Basic formula of elastic equation in displacement form:

$$\lambda\nabla(\nabla\cdot\boldsymbol{u})+G\nabla\cdot[(\nabla\boldsymbol{u})^T+\nabla\boldsymbol{u}]+\boldsymbol{F}=0 \tag{1}$$

for test function $\boldsymbol{u}$,which is zero on part$\Gamma_D\subset\partial\Omega$ weak form:

$$\lambda(\nabla(\nabla\cdot\boldsymbol{u}),\boldsymbol{v})_\Omega+G(\nabla\cdot[(\nabla\boldsymbol{u})^T+\nabla\boldsymbol{u}],\boldsymbol{v})_\Omega+(\boldsymbol{F},\boldsymbol{v})_\Omega=0. \tag{2}$$

The inner product is defined by (d=2 or 3):

$$(\boldsymbol{u},\boldsymbol{v})=\sum_{i=1}^{d}\int_\Omega \boldsymbol{u}_i(\boldsymbol{x})\boldsymbol{v}_i(\boldsymbol{x})d\boldsymbol{x} \tag{3}$$

we denote $\boldsymbol{\epsilon}(\boldsymbol{u})=\frac{1}{2}[(\nabla\boldsymbol{u})^T+\nabla\boldsymbol{u}]$, which is strain tensor. Therefore, the weak form is:

$$\lambda(\nabla(\nabla\cdot\boldsymbol{u}),\boldsymbol{v})_\Omega+2G(\nabla\cdot\boldsymbol{\epsilon}(\boldsymbol{u}),\boldsymbol{v})_\Omega+(\boldsymbol{F},\boldsymbol{v})_\Omega=0. \tag{4}$$

We integrate by parts and use divergence theorem:

$$-\lambda(\nabla\cdot\boldsymbol{u},\nabla\cdot\boldsymbol{v})_\Omega+\lambda(\boldsymbol{n}\cdot\boldsymbol{v},\nabla\cdot\boldsymbol{u})_{\partial\Omega}-2G(\boldsymbol{\epsilon}(\boldsymbol{u}),\nabla\boldsymbol{v})_\Omega+2G(\boldsymbol{\epsilon}(\boldsymbol{u}),\boldsymbol{n}\otimes\boldsymbol{v})_{\partial\Omega}+(\boldsymbol{F},\boldsymbol{v})_\Omega=0. \tag{5}$$

where $\boldsymbol{n}$ is the normal vector at boundary $\partial\Omega$ For boundary condition, notice the symmetric property of $\boldsymbol{\epsilon}(\boldsymbol{u})$ we have:

$$\lambda(\boldsymbol{n}\cdot\boldsymbol{v},\nabla\cdot\boldsymbol{u})_{\partial\Omega}+2G(\boldsymbol{\epsilon}(\boldsymbol{u}),\boldsymbol{n}\otimes\boldsymbol{v})_{\partial\Omega}=(\boldsymbol{v},((\lambda\nabla\cdot\boldsymbol{u})\boldsymbol{I}_d+2G\boldsymbol{\epsilon}(\boldsymbol{u}))\cdot\boldsymbol{n})_{\Gamma_N} \tag{6}$$

It follows:

$$(\boldsymbol{v},((\lambda\nabla\cdot\boldsymbol{u})\boldsymbol{I}_d+2G\boldsymbol{\epsilon}(\boldsymbol{u}))\cdot\boldsymbol{n})_{\Gamma_N}=(\boldsymbol{v},\boldsymbol{\sigma}\cdot\boldsymbol{n})_{\Gamma_N} \tag{7}$$

Therefore, if we prescribe stress boundary condition:

$$\boldsymbol{\sigma}\cdot\boldsymbol{n}=\boldsymbol{T}$$

, then the neumann condition contributes to the right hand side of the weak form integration$(\boldsymbol{v},\boldsymbol{T})_{\Gamma_N}$ In conclusion, the weak formulation follows:

$$\lambda(\nabla\cdot\boldsymbol{u},\nabla\cdot\boldsymbol{v})_\Omega+2G(\boldsymbol{\epsilon}(\boldsymbol{u}),\nabla\boldsymbol{v})_\Omega=(\boldsymbol{F},\boldsymbol{v})_\Omega+(\boldsymbol{v},\boldsymbol{T})_{\Gamma_N}. \tag{8}$$