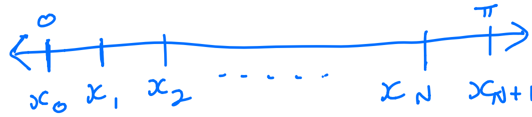
General SLEP

$$(p(x)y')' + q(x)y = \lambda r(x) y \qquad 0 \le x \le \pi$$



BC: $y(0) = y(L) = 0$

$$y'(x_i) \approx \frac{y(x_{i+1}) - y(x_i)}{\Delta x}$$

$$y''(x_i) \approx \frac{y(x_{i+1}) + y(x_{i-1}) - 2y(x_i)}{\Delta x^2}$$

$\Downarrow \qquad h = \Delta x$

$$\boxed{p(x_i)h^{-2}\left[y(x_{i+1}) + y(x_{i-1}) - 2y(x_i)\right]} + \boxed{p'(x_i)h^{-1}\left[y(x_{i+1}) - y(x_i)\right]}$$

$$+ \boxed{q(x_i)y(x_i)} = \boxed{\lambda_i r(x_i) y(x_i)}$$

part.

$$\boxed{p(x_i)h^{-2}\left[y(x_{i+1}) + y(x_{i-1}) - 2y(x_i)\right]}$$

Say $N = 5$ $\qquad y_1 = y(x_1)$. $\qquad y_0 = y_6 = 0$ (BC)

$$\begin{aligned}
&(2h^{-2}y_1 - h^{-2}y_2) \times P_1 \\
&(-h^{-2}y_1 + 2h^{-2}y_2 - h^{-2}y_3) \times P_2 \\
&(-h^{-2}y_2 + 2h^{-2}y_3 - h^{-2}y_4) \times P_3 \\
&(-h^{-2}y_3 + 2h^{-2}y_4 - h^{-2}y_5) \times P_4 \\
&(-h^{-2}y_4 + 2h^{-2}y_5) \times P_5
\end{aligned}$$

$$\Rightarrow \underbrace{h^{-2}\begin{bmatrix} P_1 & P_1 & P_1 & P_1 & P_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_5 & P_5 & P_5 & P_5 & P_5 \end{bmatrix} \cancel{\times} \bullet \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}}_{\rightarrow A} \begin{bmatrix} y_1 \\ \vdots \\ \vdots \\ \vdots \\ y_5 \end{bmatrix}$$

part

$$\boxed{p'(x_i)h^{-1}[y(x_{i+1})-y(x_i)]}$$

say $N=5$

$$\begin{aligned}
&(h^{-1}y_2 - h^{-1}y_1)\,p'_1\\
&(h^{-1}y_3 - h^{-1}{}_2)\,p'_2\\
&(h^{-1}y_4 - h^{-1}{}_3)\,p'_3\\
&(h^{-1}y_5 - h^{-1}{}_4)\,p'_4\\
&(h^{-1}\underset{0}{\cancel{y_6}} - h^{-1}{}_5)\,p'_5
\end{aligned}
\Rightarrow h^{-1}\underbrace{\begin{bmatrix} p'_1 & p'_1 & p'_1 & p'_1 & p'_1\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ p'_5 & p'_5 & p'_5 & p'_5 & p'_5 \end{bmatrix} * \begin{bmatrix} -1 & 1 & 0 & 0 & 0\\ 0 & -1 & 1 & 0 & 0\\ 0 & 0 & -1 & 1 & 0\\ 0 & 0 & 0 & -1 & 1\\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}}_{\hookrightarrow B}\begin{bmatrix} y_1\\ \vdots\\ y_5 \end{bmatrix}$$

Part

$$\boxed{q(x_i)y(x_i)}$$

say $N=5$

$$\underbrace{\begin{bmatrix} q_1 & 0 & 0 & 0 & 0\\ 0 & q_2 & 0 & 0 & 0\\ 0 & 0 & q_3 & 0 & 0\\ 0 & 0 & 0 & q_4 & 0\\ 0 & 0 & 0 & 0 & q_5 \end{bmatrix}}_{\hookrightarrow C}\begin{bmatrix} y_1\\ \vdots\\ y_5 \end{bmatrix}$$

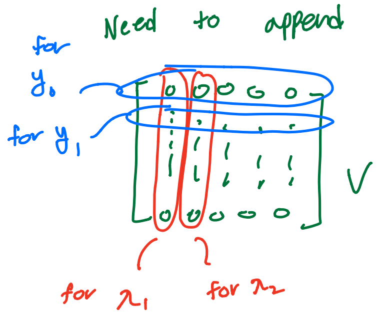
part

$$\boxed{\lambda_i r(x_i) y(x_i)}$$

$$\begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0\\ 0 & \lambda_2 & 0 & 0 & 0\\ 0 & 0 & \lambda_3 & 0 & 0\\ 0 & 0 & 0 & \lambda_4 & 0\\ 0 & 0 & 0 & 0 & \lambda_5 \end{bmatrix}\begin{bmatrix} r_1 & 0 & 0 & 0 & 0\\ 0 & r_2 & 0 & 0 & 0\\ 0 & 0 & r_3 & 0 & 0\\ 0 & 0 & 0 & r_4 & 0\\ 0 & 0 & 0 & 0 & r_5 \end{bmatrix}\begin{bmatrix} y_1\\ \vdots\\ y_5 \end{bmatrix}$$

$$A + B + C = D \longrightarrow Dy = r\lambda y.$$

Need to append BC to eigenvectors.



V = [zeros(1,N) ; V ; zeros(1,N)] ;

Neumann BC

$$\boxed{p(x_i)h^{-2}\left[y(x_{i+1})+y(x_{i-1})-2y(x_i)\right]}+\boxed{p'(x_i)h^{-1}\left[y(x_{i+1})-y(x_i)\right]}$$

$$+\boxed{q(x_i)y(x_i)}=\boxed{\lambda_i r(x_i)y(x_i)}$$

part.

$$\boxed{p(x_i)h^{-2}\left[y(x_{i+1})+y(x_{i-1})-2y(x_i)\right]}$$

Say $N=5$ $\qquad y_1=y(x_1)$. $\qquad y_0=0$ (BC)

$y_6'=0$ (BC)

$(2h^{-2}y_1-h^{-2}y_2)\times P_1$

$(-h^{-2}y_1+2h^{-2}y_2-h^{-2}y_3)\times P_2$

$(-h^{-2}y_2+2h^{-2}y_3-h^{-2}y_4)\times P_3$

$(-h^{-2}y_3+2h^{-2}y_4-h^{-2}y_5)\times P_4$

$(-h^{-2}y_4+2h^{-2}y_5-h^{-2}y_6)\times P_5$

$\underline{(-h^{-2}y_5+2h^{-2}y_6-h^{-2}y_7)\times P_6}$

$$y'(x_i)\approx\frac{y(x_{i+1})-y(x_i)}{\Delta x}$$

$$\Delta x\, y_6'\approx y_7-y_6$$

$$\to y_7\approx y_6+\Delta x\,\cancel{y_6'}$$

0

BC

$$\Rightarrow y_7\approx y_6$$

$$
\underbrace{\begin{bmatrix} P_1 & P_1 & P_1 & P_1 & P_1 & P_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P_6 & P_6 & P_6 & P_6 & P_6 & P_6 \end{bmatrix} * \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} h^{-2}}_{A} \begin{bmatrix} y_1 \\ \vdots \\ y_6 \end{bmatrix}
$$

~~[illegible]~~ part

$$
\boxed{p'(x_i)\, h^{-1} \left[y(x_{i+1}) - y(x_i) \right]}
$$

say $N = 5$

$$
\begin{aligned}
&(h^{-1}y_2 - h^{-1}y_1)\,P_1' \\
&(h^{-1}y_3 - h^{-1}y_2)\,P_2' \\
&(h^{-1}y_4 - h^{-1}y_3)\,P_3' \\
&(h^{-1}y_5 - h^{-1}y_4)\,P_4' \\
&(h^{-1}y_6 - h^{-1}y_5)\,P_5' \\
&(h^{-1}\underbrace{\cancel{y_7}}_{0} - h^{-1}y_6)\,P_6'
\end{aligned}
\Rightarrow h^{-1} \begin{bmatrix} P_1' & P_1' & P_1' & P_1' & P_1' & P_1' \\ \vdots & \vdots & \vdots & & & \\ P_6' & P_6' & P_6' & P_6' & P_6' & P_6' \end{bmatrix} * \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_6 \end{bmatrix}
$$

Part

$\boxed{q(x_i)\, y(x_i)}$

say $N=5$

$$\underbrace{\begin{bmatrix} q_1 & 0 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 & 0 \\ 0 & 0 & q_3 & 0 & 0 \\ 0 & 0 & 0 & q_4 & 0 \\ 0 & 0 & 0 & 0 & q_5 \end{bmatrix}}_{C} \begin{bmatrix} y_1 \\ \vdots \\ y_5 \end{bmatrix} \quad q_6 \quad y_6$$

part

$\boxed{\lambda_i\, r(x_i)\, y(x_i)}$

$$\begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 \end{bmatrix} \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 & 0 \\ 0 & 0 & r_3 & 0 & 0 \\ 0 & 0 & 0 & r_4 & 0 \\ 0 & 0 & 0 & 0 & r_5 \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_5 \end{bmatrix}$$

$\lambda_6 \quad r_6 \quad y_6$

Robin

$$\boxed{p(x_i)h^{-2}\left[y(x_{i+1})+y(x_{i-1})-2y(x_i)\right]}+\boxed{p'(x_i)h^{-1}\left[y(x_{i+1})-y(x_i)\right]}$$
$$+\boxed{q(x_i)y(x_i)}=\boxed{\lambda_i r(x_i)y(x_i)}$$

part.

$$\boxed{p(x_i)h^{-2}\left[y(x_{i+1})+y(x_{i-1})-2y(x_i)\right]}$$

Say $N=5$ $y_1=y(x_1)$. $y_0=0$ (BC)

$K y_6+y_6'=0$ (BC)

$$(2h^{-2}y_1-h^{-2}y_2)\times p_1$$
$$(-h^{-2}y_1+2h^{-2}y_2-h^{-2}y_3)\times p_2$$
$$(-h^{-2}y_2+2h^{-2}y_3-h^{-2}y_4)\times p_3$$
$$(-h^{-2}y_3+2h^{-2}y_4-h^{-2}y_5)\times p_4$$
$$(-h^{-2}y_4+2h^{-2}y_5-h^{-2}y_6)\times p_5$$
$$\underline{(-h^{-2}y_5+2h^{-2}y_6-h^{-2}y_7)\times p_6}$$

$$y'(x_i)\approx\frac{y(x_{i+1})-y(x_i)}{\Delta x}$$

$$\Delta x\, y_6'\approx y_7-y_6$$

$$y_7\approx y_6+\Delta x\, y_6'$$

$$y_7\approx y_6+\Delta x\, y_6'$$