PDE: $$\rho \frac{\partial^2 u}{\partial t^2} = T_0 \frac{\partial^2 u}{\partial x^2}$$

BC: $u(0,t) = 0$
$u(L,t) = 0$

IC: $u(x,0) = f(x)$
$\frac{\partial u}{\partial t}(x,0) = g(x)$

separation of variables

$\longrightarrow$

$$u(x,t) = \phi(x)\, h(t)$$

$$\frac{d^2 h}{dt^2} = -\lambda h$$

SLEP

$$T_0 \frac{d^2\phi}{dx^2} + \lambda \rho(x)\phi = 0$$

$\phi(0) = 0$

$\phi(L) = 0$

$\downarrow$ use SLEP.m to obtain ϕ_n and λ_n

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin\sqrt{\lambda_n}\, t\, \phi_n(x) + \sum_{n=1}^{\infty} b_n \cos\sqrt{\lambda_n}\, t\, \phi_n(x).$$

$$f(x) = \sum_{n=1}^{\infty} b_n \phi_n(x) \quad , \quad g(x) = \sum_{n=1}^{\infty} a_n \sqrt{\lambda_n}\, \phi_n(x)$$

$$b_n = \frac{\int_0^L f(x)\phi_n(x)\rho(x)\,dx}{\int_0^L \phi_n^2 \rho\, dx} \qquad a_n = \frac{\int_0^L g(x)\phi_n(x)\rho(x)\,dx}{\sqrt{\lambda_n}\int_0^L \phi_n^2 \rho\, dx}$$