Stat 435 Intro to Statistical Machine Learning

Week 10: Time to recap! (and a little more SVM of course)

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SVM: a summary

Binary response: $y_i \in \{-1, 1\}$

- Maximal margin classifier

$$\underset{\beta_0,\beta_1,\ldots,\beta_p}{\text{maximize}}\, M \tag{9.9}$$

$$\text{subject to } \sum_{j=1}^{p} \beta_j^2 = 1, \tag{9.10}$$

$$y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_p x_{ip}) \geq M \;\; \forall\, i = 1, \ldots, n. \tag{9.11}$$

- Support vector classifier
- Support vector classifier (alternative representation)
- Support vector machine

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$$y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_p x_{ip}) \geq M(1 - \epsilon_i), \tag{9.14}$$

$$\epsilon_i \geq 0, \quad \sum_{i=1}^{n} \epsilon_i \leq C, \tag{9.15}$$

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$$f(x) = \beta_0 + \sum_{i \in S} \alpha_i \langle x, x_i \rangle, \tag{9.19}$$

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$$f(x) = \beta_0 + \sum_{i \in \mathcal{S}} \alpha_i K(x, x_i). \tag{9.23}$$

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$$\underset{\beta_0,\beta_1,\ldots,\beta_p}{\text{minimize}} \left\{ \sum_{i=1}^{n} \max\left[0, 1 - y_i f(x_i)\right] + \lambda \sum_{j=1}^{p} \beta_j^2 \right\}, \qquad (9.25)$$

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- The second part is a ridge penalty!

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  - kernel (imagine you want to cut the skins of an orange by a linear plane)

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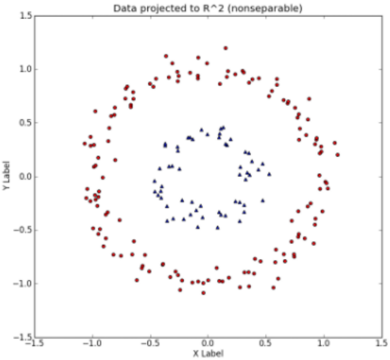
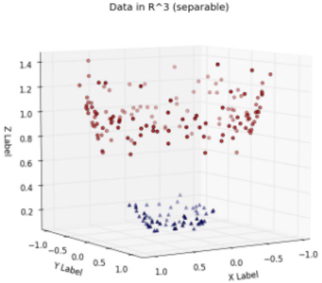
which transforms our data in $\mathbb{R}^2$ to $\mathbb{R}^5$

The kernel trick

$[x_1, x_2] \rightarrow [x_1, x_2, x_1^2 + x_2^2]$





But You don't need to look for such transformations directly (difficult in high dimensions!), thus "trick" as in "the kernel trick"

The kernel trick

- It can be used in regression
- It can be used in PCA
- It can be used in KNN
- In many algorithms where you see $x^T x$, chances are the kernel trick can be applied...

One of the last general methods in this course!

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Do we care about model interpretability or just prediction accuracy

- See figure 2.7 of text book (page 25)

Common considerations

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Related reoccurring topics (in high dimensional data)

- Simpler model is more favorable: *Penalization*
- More flexible model is more favorable (if can be computed efficiently): *Kernel methods*

A quick ride in statistical learning land

But incomplete and not meant as the full material for final!

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- Bayes classifier / Bayes error rate (*HW1, Week1*)

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- Interpretation, tests, diagnostics (*week2*)

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- Interpretation, tests, diagnostics (*week2*)
- Extensions (*week2*)
  - Interaction (interpretation!)
  - Qualitative predictors

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- (GAM)
- Classification tree
- SVM (related to logistic regression! Sec 9.5 textbook)

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- Generalized Additive Models (*HW7*)

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- Bagging, boosting, and random forest(*week9*)

SVM and unsupervised learning

Then there's the recent stuff, and they should still be fresh in your memory

There's a lot lot more to statistical learning

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Bran Stark: *Learn what?*

Three-Eyed Raven: *Hmm let's see, bias-variance tradeoff, regression, classification, cross-validation, regularization, model selection, dimension reduction splines, GAMs, trees, bagging, boosting, random forests, support vector machines, PCA, LDA, QDA, ...*

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Course evaluation closing on Friday!

https://uw.iasystem.org/survey/174515

Thank you!