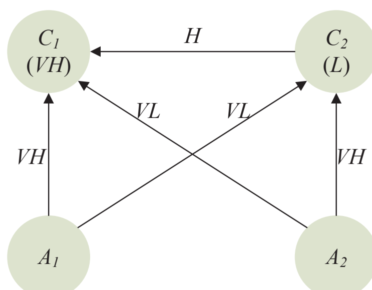

Figure 6: The causal relation graph of Example 2.

Step 4: According to the strength and influence intensities of the alternatives and criteria, the intuitionistic fuzzy direct strength-influence matrix is constructed, where the first two rows and columns represent criteria C_1 and C_2, and the last two rows and columns represent alternatives A_1 and A_2, respectively. The obtained intuitionistic fuzzy direct strength-influence matrix is as follows:

$$D = \begin{bmatrix} (0.9, 0.1) & (0.1, 0.9) & (0.1, 0.9) & (0.1, 0.9) \\ (0.7, 0.3) & (0.5, 0.5) & (0.1, 0.9) & (0.1, 0.9) \\ (0.9, 0.1) & (0.3, 0.7) & (0.1, 0.9) & (0.1, 0.9) \\ (0.3, 0.7) & (0.9, 0.1) & (0.1, 0.9) & (0.1, 0.9) \end{bmatrix}$$

Step 5: Based on the score function of the IFN, the expected intuitionistic fuzzy direct strength-influence matrix is obtained as follows:

$$\hat{D} = \begin{bmatrix} 0.9 & 0.1 & 0.5 & 0.1 \\ 0.7 & 0.5 & 0.1 & 0.1 \\ 0.9 & 0.3 & 0.1 & 0.1 \\ 0.3 & 0.9 & 0.1 & 0.1 \end{bmatrix}$$

Step 6: By normalizing the expected intuitionistic fuzzy direct strength-influence matrix, the normalized intuitionistic fuzzy direct strength-influence matrix can be calculated by:

$$\tilde{D} = \begin{bmatrix} 0.1698 & 0.0189 & 0.0189 & 0.0189 \\ 0.1321 & 0.0943 & 0.0189 & 0.0189 \\ 0.1698 & 0.0377 & 0.0189 & 0.0189 \\ 0.0566 & 0.1698 & 0.0189 & 0.0189 \end{bmatrix}$$

Step 7: Based on the normalized intuitionistic fuzzy direct strength-influence matrix, the total intuitionistic fuzzy strength-influence matrix is calculated as:

$$T = \begin{bmatrix} 0.2162 & 0.0309 & 0.0245 & 0.0245 \\ 0.1842 & 0.1138 & 0.0255 & 0.0255 \\ 0.2196 & 0.0519 & 0.0249 & 0.0249 \\ 0.1063 & 0.1956 & 0.0255 & 0.0255 \end{bmatrix}$$

Step 8: The total impact score r_i and the total receptive score c_j are obtained based on the total intuitionistic fuzzy strength-influence matrix as:

$$r = \begin{bmatrix} 0.2960 \\ 0.3489 \\ 0.3214 \\ 0.3529 \end{bmatrix}$$

$$c = \begin{bmatrix} 0.7262 & 0.3922 & 0.1004 & 0.1004 \end{bmatrix}$$

Then, the total engagement $(r_i + c_i)$ and the role $(r_i - c_i)$ are calculated as:

$$(r + c) = \begin{bmatrix} 1.0222 \\ 0.7412 \\ 0.4217 \\ 0.4533 \end{bmatrix}$$

$$(r - c) = \begin{bmatrix} -0.4302 \\ -0.0443 \\ 0.2210 \\ 0.2525 \end{bmatrix}$$

Step 9: Based on the total impact score r_i, the total engagement $(r_i + c_i)$, and the role $(r_i - c_i)$, the rankings of the alternatives can be obtained as:

$$r_i : A_2 \succ A_1$$
$$r_i + c_i : A_2 \succ A_1$$
$$r_i - c_i : A_2 \succ A_1$$

As the rankings obtained by r_i, $(r_i + c_i)$ and $(r_i - c_i)$ are identical, it can be concluded that the ranking of alternatives A_1 and A_2 is $A_2 \succ A_1$.

5. Equipment evaluation under SoS using IF-WINGS

In this section, we employ the IF-WINGS approach to address the challenges posed by the complex and interconnected nature of equipment in the SoS warfare environment.

As modern warfare increasingly relies on the integration of diverse equipment, the evaluation of their performance and contribution within an SoS becomes essential. In the context of SoS warfare, the successful achievement of the operational goal is dependent on the seamless interaction and interconnection of various equipment. This interconnectedness extends to the multiple tasks and capabilities that the equipment must fulfill. Consequently, causal relationships emerge among these tasks, capabilities, and equipment. For instance, one equipment may be unable to perform its task without the completion of its preceding task, or certain capabilities may be critical for the successful execution of specific tasks. In light of these interconnected relationships, effectively modeling the causal links among equipment components becomes crucial to ensure a reliable and accurate evaluation. By adopting the IF-WINGS method, we can incorporate intuitionistic fuzzy sets to capture uncertainty and ambiguity inherent in expert judgments. This empowers our approach to more flexibly represent the strengths and influence intensities of criteria, enabling a comprehensive evaluation of the equipment's performance and impact within the SoS.

In the equipment SoS, there are m equipment components denoted as $E = \{E_1, E_2, \ldots, E_m\}$. The SoS warfare is composed of n tasks denoted as $T = \{T_1, T_2, \ldots, T_n\}$ and k capabilities denoted as $C = \{C_1, C_2, \ldots, C_k\}$. It is important to emphasize that various tasks may have mutual influences on each other, and each task can be influenced by multiple capabilities. Similarly, each capability may be influenced by one or more equipment components. As a result, the entire equipment SoS can be effectively modeled using a causal relation graph. In the causal relation graph, the nodes represent the tasks, capabilities, and equipment components, and the interconnections between these nodes are represented by arrows. These arrows indicate the causal relationships and influences among different elements. In order to evaluate different equipment in the equipment SoS, the process of the novel equipment evaluation method based on IF-WINGS is described as follows.

Step 1: Carefully analyze the equipment SoS and its operational goals, and identify the relevant equipment components, denoted as E, tasks, denoted as T, and capabilities, denoted as C, that are involved in the equipment SoS.

Step 2: Analyze the mappings between tasks and capabilities, as well as the mappings between capabilities and equipment. Determine the interdependencies and causal relationships among tasks, capabilities, and equipment using a causal relation graph. An illustrative example of such a graph is provided in Fig 7, considering two equipment E_1 and E_2, two capabilities C_1 and C_2, and two tasks T_1 and T_2. From Fig 7, it can be observed that C_1 is influenced by both E_1 and E_2, whereas C_2 is influenced only by E_1. Moreover, T_1 is influenced by C_1 and C_2, while T_2 is influenced by T_1, C_1, and C_2.

Step 3: With the knowledge of the interrelationships among tasks, capabilities, and equipment from the causal relation graph, determine the strengths of the elements and their influence intensities using linguistic terms listed in Table 1. For example, in Fig 7, the strength and influence intensities of the equipment components, tasks, and capabilities can be assessed based on expert judgment or available data. These linguistic terms provide valuable input for the subsequent evaluation process.

Step 4: Utilize the strengths of different elements and their influence intensities to construct the intuitionistic fuzzy direct strength-influence matrix $D = [d_{ij}]_{(m+n+k)\times(m+n+k)}$. Each element in the matrix (μ_{ij}, ν_{ij}) denotes the strength and influence intensity between the ith element and the jth element, respectively, and

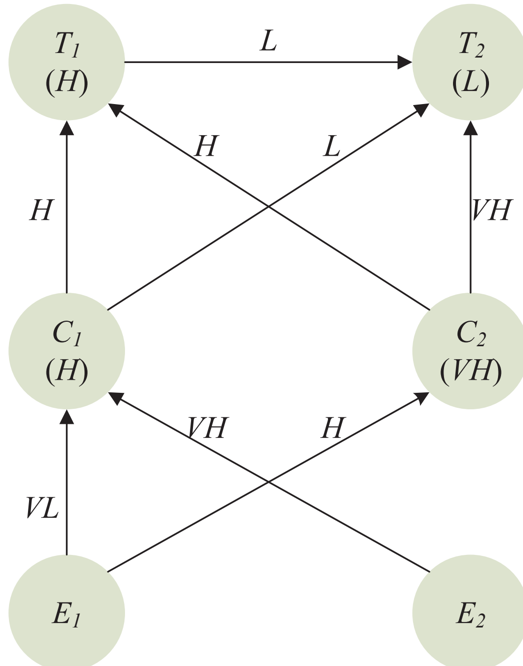

Figure 7: An illustrative example of the causal relation graph for equipment SoS.

the matrix is expressed as follows:

$$D = \begin{bmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & \cdots & (\mu_{1(m+n+k)}, \nu_{1(m+n+k)}) \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & \cdots & (\mu_{2(m+n+k)}, \nu_{2(m+n+k)}) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{(m+n+k)1}, \mu_{(m+n+k)1}) & (\mu_{(m+n+k)2}, \mu_{(m+n+k)2}) & \cdots & (\mu_{(m+n+k)(m+n+k)}, \mu_{(m+n+k)(m+n+k)}) \end{bmatrix} \tag{16}$$

where (μ_{ii}, ν_{ii}) denotes the strength of the ith element and (μ_{ij}, ν_{ij}) denotes the influence intensity of the ith element on the jth element. It should be noted that the first n rows and columns represent the tasks, the next k rows and columns represent the capabilities, and the remaining rows and columns represent the equipment.

Step 5: Construct the expected intuitionistic fuzzy direct strength-influence matrix by using the score function of the IFN as:

$$\hat{D} = [\hat{d}_{ij}]_{(m+n+k)\times(m+n+k)} \tag{17}$$

where $\hat{d}_{ij}$ is calculated using Eq. (6) as:

$$\hat{d}_{ij} = \frac{1}{2}\left(\mu_{\tilde{A}} - \nu_{\tilde{A}} - \pi_{\tilde{A}} \times \frac{\log_2(1+\pi_{\tilde{A}})}{100} + 1\right) \tag{18}$$

Step 6: Normalize the expected intuitionistic fuzzy direct strength-influence matrix to obtain the normalized intuitionistic fuzzy direct strength-influence matrix as follows:

$$\tilde{D} = [\tilde{d}_{ij}]_{(m+n+k)\times(m+n+k)} \tag{19}$$

where

$$\begin{aligned} \tilde{d}_{ij} &= \frac{\hat{d}_{ij}}{s} \\ s &= \sum_{i=1}^{m+n+k} \sum_{j=1}^{m+n+k} \hat{d}_{ij} \end{aligned} \tag{20}$$

Step 7: Use the normalized intuitionistic fuzzy direct strength-influence matrix to calculate the total intuitionistic fuzzy strength-influence matrix as:

$$T = \tilde{D} \times (I - \tilde{D})^{-1} \tag{21}$$

Step 8: Calculate the total impact score r_i and total receptive score c_j for each element based on the elements in the total intuitionistic fuzzy strength-influence matrix T. The total impact score r_i is obtained by calculating the sum of the ith row in the matrix, while the total receptive score c_j is obtained by calculating the sum of the jth column in the matrix as:

$$\begin{aligned} r_i &= \sum_{j=1}^{m+n+k} t_{ij} \\ c_j &= \sum_{i=1}^{m+n+k} t_{ij} \end{aligned} \tag{22}$$

Step 9: Based on the total engagement $(r_i + c_i)$ and the role $(r_i - c_i)$ values, determine the distance of each alternative to the best and the worst solutions using the Euclidean distance, denoted as d^+ and d^-, respectively. Then, by using $(r_i + c_i)$ as the horizontal dimension and $(r_i - c_i)$ as the vertical dimension, plot the different alternatives in a two-dimensional space. It should be noted that other distance measures such as cosine distance, Manhattan distance, Minkowski distance, or Chebyshev distance could also be applied.

The closer an alternative is to the best solution and the farther it is from the worst solution, the better the alternative is. Therefore, based on the concept of

TOPSIS, calculate the relative closeness of each alternative as follows:

$$RC_i = \frac{d^-}{d^+ + d^-} \tag{23}$$

where RC_i represents the relative closeness of the ith alternative.

Finally, rank the different equipment according to their relative closeness values, with higher values indicating better alternatives. This ranking provides a reliable and effective evaluation of the equipment under the SoS operation environment.

6. Case study

6.1. Background

In this section, we present a practical case to demonstrate the effectiveness of the proposed equipment evaluation method in an integrated air-ground confrontation equipment System-of-Systems (SoS). The equipment SoS comprises various equipment, including fighter jet (E_1), early warning aircraft (E_2), reconnaissance aircraft (E_3), surveillance UAV (E_4), combat UAV (E_5), missile (E_6), and bomber (E_7). To conduct the evaluation, experts from the aeronautical engineering and equipment acquisition fields are invited, and they are tasked with determining the interdependencies among the elements and providing evaluations on the strengths and influence intensities using the linguistic terms listed in Table 1. The following procedure is conducted for the evaluation of equipment in this equipment SoS.

6.2. Implementation

Step 1: Based on the analysis of the equipment SoS, seven tasks, including mission planning (T_1), reconnaissance (T_2), command and control (T_3), maneuver (T_4), assault (T_5), defense (T_6), and effectiveness evaluation (T_7), are identified. These tasks are used as the evaluation criteria to model and evaluate different equipment. Moreover, during the analysis of the equipment SoS, a series of capabilities are identified by the experts, including reconnaissance (C_1), communication (C_2), information sharing (C_3), information processing (C_4), command (C_5), maneuver (C_6), attack (C_7), defense (C_8), and evaluation (C_9). These capabilities are used as sub-criteria to evaluate the equipment.

Step 2: By analyzing the relations among these tasks and capabilities, the hierarchical structure of the equipment SoS that represents the relations among the elements of the equipment SoS is determined. The hierarchical structure illustrates how the tasks and capabilities interact with each other and with the equipment, as illustrated in Fig 8. The diagram shows three different kinds of relations, namely, interrelations among tasks, the influence of capabilities on tasks, and the influence of the equipment on tasks. These interdependencies help to understand the complex interactions within the equipment SoS.

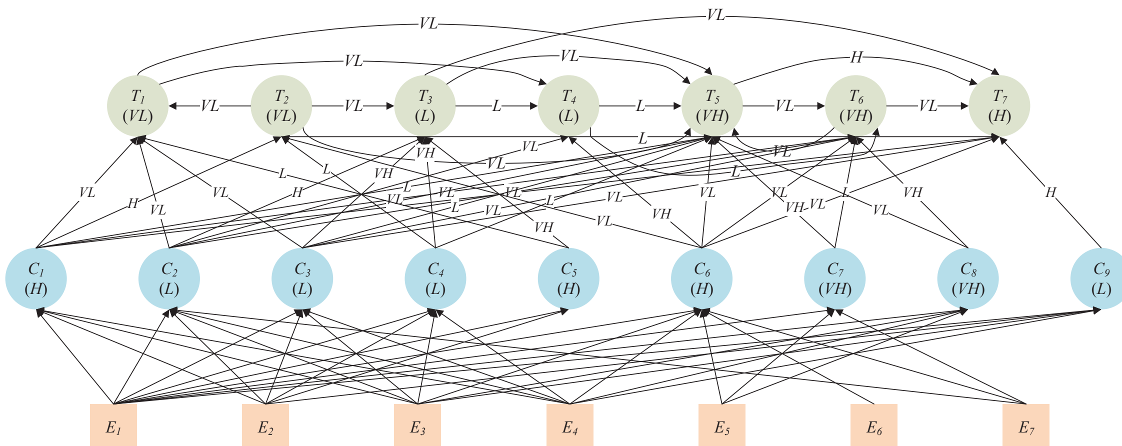

Figure 8: The hierarchical structure between tasks, capabilities and equipment.

Step 3: After analyzing the elements of the equipment SoS and their relations, the strengths of the elements and their influence intensities are determined, as depicted in Fig 8. The experts provide evaluations for each element using the linguistic terms listed in Table 1. Additionally, the influence intensities of the equipment on the capabilities are presented in Table 2.

The strengths of the elements represent their overall importance or effectiveness within the equipment SoS, while the influence intensities capture the extent to which one element affects another. These assessments are essential for the subsequent evaluation and ranking of the equipment in the equipment SoS.

Step 4: With the linguistic terms provided by the experts in Table 1, the intuitionistic fuzzy direct strength-influence matrix is constructed based on the hierarchical structure in Fig 8. The intuitionistic fuzzy direct strength-influence matrix is presented in Table 3.

The matrix reflects the strengths of different elements and their influence intensities on one another, considering both the membership and non-membership functions of the intuitionistic fuzzy sets. This matrix serves as a basis for further calculations and evaluations in the equipment SoS.

Step 5: Using Eq. (18), the expected intuitionistic fuzzy direct strength-influence matrix is calculated based on the intuitionistic fuzzy direct strength-influence matrix presented in Table 3. The expected intuitionistic fuzzy direct strength-influence

Table 2: The influence intensity of equipment on capabilities

Equipment	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
E_1	L	L	H	VL	H	VH	VH	VH	L
E_2	VH	H	H	VH	VH	N	N	N	L
E_3	VH	VL	H	L	N	L	N	VL	VH
E_4	H	VL	L	L	N	VL	N	VL	H
E_5	N	N	N	N	N	H	H	L	N
E_6	N	N	N	N	N	VH	N	N	N
E_7	N	VL	N	N	N	VH	VL	N	N
Best	VH	H	H	VH	VH	VH	VH	VH	VH
Worst	N	N	N	N	N	N	N	N	N

Table 3: The intuitionistic fuzzy direct strength-influence matrix

Row	Column								
	1	2	3	4	$\cdots$	22	23	24	25
1	(0.3,0.7)	(0.1,0.9)	(0.1,0.9)	(0.3,0.7)	$\cdots$	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
2	(0.3,0.7)	(0.3,0.7)	(0.3,0.7)	(0.1,0.9)	$\cdots$	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
3	(0.1,0.9)	(0.1,0.9)	(0.5,0.5)	(0.5,0.5)	$\cdots$	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
4	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.5,0.5)	$\cdots$	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
22	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	$\cdots$	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
23	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	$\cdots$	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
24	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	$\cdots$	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
25	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	$\cdots$	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)

matrix is shown in Table 4.

Step 6: By using Eq. (20), the expected intuitionistic fuzzy direct strength-influence matrix in Table 4 is normalized to obtain the normalized intuitionistic fuzzy direct strength-influence matrix. The normalized intuitionistic fuzzy direct strength-influence matrix is presented in Table 5.

Step 7: Using Eq. (21), the normalized intuitionistic fuzzy direct strength-influence matrix in Table 5 is used to calculate the total intuitionistic fuzzy strength-influence matrix. The total intuitionistic fuzzy strength-influence matrix is presented in Table 6.

Step 8: Using Eq. (22), the sum of rows and columns of the total intuitionistic fuzzy strength-influence matrix in Table 6 is calculated to obtain the total impact score r_i and the total receptive score c_j for each equipment. The results are presented in Table 7.

Table 4: The expected intuitionistic fuzzy direct strength-influence matrix

Row	Column										
	1	2	3	4	5	$\cdots$	21	22	23	24	25
1	0.30	0.10	0.10	0.30	0.30	$\cdots$	0.10	0.10	0.10	0.10	0.10
2	0.30	0.30	0.30	0.10	0.30	$\cdots$	0.10	0.10	0.10	0.10	0.10
3	0.10	0.10	0.50	0.50	0.30	$\cdots$	0.10	0.10	0.10	0.10	0.10
4	0.10	0.10	0.10	0.50	0.50	$\cdots$	0.10	0.10	0.10	0.10	0.10
5	0.10	0.10	0.10	0.10	0.90	$\cdots$	0.10	0.10	0.10	0.10	0.10
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
21	0.10	0.10	0.10	0.10	0.10	$\cdots$	0.10	0.10	0.10	0.10	0.10
22	0.10	0.10	0.10	0.10	0.10	$\cdots$	0.10	0.10	0.10	0.10	0.10
23	0.10	0.10	0.10	0.10	0.10	$\cdots$	0.10	0.10	0.10	0.10	0.10
24	0.10	0.10	0.10	0.10	0.10	$\cdots$	0.10	0.10	0.10	0.10	0.10
25	0.10	0.10	0.10	0.10	0.10	$\cdots$	0.10	0.10	0.10	0.10	0.10

Table 5: The normalized intuitionistic fuzzy direct strength-influence matrix

Row	Column										
	1	2	3	4	5	$\cdots$	21	22	23	24	25
1	0.0027	0.0009	0.0009	0.0027	0.0027	$\cdots$	0.0009	0.0009	0.0009	0.0009	0.0009
2	0.0027	0.0027	0.0027	0.0009	0.0027	$\cdots$	0.0009	0.0009	0.0009	0.0009	0.0009
3	0.0009	0.0009	0.0044	0.0044	0.0027	$\cdots$	0.0009	0.0009	0.0009	0.0009	0.0009
4	0.0009	0.0009	0.0009	0.0044	0.0044	$\cdots$	0.0009	0.0009	0.0009	0.0009	0.0009
5	0.0009	0.0009	0.0009	0.0009	0.0080	$\cdots$	0.0009	0.0009	0.0009	0.0009	0.0009
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
21	0.0009	0.0009	0.0009	0.0009	0.0009	$\cdots$	0.0009	0.0009	0.0009	0.0009	0.0009
22	0.0009	0.0009	0.0009	0.0009	0.0009	$\cdots$	0.0009	0.0009	0.0009	0.0009	0.0009
23	0.0009	0.0009	0.0009	0.0009	0.0009	$\cdots$	0.0009	0.0009	0.0009	0.0009	0.0009
24	0.0009	0.0009	0.0009	0.0009	0.0009	$\cdots$	0.0009	0.0009	0.0009	0.0009	0.0009
25	0.0009	0.0009	0.0009	0.0009	0.0009	$\cdots$	0.0009	0.0009	0.0009	0.0009	0.0009

Based on the total impact score r_i and total receptive score c_j, the total engagement $(r_i + c_i)$ and the role $(r_i - c_i)$ of each equipment are also computed and shown in Table 7. These scores provide valuable information on the overall importance and influence of each equipment in the equipment SoS.

Step 9: To facilitate the analysis of the results, the Euclidean distance measure is used to compare the values of $(r_i + c_i)$ and $(r_i - c_i)$. Each equipment is plotted in a two-dimensional space with its $(r_i + c_i)$ value on the x-axis and its $(r_i - c_i)$ value on the y-axis. Additionally, the positions of the best and worst solutions are marked

Table 6: The total intuitionistic fuzzy strength-influence matrix

Row	Column										
	1	2	3	4	5	⋯	21	22	23	24	25
1	0.0027	0.0009	0.0009	0.0027	0.0027	⋯	0.0009	0.0009	0.0009	0.0009	0.0009
2	0.0027	0.0027	0.0027	0.0009	0.0027	⋯	0.0009	0.0009	0.0009	0.0009	0.0009
3	0.0009	0.0009	0.0045	0.0045	0.0028	⋯	0.0009	0.0009	0.0009	0.0009	0.0009
4	0.0009	0.0009	0.0009	0.0045	0.0045	⋯	0.0009	0.0009	0.0009	0.0009	0.0009
5	0.0009	0.0009	0.0010	0.0009	0.0081	⋯	0.0009	0.0009	0.0009	0.0009	0.0009
⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋮
21	0.0009	0.0009	0.0010	0.0010	0.0010	⋯	0.0009	0.0009	0.0009	0.0009	0.0009
22	0.0009	0.0009	0.0009	0.0010	0.0010	⋯	0.0009	0.0009	0.0009	0.0009	0.0009
23	0.0009	0.0009	0.0010	0.0010	0.0010	⋯	0.0009	0.0009	0.0009	0.0009	0.0009
24	0.0010	0.0010	0.0012	0.0010	0.0012	⋯	0.0010	0.0010	0.0010	0.0010	0.0010
25	0.0009	0.0009	0.0009	0.0009	0.0009	⋯	0.0009	0.0009	0.0009	0.0009	0.0009

Table 7: The r_i, c_j, $(r_i + c_i)$, $(r_i - c_i)$ values

Equipment	Total impact	Total receptive	Total engagement	Role
E_1	0.0690	0.0360	0.1050	0.0331
E_2	0.0598	0.0360	0.0958	0.0239
E_3	0.0543	0.0360	0.0903	0.0183
E_4	0.0470	0.0360	0.0829	0.0110
E_5	0.0378	0.0360	0.0737	0.0018
E_6	0.0304	0.0360	0.0664	-0.0056
E_7	0.0341	0.0360	0.0701	-0.0019
Best	0.0856	0.0360	0.1216	0.0496
Worst	0.0230	0.0360	0.0590	-0.0130

on the plot. The equipment plot results are illustrated in Fig 9. The closer an equipment point is to the best solution and the farther it is from the worst solution, the more favorable its evaluation.

6.3. Results

From the results in Fig 9, it can be observed that each equipment has different distance from the best and worst solutions. Theoretically, the best equipment should be the one that is closest to the best solution and farthest from the worst solution. By using the Euclidean distance formula, the distance from each equipment to the best and worst solutions is calculated, as shown in Table 8. Subsequently, the relative closeness of each equipment can be computed using Eq. (23), and the results are also presented in Table 8.

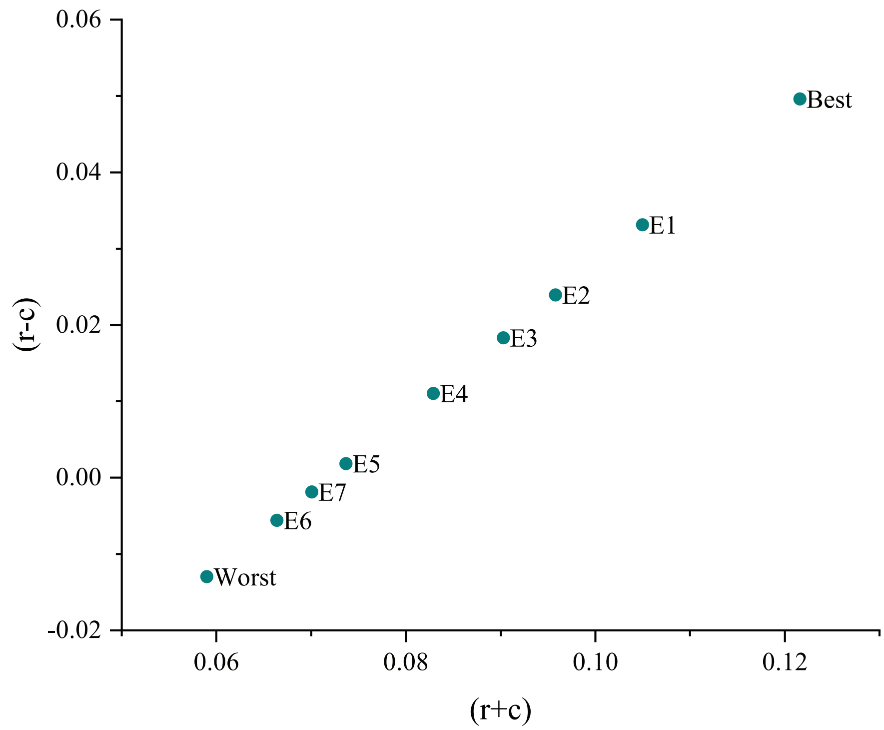

Figure 9: The graphical presentation of the equipment in the two-dimension space.

Based on the relative closeness values, the equipment can be ranked according to their importance and performance in the integrated air-ground confrontation equipment SoS. The higher the relative closeness value, the more favorable the evaluation for that equipment, indicating its greater significance and effectiveness in contributing to the success of the equipment SoS operation.

Table 8: Equipment evaluation results

Equipment	$(r+c)$	$(r-c)$	d^+	d^-	Relative closeness	Rank
E_1	0.1050	0.0331	0.0234	0.0651	0.7354	1
E_2	0.0958	0.0239	0.0364	0.0521	0.5884	2
E_3	0.0903	0.0183	0.0442	0.0443	0.5002	3
E_4	0.0829	0.0110	0.0547	0.0338	0.3824	4
E_5	0.0737	0.0018	0.0677	0.0208	0.2354	5
E_6	0.0664	-0.0056	0.0781	0.0104	0.1179	7
E_7	0.0701	-0.0019	0.0729	0.0156	0.1767	6

From Table 8, the ranking of the equipment can be determined as follows: $E_1 \succ E_2 \succ E_3 \succ E_4 \succ E_5 \succ E_7 \succ E_6$. This means that the fighter jet (E_1) is evaluated to be the best equipment in the integrated air-ground confrontation equipment SoS, followed by the early warning aircraft (E_2), reconnaissance aircraft (E_3), surveillance UAV (E_4), combat UAV (E_5), bomber (E_7), and missile (E_6), respectively. Therefore, from the results, it can be observed that different equipment has different importance with regard to the operation of the equipment SoS, nevertheless, the performance of the equipment can be indicated by their relative closeness, where higher values represent higher importance. Clearly, fighter jet (E_1) has the best performance in SoS warfare, as it contributes to capabilities such as attack, maneuver, and command. On the other hand, missile (E_6) has the most insignificant importance in comparison with other equipment, as indicated by its low relative closeness. By using relative closeness to evaluate the performance of the equipment, a more balanced and reliable result could be obtained.

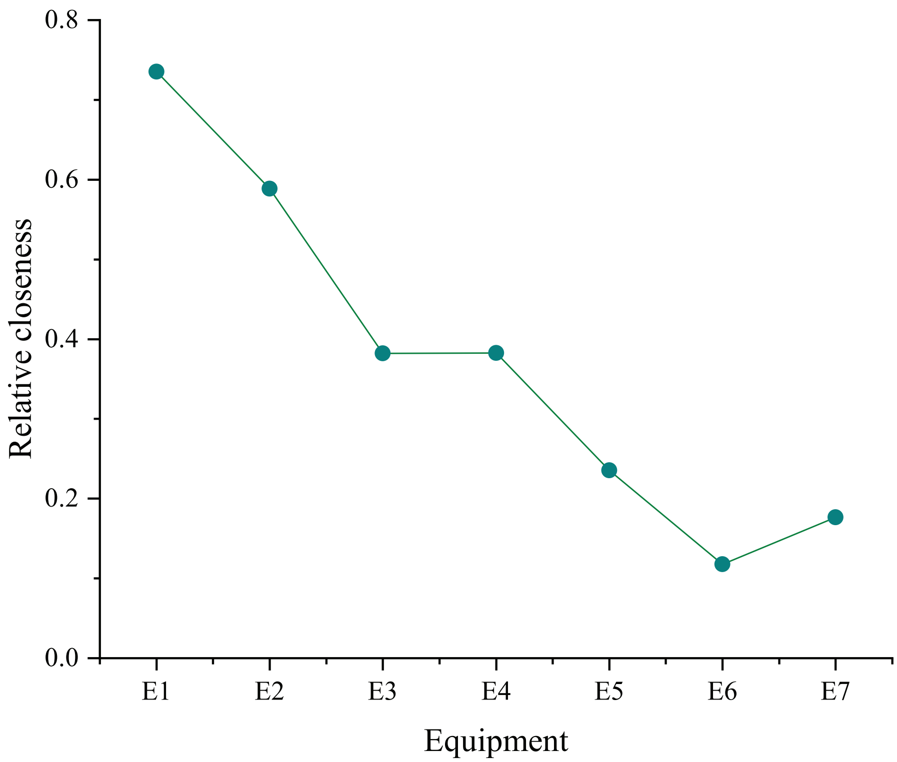

Figure 10: Equipment evaluation results.

In order to better demonstrate the evaluation results, the relative closeness of different equipment is plotted in Fig 10, where the x-axis shows different equipment and the y-axis shows the corresponding relative closeness. Clearly, higher relative closeness indicates better performance, and it can be observed from Fig 10 that fighter jet (E_1) has the highest relative closeness value, whereas missile (E_6) has the lowest relative closeness value, corresponding to their rankings. Hence, Fig 10 could provide a more illustrative way to demonstrate the results.

7. Discussion

7.1. Validity analysis

In order to validate the proposed method, a validity analysis is conducted in this section. The analysis evaluates the performance of the proposed method in the following aspects.

Aspect 1: An effective decision-making method should have the same optimal alternative if an insufficient alternative is replaced by a worse alternative without other changes.

Aspect 2: An effective decision-making method should satisfy the property of transitivity, that is, if alternative A is preferred to alternative B, and alternative B is preferred to alternative C, then alternative A should be preferred to alternative C.

Aspect 3: If a complex decision-making problem is decomposed into several sub-problems, then the combined ranking results obtained from these sub-problems using the decision-making method should be the same as the ranking result of the original problem.

In order to determine the validity of the proposed method, the following tests are conducted.

Aspect 1: To validate the proposed method under Aspect 1, the insufficient alternative E_3 is replaced by a worse alternative E_3', and the influence intensity of E_3' on capabilities is listed in Table 9.

Table 9: The influence intensity E_3' on capabilities

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
E_3	H	VL	VL	VL	N	L	N	VL	VH

Then, by using the proposed method, the ranking of the alternatives is obtained as $E_1 \succ E_2 \succ E_4 \succ E_3' \succ E_5 \succ E_7 \succ E_6$, where E_1 remains the optimal alternative. Therefore, from the results, it can be found that when an insufficient alternative (E_3) is replaced by a worse alternative (E_3'), the optimal alternative remains the same.

The same conclusion could be obtained for other insufficient alternatives such as E_2, E_4, E_5, E_6 and E_7. Thus, this indicates that the proposed method meets the first aspect.

Aspects 2 and 3: In order to test the performance of the proposed method under Aspects 2 and 3, the original decision-making problem is decomposed into four sub-problems that contain $\{E_1, E_2, E_4, E_6\}$, $\{E_2, E_3, E_5, E_6\}$, $\{E_4, E_5, E_6, E_7\}$ and $\{E_2, E_4, E_6, E_7\}$, respectively. By using the proposed method to solve these sub-problems, the ranking orders of these problems can be obtained as:

$$\begin{gathered} E_1 \succ E_2 \succ E_4 \succ E_6 \\ E_2 \succ E_3 \succ E_5 \succ E_6 \\ E_4 \succ E_5 \succ E_7 \succ E_6 \\ E_2 \succ E_4 \succ E_7 \succ E_6 \end{gathered}$$

By combining the rankings of these sub-problems, the overall ranking could be obtained as:

$$E_1 \succ E_2 \succ E_3 \succ E_4 \succ E_5 \succ E_7 \succ E_6$$

which is in line with the ranking of the original problem. Therefore, the proposed method is validated through aspects 2 and 3.

7.2. Comparative analysis

To show the effectiveness and reliability of the proposed IF-WINGS method, we conducted a comparative analysis with several existing methods using the same case study. The results obtained by the proposed method and the comparative methods, including fuzzy DEMATEL (Li et al., 2020), intuitionistic fuzzy DEMATEL (Ocampo & Yamagishi, 2020), conventional WINGS (Michnik, 2013), and fuzzy WINGS (Tavana et al., 2021), are listed in Table 10.

Table 10: Comparison results

Method	Ranking	Best equipment
Fuzzy DEMATEL	$E_1 \succ E_2 \succ E_3 \succ E_4 \succ E_5 \succ E_6 \succ E_7$	E_1
Intuitionistic fuzzy DEMATEL	$E_1 \succ E_2 \succ E_3 \succ E_5 \succ E_4 \succ E_7 \succ E_6$	E_1
Conventional WINGS	$E_1 \succ E_2 \succ E_3 \succ E_4 \succ E_5 \succ E_7 \succ E_6$	E_1
Fuzzy WINGS	$E_1 \succ E_2 \succ E_3 \succ E_4 \succ E_5 \succ E_7 \succ E_6$	E_1
Proposed method	$E_1 \succ E_2 \succ E_3 \succ E_4 \succ E_5 \succ E_7 \succ E_6$	E_1

From the comparative analysis in Table 10, it is evident that all the methods, including the proposed IF-WINGS method, consistently rank equipment E_1 as the best equipment in the equipment SoS. This consistency in the rankings across different methods highlights the reliability and validity of the proposed method. However, it is worth noting that there are some variations in the rankings of the lower-ranked alternatives, particularly between fuzzy DEMATEL and intuitionistic fuzzy DEMATEL. These variations can be attributed to the different approaches used for uncertainty representation and alternative evaluation in these methods. In general, the comparison results demonstrate that the proposed IF-WINGS method provides reliable and effective results, as it consistently identifies E_1 as the best alternative, aligning with the rankings obtained by other methods. The results of these methods are further illustrated in Fig 11. The consistency of the proposed method with other existing methods in ranking the best alternative and its robustness to variations validate the effectiveness of the IF-WINGS approach in evaluating equipment in the equipment SoS. The proposed method shows promising potential in real-world decision-making problems, especially in complex system-of-systems scenarios, where uncertainties and interdependencies exist among different elements.

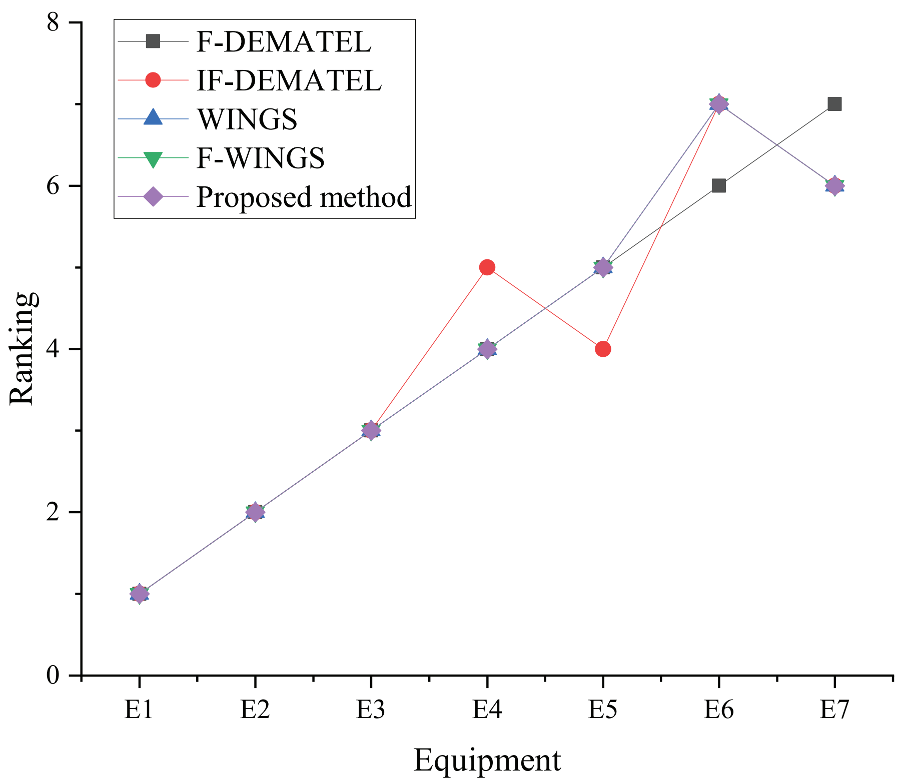

Figure 11: Comparative analysis results.

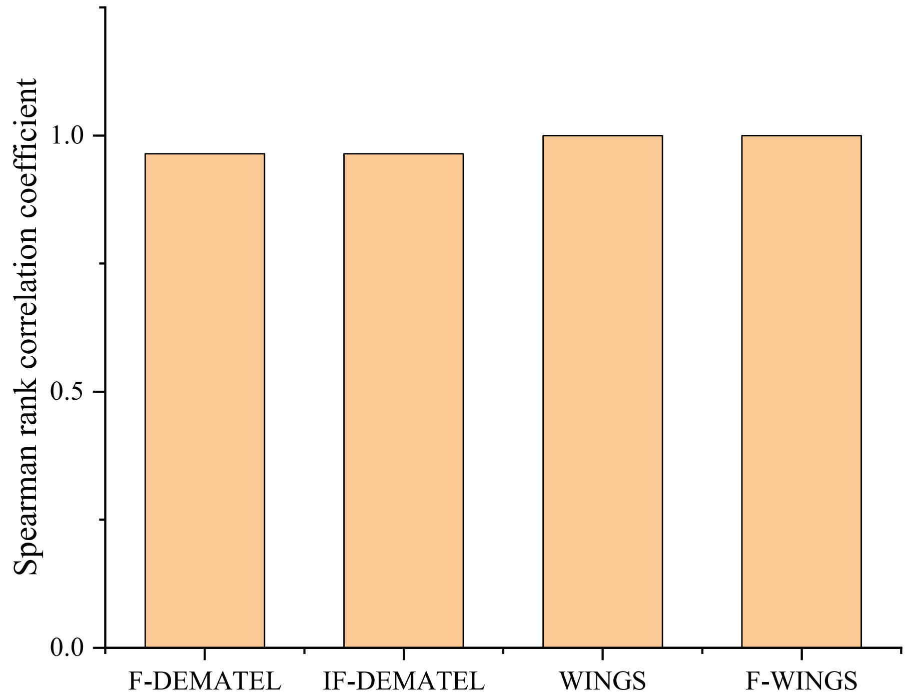

Figure 12: Spearman's rank correlation coefficient of different methods.

In addition, to better evaluate the similarity of the ranking orders obtained by the proposed method and the comparative methods, Spearman's rank correlation coefficients are computed, and the results are shown in Fig 12. The rank correlation coefficients between the proposed method and the comparative method are obtained as $(0.96, 0.96, 1.00, 1.00)$, which are close to $+1$. These coefficients indicate that there is a strong positive correlation between the results obtained by the proposed method and the comparative methods. This further validates the consistency and reliability of the proposed IF-WINGS method. Based on the comparative analysis, several advantages of the proposed method can be summarized as follows:

(1) The proposed method and intuitionistic fuzzy DEMATEL are developed based on IFS, which offers a more flexible and powerful tool to represent uncertain, linguistic, and subjective information compared to traditional fuzzy sets. This enhanced representation allows decision-makers to express their judgments more accurately and comprehensively, increasing the reliability of the proposed method.

(2) The proposed method is based on the WINGS method, which considers not only the influence intensity of the criteria but also their strength. In contrast, DEMATEL methods may not fully capture the importance of different criteria in complex decision-making problems. For equipment evaluation in the equipment SoS, where

tasks and capabilities may have varying levels of significance, the proposed method is better suited to provide more effective and accurate representations.

(3) The proposed method extends the WINGS method by incorporating the idea of TOPSIS. By calculating the distance of each alternative from the best and worst solutions, the proposed method provides a more effective and reliable evaluation of the alternatives, instead of merely comparing them based on r_i, $(r_i + c_i)$, and $(r_i - c_i)$. This enhancement allows for more meaningful and robust evaluations of the alternatives.

In conclusion, the comparative analysis highlights the superiority of the proposed IF-WINGS method over other existing methods, particularly in terms of flexibility, accuracy in representing importance, and effective evaluation of alternatives. The proposed method shows great promise for decision-making in complex system-of-systems scenarios, where uncertainty and interdependencies play significant roles.

7.3. Managerial implications

In this study, we propose the IF-WINGS method for evaluating different equipment under SoS warfare environment. The IF-WINGS method combines the advantages of IFS and the WINGS method, providing a reliable and effective evaluation of equipment in complex scenarios.

The key managerial implications derived from this study are as follows:

(1) SoS warfare presents unique challenges due to its interconnectivity and emergent nature. Traditional data-based methods are insufficient in capturing the overall performance of equipment in such environments. Recently, Han et al. (2022) presented the idea of treating the equipment assessment problem as a MCDM problem, however, current research fails to comprehensively analyze the interrelationships among different elements of the equipment SoS. In comparison, the IF-WINGS method considers the interdependencies among tasks, capabilities, and equipment in the SoS, resulting in more comprehensive and reliable evaluations from an operational perspective.

(2) Intuitionistic fuzzy sets offer a flexible and powerful tool for representing uncertain, linguistic, and subjective information. Compared with fuzzy sets, the IFS could be used as a more effective and powerful tool, especially for representing uncertain information in MCDM, as been discussed by Xie et al. (2022); Alcantud et al. (2020); Garg & Rani (2022). In equipment evaluation, precise information may be limited, and expert judgments are often subjective and uncertain. The IFS allows for accurate modeling of evaluations using membership and non-membership functions, leading to more reliable and accurate results.

(3) The WINGS method's advantage lies in effectively modeling interrelationships among elements based on both strength and influence intensity. However, its final ranking results may lack quantitative analysis. Compared with the previous extensions on the WINGS method in (Michnik, 2013; Tavana et al., 2021, 2023), the IF-WINGS method addresses this by integrating the concept of TOPSIS, enabling a more robust evaluation and ranking of alternatives based on their distances from the best and worst solutions.

(4) The comparative analysis validates the superiority of the proposed IF-WINGS method over other existing methods, including fuzzy DEMATEL (Li et al., 2020), intuitionistic fuzzy DEMATEL (Ocampo & Yamagishi, 2020), and conventional WINGS (Michnik, 2013). The consistent identification of the best equipment in the equipment SoS underscores the method's reliability and validity in decision-making.

(5) Beyond equipment evaluation in SoS warfare, the IF-WINGS method's flexibility, accuracy in representing importance, and effective evaluation of alternatives make it a valuable tool for various complex decision-making problems involving uncertainty, interdependencies, and linguistic judgments.

In conclusion, the IF-WINGS method is a promising approach for evaluating equipment in system-of-systems warfare environments. Its incorporation of IFS, the WINGS method, and TOPSIS results in a comprehensive, accurate, and reliable evaluation process. Moreover, its applicability extends to other decision-making scenarios involving complex systems and uncertain data. The comparative analysis further supports the method's superiority over existing approaches, reinforcing its potential as a valuable tool for decision-making in various domains.

8. Conclusion

This study proposes an intuitionistic fuzzy decision-making approach for equipment evaluation under SoS warfare environment. The method introduces a novel framework based on the WINGS method under the intuitionistic fuzzy environment, considering both the strength and influence intensity of elements within the SoS. Uncertain evaluation information is represented using IFSs, and a causal relation diagram is constructed to analyze the interrelations among elements. The proposed method determines the evaluation and ranking of alternatives by calculating the distance from each alternative to the best and worst solutions, enabling more reliable and reasonable evaluations. Compared with existing literature, this study is the first of its kind that considers the complex interrelationships among different elements in equipment SoS for equipment evaluation. By extending the WINGS method with IFS and adopting the proposed IF-WINGS method for equipment evaluation, this study could present a more reliable, reasonable, and flexible way for equipment evaluation under SoS warfare environment.

The contributions of this study are as follows:

(1) A novel approach for equipment evaluation under SoS warfare environment is introduced. By considering the tasks, capabilities, and equipment, along with their relations, the proposed method offers more reliable and reasonable evaluations. Additionally, accounting for both the strength and influence intensity of tasks and capabilities enhances the accuracy and reliability of the results.

(2) The WINGS method is extended with IFS, and the IF-WINGS method is proposed. The incorporation of IFS allows for more effective handling of uncertain expert judgments during decision-making, offering greater flexibility and reliability compared to conventional WINGS.

(3) The concept of TOPSIS is introduced into the WINGS method, and a novel alternative ranking approach is introduced. By considering the distance of each alternative to the best and worst solutions, the proposed method provides quantitative analysis for more reliable decision-making. The use of relative closeness for evaluating and ranking alternatives enhances the reliability and reasonability of the results.

There are some limitations to this study. Firstly, the case study is conducted with a relatively small group of experts, and the inclusion of a large group of experts could be studied in the future. Secondly, in the proposed method, the interrelationships among neighboring layers are analyzed and quantified, however, how each equipment affects the overall operational effectiveness is not fully considered, which could be further investigated. Thirdly, like many previous studies, the optimality of the obtained results is not theoretically proven in this study, which could limit its application, and it is necessary to develop a more systemic approach to analyze the optimality of the solution, not just for this study, but for other MCDM problems as well.

Future research could explore the following areas: Firstly, the proposed method could also be integrated with other decision-making methods such as VIKOR and PROMETHEE to further enhance the reliability of the results. Secondly, it is also worth noting that the case study only considers a relatively simple case, the application of the proposed method to more complex problems could be studied.

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Appendix A. Complete tables for the case study

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Table A.1: The intuitionistic fuzzy direct strength-influence matrix (Table 2)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	(0.3,0.7)	(0.1,0.9)	(0.1,0.9)	(0.3,0.7)	(0.3,0.7)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
2	(0.3,0.7)	(0.3,0.7)	(0.3,0.7)	(0.1,0.9)	(0.3,0.7)	(0.1,0.9)	(0.5,0.5)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
3	(0.1,0.9)	(0.1,0.9)	(0.5,0.5)	(0.5,0.5)	(0.3,0.7)	(0.1,0.9)	(0.3,0.7)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
4	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.5,0.5)	(0.5,0.5)	(0.5,0.5)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
5	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.9,0.1)	(0.3,0.7)	(0.7,0.3)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
6	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.3,0.7)	(0.9,0.1)	(0.3,0.7)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
7	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.7,0.3)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
8	(0.3,0.7)	(0.7,0.3)	(0.1,0.9)	(0.1,0.9)	(0.5,0.5)	(0.3,0.7)	(0.5,0.5)	(0.7,0.3)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
9	(0.3,0.7)	(0.1,0.9)	(0.7,0.3)	(0.3,0.7)	(0.3,0.7)	(0.3,0.7)	(0.1,0.9)	(0.1,0.9)	(0.5,0.5)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
10	(0.3,0.7)	(0.1,0.9)	(0.9,0.1)	(0.1,0.9)	(0.5,0.5)	(0.3,0.7)	(0.3,0.7)	(0.1,0.9)	(0.1,0.9)	(0.5,0.5)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
11	(0.1,0.9)	(0.5,0.5)	(0.9,0.1)	(0.1,0.9)	(0.5,0.5)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.5,0.5)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
12	(0.5,0.5)	(0.1,0.9)	(0.9,0.1)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.7,0.3)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
13	(0.1,0.9)	(0.3,0.7)	(0.1,0.9)	(0.9,0.1)	(0.3,0.7)	(0.3,0.7)	(0.3,0.7)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.7,0.3)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
14	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.9,0.1)	(0.5,0.5)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.9,0.1)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
15	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.3,0.7)	(0.9,0.1)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.9,0.1)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
16	(0.1,0.9)	(0.1,0.9)	(0.5,0.5)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.7,0.3)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.5,0.5)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
17	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.5,0.5)	(0.5,0.5)	(0.7,0.3)	(0.3,0.7)	(0.7,0.3)	(0.9,0.1)	(0.9,0.1)	(0.9,0.1)	(0.5,0.5)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
18	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.9,0.1)	(0.7,0.3)	(0.7,0.3)	(0.9,0.1)	(0.9,0.1)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.5,0.5)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
19	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.9,0.1)	(0.3,0.7)	(0.7,0.3)	(0.5,0.5)	(0.1,0.9)	(0.5,0.5)	(0.1,0.9)	(0.3,0.7)	(0.9,0.1)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
20	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.7,0.3)	(0.3,0.7)	(0.5,0.5)	(0.5,0.5)	(0.1,0.9)	(0.3,0.7)	(0.1,0.9)	(0.3,0.7)	(0.7,0.3)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
21	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.7,0.3)	(0.7,0.3)	(0.5,0.5)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
22	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.9,0.1)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
23	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.3,0.7)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.9,0.1)	(0.3,0.7)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
24	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.9,0.1)	(0.7,0.3)	(0.7,0.3)	(0.9,0.1)	(0.9,0.1)	(0.9,0.1)	(0.9,0.1)	(0.9,0.1)	(0.9,0.1)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)
25	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)	(0.1,0.9)

Table A.2: The expected intuitionistic fuzzy direct strength-influence matrix (Table 4)

Row	Column																								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0.30	0.10	0.10	0.30	0.30	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
2	0.30	0.30	0.30	0.10	0.30	0.10	0.50	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
3	0.10	0.10	0.50	0.50	0.30	0.10	0.30	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
4	0.10	0.10	0.10	0.50	0.50	0.50	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
5	0.10	0.10	0.10	0.10	0.90	0.30	0.70	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
6	0.10	0.10	0.10	0.10	0.30	0.90	0.30	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
7	0.10	0.10	0.10	0.10	0.10	0.10	0.70	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
8	0.30	0.70	0.10	0.10	0.50	0.30	0.50	0.70	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
9	0.30	0.10	0.70	0.30	0.30	0.30	0.10	0.10	0.50	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
10	0.30	0.10	0.90	0.10	0.50	0.30	0.30	0.10	0.10	0.50	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
11	0.10	0.50	0.90	0.10	0.50	0.10	0.10	0.10	0.10	0.10	0.50	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
12	0.50	0.10	0.90	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.70	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
13	0.10	0.30	0.10	0.90	0.30	0.30	0.30	0.10	0.10	0.10	0.10	0.10	0.70	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
14	0.10	0.10	0.10	0.10	0.90	0.50	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.90	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
15	0.10	0.10	0.10	0.10	0.30	0.90	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.90	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
16	0.10	0.10	0.50	0.10	0.10	0.10	0.70	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.50	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
17	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.50	0.50	0.70	0.30	0.70	0.90	0.90	0.90	0.50	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
18	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.90	0.70	0.70	0.90	0.90	0.10	0.10	0.10	0.50	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
19	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.90	0.30	0.70	0.50	0.10	0.50	0.10	0.30	0.90	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
20	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.70	0.30	0.50	0.50	0.10	0.30	0.10	0.30	0.70	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
21	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.70	0.70	0.50	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
22	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.90	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
23	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.30	0.10	0.10	0.10	0.90	0.30	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
24	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.90	0.70	0.70	0.90	0.90	0.90	0.90	0.90	0.90	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
25	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10

Table A.3: The normalized intuitionistic fuzzy direct strength-influence matrix (Table 5)

Row	Column																								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0.0027	0.0009	0.0009	0.0027	0.0027	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
2	0.0027	0.0027	0.0027	0.0009	0.0027	0.0009	0.0044	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
3	0.0009	0.0009	0.0044	0.0044	0.0027	0.0009	0.0027	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
4	0.0009	0.0009	0.0009	0.0044	0.0044	0.0044	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
5	0.0009	0.0009	0.0009	0.0009	0.0080	0.0027	0.0062	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
6	0.0009	0.0009	0.0009	0.0009	0.0027	0.0080	0.0027	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
7	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0062	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
8	0.0027	0.0062	0.0009	0.0009	0.0044	0.0027	0.0044	0.0062	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
9	0.0027	0.0009	0.0062	0.0027	0.0027	0.0027	0.0009	0.0009	0.0044	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
10	0.0027	0.0009	0.0080	0.0009	0.0044	0.0027	0.0027	0.0009	0.0009	0.0044	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
11	0.0009	0.0044	0.0080	0.0009	0.0044	0.0009	0.0009	0.0009	0.0009	0.0009	0.0044	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
12	0.0044	0.0009	0.0080	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0062	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
13	0.0009	0.0027	0.0009	0.0080	0.0027	0.0027	0.0027	0.0009	0.0009	0.0009	0.0009	0.0009	0.0062	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
14	0.0009	0.0009	0.0009	0.0009	0.0080	0.0044	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0080	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
15	0.0009	0.0009	0.0009	0.0009	0.0027	0.0080	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0080	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
16	0.0009	0.0009	0.0044	0.0009	0.0009	0.0009	0.0062	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0044	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
17	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0044	0.0044	0.0062	0.0027	0.0062	0.0080	0.0080	0.0080	0.0044	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
18	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0080	0.0062	0.0062	0.0080	0.0080	0.0009	0.0009	0.0009	0.0044	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
19	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0080	0.0027	0.0062	0.0044	0.0009	0.0044	0.0009	0.0027	0.0080	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
20	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0062	0.0027	0.0044	0.0044	0.0009	0.0027	0.0009	0.0027	0.0062	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
21	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0062	0.0062	0.0044	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
22	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0080	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
23	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0027	0.0009	0.0009	0.0009	0.0080	0.0027	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
24	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0080	0.0062	0.0062	0.0080	0.0080	0.0080	0.0080	0.0080	0.0080	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
25	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009

Table A.4: The total intuitionistic fuzzy strength-influence matrix (Table 6)

Row	Column																								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0.0027	0.0009	0.0009	0.0027	0.0027	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
2	0.0027	0.0027	0.0027	0.0009	0.0027	0.0009	0.0045	0.0009	0.0009	0.0009	0.0009	0.0009	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
3	0.0009	0.0009	0.0045	0.0045	0.0028	0.0010	0.0027	0.0009	0.0009	0.0009	0.0009	0.0009	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
4	0.0009	0.0009	0.0009	0.0045	0.0045	0.0045	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
5	0.0009	0.0009	0.0010	0.0009	0.0081	0.0027	0.0063	0.0010	0.0009	0.0009	0.0009	0.0009	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
6	0.0009	0.0009	0.0009	0.0009	0.0027	0.0081	0.0027	0.0009	0.0009	0.0009	0.0009	0.0009	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
7	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0063	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
8	0.0027	0.0063	0.0010	0.0009	0.0046	0.0027	0.0046	0.0063	0.0009	0.0010	0.0009	0.0009	0.0010	0.0010	0.0010	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
9	0.0027	0.0009	0.0063	0.0027	0.0028	0.0027	0.0010	0.0010	0.0045	0.0009	0.0009	0.0009	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
10	0.0027	0.0009	0.0081	0.0010	0.0046	0.0027	0.0028	0.0010	0.0009	0.0045	0.0009	0.0009	0.0010	0.0009	0.0009	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
11	0.0009	0.0045	0.0081	0.0010	0.0046	0.0010	0.0010	0.0010	0.0009	0.0009	0.0045	0.0009	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
12	0.0045	0.0009	0.0081	0.0010	0.0010	0.0009	0.0010	0.0010	0.0009	0.0009	0.0009	0.0063	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
13	0.0009	0.0027	0.0010	0.0081	0.0028	0.0028	0.0028	0.0010	0.0009	0.0009	0.0009	0.0009	0.0063	0.0009	0.0009	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
14	0.0009	0.0009	0.0010	0.0009	0.0081	0.0045	0.0010	0.0010	0.0009	0.0009	0.0009	0.0009	0.0010	0.0081	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
15	0.0009	0.0009	0.0010	0.0009	0.0028	0.0081	0.0010	0.0010	0.0009	0.0009	0.0009	0.0009	0.0010	0.0009	0.0081	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
16	0.0009	0.0009	0.0045	0.0009	0.0010	0.0009	0.0063	0.0009	0.0009	0.0009	0.0009	0.0009	0.0010	0.0009	0.0009	0.0045	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
17	0.0010	0.0010	0.0011	0.0010	0.0011	0.0011	0.0010	0.0045	0.0045	0.0063	0.0027	0.0063	0.0081	0.0081	0.0081	0.0045	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
18	0.0010	0.0010	0.0012	0.0010	0.0011	0.0010	0.0010	0.0081	0.0063	0.0063	0.0081	0.0081	0.0010	0.0010	0.0010	0.0045	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
19	0.0010	0.0010	0.0011	0.0010	0.0010	0.0010	0.0010	0.0081	0.0027	0.0063	0.0045	0.0010	0.0045	0.0010	0.0027	0.0081	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
20	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0063	0.0027	0.0045	0.0045	0.0009	0.0027	0.0010	0.0027	0.0063	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
21	0.0009	0.0009	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0009	0.0009	0.0009	0.0009	0.0063	0.0063	0.0045	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
22	0.0009	0.0009	0.0009	0.0010	0.0010	0.0010	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0081	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
23	0.0009	0.0009	0.0010	0.0010	0.0010	0.0010	0.0010	0.0009	0.0027	0.0009	0.0009	0.0009	0.0081	0.0027	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
24	0.0010	0.0010	0.0012	0.0010	0.0012	0.0011	0.0011	0.0081	0.0063	0.0063	0.0081	0.0081	0.0081	0.0081	0.0081	0.0081	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
25	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009