Referee Report on "On the identifiability of interaction functions" by Li, Lu, Maggioni, Tang and Zhang

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1 General Comments

The paper under review studies the identifiability or learnability of the interaction kernels of some possible large systems of interacting particles. The authors consider the N particle dynamics given by the following SDE's

$$\mathrm{d}X_t^i = \frac{1}{N}\sum_{j\neq i}\phi(|X_t^j - X_t^i|)\frac{X_t^j - X_t^i}{|X_t^j - X_t^i|} + \sigma \mathrm{d}B_t^i, \quad i = 1, 2, \cdots, N. \tag{1}$$

By taking a radial symmetric function Ψ with $\Psi'(r) = \phi(r)$, the interaction term above reads in the gradient form

$$\frac{1}{N}\sum_{j\neq i}\nabla\Psi(|X_t^j - X_t^i|).$$

The goal of this paper is to establish some reasonable criteria of the learnability of the so called interaction function $\phi : \mathbb{R}_+ \to \mathbb{R}$. This is an interesting and challenging topic. The authors express the identifiability problem to the coercivity condition in the previous works by some of the authors here, which is further shown to be equivalent to the condition that some associated integral operators are strictly positive definite. In the end, the authors first establish the identifiablity for the linear case, that is $\phi(r) = \theta r$ under the assumption that the initial law is Gaussian. Secondly, for the nonlinear case, the authors proved the coercivity for the particle system with the number of particles $N = 3$ and also the systems should start from some stationary measure. Even though the results seem still not polished enough, considering the difficulty of the inverse type problems, I still suggest recommend accepting the paper for publication provided that the authors properly address the questions specified in the following sections.

2 Suggested Corrections

1. I would suggest that the authors to add the assumptions on their initial law of $\mathbf{X}_0$ in the abstract. Some discussions on the possible universality of the initial laws would be welcome as well.

2. In the paragraph after Eq. (1.2), could you specify "$\phi(r)$ dominated by r^α" by formulas? Typo in the same paragraph, change "arisen" to "arising".

3. It would be expanding the introduction a bit. Explain in more detailed way of your main results and strategies.

4. In the linear case, what we really need to learn is the parameter θ. Could you discuss a bit some existing works on directly estimating the parameter θ, instead of going through the functional framework as used in the current paper?

5. I have some concern for the assumptions (H_1) and (H_2) after Eq. (2.2) in page 3. I would prefer the authors to discussion what kind of Φ can lead to (H_1) and (H_2)? And also, these conditions hold only for the initial data or for any time $t > 0$?

6. In the beginning of Sec. 2.1, please details the definition $\mathcal{E}_X(\varphi)$? Why is this the right object? Instead of just citing a book/paper, recalling some computations in your previous papers might be better.

7. I believe that the 1st equation in page 4 is not correct. Please check and fix that! (The 2nd last equation before Eq. (2.4)).

8. I would suggest to use time as lower subscript. Please make notations consistent. For instance X^0 and $\mathbf{X}^0$ are both used...