# Foundations of Machine Learning: Assignment 1

Instructor: *Prof. Xiangyang Ji*

## Problem 1

(Exercise 2.3 in *Foundations of Machine Learning*) Concentric circles. Let $X = \mathbb{R}^2$ and consider the set of concepts of the form $c = \{(x, y) : x^2 + y^2 \leq r^2\}$ for some real number $r$. Show that this class can be $(\epsilon, \delta)$-PAC-learned from training data of size $m \geq (1/\epsilon) log(1/\delta)$.

## Problem 2

(Exercise 2.4 in *Foundations of Machine Learning*) Non-Concentric circles. Let $X = \mathbb{R}^2$ and consider the set of concepts of the form $c = \{x \in \mathbb{R}^2 : ||x - x_0|| \leq r\}$ for some point $x_0 \in \mathbb{R}^2$ and real number $r$. Gertrude, an aspiring machine learning researcher, attempts to show that this class of concepts may be $(\epsilon, \delta)$-PAC-learned with sample complexity $m \geq (3/\epsilon) log(3/\delta)$, but she is having trouble with her proof. Her idea is that the learning algorithm would select the smallest circle consistent with the training data. She has drawn three regions $r_1, r_2, r_3$ around the edge of concept $c$, with each region having probability $\epsilon/3$ (see Figure 1). She wants to argue that if the generalization error is greater than or equal to $\epsilon$, then one of these regions must have been missed by the training data, and hence this event will occur with probability at most $\delta$. Can you tell Gertrude if her approach works?

## Problem 3

(Exercise 2.9 in *Foundations of Machine Learning*) Consistent hypotheses. In this chapter, we showed that for a finite hypothesis set $H$, a consistent learning algorithm $\mathcal{A}$ is a PAC-learning algorithm. Here, we consider a converse question. Let $Z$ be a finite set of m labeled points. Suppose that you are given a PAC-learning algorithm $\mathcal{A}$. Show that you can use $\mathcal{A}$ and a finite training sample $S$ to find a hypothesis $h \in H$

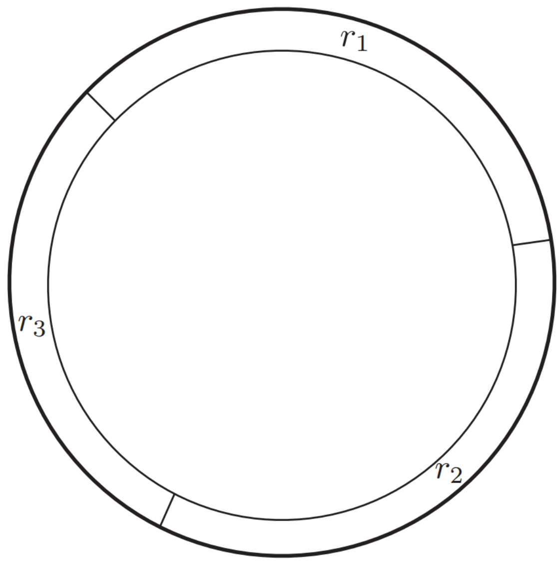


Figure 1: Gertrude's regions $r_1, r_2, r_3$.

that is consistent with $Z$, with high probability. (*Hint*: you can select an appropriate distribution $D$ over $Z$ and give a condition on $R(h)$ for $h$ to be consistent.)
**Note**: $S$ represents the training dataset, and $Z$ represents the set of *all* possible examples in this problem. You should focus on proving $S$ is finite and $h$ is consistent on $Z$.

## Problem 4

Let $X$ have mean $\mu$ and variance $\sigma^2$. Prove that

$$P(X - \mu \geq t) \leq \frac{\sigma^2}{t^2 + \sigma^2}$$

for any $t \geq 0$.
**Hint**: Introduce $\lambda$ to both sides of the inequality $X - \mu \geq t$ and then apply Markov's inequality (Review the Appendix C in *Foundations of Machine Learning*).