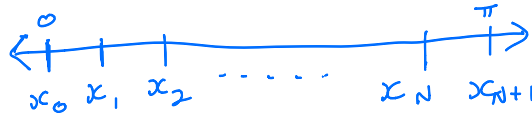
General SLEP

$0 \le x \le \pi$

$$(p(x)y')' + q(x)y = \lambda r(x) y$$



BC: $y(0) = y(L) = 0$

$$y'(x_i) \approx \frac{y(x_{i+1}) - y(x_i)}{\Delta x}$$

$$y''(x_i) \approx \frac{y(x_{i+1}) + y(x_{i-1}) - 2y(x_i)}{\Delta x^2}$$

$\Downarrow$ $h = \Delta x$

$$\boxed{p(x_i) h^{-2} \left[y(x_{i+1}) + y(x_{i-1}) - 2y(x_i) \right]} + \boxed{p'(x_i) h^{-1} \left[y(x_{i+1}) - y(x_i) \right]}$$

$$+ \boxed{q(x_i) y(x_i)} = \boxed{\lambda_i r(x_i) y(x_i)}$$

part.

$$\boxed{p(x_i) h^{-2} \left[y(x_{i+1}) + y(x_{i-1}) - 2y(x_i) \right]}$$

Say $N = 5$ $\qquad y_1 = y(x_1)$. $\qquad y_0 = y_6 = 0$ (BC)

$$\begin{aligned}
&(2h^{-2}y_1 - h^{-2}y_2) \times P_1 \\
&(-h^{-2}y_1 + 2h^{-2}y_2 - h^{-2}y_3) \times P_2 \\
&(-h^{-2}y_2 + 2h^{-2}y_3 - h^{-2}y_4) \times P_3 \\
&(-h^{-2}y_3 + 2h^{-2}y_4 - h^{-2}y_5) \times P_4 \\
&(-h^{-2}y_4 + 2h^{-2}y_5) \times P_5
\end{aligned}$$

$$\Rightarrow \underbrace{h^{-2} \begin{bmatrix} P_1 & P_1 & P_1 & P_1 & P_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_5 & P_5 & P_5 & P_5 & P_5 \end{bmatrix} \bullet \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}}_{\to A} \begin{bmatrix} y_1 \\ \vdots \\ \vdots \\ y_5 \end{bmatrix}$$

part

$$\boxed{p'(x_i)h^{-1}\left[y(x_{i+1})-y(x_i)\right]}$$

say $N=5$

$$
\begin{aligned}
&(h^{-1}y_2 - h^{-1}y_1)p'_1\\
&(h^{-1}y_3 - h^{-1}{}_2)p'_2\\
&(h^{-1}y_4 - h^{-1}{}_3)p'_3\\
&(h^{-1}y_5 - h^{-1}{}_4)p'_4\\
&(\cancel{h^{-1}y_6}_{\,0} - h^{-1}{}_5)p'_5
\end{aligned}
\Rightarrow h^{-1}
\underbrace{
\begin{bmatrix}
p'_1 & p'_1 & p'_1 & p'_1 & p'_1\\
\vdots & \vdots & \vdots & \vdots & \vdots\\
p'_5 & p'_5 & p'_5 & p'_5 & p'_5
\end{bmatrix}
\odot
\begin{bmatrix}
-1 & 1 & 0 & 0 & 0\\
0 & -1 & 1 & 0 & 0\\
0 & 0 & -1 & 1 & 0\\
0 & 0 & 0 & -1 & 1\\
0 & 0 & 0 & 0 & -1
\end{bmatrix}
}_{B}
\begin{bmatrix}
y_1\\ \vdots\\ y_5
\end{bmatrix}
$$

Part

$$\boxed{q(x_i)y(x_i)}$$

say $N=5$

$$
\underbrace{
\begin{bmatrix}
q_1 & 0 & 0 & 0 & 0\\
0 & q_2 & 0 & 0 & 0\\
0 & 0 & q_3 & 0 & 0\\
0 & 0 & 0 & q_4 & 0\\
0 & 0 & 0 & 0 & q_5
\end{bmatrix}
}_{C}
\begin{bmatrix}
y_1\\ \vdots\\ y_5
\end{bmatrix}
$$

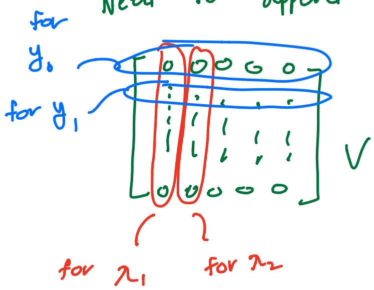
part

$$\boxed{\lambda_i r(x_i) y(x_i)}$$

$$
\begin{bmatrix}
\lambda_1 & 0 & 0 & 0 & 0\\
0 & \lambda_2 & 0 & 0 & 0\\
0 & 0 & \lambda_3 & 0 & 0\\
0 & 0 & 0 & \lambda_4 & 0\\
0 & 0 & 0 & 0 & \lambda_5
\end{bmatrix}
\begin{bmatrix}
r_1 & 0 & 0 & 0 & 0\\
0 & r_2 & 0 & 0 & 0\\
0 & 0 & r_3 & 0 & 0\\
0 & 0 & 0 & r_4 & 0\\
0 & 0 & 0 & 0 & r_5
\end{bmatrix}
\begin{bmatrix}
y_1\\ \vdots\\ y_5
\end{bmatrix}
$$

$$A + B + C = D \longrightarrow Dy = r\lambda y.$$

Need to append BC to eigenvectors.



V = [zeros(1,N); V; zeros(1,N)];