HW0-b

B.1

As $X_1, ..., X_n$ are i.i.d
We can write cdf

$$
\begin{gathered}
F(Y=y) = F(max(X_1, ..., X_n \leq y) \\
= F(X_1 \leq y, X_2 \leq y, ..., X_n \leq y) \\
= F(X_1 \leq y)F(X_2 \leq y)...F(X_n \leq y) \\
= (\frac{y-0}{1-0})^n = y^n
\end{gathered}
$$

$$
P(Y) = \frac{d}{dy}F(Y) = n * y^{n-1}
$$

Therefore,

$$
\begin{gathered}
E(y) = \int_0^1 y * P(y) dy \\
= \int_0^1 y * n * y^{n-1} dy \\
= n \int_0^1 y^n dy \\
= n * \frac{1}{n+1} \\
= \frac{n}{n+1}
\end{gathered}
$$

B.2

$$
Tr(A) = \sum_{i=1}^{n} A_{ii}
$$

$$
\begin{gathered}
Tr(AB) = \sum_{j=1}^{n} A_{1j}B_{j1} + \sum_{j=1}^{n} A_{2j}B_{j2} + ... + \sum_{j=1}^{n} A_{nj}B_{jn} \\
= \sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}B_{ij} \\
= \sum_{j=1}^{n}\sum_{i=1}^{n} B_{ij}A_{ij} = Tr(BA)
\end{gathered}
$$

B.3

a)

Dimension of $v_i v_i^T$ is dx1x1xd= dxd.
So the maximum rank is d
As v_i is non-zero vector, the minmum rank is 1. (Consider all v_i are identical vector with 1 on one entry and 0 elsewhere)

b)

Dimension of V is dxn.
So the maximum rank is min(d,n)
The minmum rank is 1. (Consider all v_i are identical vector with 1 on one entry and 0 elsewhere)

c)

Diemsnion of $(Av_i)(Av_i)^T$ is (Dxdxdx1)(Dxdxdx1)^{T}=Dx1x1xD=DxD
The min rank is 0 as A can be zero matrix which will cancel out everything.
The max rank is D.

d)

Dimension of AV is Dxdxdxn=Dxn.
The min rank is 0 as A can be zero matrix which will cancel out everything.
The max rank is min(D,n).