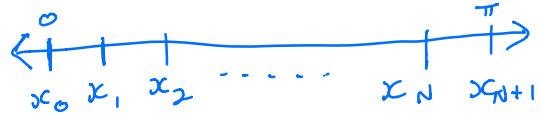


General step

$$-(p(x)y') - q(x)y = \lambda r(x)y$$

$$0 \leq x \leq \pi$$

$$BC: y(0) = y(L) = 0$$



$$y'(x_i) \approx \frac{y(x_{i+1}) - y(x_i)}{\Delta x}$$

$$y''(x_i) \approx \frac{y(x_{i+1}) + y(x_{i-1}) - 2y(x_i)}{\Delta x^2}$$

$$\Downarrow \quad h = \Delta x$$

$$-p(x_i)h^{-2} [y(x_{i+1}) + y(x_{i-1}) - 2y(x_i)] - p'(x_i)h^{-1} [y(x_{i+1}) - y(x_i)] - q(x_i)y(x_i) = \lambda_i r(x_i)y(x_i)$$

• Dirichlet BC

part.

$$-p(x_i)h^{-2} [y(x_{i+1}) + y(x_{i-1}) - 2y(x_i)]$$

$$\text{Say } N=5 \quad y_1 = y(x_1) \quad y_0 = y_6 = 0 \quad (BC)$$

$$\begin{aligned} & (2h^{-2}y_1 - h^{-2}y_2) \times p_1 \\ & (-h^{-2}y_1 + 2h^{-2}y_2 - h^{-2}y_3) \times p_2 \\ & (-h^{-2}y_2 + 2h^{-2}y_3 - h^{-2}y_4) \times p_3 \\ & (-h^{-2}y_3 + 2h^{-2}y_4 - h^{-2}y_5) \times p_4 \\ & (-h^{-2}y_4 + 2h^{-2}y_5) \times p_5 \end{aligned}$$

$$\Rightarrow h^{-2} \begin{bmatrix} p_1 & p_1 & p_1 & p_1 & p_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_5 & p_5 & p_5 & p_5 & p_5 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_5 \end{bmatrix}$$

$\rightarrow A$

part

$$-P'(x_i)h^{-1}[y(x_{i+1}) - y(x_i)]$$

say $N=5$

$$\begin{aligned} & (h^{-1}y_2 - h^{-1}y_1)P'_1 \\ & (h^{-1}y_3 - h^{-1}y_2)P'_2 \\ & -(h^{-1}y_4 - h^{-1}y_3)P'_3 \\ & (h^{-1}y_5 - h^{-1}y_4)P'_4 \\ & (h^{-1}y_6 - h^{-1}y_5)P'_5 \\ & 0 \end{aligned} \Rightarrow -h^{-1} \begin{bmatrix} P'_1 & P'_1 & P'_1 & P'_1 & P'_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P'_5 & P'_5 & P'_5 & P'_5 & P'_5 \end{bmatrix} * \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_5 \end{bmatrix}$$

$\hookrightarrow B$

part

$$-q(x_i)y(x_i)$$

say $N=5$

$$- \begin{bmatrix} q_1 & 0 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 & 0 \\ 0 & 0 & q_3 & 0 & 0 \\ 0 & 0 & 0 & q_4 & 0 \\ 0 & 0 & 0 & 0 & q_5 \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_5 \end{bmatrix}$$

part

$$\lambda_i r(x_i)y(x_i)$$

$$\begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 \end{bmatrix} \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 & 0 \\ 0 & 0 & r_3 & 0 & 0 \\ 0 & 0 & 0 & r_4 & 0 \\ 0 & 0 & 0 & 0 & r_5 \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_5 \end{bmatrix}$$

$$A + B + C = D \rightarrow Dy = r\lambda y.$$

Need to append BC to eigenvectors.

for y_1

for λ_1

for λ_2

V

$$V = [\text{zeros}(1,N); V; \text{zeros}(1,N)];$$

Neumann BC

$$-P(x_i)h^{-2} [y(x_{i+1}) + y(x_{i-1}) - 2y(x_i)] - P'(x_i)h^{-1} [y(x_{i+1}) - y(x_i)]$$

$$- [q(x_i)y(x_i)] = \lambda_i r(x_i)y(x_i)$$

$$\left. \begin{aligned} y'(a) &= 0 \\ y'(b) &= 0 \end{aligned} \right\} \text{BC}$$

part.

$$-P(x_i)h^{-2} [y(x_{i+1}) + y(x_{i-1}) - 2y(x_i)]$$

say $N=5$ (inner points)

$$\begin{aligned} & (-h^{-2}y_1 - \cancel{h^{-2}y_0} + 2h^{-2}y_0)P_0 \\ & (-h^{-2}y_2 - h^{-2}y_0 + 2h^{-2}y_1)P_1 \\ & (-h^{-2}y_3 - h^{-2}y_1 + 2h^{-2}y_2)P_2 \\ & (-h^{-2}y_4 - h^{-2}y_2 + 2h^{-2}y_3)P_3 \\ & (-h^{-2}y_5 - h^{-2}y_3 + 2h^{-2}y_4)P_4 \\ & (-h^{-2}y_6 - h^{-2}y_4 + 2h^{-2}y_5)P_5 \\ & (-\cancel{h^{-2}y_7} - h^{-2}y_5 + 2h^{-2}y_6)P_6 \\ & -h^{-2}y_6 \end{aligned}$$

$$y'(x_i) \approx \frac{y(x_{i+1}) - y(x_i)}{\Delta x} \quad \Delta x y'_6 \approx y_7 - y_6 \quad \overset{0}{\text{BC}} \\ \rightarrow y_7 \approx y_6 + \Delta x y'_6 \quad \Rightarrow y_7 \sim y_6$$

$$\rightarrow h^{-2} \begin{bmatrix} P_0 & \dots & P_6 \\ \vdots & & \vdots \\ P_6 & \dots & P_6 \end{bmatrix} * \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

part

$$-P'(x_i)h^{-1}[y(x_{i+1})-y(x_i)]$$

$$(-h^{-1}y_1 + h^{-1}y_0)P'_0$$

$$(-h^{-1}y_2 + h^{-1}y_1)P'_1$$

$$(-h^{-1}y_3 + h^{-1}y_2)P'_2$$

$$(-h^{-1}y_4 + h^{-1}y_3)P'_3$$

$$(-h^{-1}y_5 + h^{-1}y_4)P'_4$$

$$(-h^{-1}y_6 + h^{-1}y_5)P'_5$$

$$\cancel{(-h^{-1}y_7 + h^{-1}y_6)P'_6} = 0$$

$$\rightarrow h^{-1} \begin{bmatrix} P'_0 & \dots & P'_0 \\ \vdots & & \vdots \\ P'_6 & \dots & P'_6 \end{bmatrix} * \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$-q(x_i)y(x_i)$$

$$- \begin{bmatrix} q_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & q_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & q_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & q_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_6 \end{bmatrix}$$

$$\lambda_i r(x_i) y(x_i)$$

$$\begin{bmatrix} r_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & \ddots & & & & & \\ & & r_6 & & & & \end{bmatrix} \begin{bmatrix} r_0 \\ \vdots \\ r_6 \end{bmatrix}$$

Mixed BC

$$-p(x_i)h^{-2} [y(x_{i+1}) + y(x_{i-1}) - 2y(x_i)] - p'(x_i)h^{-1} [y(x_{i+1}) - y(x_i)]$$

$$- [q(x_i)y(x_i)] = \lambda_i r(x_i)y(x_i)$$

part.

$$-p(x_i)h^{-2} [y(x_{i+1}) + y(x_{i-1}) - 2y(x_i)]$$

Say $N=5$

$$y_1 = y(x_1)$$

$$y_0 = 0 \text{ (BC)}$$

$$y'_6 = 0 \text{ (BC)}$$

$$\begin{aligned} & (2h^2 y_1 - h^2 y_2) \times p_1 \\ & (-h^2 y_1 + 2h^2 y_2 - h^2 y_3) \times p_2 \\ & (-h^2 y_2 + 2h^2 y_3 - h^2 y_4) \times p_3 \\ & (-h^2 y_3 + 2h^2 y_4 - h^2 y_5) \times p_4 \\ & (-h^2 y_4 + 2h^2 y_5 - h^2 y_6) \times p_5 \\ & (-h^2 y_5 + 2h^2 y_6 - h^2 y_7) \times p_6 \end{aligned}$$

$$y'(x_i) \approx \frac{y(x_{i+1}) - y(x_i)}{\Delta x}$$

$$\Delta x y'_6 \approx y_7 - y_6$$

$$y_7 \approx y_6 + \Delta x y'_6$$

$$\Rightarrow y_7 \sim y_6$$

$$\underbrace{\begin{bmatrix} p_1 & p_1 & p_1 & p_1 & p_1 & p_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ p_6 & p_6 & p_6 & p_6 & p_6 & p_6 \end{bmatrix} \star \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}}_A h^{-2} \begin{bmatrix} y_1 \\ \vdots \\ y_6 \end{bmatrix}$$

~~part~~ part

$$-P'(x_i)h^{-1}[y(x_{i+1}) - y(x_i)]$$

say $N=5$

$$\begin{aligned} & (h^{-1}y_2 - h^{-1}y_1)P'_1 \\ & (h^{-1}y_3 - h^{-1}y_2)P'_2 \\ & - (h^{-1}y_4 - h^{-1}y_3)P'_3 \\ & (h^{-1}y_5 - h^{-1}y_4)P'_4 \\ & (h^{-1}y_6 - h^{-1}y_5)P'_5 \\ & (\cancel{h^{-1}y_7 - h^{-1}y_6})P'_6 \\ & 0 \end{aligned} \Rightarrow -h^{-1} \begin{bmatrix} P'_1 & P'_1 & P'_1 & P'_1 & P'_1 & P'_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P'_6 & P'_6 & P'_6 & P'_6 & P'_6 & P'_6 \end{bmatrix} \star \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_6 \end{bmatrix}$$

part

$$q(x_i) y(x_i)$$

say $N=5$

$$\begin{bmatrix} q_1 & 0 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 & 0 \\ 0 & 0 & q_3 & 0 & 0 \\ 0 & 0 & 0 & q_4 & 0 \\ 0 & 0 & 0 & 0 & q_5 \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_5 \end{bmatrix}$$

$\xrightarrow{C} \begin{matrix} q_6 & y_6 \end{matrix}$

part

$$\lambda_i r(x_i) y(x_i)$$

$$\begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 \end{bmatrix} \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 & 0 \\ 0 & 0 & r_3 & 0 & 0 \\ 0 & 0 & 0 & r_4 & 0 \\ 0 & 0 & 0 & 0 & r_5 \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_5 \end{bmatrix}$$

$\begin{matrix} \lambda_6 & r_6 & y_6 \end{matrix}$