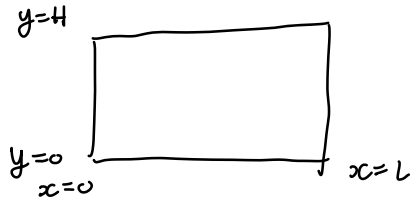
$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$



BC

$u(0,y,t) = 0 \qquad u(x,0,t) = 0$

$u(L,y,t) = 0 \qquad u(x,H,t) = 0$

IC

$u(x,y,0) = \alpha(x,y)$

$\frac{\partial u}{\partial t}(x,y,0) = \beta(x,y)$

separation of variables

$\Rightarrow \quad u(x,y,t) = h(t)\,\phi(x,y)$

$$\frac{d^2h}{dt^2} = -\lambda c^2 h$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = -\lambda\phi$$

$\phi(0,y) = 0 \qquad \phi(x,0) = 0$

$\phi(L,y) = 0 \qquad \phi(x,H) = 0$

set $\quad \phi(x,y) = f(x)\,g(y)$

then $\quad u(x,y,t) = f(x)\,g(y)\,h(t)$

SLEP!

$$\frac{d^2f}{dx^2} = -\mu f \quad , \quad \frac{d^2g}{dy^2} = -(\lambda-\mu)g$$

$f(0) = 0 \qquad f(L) = 0$

$g(0) = 0 \qquad g(H) = 0$

$$\Rightarrow \quad \mu_n = \left(\frac{n\pi}{L}\right)^2, \quad n = 1,2,3,\ldots,$$

$$f_n(x) = \sin\frac{n\pi x}{L}$$

$$\Rightarrow \quad \lambda_{nm} - \mu_n = \left(\frac{m\pi}{H}\right)^2, \quad m = 1, 2, 3, \cdots,$$

$$g_{nm}(y) = \sin\frac{m\pi y}{H}$$

$$\lambda_{nm} = \mu_n + \left(\frac{m\pi}{H}\right)^2 = \left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{H}\right)^2$$

$$\phi_{nm}(x,y) = \sin\frac{n\pi x}{L}\sin\frac{m\pi y}{H}$$

$$\rightarrow \quad u(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{nm}\sin\frac{n\pi x}{L}\sin\frac{m\pi y}{H}\cos c\sqrt{\lambda_{nm}}\,t + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{nm}\sin\frac{n\pi x}{L}\sin\frac{m\pi y}{H}\sin c\sqrt{\lambda_{nm}}\,t$$

$$u(x,y,0) = \alpha(x,y)$$

$$\alpha(x,y) = \sum_{m=1}^{\infty}\left(\sum_{n=1}^{\infty} A_{nm}\sin\frac{n\pi x}{L}\right)\sin\frac{m\pi y}{H}$$

$$\sum_{n=1}^{\infty} A_{nm}\sin\frac{n\pi x}{L} = \frac{2}{H}\int_0^H \alpha(x,y)\sin\frac{m\pi y}{H}\,dy$$

$$A_{nm} = \frac{2}{L}\int_0^L\left[\frac{2}{H}\int_0^H \alpha(x,y)\sin\frac{m\pi y}{H}\,dy\right]\sin\frac{n\pi x}{L}\,dx$$

$$\frac{\partial u}{\partial t}(x,y,0) = \beta(x,y)$$

$$\beta(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}\,B_{nm}\sin\frac{n\pi x}{L}\sin\frac{m\pi y}{H}$$

$$c\sqrt{\lambda_{nm}}\,B_{nm} = \frac{4}{LH}\int_0^L\int_0^H \beta(x,y)\sin\frac{m\pi y}{H}\sin\frac{n\pi x}{L}\,dy\,dx$$

$$A_{nm} = \frac{2}{L}\int_0^L \left[\frac{2}{H}\int_0^H \alpha(x,y)\sin\frac{m\pi y}{H}\,dy\right]\sin\frac{n\pi x}{L}\,dx$$

```
Sum ( sum(alphaxy .* V2(:,m))*dy .* V1(:,n) )*dx
    ------------------------------------------
        dot(V2(:,m).^2, r2)*dy
    ------------------------------
        dot(V1(:,n).^2, r1)*dx
```

$$u(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{nm}\sin\frac{n\pi x}{L}\sin\frac{m\pi y}{H}\cos c\sqrt{\lambda_{nm}}\,t + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{nm}\sin\frac{n\pi x}{L}\sin\frac{m\pi y}{H}\sin c\sqrt{\lambda_{nm}}\,t$$

```
for n = 1:K
    for m: 1:K
        [V1, V2] = meshgrid(V1(:,n), V2(:,m))
u = u + A(n,m) * V1 .* V2 * cos c√λnm t
```