0 积分公式

1. $$\int_a^b f(x)\,g'(x)\,dx = \big[f(x)g(x)\big]_a^b - \int_a^b f'(x)\,g(x)\,dx$$

2. $$J_F(x,y) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix}.$$

3. $d\tan x = \sec^2 x$

   $d\sec x = \sec x \tan x$

   $d\cot x = -\csc^2 x$

   $d\csc x = -\csc x \cot x$

   $d\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

   $d\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$

   $d\tan^{-1} x = \frac{1}{1+x^2}$

4. Taylor series

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n, \qquad |x| < 1.$$

$$\frac{1}{1+x} = 1 - x + x^2 - \cdots + (-x)^n + \cdots = \sum_{n=0}^{\infty} (-1)^n x^n, \qquad |x| < 1.$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \qquad x \in \mathbb{R}.$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \qquad x \in \mathbb{R}.$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, \qquad x \in \mathbb{R}.$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}, \qquad -1 < x \le 1.$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, \qquad |x| \le 1.$$

5.

1 Basic Ideas in Probability

1. **容斥原理**:
   $P(A_1 \cap A_2 \cap \ldots \cap A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{1 \le i < j \le n} P(A_i A_j) + \ldots (-1)^{n-1} P(A_1 A_2 \ldots A_n)$

2. $C_n^k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$

2 Random Variables and Distribution

1. **琴生**:
凸: $E(g(X)) \geq g(E(X))$
凹: $E(g(X)) \leq g(E(X))$

- $E(|X|) \geq |E(X)|\ (g(x) = |x|)$
- $E(X^2) \geq (E(X))^2\ (g(x) = x^2)$
- $E(|X|^p) \geq |E(X)|^p$ 对于 $p \geq 1\ (g(x) = |x|^p, p \geq 1)$
- $E(e^{cX}) \geq e^{cE(X)}\ (g(x) = e^{cX})$

2. $Y = g(X)$, $h(y) = g^{-1}(y)$,
$\Rightarrow f_Y(y) = \begin{cases} |h'(y)| \cdot f_X(h(y)), 在h(y)有定义处 \\ 0，否则 \end{cases}$

3.

分布	PDF/PMF	期望	方差
均匀 (a,b)	$f(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a < x < b \\ 0, & \text{otherwise} \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
正态 (μ, σ^2)	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ^2
指数 (λ)	$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
几何 (p)(无记忆离散)	$p(x) = p(1-p)^{x-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
泊松(λ)	$p(x) = \frac{\lambda^x}{x!} e^{-\lambda}$	λ	λ
二项(n,p)	$p(x) = C_n^x p^x (1-p)^{n-x}$	np	np(1-p)
伯努利(p)	$p(x) = p^x (1-p)^{1-x}$	p	p(1-p)

4. 标准化：$Z = \frac{X-\mu}{\sigma}$
5. $E(X) = \int_{-\infty}^{\infty} x f(x) dx$
6. $E(Y) = E(g(X)) = \int_{-\infty}^{\infty} g(x) f(x) dx$
7. $\text{Var(X)} = E(X - E(X))^2 = E(X^2) - [E(X)]^2 = \int_{-\infty}^{\infty} [x - E(X)]^2 f(x) dx$

3 Joint Distributions

1. $\text{Cov}(X,Y) = E(XY) - E(X)E(Y) = E[(X - E(X))(Y - E(Y))]$
2. $\text{Cov}(aX, bY) = ab\text{Cov}(X,Y)$
3. $\text{Var}(X \pm Y) = \text{Var}(X) + \text{Var}(Y) \pm 2\text{Cov}(X,Y)$
4. $\text{Cov}(\sum_{i=1}^n a_i X_i, \sum_{j=1}^m b_j Y_j) = \sum_{i=1}^n \sum_{i=1}^m a_i b_j \text{Cov}(X_i, Y_j)$
5. $\rho_{XY} = Cor(X,Y) = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$
6. **MSE**:

$b_0 = \frac{\mathrm{Cov}(X,Y)}{\mathrm{Var}(X)}$
$a_0 = E(Y) - \frac{\mathrm{Cov}(X,Y)}{\mathrm{Var}(X)} E(X)$
$\min\limits_{a,b} MSE = \mathrm{Var}(Y)(1 - \rho_{XY}^2)$

7. **Law of total expectation**: $E(X) = E((X|Y))$
   $E(X) = \int_{-\infty}^{\infty} E(X|Y = y) \cdot f_Y(y) dy$