第十一章　　二端口网络

11-1 求图题 11-1 所示二端口网络的 A 参数矩阵。

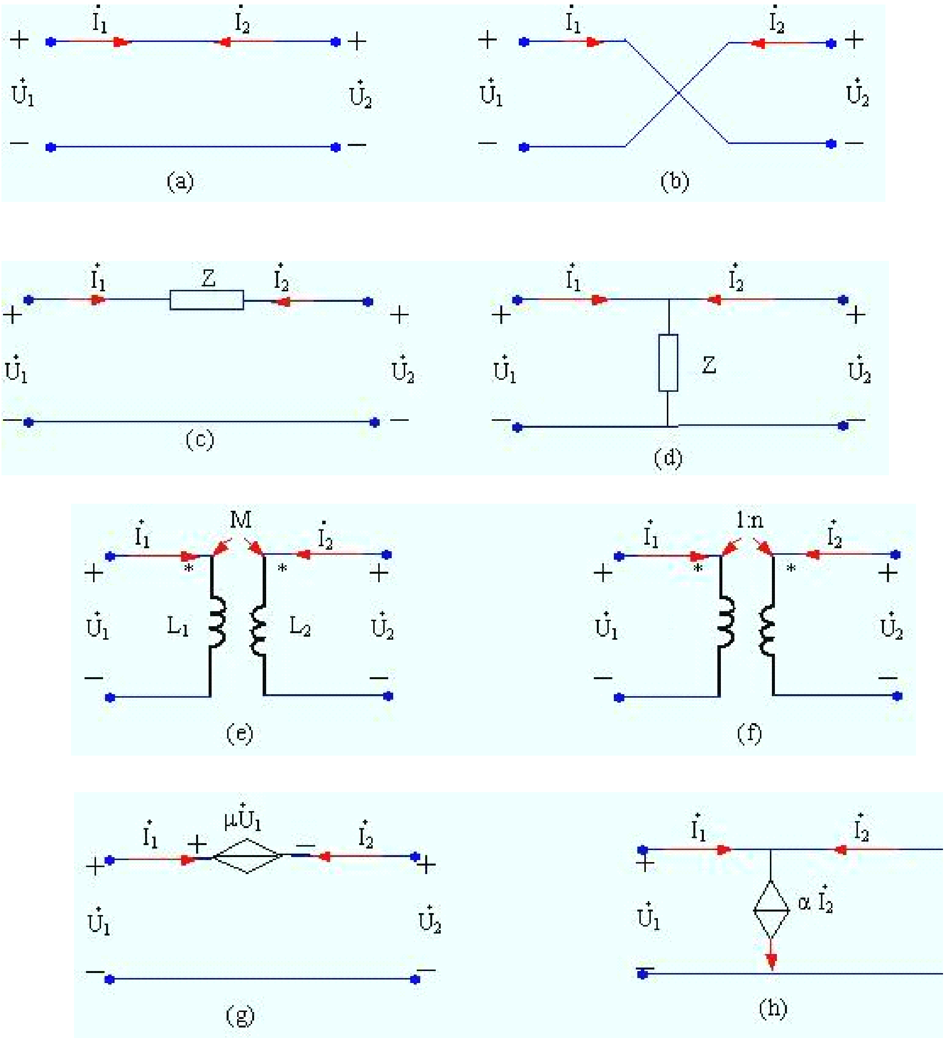

图　题 11-1

答案

解：（a）$\because \begin{cases}\dot{U}_1 = \dot{U}_2 \\ \dot{I}_1 = -\dot{I}_2\end{cases}$ $\therefore A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

（b）$\because \begin{cases}\dot{U}_1 = -\dot{U}_2 \\ \dot{I}_1 = \dot{I}_2\end{cases}$ $\therefore A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

（c）

$\because \begin{cases}\dot{U}_1 = \dot{U}_2 + Z(-\dot{I}_2) \\ \dot{I}_1 = -\dot{I}_2\end{cases}$ $\therefore A = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

（d）

$\because \begin{cases}\dot{U}_1 = \dot{U}_2 \\ \dot{I}_1 = \dot{U}/Z - \dot{I}_2\end{cases}$ $\therefore A = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$

（e）$\because \left.\begin{aligned}\dot{U}_1 &= j\omega L_1\dot{I}_1 + j\omega M\dot{I}_2 \\ \dot{U}_2 &= j\omega M\dot{I}_1 + j\omega L_2\dot{I}_2\end{aligned}\right\}$

有：$\dot{U}_1 = \frac{L_1}{M}\dot{U}_2 - j\omega(\frac{L_1L_2}{M} - M)\dot{I}_2$

$$\dot{I}_1 = \frac{1}{j\omega M}\dot{U}_2 - \frac{L_2}{M}\dot{I}_2$$

$$\therefore A = \begin{bmatrix} \frac{L_1}{M} & j\omega(\frac{L_1L_2}{M} - M) \\ \frac{1}{j\omega M} & \frac{L_2}{M} \end{bmatrix}$$

（f）$\because \begin{cases}\dot{U}_1 = \frac{1}{n}\dot{U}_2 \\ \dot{I}_1 = -n\dot{I}_2\end{cases}$ $\therefore A = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$

（g）$\because \begin{cases}\dot{U}_1 = \mu\dot{U}_1 + \dot{U}_2 \\ \dot{I}_1 = -\dot{I}_2\end{cases}$ $\therefore A = \begin{bmatrix} \frac{1}{1-\mu} & 0 \\ 0 & 1 \end{bmatrix}$

（h）$\because \begin{cases}\dot{U}_1 = \dot{U}_2 \\ \dot{I}_1 = \alpha\dot{I}_2 - \dot{I}_2\end{cases}$ $\therefore A = \begin{bmatrix} 1 & 0 \\ 0 & 1-\alpha \end{bmatrix}$

11-2 求图题 11-2 所示二端口网络的 Z、Y、A 参数矩阵。

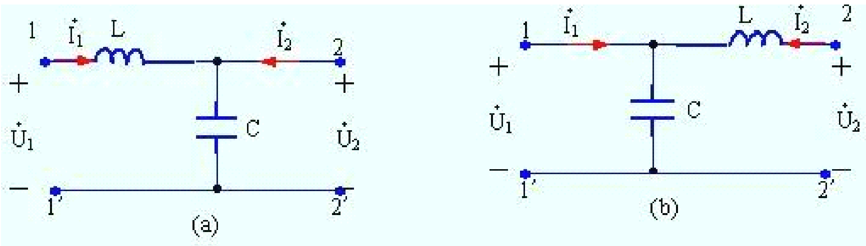

图 题 11—2

答案

解：（a）$\because \dot{U}_1 = j\omega L\dot{I}_1 + \dfrac{1}{j\omega C}(\dot{I}_1 + \dot{I}_2)$

$$\dot{U}_2 = \frac{1}{j\omega C}(\dot{I}_1 + \dot{I}_2)$$

$$\therefore Z = \begin{bmatrix} j\omega L + \dfrac{1}{j\omega C} & \dfrac{1}{j\omega C} \\ \dfrac{1}{j\omega C} & \dfrac{1}{j\omega C} \end{bmatrix}$$

$$\because Y_{11} = \frac{\dot{I}_1}{\dot{U}_1}\bigg|_{\dot{U}_2=0} = \frac{1}{j\omega L} \qquad Y_{12} = \frac{\dot{I}_1}{\dot{U}_2}\bigg|_{\dot{U}_1=0} = -\frac{1}{j\omega L}$$

$$Y_{21} = \frac{\dot{I}_2}{\dot{U}_1}\bigg|_{\dot{U}_2=0} = -\frac{1}{j\omega L} \qquad Y_{21} = \frac{\dot{I}_2}{\dot{U}_2}\bigg|_{\dot{U}_1=0} = j\omega C + \frac{1}{j\omega L}$$

$$\therefore Y = \begin{bmatrix} \dfrac{1}{j\omega L} & -\dfrac{1}{j\omega L} \\ -\dfrac{1}{j\omega L} & j\omega C + \dfrac{1}{j\omega L} \end{bmatrix}$$

或 $Y = Z^{-1}$

$$\because \dot{U}_1 = \dot{U}_2 + j\omega L(\dot{U}_2 j\omega C - \dot{I}_2) \quad \therefore A = \begin{bmatrix} 1-\omega^2 LC & j\omega L \\ j\omega C & 1 \end{bmatrix}$$

$$\dot{I}_1 = j\omega C\dot{U}_2 - \dot{I}_2$$

（b）同理可得：

$$Z = \begin{bmatrix} \frac{1}{j\omega C} & \frac{1}{j\omega C} \\ \frac{1}{j\omega C} & j\omega L + \frac{1}{j\omega C} \end{bmatrix} \qquad Y = \begin{bmatrix} j\omega C + \frac{1}{j\omega L} & -\frac{1}{j\omega L} \\ -\frac{1}{j\omega L} & \frac{1}{j\omega L} \end{bmatrix}$$

$$\because \left.\begin{aligned} \dot{U}_1 &= \dot{U}_2 - j\omega L\dot{I}_2 \\ \dot{I}_1 &= j\omega C\dot{U}_1 - \dot{I}_2 \end{aligned}\right\} \qquad A = \begin{bmatrix} 1 & j\omega L \\ j\omega C & 1-\omega^2 LC \end{bmatrix}$$

11-3 求图题 11-3 所示二端口网络的 Y、Z 参数矩阵。

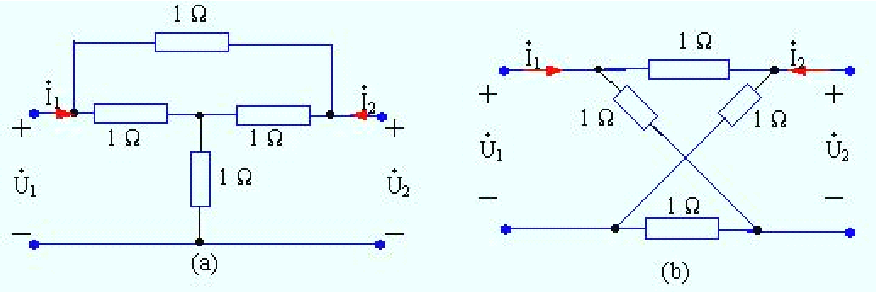

图 题 11—3

[答案](#)

解：（a）$Z_{11} = \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{I}_2=0} = 1 + \frac{2}{3} = \frac{5}{3}\Omega$

$$Z_{12}=\frac{\dot{U}_1}{\dot{I}_2}\bigg|_{\dot{I}_1=0}=1+\frac{1}{3}=\frac{4}{3}\Omega$$

$$Z_{21}=\frac{\dot{U}_2}{\dot{I}_1}\bigg|_{\dot{I}_2=0}=Z_{12}=\frac{4}{3}\Omega$$

$$Z_{22}=\frac{\dot{U}_2}{\dot{I}_2}\bigg|_{\dot{I}_1=0}=\frac{5}{3}\Omega$$

$$Z=\begin{bmatrix}\frac{5}{3} & \frac{4}{3}\\ \frac{4}{3} & \frac{5}{3}\end{bmatrix} \quad Y=Z^{-1}=\frac{1}{3}\begin{bmatrix}5 & -4\\ -4 & 5\end{bmatrix}$$

(b) $Z_{11}=\frac{\dot{U}_1}{\dot{I}_1}\bigg|_{\dot{I}_2=0}=1\Omega \quad Z_{12}=\frac{\dot{U}_1}{\dot{I}_2}\bigg|_{\dot{I}_1=0}=0\Omega$

$$Z_{12}=Z_{21}=0 \qquad Z_{22}=Z_{11}=1\Omega$$

$$\therefore\ Z=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \qquad Y=Z^{-1}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

11-4 求图题 11-4 所示网络的 Y 参数矩阵。

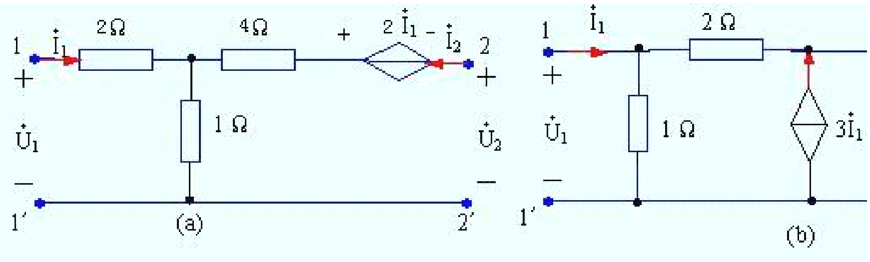

图 题 11—4

答案

解：（a）
$$\left.\begin{aligned}\dot{U}_1 &= 2\dot{I}_1 + 1(\dot{I}_1+\dot{I}_2)\\ \dot{U}_2 &= 2\dot{I}_1+4\dot{I}_2+1(\dot{I}_1+\dot{I}_2)\end{aligned}\right\}$$

有：
$$Z=\begin{bmatrix}3 & 1\\ 3 & 5\end{bmatrix}$$

$$\therefore Y=Z^{-1}=\begin{bmatrix}5/12 & -1/12\\ -1/4 & 1/4\end{bmatrix}$$

(b)
$$\dot{I}_1=\dot{U}_1+(\dot{U}_1-\dot{U}_2)/2$$

$$\dot{I}_2=\dot{U}_2-3\dot{I}_1-(\dot{U}_1-\dot{U}_2)/2$$

$$\therefore Y=\begin{bmatrix}3/2 & -1/2\\ -5 & 3\end{bmatrix}$$

11-5 求图题 11-5 所示网络的 H 参数矩阵。

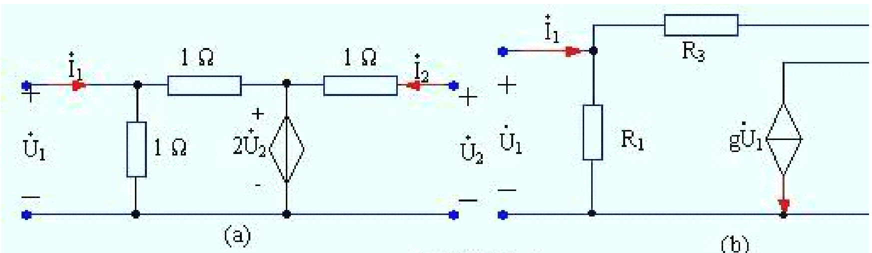

图 题 11—5

答案

解：（a）
$$\because \dot{U}_1=1(\dot{I}_1-(\dot{U}_1-2\dot{U}_2)/1)=\dot{I}_1+2\dot{U}_2-\dot{U}_1$$

$$\dot{I}_2=(\dot{U}_2-2\dot{U}_2)/1=-\dot{U}_2$$

$$\therefore H=\begin{bmatrix}1/2 & 1\\ 0 & -1\end{bmatrix}$$

(b) $$\because \dot{U}_1=R_1[\dot{I}_1+(\dot{U}_2-\dot{U}_1)\big/R_3]=R_1\dot{I}_1+\frac{R_1}{R_3}\dot{U}_2-\frac{R_1}{R_3}\dot{U}_1$$

$$\dot{I}_2=g\dot{U}_1+\frac{\dot{U}_2}{R_2}+\frac{\dot{U}_2-\dot{U}_1}{R_3}$$

有 $$\dot{U}_1=\frac{R_1R_3}{R_1+R_3}\dot{I}_1+\frac{R_1}{R_1+R_3}\dot{U}_2$$

$$\dot{I}_2=\frac{R_1(R_3g-1)}{R_1+R_3}\dot{I}_1+[\frac{1}{R_2}+\frac{1}{R_3}+\frac{R_1(R_3g-1)}{R_3(R_1+R_3)}]\dot{U}_2$$

$$\therefore H=\begin{bmatrix}\dfrac{R_1R_3}{R_1+R_3} & \dfrac{R_1R_3}{R_1+R_3}\\ \dfrac{R_1(R_3g-1)}{R_1+R_3} & \dfrac{1}{R_2}+\dfrac{1}{R_3}+\dfrac{R_1(R_3g-1)}{R_3(R_1+R_3)}\end{bmatrix}$$

11-6　图题 11-6 所示网络，已知网络 P_1 的 A 参数矩阵为 $A_1=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}$，求总网络 A 的矩阵[A]。

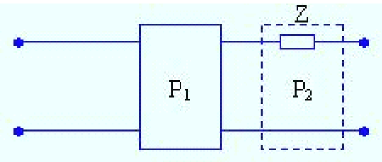

图 题 11—6

答案

解：图示网络可看成 P_1 与 P_2 网络的级联形式。

$$\because A_1=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}$$

$$A_2=\begin{bmatrix}1 & Z\\ 0 & 1\end{bmatrix}$$

$$[A]=[A_1][A_2]=\begin{bmatrix}A_{11} & A_{11}Z+A_{12}\\ A_{21} & A_{21}Z+A_{22}\end{bmatrix}$$

11-7　图题 11-7 所示网络，已知网络 P_1 的 A 参数矩阵为 $[A_1]=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}$。求总网络的[A]矩阵。

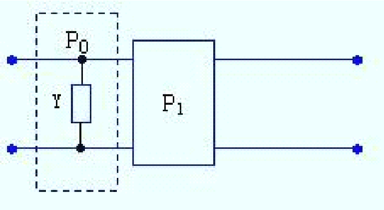

图 题 11—7

答案

解：图示网络可看成 P_0 与 P_1 网络的级联形式。

$$\because\ [A_0]=\begin{bmatrix}1 & 0\\ Y & 1\end{bmatrix}$$

$$[A_1]=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}$$

$$\therefore A=[A_0][A_1]=\begin{bmatrix}A_{11} & A_{12}\\ A_{21}+A_{11}Y & A_{12}Y+A_{22}\end{bmatrix}$$

11-8　图题 11-8 所示网络，已知网络 P_1 的 Y 矩阵为 $Y_1=\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}$，求总网络的 Y 矩阵 [Y]。

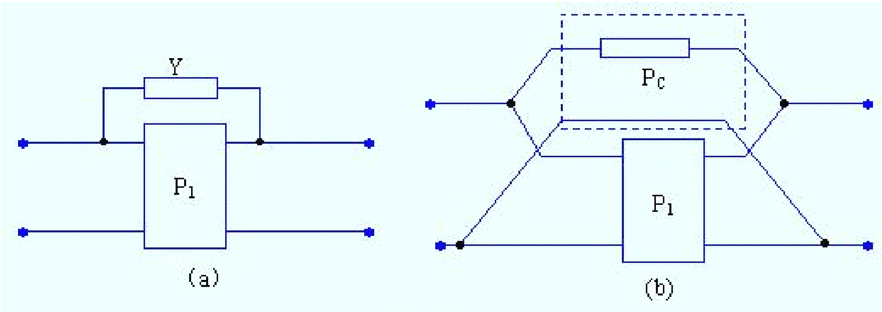

图 题 11—8

答案

解：若网络P_1互易，则图示网络可看作P_1、P_0两个网络的并联联形式，如(b)所示。

$$\because Y_1=\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}\ [Y_0]=\begin{bmatrix} Y & -Y \\ -Y & Y \end{bmatrix}$$

$$\therefore [Y]=[Y_0]+[Y_1]=\begin{bmatrix} Y+Y_{11} & Y_{12}-Y \\ A_{21}-Y & Y+A_{22} \end{bmatrix}$$

11-9 求图题 11-9 所示网络的 Y、Z 参数。

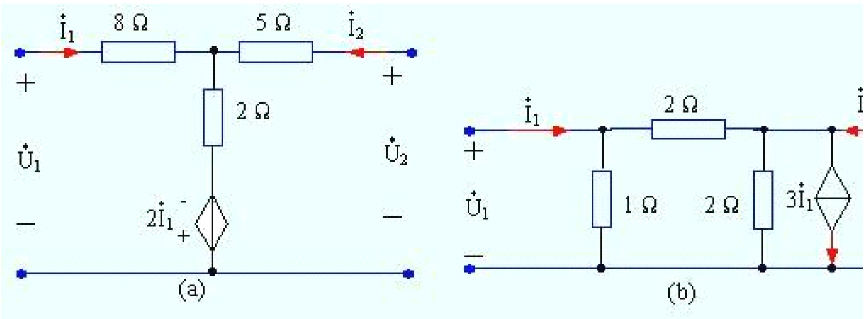

图 题 11—9

答案

解：（a） $\because \dot{U}_1 = 8\dot{I}_1 - 2\dot{I}_1 + 2(\dot{I}_1 + \dot{I}_2)$

$$\dot{U}_2 = -2\dot{I}_1 + 2(\dot{I}_1 + \dot{I}_2) + 5\dot{I}_2$$

$$\therefore [Z] = \begin{bmatrix} 8 & 2 \\ 0 & 7 \end{bmatrix} \qquad [Y] = [Z]^{-1} = \begin{bmatrix} \frac{1}{8} & -\frac{1}{28} \\ 0 & \frac{1}{7} \end{bmatrix}$$

(b) $\because \dot{I}_1 = \dot{U}_1 + (\dot{U}_1 - \dot{U}_2)/2$

$$\dot{I}_2 = 3\dot{I}_1 + \frac{\dot{U}_2}{2} + \frac{(\dot{U}_2 - \dot{U}_1)}{2}$$

$$\therefore [Y] = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ 4 & -\frac{1}{2} \end{bmatrix} \qquad [Z] = [Y]^{-1} = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix}$$

11-10　求图题 11-10 所示网络的 A、H 参数。

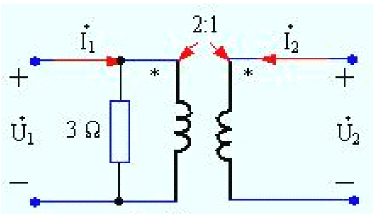

图 题 11—10

答案

解： $\because [A_1] = \begin{bmatrix} 1 & 0 \\ 1/3 & 1 \end{bmatrix}$

$$[A_2] = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$\therefore [A]=[A_1][A_2]=\begin{bmatrix} 2 & 0 \\ 2/3 & 1/2 \end{bmatrix}$$

又 $\because \dot{U}_1 = 2\dot{U}_2$

$$\dot{I}_2 = -2(\dot{I}_1 - \frac{\dot{U}_1}{3})$$

$$\therefore [H]=\begin{bmatrix} 0 & 2 \\ -2 & \frac{4}{3} \end{bmatrix}$$

11-11 求图题 11-11 所示相移网络的特性阻抗。

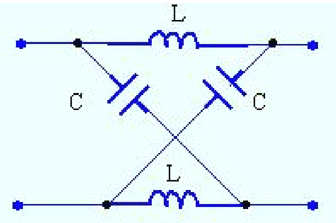

图 题 11－11

答案

解：$Z_\infty = \frac{1}{2}(j\omega L + \frac{1}{j\omega C})$

$$Z_0 = 2\frac{L/C}{j\omega L + \frac{1}{j\omega C}}$$

$$\therefore Z_{C_1} = Z_{C_2} = \sqrt{Z_0 Z_\infty} = \sqrt{\frac{L}{C}}$$

11-12　　图题 11-12 所示网络，已知 $R=\sqrt{\dfrac{L}{C}}$。求输入阻抗 Z_{in}。

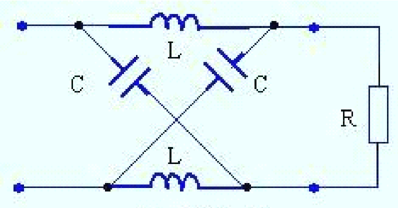

图 题 11－12

答案

解： 相移二端口网络的特性阻抗

$$Z_C=\sqrt{\frac{L}{C}}$$　　（上题）

又 $$R=\sqrt{\frac{L}{C}}=Z_C$$

$$\therefore Z_{in}=Z_{C}=\sqrt{\frac{L}{C}}=R$$

11-13　　图题 11-13 所示，已知网络的 A 矩阵为 $[A]=\begin{bmatrix}4 & 6\\0.5 & 1\end{bmatrix}$，求 $[A_\alpha]$。

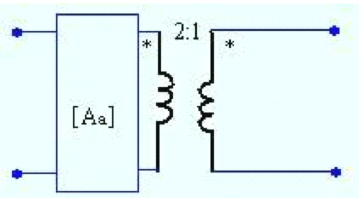

图 题 11－13

答案

解：$\because [A]=[A_a][A_b]$

$$[A_b]=\begin{bmatrix}2 & 0\\ 0 & \frac{1}{2}\end{bmatrix}$$

$$[A_a]=[A][A_b]^{-1}=\begin{bmatrix}4 & 6\\ 0.5 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{2} & 0\\ 0 & 2\end{bmatrix}=\begin{bmatrix}2 & 12\\ 0.25 & 2\end{bmatrix}$$

11-14　欲使图题 11-14 所示网络为互易网络，则 μ 和 α 之间应满足何关系？

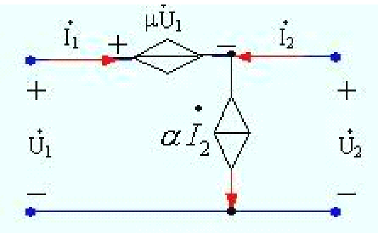

图 题 11—14

答案

解：$\because \dot{U}_1=\mu\dot{U}_1+\dot{U}_2$

$\dot{I}_1=(\alpha-1)\dot{I}_2$

$$[A]=\begin{bmatrix}\frac{1}{1-\mu} & 0\\ 0 & 1-\alpha\end{bmatrix}$$

若网络为互易网络，则应：$|A|=\dfrac{1-\alpha}{1-\mu}=1$

$\therefore \alpha=\mu$ 且 $\alpha=\mu\neq 1$

11-15　求图题 11-15 所示网络的开路电压传输函数 $H(j\omega)=\dfrac{\dot{U}_2}{\dot{U}_1}$，并画出模频与相频特性。

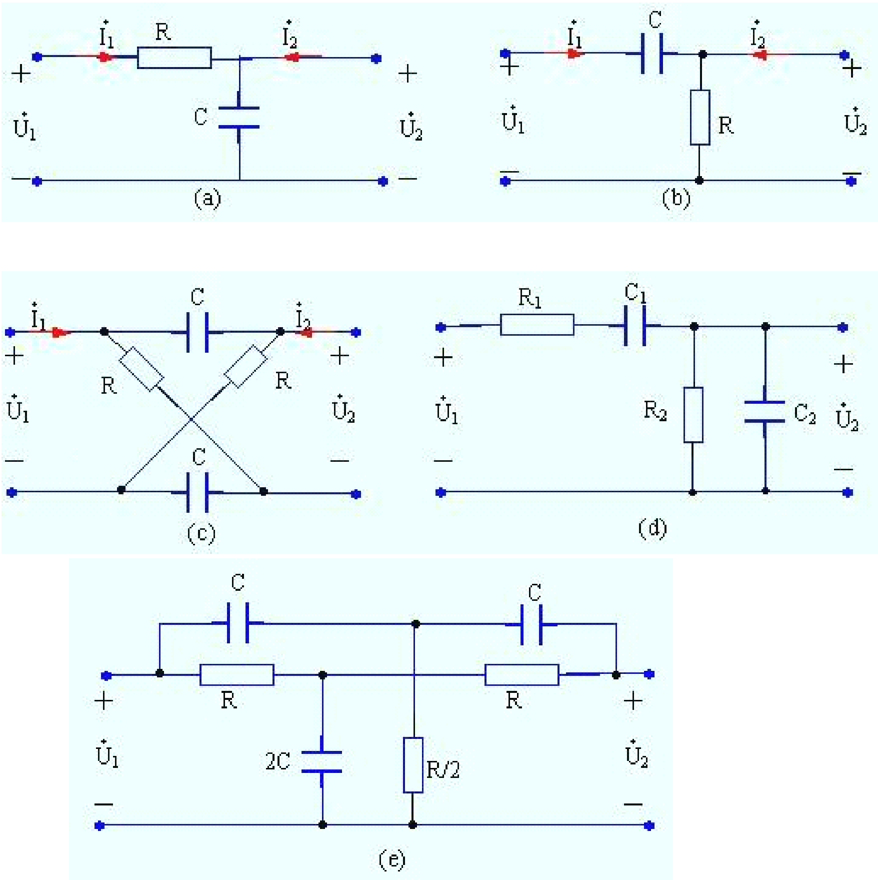

图 题 11—15

答案

解：（a）　$H(j\omega)=\dfrac{\dot{U}_2}{\dot{U}_1}=\dfrac{1}{1+j\omega CR}$

$$\therefore\ H(\omega)=\frac{1}{\sqrt{1+(\omega CR)^2}}\qquad \varphi(\omega)=-arctg\omega CR$$

（b） $$H(j\omega)=\frac{\dot{U}_2}{\dot{U}_1}=\frac{j\omega CR}{1+j\omega CR}$$

$$\therefore H(\omega)=\frac{\omega CR}{\sqrt{1+(\omega CR)^2}} \qquad \varphi(\omega)=\frac{\pi}{2}-arctg\omega CR$$

（c） $$H(j\omega)=\frac{\dot{U}_2}{\dot{U}_1}=\frac{j\omega CR-1}{j\omega CR+1}$$

$$\therefore H(\omega)=1 \qquad \varphi(\omega)=\pi-2\mathrm{arctg}\omega\mathrm{CR}$$

（d） $$H(j\omega)=\frac{\dot{U}_2}{\dot{U}_1}=\frac{1}{(\frac{R_1}{R_2}+\frac{C_2}{C_1}+1)+j(\omega R_1C_2-}$$

$$\therefore H(\omega)=\frac{1}{\sqrt{(\frac{R_1}{R_2}+\frac{C_2}{C_1}+1)^2+(\omega R_1C_2-\frac{1}{\omega R_2C_1})^2}} \qquad \varphi(\omega)=-arctg\frac{(\omega R_1C_2-\frac{}{\omega}}{(\frac{R_1}{R_2}+\frac{C_2}{C_1}}$$

（e）图示网络可看成是两个 T 形网络的并联。

对于上 T 形： $$Y_{21}'=\frac{\frac{R}{2}\omega^2C^2}{1+j\omega CR} \qquad Y_{22}'=\frac{\frac{R^2C\omega^2}{2}+j\omega C}{1+j\omega CR}$$

对于下 T 形： $$Y_{21}''=\frac{-1}{2R+j2\omega CR^2} \quad Y_{22}''=\frac{(1+j2\omega CR)}{2R(1+j\omega CR)}$$

双 T 形网络中：

$$Y_{21}=Y_{21}'+Y_{21}''=\frac{-(1+j\omega CR)+R^2C^2\omega^2(1+j\omega CR)}{2R(1+j\omega CR)(1+j\omega CR)}=\frac{-1+\omega^2C^2R^2}{2R(1+j\omega CR)}$$

$$Y_{22}=Y_{22}'+Y_{22}''=\frac{-(R^2C^2\omega^2-j2CR\omega)+(1+j2CR\omega)}{2R(1+j\omega CR)}=\frac{1-R^2C^2\omega^2+j4CR\omega}{2R(1+j\omega CR)}$$

又 当 $\dot{I}_2=0$ 时

$$\because \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2$$

$$H(j\omega) = \frac{\dot{U}_2}{\dot{U}_1} = -\frac{Y_{21}}{Y_{22}} = \frac{(R^2C^2\omega^2 - 1)R}{(R^2C^2\omega^2 - 1) - j4CR\omega} = \frac{R}{1 - j\dfrac{4CR\omega}{R^2C^2\omega^2 - 1}}$$

$$\therefore H(\omega) = \frac{R}{\sqrt{1 + \left(\dfrac{4CR\omega}{R^2C^2\omega^2 - 1}\right)^2}} \qquad \varphi(\omega) = arctg\frac{4CR\omega}{R^2C^2\omega^2 - 1}$$