PDE: $c\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(K_0 \frac{\partial u}{\partial x}\right)$

BC: $u(0,t) = 0$

$\frac{\partial u}{\partial x}(L,t) = 0$

IC: $u(x,0) = f(x)$

$u(x,t) = \phi(x) h(t)$ → SLEP

$\frac{dh}{dt} = -\lambda h$

$\frac{d}{dx}\left(K_0 \frac{d\phi}{dt}\right) + \lambda c \rho \phi = 0$

$\phi(0) = 0$

$\frac{d\phi}{dx}(L) = 0$

use SLEP.m with mixed BC to obtain λ and ϕ

$h(t) = c e^{-\lambda_n t}$

$u(x,t) = \phi_n(x) e^{-\lambda_n t}$

$u(x,t) = \sum_{n=1}^{\infty} a_n \phi_n(x) e^{-\lambda_n t}$ → for u= u+ an* ϕ* exp(-λn t) end

$u(x,0) = f(x) = \sum_{n=1}^{\infty} a_n \phi_n(x)$

$\int_0^L \phi_n(x) \phi_m(x) c(x) \rho(x)\, dx = 0$ for $n \neq m$

$a_n = \frac{\int_0^L f(x) \phi_n(x) c(x) \rho(x)\, dx}{\int_0^L \phi_n^2(x) c(x) \rho(x)\, dx} = \frac{\text{sum}(f.*\phi_n.*c\rho)*dx}{\text{sum}(\phi.\text{^}2.*c\rho)*dx}$