Machine Learning for Image Analysis, SoSe 2024
Exercise Sheet #3: Backpropagation
Due date: May 7, 2024, before 11 am

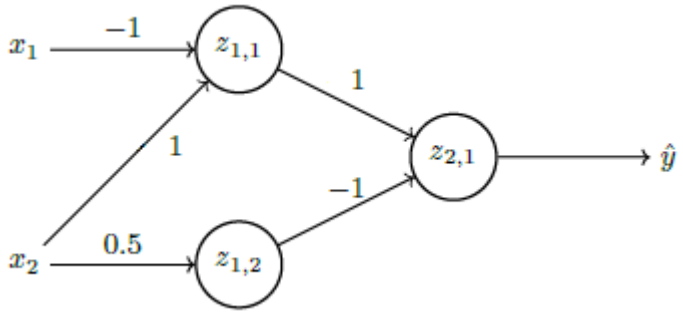
Problem 1 (Properties of the Logistic Sigmoid) (9 pt).
A more common notation for the logistic sigmoid function uses σ and is defined as

$$\sigma(a) = \frac{1}{1 + e^{-a}} \tag{1.1}$$

(a) (4 pt) Show: $\sigma(-a) = 1 - \sigma(a)$

(b) (5 pt) Show: $\frac{d}{da}\sigma(a) = (1 - \sigma(a))\sigma(a)$

Problem 2 (Backpropagation in Neural Networks) (16 pt).
Consider the Neural Network depicted below. Weights are shown at arrows and bias terms are omitted in this exercise.



(a) (2pt) Perform the forward pass for the single input datapoint $x = (1, 2)$ using the ReLU activation function at the hidden layer and the sigmoid activation function for the final output.

(b) (3pt) Given the single training instance $x = (1, 2)$, $y = 1$ we want to perform backpropagation to update all the weights. We are using the log-likelihood objective function $J = -y \ln(\hat{y}) - (1 - y) \ln(1 - \hat{y})$, the learning rate $\lambda = 1$ and do without any regularization. Draw the corresponding compute graph with weights as inputs.

(c) (6pt) Update all the weights once via back-propagation.
Hint: Problem 1 might be useful here.

(d) (3pt) Perform another forward pass, using the updated weights, and compute $\hat{y}$ and the resulting loss.

(e) (2pt) What result in (d) is expected? You can answer this question even if your calculations might turn out wrong.