

# *Theory of Relational Database*

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**Database Systems-Design and Application**

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# Outline

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- Potential Problems in a Base Relation
- The concept of Normalization
- Functional Dependencies
- First Normal Form (1NF)
- Second Normal Form (2NF)
- Third Normal Form (3NF)
- Review 1NF to 3NF
- General Definitions of 2NF and 3NF
- Boyce-Codd Normal Form (BCNF)

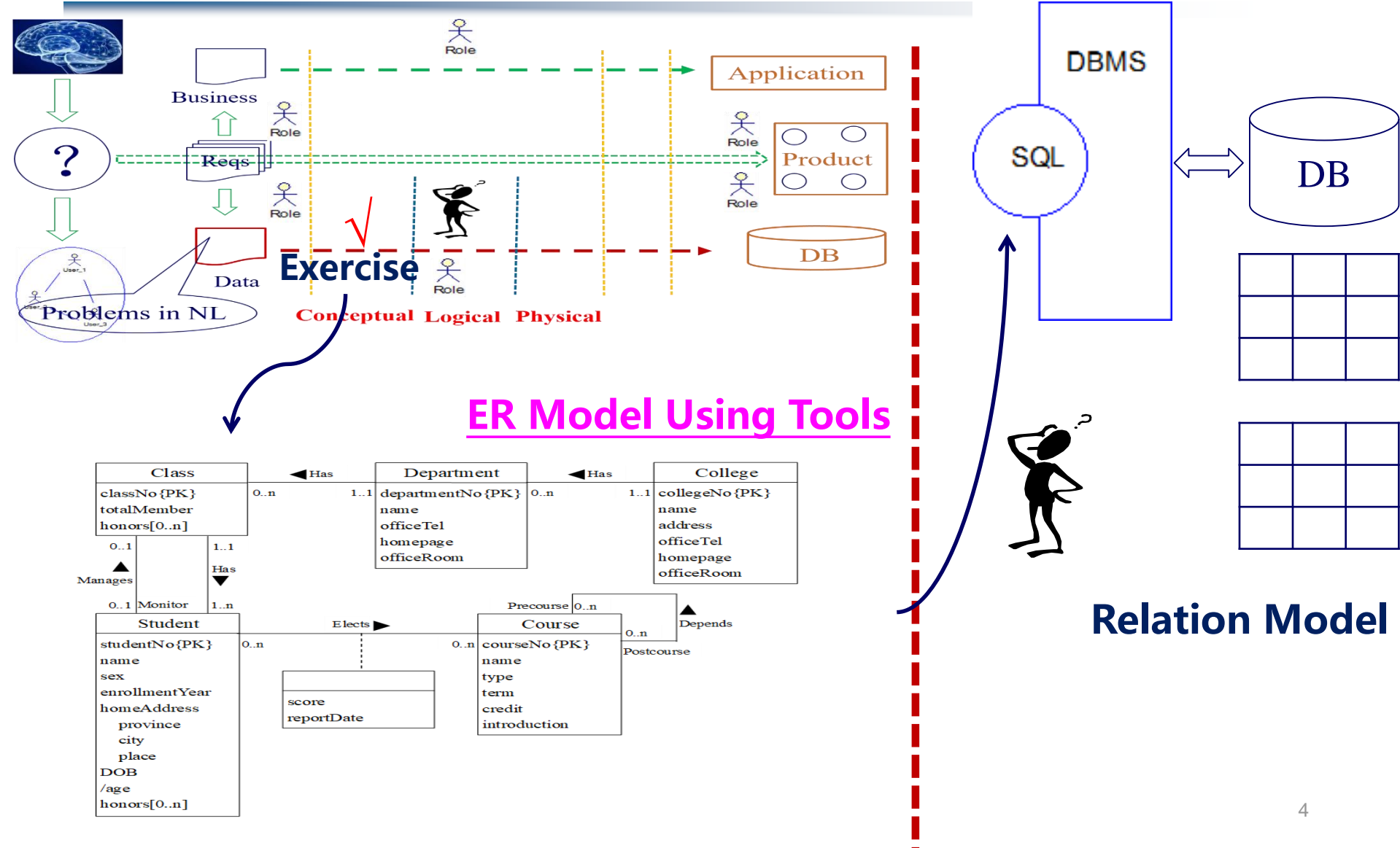
# Outline

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- Engineering Consideration about Normalization
- Fourth Normal Form (4NF)
- Discuss

## 课程内容组织与实施计划

# Problem we will discuss



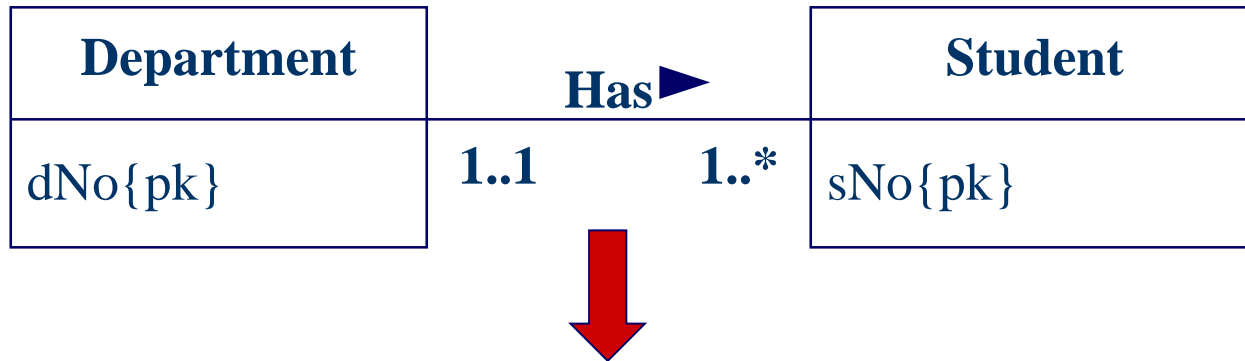
# Problem we will discuss

Elements in ER Model
entity
strong, weak
attribute
simple, composite, mutli-value
relationship
1:1, 1:N, M:N
N-ary
recursive
value set
key attribute



Elements in Relation Model
relation(table)
attribute(column)
foreign key
domain
primary key (alternate key)

# Problem we will discuss



D-S(dNo,dName, officeRoom, homepage,sNo, sName, sex, age)

D(dNo,dName, officeRoom, homepage,sNo)

S(sNo, sName, sex, age)

D(dNo,dName, officeRoom, homepage)

S(sNo, sName, sex, age,dNo)

D-1(dNo), D-2(dName), D-3(officeRoom), ...

S-1(sNo), S-2(sName), S-3(sex), ...



# Potential Problems in a Base Relation

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dNo	dName	homepage	office	address
01	Software	<a href="http://www.software.edu.cn">www.software.edu.cn</a>	1#301	A1
02	Computer	<a href="http://www.computer.edu.cn">www.computer.edu.cn</a>	2#101	A2
03	Math	<a href="http://www.math.edu.cn">www.math.edu.cn</a>	3#201	A3

# Potential Problems in a Base Relation

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tNo	tName	address	cNo	cName
T1	Wangbin	A1	C1	N1
T1	Wangbin	A1	C2	N2
T1	Wangbin	A1	C3	N3
T2	Lina	A2	C4	N4
T2	Lina	A2	C5	N5
T3	Zhaole	A3	C6	N6



# Potential Problems in a Base Relation

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<u>sNo</u>	sName	dept	deptAddr	deptMn	<u>cNo</u>	cName	score
s01	Lucy	CS	Building 1	Zhaobin	c01	DB	80
s01	Lucy	CS	Building 1	Zhaobin	c02	OS	90
s02	Jam	IS	Building 2	Qiansi	c01	DB	70
s02	Jam	IS	Building 2	Qiansi	c03	DS	85
s03	Susan	Math	Building 3	Sunwu	c01	DB	75
s04	Tom	CS	Building 1	Zhaobin	c02	OS	85

# Potential Problems in a Base Relation

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- What problems do often exist in relations?
- Why are there problems in some relations but not in other relations?
- How to resolve these problems?
- How to measure the appropriateness of attributes groupings in relations?
- How to achieve a better relation schema?
- **Normalization** is helpful for resolving these problems.

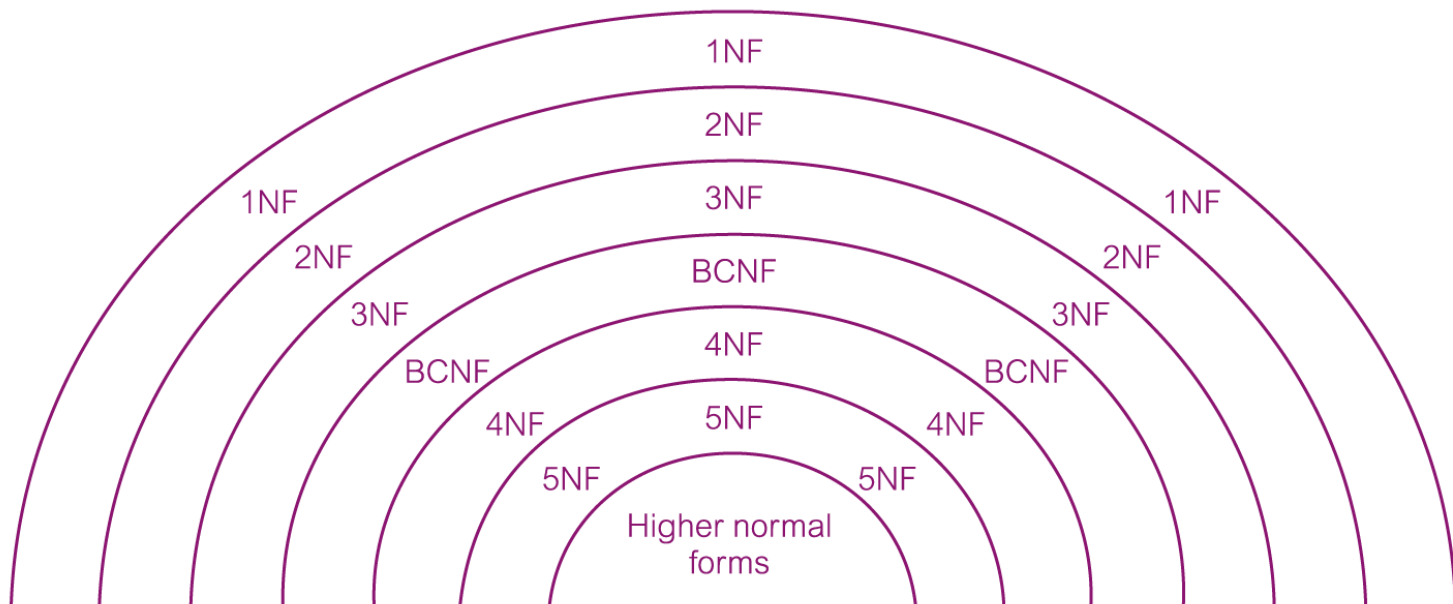
# The concept of Normalization

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- Normalization
  - A technique for producing a set of relations with desirable properties, given the data requirements of an enterprise.
  - The process of normalization is a formal method that identifies relations based on their primary or candidate keys and the **data dependencies** among their attributes.
- The process of normalization was first developed by E.F.Codd.

# The concept of Normalization

- Normalization is often performed as a series of tests on a relation to determine whether it satisfies or violates the requirements of a given normal form.



# Functional Dependencies

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- Data dependency
  - The relationships between attributes in a relation that describe the dependency or restriction on each other.
- The types of data dependency
  - Functional dependency(FD)
  - Multi-valued dependency(MVD)
  - Join dependency(JD)

# Functional Dependencies

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- Functional dependency(FD)
  - Describes the relationship between attributes in a relation.
  - If A and B are attributes of relation R, B is functionally dependent on A(denoted  $A \rightarrow B$ ), if each value of A is associated with exactly one value of B. A is **determinant** of B
  - Functional dependency is a property of the meaning or semantics of the attributes in a relation.

# Functional Dependencies

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- Functional dependency (FD)
  - If  $A \rightarrow B$  and  $B \rightarrow A$ , then we denote  $A \leftrightarrow B$
- Trivial functional dependency (平凡函数依赖)
  - If  $A \rightarrow B$  and  $B$  is a subset of  $A$  ( $B \subseteq A$ )
- Nontrivial functional dependency
  - If  $A \rightarrow B$  and  $B$  is not a subset of  $A$  ( $B \not\subseteq A$ )
- We are normally more interested in nontrivial dependencies because they represent integrity constraints for the relation.

# Functional Dependencies

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- Identifying a functional dependency
  - When identifying functional dependencies between attributes in a relation it is important to distinguish clearly between the values held by an attribute at a given point in time and **the set of all possible values** that an attribute may hold at different times.
  - A functional dependency is a property of a relational schema(intension) and not a property of a particular instance of the schema(extension).



# Functional Dependencies

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- Generally we can identify
  - If the relationship between  $X$  and  $Y$  is one-to-one(1:1), then  $X \leftrightarrow Y$
  - If the relationship between  $X$  and  $Y$  is one-to-many(1:\*), then  $Y \rightarrow X$
  - If the relationship between  $X$  and  $Y$  is many-to-many(\*:\*), then maybe there is no functional dependency between  $X$  and  $Y$

# Functional Dependencies

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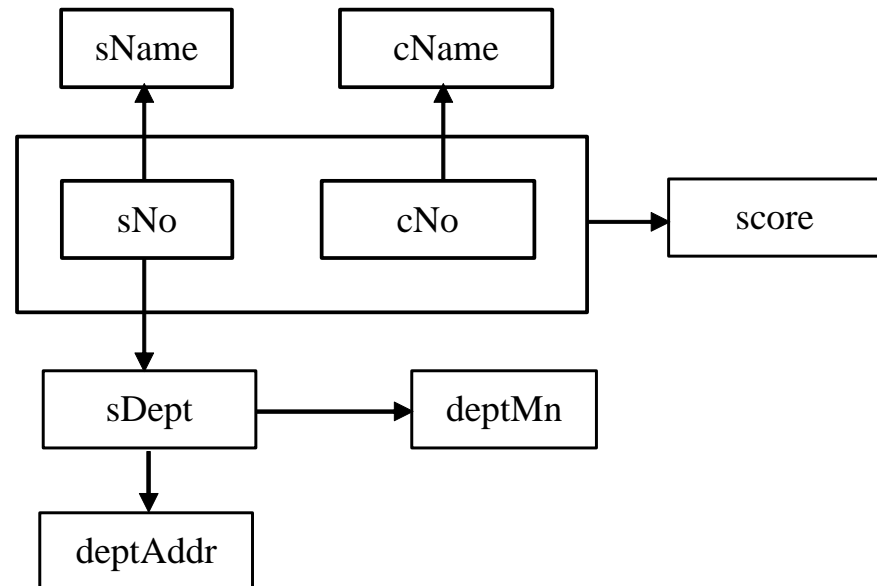
- Formal description of a relation schema
- $R(U, D, \text{dom}, F)$ 
  - R-relation name
  - U-attribute set
  - D-domains of all attributes
  - Dom-mapping between attributes and domains
  - F-data dependency between attributes
  - D and dom is not important for relation schema, so we can simplify it as
- $R(U, F)$

# Functional Dependencies

Given relation schema  $R(U, F)$

$U = \{ sNo, sName, sDept, deptAddr, deptMn, cNo, cName, score \}$

$F = \{ sNo \rightarrow sName, sNo \rightarrow sDept, sDept \rightarrow deptAddr, deptMn, cNo \rightarrow cName, (sNo, cNo) \rightarrow score \}$



# Functional Dependencies

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- The complete set of functional dependencies for a given relation can be very large.
- It is important to find an approach that can reduce that set to a manageable size.
- The set of all functional dependencies that are implied by a given set of functional dependencies  $X$  is called the **closure** of  $X$ , written  $X^+$ .
- A set of inference rules, called **Armstrong's axioms**, can help to compute  $X^+$  from  $X$ .

# Functional Dependencies

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- Armstrong's axioms are as follows:
  - Reflexivity: If  $B$  is a subset of  $A$ , then  $A \rightarrow B$
  - Augmentation: If  $A \rightarrow B$ , then  $A, C \rightarrow B, C$
  - Transitivity: If  $A \rightarrow B$  and  $B \rightarrow C$ , then  $A \rightarrow C$ .
- Self-fetermination:  $A \rightarrow A$
- Decomposition: If  $A \rightarrow B, C$ , then  $A \rightarrow B$  and  $A \rightarrow C$ .
- Union: If  $A \rightarrow B$  and  $A \rightarrow C$ , then  $A \rightarrow B, C$
- Composition: If  $A \rightarrow B$  and  $C \rightarrow D$ , then  $A, C \rightarrow B, D$

# Functional Dependencies

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- Minimal Sets of Functional Dependencies( $X_{\min}$ )
- A minimal set of dependencies should be in a standard form with no redundancies.
- There can be several minimal covers for a set of functional dependencies.
- Try to identify the minimal set of a functional dependency using Armstrong axioms.

# Functional Dependencies

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Given relation schema  $R(U,F)$

$U = \{sNo, sName, sDept, deptAddr, deptMn, cNo, cName, score\}$

$F = \{sNo \rightarrow sName, sNo \rightarrow sDept, sNo \rightarrow deptMn, sDept \rightarrow deptAddr, deptMn, cNo \rightarrow cName, (sNo, cNo) \rightarrow score\}$

Is  $F$  a minimal cover?

If  $F$  is not a minimal cover, try to give a  $F_{min}$ .

# First Normal Form (1NF)

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- Definition of 1NF
  - A relation in which the intersection of each row and column contains one and only one value.
- For the relational data model, it is important to recognize that it is only 1NF that is critical in creating relations. All the subsequent normal forms are optional.



# First Normal Form (1NF)

- Example

sNo	sName	addr	phoneNo
s01	Lucy	No.1 xxx	02988451234 02886654321
s02	Jam	No.2 xxx	02988451234
s03	Susan	No.3 xxx	02988451234 02168654321

# First Normal Form (1NF)

---

sNo	sName	addr	phoneNo
s01	Lucy	No.1 xxx	02988451234
s01	Lucy	No.1 xxx	02886654321
s02	Jam	No.2 xxx	02988451234
s03	Susan	No.3 xxx	02988451234
s03	Susan	No.3 xxx	02168654321

# First Normal Form (1NF)

---

sNo	sName	addr	phoneNo1	phoneNo2
s01	Lucy	No.1 xxx	02886654321	02988451234
s02	Jam	No.2 xxx	02988451234	
s03	Susan	No.3 xxx	02168654321	02988451234

# First Normal Form (1NF)

---

sNo	sName	addr
s01	Lucy	No.1 xxx
s02	Jam	No.2 xxx
s03	Susan	No.3 xxx

sNo	phoneNo
s01	02988451234
s01	02886654321
s02	02988451234
s03	02988451234
s03	02168654321

# First Normal Form (1NF)

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- Modify a table to 1NF
  - Add rows
  - Add columns
  - Placing the repeating data along with a copy of the original key attribute(s) into a separate relation.
- Add columns to restrict each cell contains only one value if we can identify the number of values may appear in a cell. Or adding columns cannot resolve the problem.

# Second Normal Form (2NF)

- Problems in a relation that is in 1NF

<u>sNo</u>	<u>cNo</u>	score	credit
s01	c01	80	2
s01	c02	90	3
s02	c01	70	2
s02	c03	85	1
s03	c01	75	2
s04	c02	85	3

# Second Normal Form (2NF)

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- 2NF is based on the concept of full functional dependency so we introduce full and partial functional dependency in first.

# Second Normal Form (2NF)

- Full/Partial Functional Dependency
  - Indicates that if A and B are attributes of a relation, B is fully functionally dependent on A if B is functionally dependent on A, but not on any proper subset of A.
  - $X \rightarrow Y$  is a FD in  $R(U, F)$ , if  $\neg \exists X' \subset X, X' \rightarrow Y$ , then Y is fully functionally dependent on X. Or Y is partially functionally dependent on X.
  - We can write Partial/Full functional dependency as

$$X \xrightarrow{P} Y \quad X \xrightarrow{F} Y$$



# Second Normal Form (2NF)

---

- Definition of 2NF
  - A relation that is in 1NF and every non-primary-key attribute is fully functionally dependent on the primary key.
  - 2NF applies to relations with composite keys, that is, relations with a primary key composed of two or more attributes.
  - A relation with a single attribute primary key is automatically in at least 2NF.

# Second Normal Form (2NF)

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- The normalization of 1NF relations to 2NF involves the removal of partial dependencies.
  - If a partial dependency exists, we remove the partially functionally dependent attribute(s) from the relation by placing them in a new relation along with a copy of their determinant.

# Second Normal Form (2NF)

## ■ Example

Given relation schema  $R(U,F)$

$U = \{sNo, cNo, score, credit\}$

$F = \{ (sNo, cNo) \rightarrow score, cNo \rightarrow credit \}$

<u>sNo</u>	<u>cNo</u>	score	credit
s01	c01	80	2
s01	c02	90	3
s02	c01	70	2
s02	c03	85	1
s03	c01	75	2
s04	c02	85	3

# Second Normal Form (2NF)

## ■ Example

We can decompose R to R1 and R2

R1:  $U=\{sNo, cNo, score\}$   $F=\{(sNo, cNo) \rightarrow score\}$

R2:  $U=\{cNo, credit\}$   $F=\{cNo \rightarrow credit\}$

<u>sNo</u>	<u>cNo</u>	score
s01	c01	80
s01	c02	90
s02	c01	70
s02	c03	85
s03	c01	75
s04	c02	85

<u>cNo</u>	credit
c01	2
c02	3
c03	1

# Third Normal Form (3NF)

- Problems in a relation that is in 2NF

<u>sNo</u>	sName	dept	deptAddr	deptMn
s01	Lucy	CS	Building 1	Zhaobin
s02	Jam	IS	Building 2	Qiansi
s03	Susan	Math	Building 3	Sunwu
s04	Tom	CS	Building 1	Zhaobin

# Third Normal Form (3NF)

---

- 3NF is based on the concept of transitive dependency so we introduce transitive dependency in first.
- Transitive Dependency
  - A condition where A, B, and C are attributes of a relation such that if  $A \rightarrow B$  and  $B \rightarrow C (C \not\subseteq B)$ , then C is transitively dependent on A via B (provided that A is not functionally dependent on B or C).

# Third Normal Form (3NF)

---

- Definition of 3NF
  - A relation that is in 1NF and 2NF and in which no non-primary-key attribute is transitively dependent on the primary key.
- The normalization of 2NF relations to 3NF involves the removal of transitive dependencies.
  - If a transitive dependency exists, we remove the transitively dependent attribute(s) from the relation by placing them in a new relation along with a copy of their determinant.

# Third Normal Form (3NF)

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## ■ Example

Relation StuDept(U,F)

$U = \{sNo, sName, dept, deptAddr, deptMn\}$

$F = \{sNo \rightarrow sName, sNo \rightarrow dept, dept \rightarrow deptAddr, dept \rightarrow deptMn\}$

We can decompose StuDept to Student and Department

Student:  $U = \{sNo, sName, dept\}$

$F = \{sNo \rightarrow sName, sNo \rightarrow dept\}$

Department:  $U = \{dept, deptAddr, deptMn\}$

$F = \{dept \rightarrow deptAddr, dept \rightarrow deptMn\}$



<u>sNo</u>	sName	dept	deptAddr	deptMn
s01	Lucy	CS	Building 1	Zhaobin
s02	Jam	IS	Building 2	Qiansi
s03	Susan	Math	Building 3	Sunwu
s04	Tom	CS	Building 1	Zhaobin



<u>sNo</u>	sName	dept
s01	Lucy	CS
s02	Jam	IS
s03	Susan	Math
s04	Tom	CS

<u>dept</u>	deptAddr	deptMn
CS	Building 1	Zhaobin
IS	Building 2	Qiansi
Math	Building 3	Sunwu

# Review 1NF to 3NF

<u>sNo</u>	sName	dept	deptAddr	deptMn	<u>cNo</u>	cName	score
s01	Lucy	CS	Building 1	Zhaobin	C01	DB	80
					c02	OS	90
s02	Jam	IS	Building 2	Qiansi	C01	DB	70
					c03	DS	85
s03	Susan	Math	Building 3	Sunwu	c01	DB	75
s04	Tom	CS	Building 1	Zhaobin	c02	OS	85

# Review 1NF to 3NF

---

<u>sNo</u>	sName	dept	deptAddr	deptMn	<u>cNo</u>	cName	score
s01	Lucy	CS	Building 1	Zhaobin	c01	DB	80
s01	Lucy	CS	Building 1	Zhaobin	c02	OS	90
s02	Jam	IS	Building 2	Qiansi	c01	DB	70
s02	Jam	IS	Building 2	Qiansi	c03	DS	85
s03	Susan	Math	Building 3	Sunwu	c01	DB	75
s04	Tom	CS	Building 1	Zhaobin	c02	OS	85

# Review 1NF to 3NF

<u>cNo</u>	cName
c01	DB
c02	OS
c03	DS

<u>sNo</u>	sName	dept	deptAddr	deptMn
s01	Lucy	CS	Building 1	Zhaobin
s02	Jam	IS	Building 2	Qiansi
s03	Susan	Math	Building 3	Sunwu
s04	Tom	CS	Building 1	Zhaobin

<u>sNo</u>	<u>cNo</u>	score
s01	c01	80
s01	c02	90
s02	c01	70
s02	c03	85
s03	c01	75
s04	c02	85

# Review 1NF to 3NF

<u>cNo</u>	cName
c01	DB
c02	OS
c03	DS

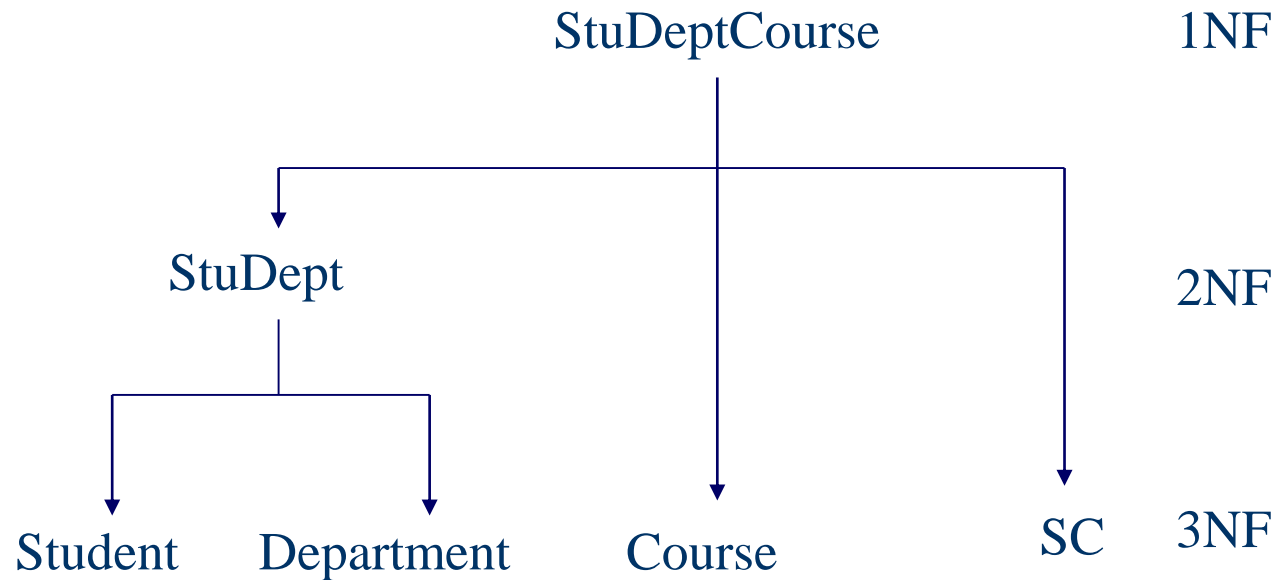
<u>sNo</u>	<u>cNo</u>	score
s01	c01	80
s01	c02	90
s02	c01	70
s02	c03	85
s03	c01	75
s04	c02	85

<u>sNo</u>	sName	dept
s01	Lucy	CS
s02	Jam	IS
s03	Susan	Math
s04	Tom	CS

dept	deptAddr	deptMn
CS	Building 1	Zhaobin
IS	Building 2	Qiansi
Math	Building 3	Sunwu

# Review 1NF to 3NF

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# General Definitions of 2NF and 3NF

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- The 2NF and 3NF we discuss above disallow partial or transitive dependencies on the primary key of relations to avoid the update anomalies. However, these definitions do not take into account other candidate keys of a relation, if any exist.
- We can present more general definitions for 2NF and 3NF that take into account candidate keys of a relation.

# General Definitions of 2NF and 3NF

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- For the general definitions, we define that a primary-key attribute is part of any candidate key.
- Definition of 2NF in general
  - A relation that is in first normal form and every **non-primary-key attribute** is fully functionally dependent on **any candidate key**.
- Definition of 3NF in general
  - A relation that is in first and second normal form and in which no **non-primary-key attribute** is transitively dependent on **any candidate key**.



# General Definitions of 2NF and 3NF

---

- Using the general definition of 2NF and 3NF can make the process of normalization more complex, but it can identify hidden redundancy in relations that could be missed.
- In fact, it is often the case that whether we use the definitions based on primary keys or the general definitions of 2NF and 3NF, the decomposition of relations is the same.

# Boyce-Codd Normal Form (BCNF)

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- Partial and transitive dependencies are the most important reasons for update anomalies. The 3NF disallow partial and transitive dependencies on the primary key so if all database relations is in 3NF we can think the result may be good.
- But 3NF does not remove the transitive dependencies of primary-key attributes on andidate keys and this can still cause redundancy.

# Boyce-Codd Normal Form (BCNF)

---

- Definition of BCNF
  - A relation is in BCNF, if and only if, every determinant is a candidate key.
  - Or no attribute in a relation is transitively dependent on any candidate key.
  - The difference between 3NF and BCNF is that for a functional dependency  $A \rightarrow B$ , 3NF allows this dependency in a relation if B is a primary-key attribute and A is not a candidate key, whereas BCNF insists that for this dependency to remain in a relation.

# Boyce-Codd Normal Form (BCNF)

## ■ Example

Relation SAG(sid, gid, aid, num)

sid - storehouseId, gid - goodsId, aid - administratorId, num - goodsNum

One administrator only work for one storehouse.

$F = \{ (sid, gid) \rightarrow aid, num, (gid, aid) \rightarrow sid, num, aid \rightarrow sid \}$

(sid, gid) and (gid, aid) are all candidate keys.

$SAG \in 3NF$        $SAG \notin BCNF$

We can decompose STC to SC and TC

AG:  $U = \{ aid, gid, num \}$

AS:  $U = \{ aid, sid \}$

# Boyce-Codd Normal Form (BCNF)

sid	gid	aid	num
1	1	1	10
1	2	2	20
2	1	3	20
2	2	4	10



aid	gid	num
1	1	10
2	2	20
3	1	20
4	2	10

aid	sid
1	1
2	1
3	2
4	2

# Boyce-Codd Normal Form (BCNF)

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- Violation of BCNF is quite rare, since it may only happen under specific conditions. The potential to violate BCNF may occur in a relation that:
  - Contains two (or more) composite candidate keys;
  - The candidate keys overlap, that is have at least one attribute in common.

# Engineering Consideration about Normalization

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- We can decompose any relation that is not in BCNF into BCNF. However, it may not always be desirable to transform a relation into BCNF because some function dependencies may not be preserved when we perform the decomposition.
- When we decompose a relation stop at 3NF, all dependencies can be preserved.

# Engineering Consideration about Normalization

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- When a relation is in BCNF, it has been decomposed completely in the category of functional dependency and anomalies of insertion and modification should have been removed.
- How about efficiency?



# Fourth Normal Form (4NF)

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- Although BCNF removes anomalies due to functional dependencies, another type of dependency called a multi-valued dependency (MVD) can also cause data redundancy.
- Possible existence of multi-valued dependencies in a relation is due to 1NF, which disallows an attribute in a tuple from having a set of values.

# Fourth Normal Form (4NF)

- Example

course	teacher	references
physics	李勇 王军	普通物理学 光学原理 物理习题集
math	李勇 张萍	数学分析 高等数学

# Fourth Normal Form (4NF)

## ■ Example

course	teacher	references
physics	李勇	普通物理学
physics	李勇	光学原理
physics	李勇	物理习题集
physics	王军	普通物理学
physics	王军	光学原理
physics	王军	物理习题集
math	李勇	数学分析
...	...	...

# Fourth Normal Form (4NF)

---

- Multi-valued dependency (MVD)
  - Dependency between attributes (for example, A, B, and C) in a relation, such that for each value of A there is a set of values for B and a set of values for C. However, the set of values for B and C are independent of each other.
  - MVD can be written as
$$A \twoheadrightarrow B$$
$$A \twoheadrightarrow C$$

# Fourth Normal Form (4NF)

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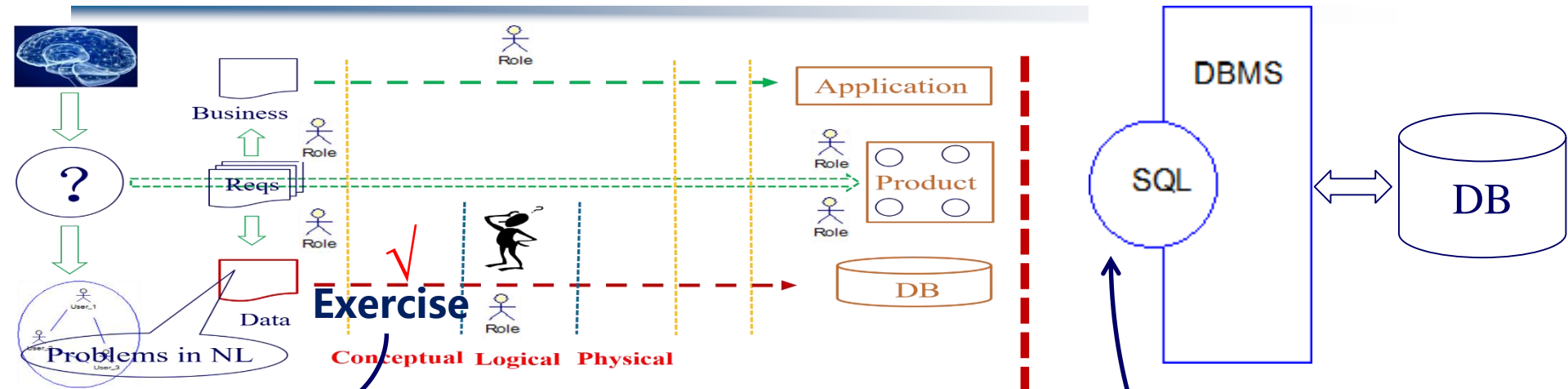
- Trivial/nontrivial MVD
  - A MVD  $A \twoheadrightarrow B$  in relation R is defined as being trivial if (a), B is a subset of A or (b),  $A \cup B = R$ .
  - A MVD is defined as being nontrivial if neither (a) nor (b) is satisfied.

# Fourth Normal Form (4NF)

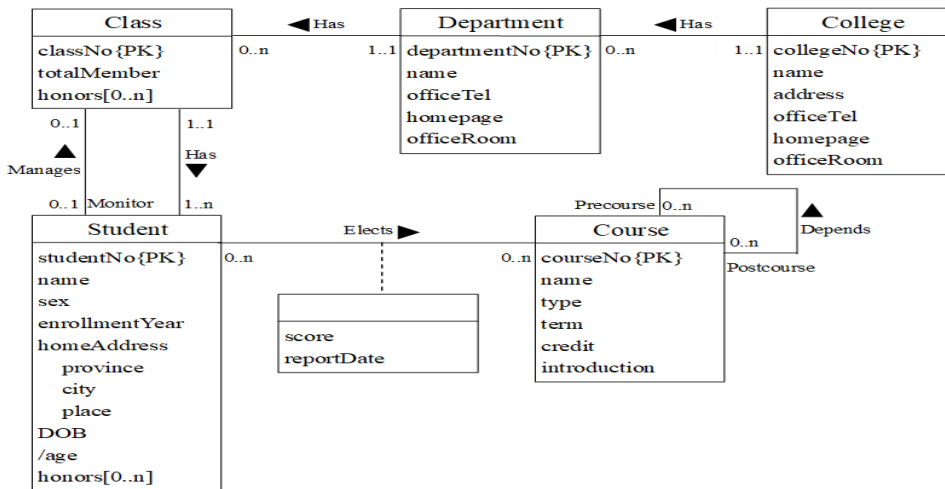
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- Definition of 4NF
  - A relation that is in BCNF and contains no nontrivial MVDs.
- The normalization of BCNF relations to 4NF involves the removal of the MVD from the relation by placing the attribute(s) in a new relation along with a copy of the determinant(s).

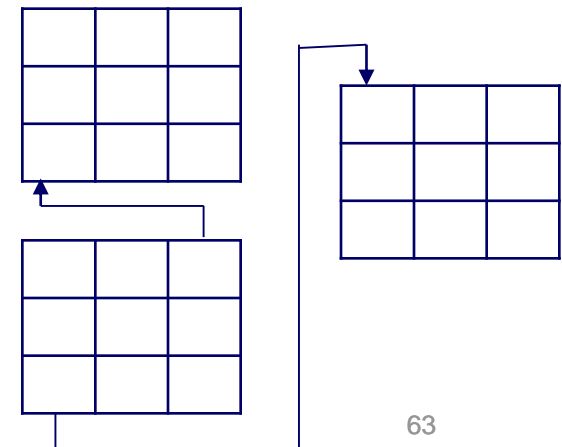
# Discuss



## ER Model Using Tools

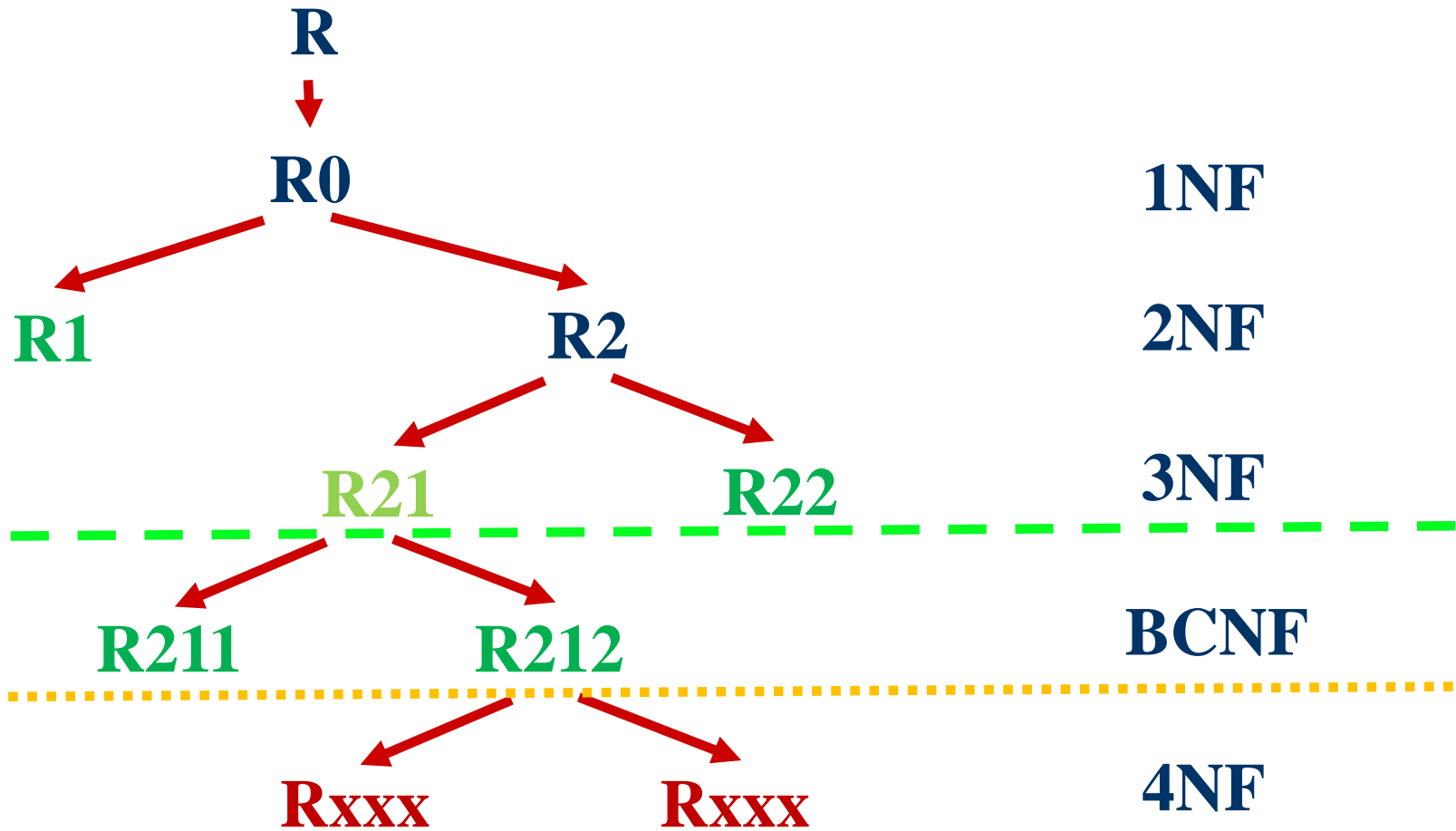


## Relation Model



## Theory of Relation Schema

# Discuss





# Summary

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- In this chapter you should have learned:
  - The purpose of normalization
  - The problems associated with redundant data
  - The concept of functional dependency
  - How to identify the most commonly used forms: 1NF, 2NF, 3NF and BCNF
  - How to use the forms to normalize relation.



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给定关系模式 $R(U, F)$ ,  $U=\{A, B, C, D, E, F, G\}$ ,  
 $F=\{A\rightarrow B, C; D\rightarrow E; AD\rightarrow F; E\rightarrow G\}$ 。

(1) 求该关系模式候选码

(2) 将关系模式 $R$ 一步一步规范化为满足BCNF的关系模式。