Swing-Up Control

Topics Covered

- Energy control.
- Nonlinear control.
- Control switching logic.

Prerequisites

- Filtering laboratory experiment.
- Rotary Pendulum Modeling laboratory experiment.
- Rotary pendulum module is attached to the QUBE-Servo 2.

1 Background

1.1 Energy Control

In theory, if the arm angle is kept constant and the pendulum is given an initial perturbation, the pendulum will keep on swinging with constant amplitude. The idea of energy control is based on the preservation of energy in ideal systems: The sum of kinetic and potential energy is constant. However, friction will be damping the oscillation in practice and the overall system energy will not be constant. It is possible to capture the loss of energy with respect to the pivot acceleration, which in turn can be used to find a controller to swing up the pendulum.

The dynamics of the pendulum can be redefined in terms of the pivot acceleration u as

$$J_p\ddot{\alpha} + \frac{1}{2}M_p g L_p \sin\alpha = \frac{1}{2}M_p L_p u \cos\alpha. \tag{1.1}$$

Here, u is the linear acceleration of the pendulum.

The potential energy of the pendulum is

$$E_p = \frac{1}{2}M_p g L_p \left(1 - \cos\alpha\right),$$

and the kinetic energy is

$$E_k = \frac{1}{2}J_p\dot{\alpha}^2.$$

The pendulum angle α and the lengths of the pendulum are illustrated in the free body diagram in Figure 1.1.

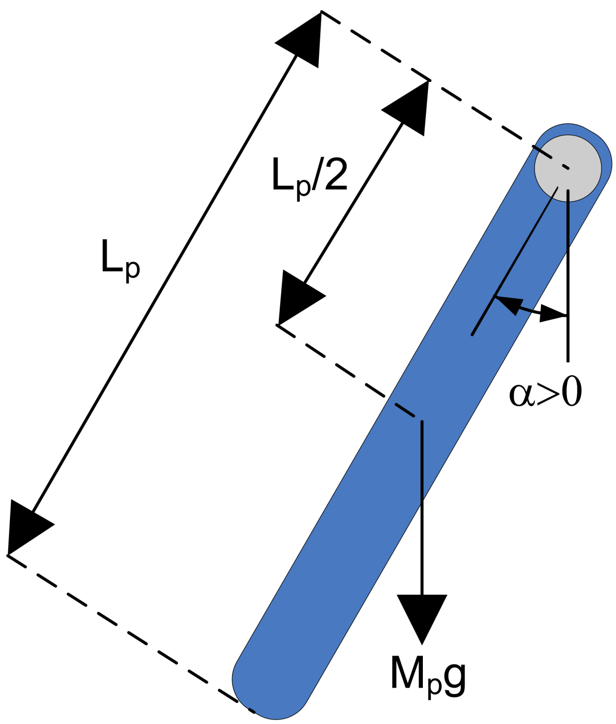

Figure 1.1: Free-body diagram of pendulum

The potential energy is zero when the pendulum is at rest at $\alpha = 0$ and equals $M_p g L_p$ when the pendulum is upright at $\alpha = \pm\pi$. The sum of the potential and kinetic energy of the pendulum is

$$E = \frac{1}{2}J_p\dot{\alpha}^2 + \frac{1}{2}M_p g L_p \left(1 - \cos\alpha\right). \tag{1.2}$$

Differentiating Equation 1.2 yields

$$\dot{E} = \dot{\alpha}\left(J_p\ddot{\alpha} + \frac{1}{2}M_p g L_p \sin\alpha\right). \tag{1.3}$$

Recalling Equation 1.1 and rearranging terms as

$$J_p\ddot{\alpha} = -\frac{1}{2}M_p g L_p \sin\alpha + \frac{1}{2}M_p u L_p \cos\alpha$$

eventually yields

$$\dot{E} = \frac{1}{2}M_p u L_p \dot{\alpha}\cos\alpha.$$

Since the acceleration of the pivot is proportional to current driving the arm motor and thus also proportional to the drive voltage, it is possible to control the energy of the pendulum with the proportional control law

$$u = (E_r - E)\dot{\alpha}\cos\alpha. \tag{1.4}$$

By setting the reference energy to the pendulum potential energy ($E_r = E_p$), the control law will swing the link to its upright position. Notice that the control law is nonlinear because the proportional gain depends on the cosine of the pendulum angle α. Further, the control changes sign when $\dot{\alpha}$ changes sign and when the angle is ± 90 degrees.

For the system energy to change quickly, the magnitude of the control signal must be large. As a result the following swing-up controller is implemented in the controller as

$$u = \text{sat}_{u_{max}}\left(\mu(E_r - E)\text{sign}(\dot{\alpha}\cos\alpha)\right) \tag{1.5}$$

where μ is a tunable control gain and the $\text{sat}_{u_{max}}$ function saturates the control signal at the maximum acceleration of the pendulum pivot, u_{max}. The expression sign$(\dot{\alpha}\cos\alpha)$ is used to enable faster control switching.

1.2 Hybrid Swing-Up Control

The energy swing-up control in Equation 1.4 (or Equation 1.5) can be combined with the balancing control law from the Balance Control Lab to obtain a control law that swings up the pendulum and then balances it.

Similarly as described in the Rotary Pendulum Modeling laboratory experiment, the balance control is to be enabled when the pendulum is within ± 20 degrees. When it is not enabled, the swing-up control is engaged. Thus the switching can be described mathematically by:

$$u = \begin{cases} u_{bal} & \text{if } |\alpha| - \pi \leq 20^\circ \\ u_{swing\_up} & \text{otherwise} \end{cases} \tag{1.6}$$

2 In-Lab Exercises

The controller in QUARC® is shown in Figure 2.1 that swings-up and balances the pendulum on the QUBE-Servo 2 rotary pendulum system. The *Swing-Up Control* subsystem implements the energy control described in Section 1.

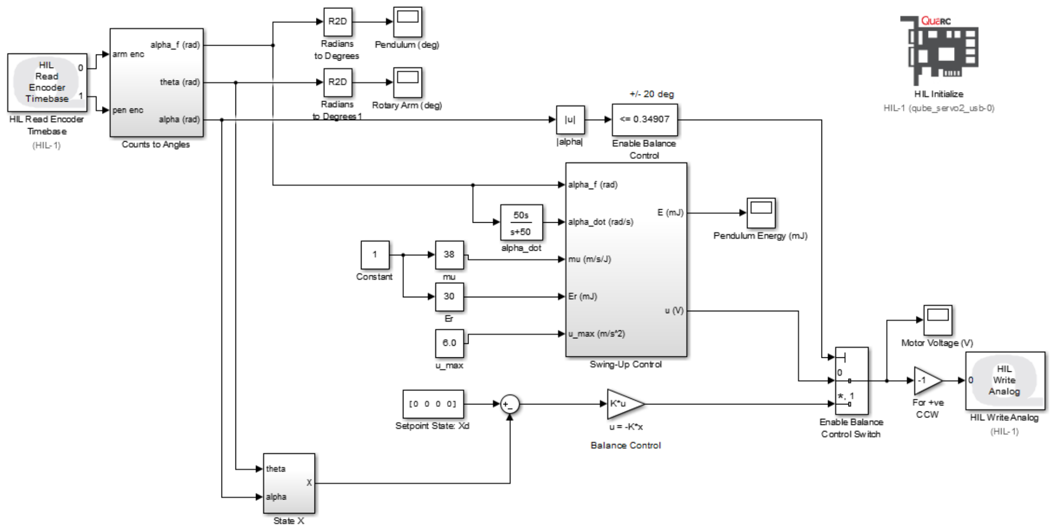

Figure 2.1: Simulink® model used with QUARC® to run swing-up controller

2.1 Energy Control

1. Open the `q_qube_swingup.mdl` Simulink® model.
2. Run the `setup_qube_rotpen.m` Matlab® script. This loads the pendulum parameters that is used by the Simulink® model.
3. To turn the swing-up control off, set the Slider Gain block called *mu* to 0.
4. Build and run the QUARC® controller.
5. Manually rotate the pendulum at different levels and examine the pendulum angle and energy in the *Pendulum (* deg*)* and *Pendulum Energy (* mJ*)* scopes.
6. B-5, B-9 What do you notice about the energy when the pendulum is moved at different positions? Record the energy when the pendulum is being balanced (i.e. fully inverted in the upright vertical position). Does this reading make sense in terms of the equations developed in Section 1?

   Answer 2.1

Outcome	**Solution**
B-5	If they followed the procedure correctly, they should be able to perform the following analysis as well as measure the energy.
B-9	The pendulum energy increases proportionally with the pendulum angle. When being balanced, the energy read is 30 mJ. As discussed in Section 1, the reading corresponds to the potential energy of the pendulum: $E_r = M_p g L_p$.

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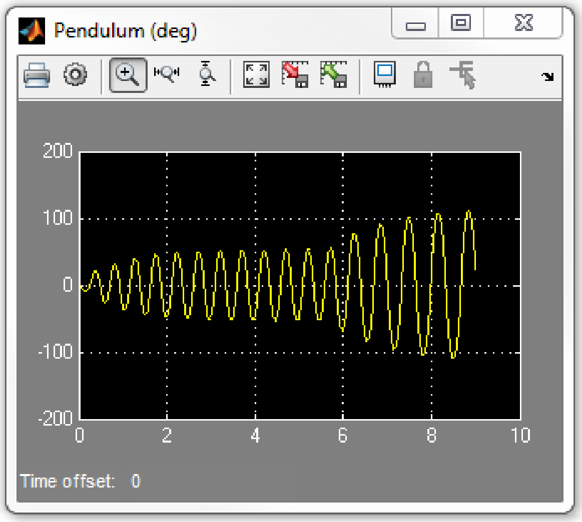
7. Click on the Stop button to bring the pendulum down to the initial, downward position.

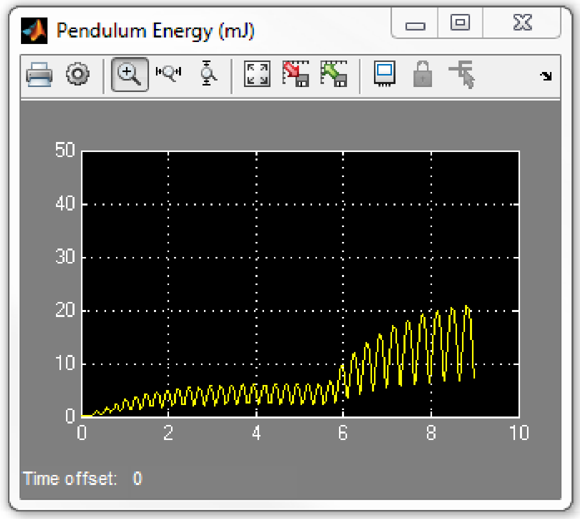
8. Set the swing-up control parameters (i.e. the Constant and Gain blocks connected to the inputs of the Swing-Up Control subsystem) to the following:
   - mu = 50 m/s/J
   - Er = 10.0 mJ
   - u_max = 6 m/s^2
9. If the pendulum is not moving, gently perturb the pendulum with your hand to get it going.
10. B-7, K-2 Vary the reference energy, *Er*, between 10.0 mJ and 20.0 mJ. As it is changed, examine the pendulum angle and energy response in *Pendulum (deg)* and the *Pendulum Energy (mJ)* scopes and the control signal in the *Motor Voltage (V)* scope. Attach the responses showing how changing the reference energy affects the system.

Answer 2.2

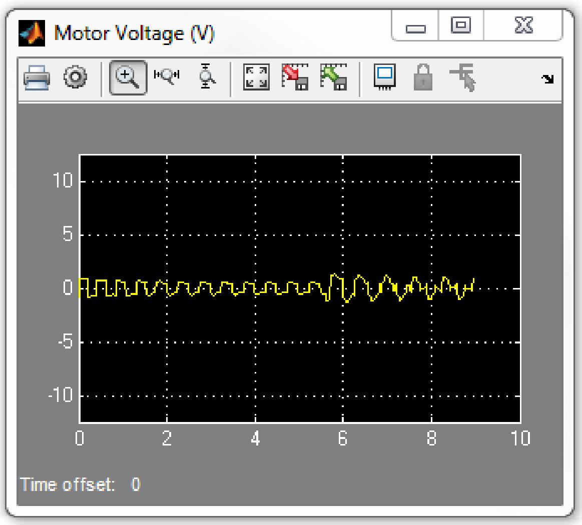
Outcome	Solution
B-7	The larger the reference energy, the large the amplitude of the control signal.
K-2	The responses in Figure Ans.2.1 show the response when increasing the reference energy from 10 to 20 mJ.



(a) Pendulum Angle



(b) Pendulum Energy



(c) Motor Voltage

Figure Ans.2.1: Pendulum response when changing reference energy

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11. B-7 Fix *Er* to 20.0 mJ and vary the swing-up control gain *mu* between 20 and 60 m/s/J. Describe how this changes the performance of the energy control.

Answer 2.3

Outcome	**Solution**
B-7	As the *mu* gain increases the amplitude of the pendulum swings become larger. Recall from swing-up controller given in Equation 1.5 that *mu* is the proportional gain.

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12. Stop the QUARC® controller.

2.2 Hybrid Swing-Up Control

1. Open the `q_qube_swing_up.mdl` Simulink® model.
2. Run the `setup_qube_rotpen.m` Matlab® script. This loads the pendulum parameters that is used by the Simulink® model.
3. Set the swing-up control parameters to the following:
   - mu = 20 m/s/J
   - u_max = 6 m/s^2
4. B-7 Based on your observations in the previous lab, 2.1, what should the reference energy be set to?

Answer 2.4

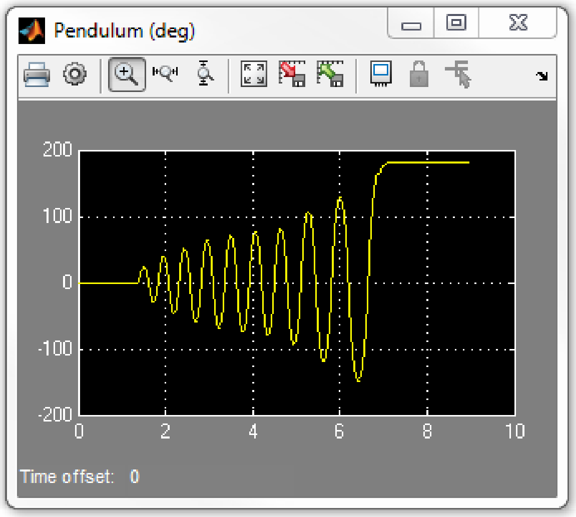
Outcome	Solution
B-7	The reference energy should be set to 30 mJ, i.e. $E_r = 30$. This is the energy that measured in Step 6 in 2.1 when the pendulum was in the vertically upwards, balanced position.

□□□

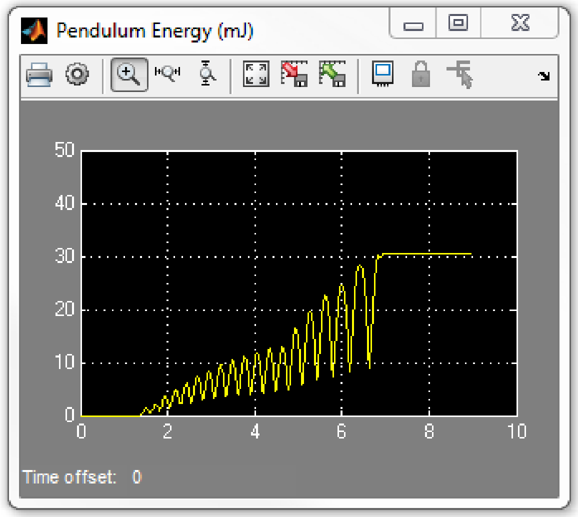
5. Make sure the pendulum is hanging down motionless and the encoder cable is not interfering with the pendulum.
6. Build and run the QUARC® controller.
7. The pendulum should begin going back and forth. If not, manually perturb the pendulum with your hand. **Click on the Stop button in the Simulink® tool bar if the pendulum goes unstable.**
8. B-5, K-2 Gradually increase the swing-up gain, μ, denoted as the *mu* Slider Gain block, until the pendulum swings up to the vertical position. Capture a response of the swing-up and record the swing-up gain that was required. Show the pendulum angle, pendulum energy, and motor voltage.

Answer 2.5

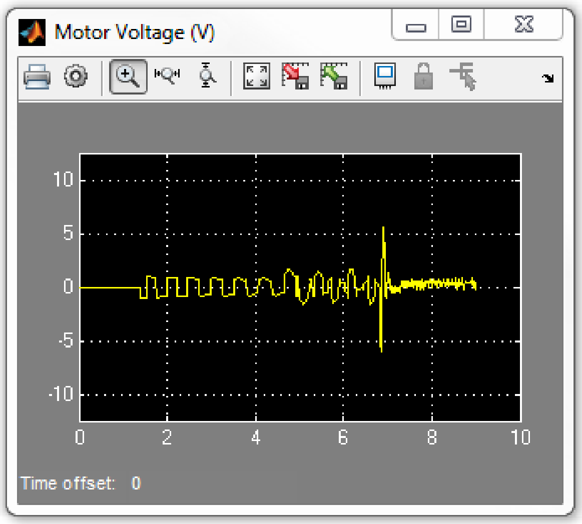
Outcome	Solution
B-5	Setting the reference energy E_r to 30 mJ should be adequate to swing-up the pendulum to its vertical position.
K-2	The responses shown in Figure Ans.2.2 are using hybrid swing-up control detailed in Section 1 using $mu = 38$ m/s/J, $Er = 30$ mJ, and $u\_max = 6$ m/s^2. The pendulum catch angle is set to $\pm 20°$.



(a) Pendulum Angle



(b) Pendulum Energy



(c) Motor Voltage

Figure Ans.2.2: Sample hybrid swing-up control response

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9. Stop the QUARC® controller.
10. Power *OFF* the QUBE-Servo 2.

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