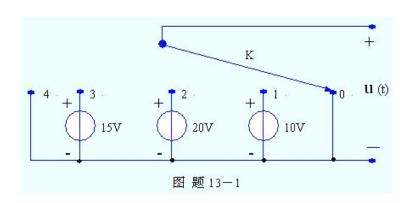
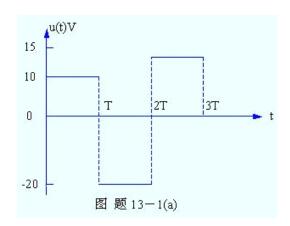
第十三章 一阶电路时域分析

13-1 图题 13-1 所示电路,t<0 时 K 一直在 0 点。今从 t=0 时刻开始。每隔 T 秒,依次将 K 向左扳动,扳道 4 点是长期停住。试画出 u(t) 的波形,并用阶 跃函数将 u(t) 表示出来。



<u>答案</u>

解: u(t)的波形如图 13-1 (a) 所示。

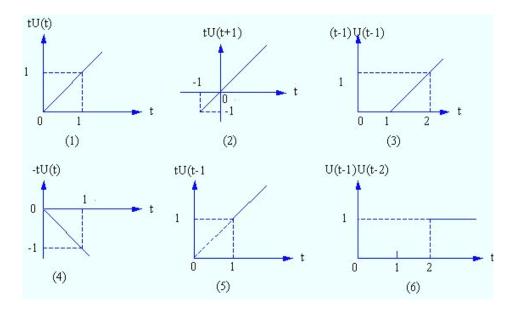


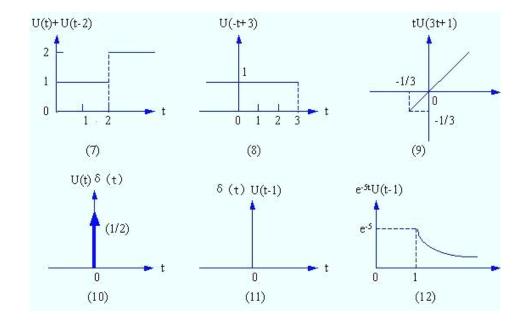
13-2 粗略画出下列时间函数的波形。 (2) tU(t+1);

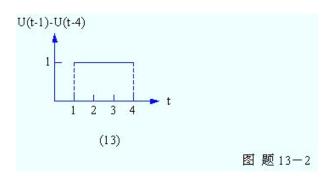
 $(3) (t-1) U(t-1) ; \qquad (4)-t U(t) ; \qquad (5) t U(t-1) \\ (6) U(t-1) U(t-2) ; \qquad (8) U(-t+3) ; \qquad (9) t U(3t+1) \\ (10) \delta(t) U(t)$ $(10) \delta(t) U(t)$ $(12) e^{-5t} U(t-1) ; \qquad (13) U(t-1) - U(t-4) .$

<u>答案</u>

解: 各波形相应如图题 13-2 所示。







13-3 求下列导数:

(1)
$$\frac{d}{dt}[u(t)-U(t-1)];$$

(2)
$$\frac{d}{dt}[u(t)\bullet U(t-1)]$$
;

(1)
$$\frac{d}{dt}[u(t)-U(t-1)];$$
 (2) $\frac{d}{dt}[u(t)\bullet U(t-1)];$ (3) $\frac{d}{dt}[e^{-at}U(t)];$ (4) $\frac{d}{dt}[e^{-5t}U(t-4)];$

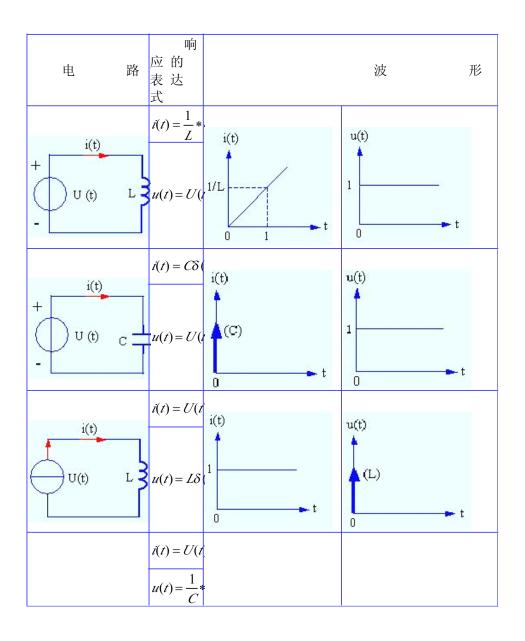
$$(4) \quad \frac{d}{dt} \left[e^{-5t} U(t-4) \right]$$

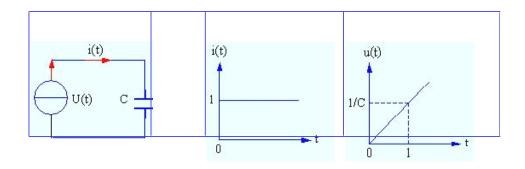
$$(5) \qquad \frac{d^2}{dt^2} [tU(t)]$$

答案

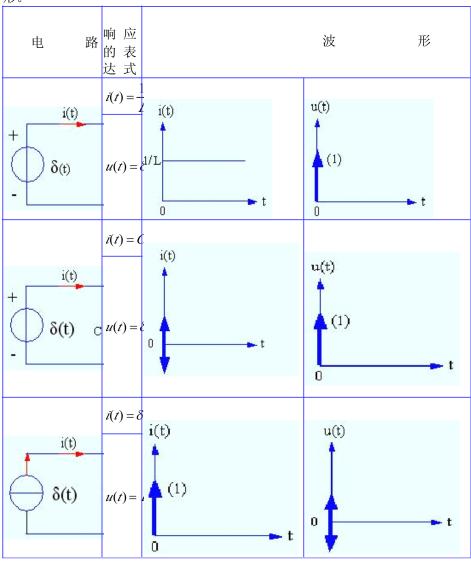
解: (1)
$$\delta(t) - \delta(t-1)$$
 ; (2) $\delta(t-1)$; (3) $\delta(t) - \alpha e^{-\alpha t} U(t)$; (4) $e^{-5t} \delta(t-4) - 5e^{-5t} U(t-4)$; (5) $\delta(t)$ 。

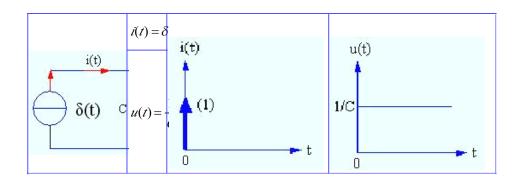
13-4 写出下表格单一元件电路的单位阶跃响应 i(t)、u(t)的表达式。画出波形。



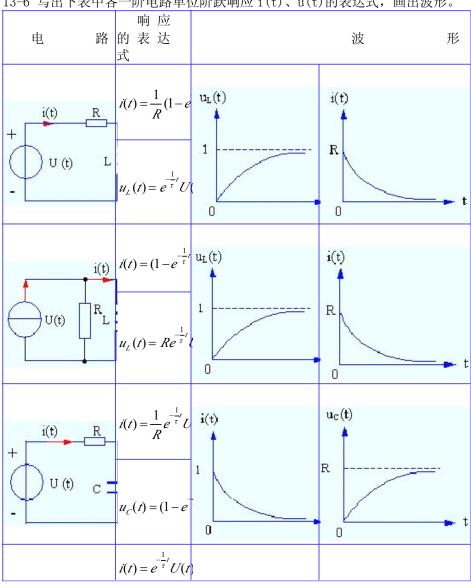


13-5 写出下表中个单一元件电路的单位冲激响应 i(t)、u(t)的表达式,画出波形。





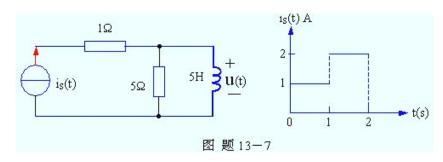
13-6 写出下表中各一阶电路单位阶跃响应 i(t)、u(t)的表达式, 画出波形。



$$u_{\mathcal{C}}(t) = R(1 - \epsilon)$$

13-7 图题 13-7 电路,激励 $i_s(t)$ 的波形如图所

示, $i(0^-)=0$ 。 求响应 u(t)。



<u>答案</u>

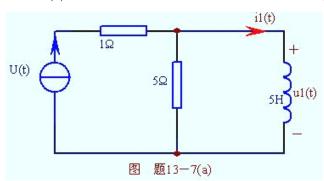
解: $i_s(t) = U(t) + U(t-2)A$ 。激励 U(t)产生的响应 $u_1(t)$ 按图 题 13-7 (a) 用三要素法求,即

$$u_1(0^-) = 5V$$
, $u_1(\infty) = 0$, $\tau = \frac{5}{5} = 1S$

故
$$u_1(t) = 5e^{-t}U(t)V$$
。

故 $i_s(t)$ 产生的响应u(t)为

$$u(t) = 5e^{-t}U(t) + 5e^{-(t-1)}U(t-1) - 10e^{-(t-2)}U(t-2)V_{o}$$



13-8 已知一阶线性定常电路,在相同的初始条件下,当激励为 f(t)时, [t<0时, f(t)=0],其全响应为

$$y_1(t) = 2e^{-t} + \cos 2t$$
: $t \ge 0$

当激励为 2f(t)时,其全响应为

$$y_2(t) = e^{-t} + 2\cos 2t$$
; $t \ge 0$

求激励为 4f(t)时的全响应 y(t)。

答案

解: 设零输入响应为 $\mathcal{Y}_x(t)$,激励 f(t)的零状态响应为 $\mathcal{Y}_x(t)$ 。则激励为 2f(t)产生的零状态响应为 $2\mathcal{Y}_x(t)$ 。故有

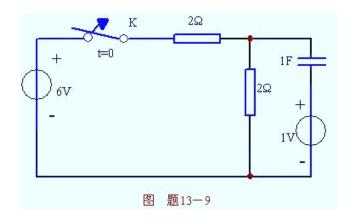
$$y_x(t) + y_1(t) = 2e^{-t} + \cos 2t$$

 $y_x(t) + 2y_1(t) = e^{-t} + 2\cos 2t$
联解得 $y_x(t) = 3e^{-t}U(t)$. $y_1(t) = -e^{-t} + \cos 2t$

激励为
$$4f(t)$$
时,全响应为
$$y(t) = y_x(t) + 4y_1(t) = 3e^{-t} + 4(-e^{-t} + \cos 2t)$$

$$= (-e^{-t} + 4\cos 2t)U(t)$$

13-9 图题 13-9 电路, t<0 时, K 闭合, 电路已达稳定状态.今于 t=0 时断开开关 K, 求 t>0 时的全响应 $u_c(t)$ 及 $u_c(t)$ 经过零值得时刻 t_o 。



答案

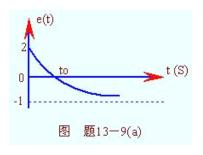
解:t<0 时,K 闭合,电路已达稳定状态,故有 $u_c(0^-)=3V$ 。t>0 时 K 打开,故有 $u_c(0^+)=u_c(0^-)=3V \ , \ u_c(\infty)=-1V \ \tau=RC=2S \ .$ 故得

$$u_c(t) = (-1 + 3e^{-0.5t})U(t)V$$

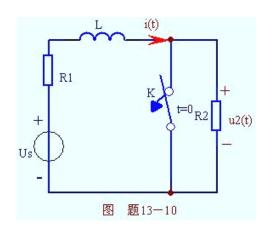
$$\chi$$
 $u_c(t_o) = 0 = -1 + 3e^{-0.5t_o}$

解得 $t_o = 2.2S$ 。

 $u_c(t)$ 的波形如图题 13-9-(a)所示。



13-10 图题 13-10 电路,t<0 时 K 闭合,电路已达稳定状态。求 t>0 时的响应 $u_2(t)$,并画出波形。



答案

解: t<0时, K闭合,电路已达稳定状态,故

$$i(0^-) = \frac{U_s}{R_1}$$

t>0 时, K 打开, 故有

$$i(0^+) = i(0^-) = \frac{U_s}{R_1}$$

$$u_2(0^+) = R_2 i(0^+) = \frac{R_2}{R_1} U_s$$

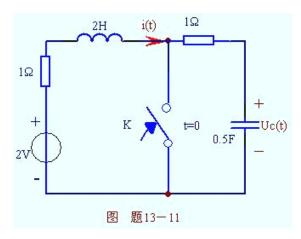
$$u_2(\infty) = \frac{R_2}{R_1 + R_2} U_s$$

北

$$u_2(t) = \frac{R_2}{R_1 + R_2} U_s (1 - e^{\frac{1}{\tau'}}) U(t) + \frac{R_2}{R_1} U_s e^{\frac{1}{\tau'}} U(t) V$$

13-11 图题 13-11 电路, t<0 时 K 打开, 电路已达稳定状态。今于 t=0 时闭合

K, 求 \triangleright 0 时的响应 $u_c(t)$ 、 $i_2(t)$ 、 i(t) , 画出波形。



<u>答案</u>

 $m: t \le 0$ 时,K 打开,电路已达稳定,故有

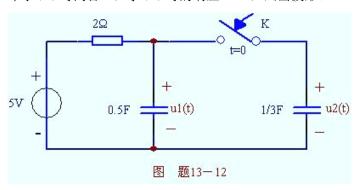
$$i_L(\infty) = \frac{2}{1} = 2A$$
, $u_c(\infty) = 0$;
 $\tau_1 = \frac{2}{1} = 2S$ $\tau_2 = 1 \times 0.5 = 0.5S$

故得
$$i_L(t) = (1 - e^{-\frac{1}{\tau_1}t})U(t)A$$
 $u_c(t) = 2e^{-\frac{1}{\tau_2}t})U(t)V$

$$i_C(t) = C\frac{du_c(t)}{dt} = -2e^{-2t}A$$

$$i(t) = i_L(t) - i_C(t) = \left[2(1 - 2e^{\frac{-1}{\tau_1}t}) + 2e^{-2t}\right]U(t)A$$

13-12 图题 13-12 电路,t<0 时 K 打开,电路已达稳定状态,且设 $u_2(0^-)=0$ 。 今于 t=0 时闭合 K,求 t>0 时的响应 $u_2(t)$,画出波形。



答案

解: t<0 时 K 打开, 电路已达稳定状态, 故有

$$u_1(0^-) = 5V$$
: $u_2(0^-) = 0$

t>0时 K闭合,此时有

$$u_1(0^+) = u_2(0^+)$$

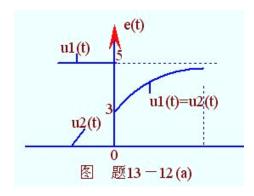
$$C_1u_1(0^+)+C_2u_2(0^+)=C_1u_1(0^-)+C_2u_2(0^-)$$

联解并代入数据得 $u_1(0^+) = u_2(0^+) = 3V_o$

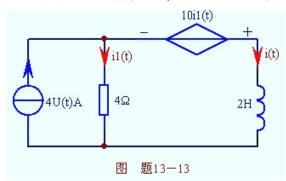
$$\chi$$
 $u_1(\infty) = u_2(\infty) = 5V$, $\tau = R(C_1 + C_2) = \frac{5}{3}S$

故得
$$u_1(t) = u_2(t) = (5 - 2e^{-0.6t})U(t)V_{\circ}$$

其波形图如图题 13-12(a) 所示。



13-13 图题 13-13 所示电路,已知 $i(0^-)=0$ 。求i(t),画出波形图。

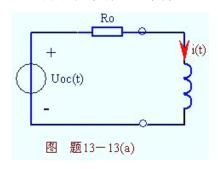


<u>答案</u>

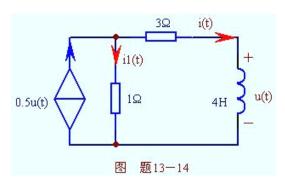
解: 求 i(t)的等效电路电压源电路如图题 13-13(a)所示。其中

$$u_{oc}(t) = 56U(t) R_0 = 14\Omega_{\circ}$$

于是用三要素法可求得 $i(t) = 4(1 - e^{-7t})U(t)A_{o}$



13-14 图题 13-14 所示电路,已知 $i(0^-)=2A$ 。求 $\mathbf{u}(\mathbf{t})$ 和 $i^-(t)$,画出波形图。



答案

解:
$$i(0^+) = i(0^-) = 2 A i(\infty) = 0$$
;由图题 13-14(a)有:

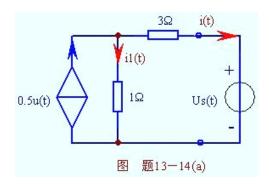
$$U_s = 3I_s + (0.5U_s + I_s) \times 1$$

故得
$$R_0 = \frac{U_s}{I_s} = 8\Omega, \quad \tau = \frac{L}{R_0} = 0.5S.$$

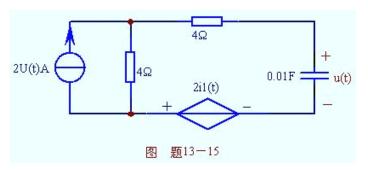
故得
$$i(t) = 2e^{-2t}U(t)A$$

$$u(t) = L \frac{di(t)}{dt} = -16e^{-2t}U(t)V$$

$$i_1(t) = 0.5u(t) - i(t) = -10e^{-2t}U(t)A$$



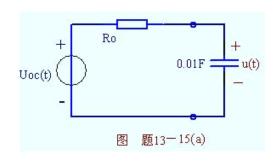
13-15 图题 13-15 所示电路,已知 $u(0^-)=0$,求 u(t)。



答案

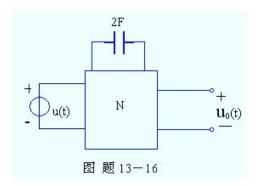
解:求 u(t)的等效电压源电路如图题 13-15(a)所示。其中 $u_{oc}(t)=12U(t)V$, $R_0=10\Omega$ 。于是用三要素法可求得

$$u(t) = 12(1 - e^{-10t})U(t)_{V_0}$$



13-16 图题 13-16 所示电路,N 内部只含有点源与电阻,零状态响应 $u_o(t) = (\frac{1}{2} + \frac{1}{8}e^{-0.25t})U(t)$ 伏。今把 2F 得电容换成 2H 电感,问响应 $u_o(t) = ?$

提示:接电感时的初始值与接电容式的稳态值相等。



<u>答案</u>

得

解: 因有
$$u_o(0^+) = u_o(\infty) = \frac{1}{2}V$$

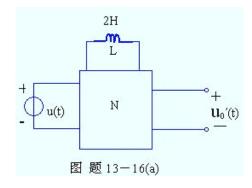
$$u_o(\infty) = u_o(0^+) = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}V$$

又因有
$$\tau = RC = \frac{1}{0.25}$$

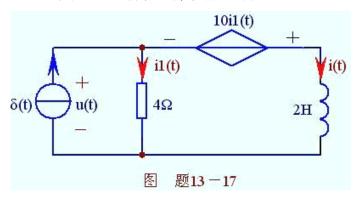
故得
$$R = \frac{1}{0.25C} = 2\Omega$$

故又得
$$\tau' = \frac{L}{R} = \frac{2}{2} = 1S$$

$$u_o'(t) = u_o'(\infty) - [u_o'(\infty) - u_o'(0^+)]e^{-\frac{1}{\tau'}t} = (\frac{5}{8} - \frac{1}{8}e^{-t})U(t)V$$



13-17 图题 13-17 所示电路, 求响应 i(t)。

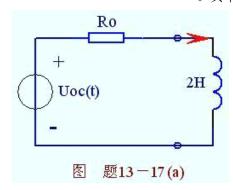


答案

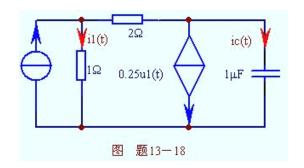
解: 求 i(t)的等效电压源电路如图题 14-17(a)所示,其中 $u_{oc}(t)=14\delta(t)V$, $R_0=14\Omega$ 故得

$$i(t) = i(0^+)e^{-\frac{1}{\tau}t} = 14 \times \frac{1}{L}e^{-7t}U(t)A$$

$$\tau = \frac{L}{R_0} = \frac{1}{7S}$$



13-18 图题 13-18 所示电路, $u_c(0^-)=2$, 求全响应 $\dot{i}(t)$ 、 $\dot{i}_c(t)$ 、 $u_c(t)$ 。



答案

解: 按图题 13-18(a)和(b)分别求等效电压源的电压 $u_{oc}(t)$ 和 R_{0} ,即

$$u_{oc}(t)=4U(t)$$
, $R_0=\frac{U_s}{I_s}=2.4\Omega$ 。 其等效电压源电路如图题 13-18(c)所示。于是用三要素法可求得

$$u_{c}(t) = u_{c}(\infty) - [u_{c}(\infty) - u_{c}(0^{+})]e^{-\frac{1}{\tau}t}$$

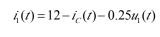
$$= (4 - 2e^{-\frac{1}{\tau}t})U(t)V$$

$$\ddagger$$

$$\tau = R_{0}C = 2.4 \times 1 \times 10^{-6} = 2.4 \times 10^{-6} S$$

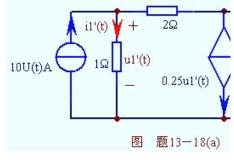
$$X$$

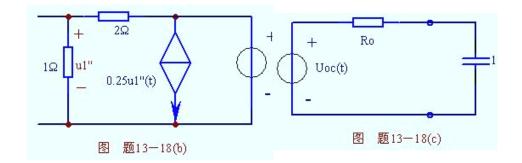
$$i_{c}(t) = C\frac{du_{c}(t)}{dt} = 0.8333e^{-\frac{1}{\tau}t}U(t)A$$



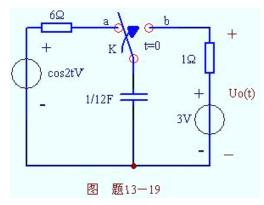
$$=10-i_{C}(t)-0.25i_{1}(t)$$

故 得 $\dot{q}(t) = (8 - 0.667e^{-\frac{1}{\tau}t})U(t)A$ 。





13-19 图题 13-19 所示电路,t<0 时 K 在 a 点,电路已达稳定.今于 t=0 时将 K 扳道 b 点,求 t>0 时的全响应 $u_0(t)$ 。



答案

解 t<0时,K在a点,电路已达稳

定.

$$\dot{U}_s = \frac{1}{\sqrt{2}} \angle 0^{\circ} V$$

$$Z = R + \frac{1}{j\omega C} = 6 - j6$$

$$= 6\sqrt{2} \angle - 45^{\circ} \Omega$$

$$\dot{U}_C = \frac{\dot{U}_s}{Z} \times \frac{1}{j\omega C} = 0.5 \angle -45^{\circ} V$$

$$u_c(t) = 0.5\sqrt{2}\cos(2t - 45^\circ)V$$

故
$$u_c(0^-) = 0.5\sqrt{2}\cos(-45^\circ) = 0.5V$$

t>0时, K扳到b点. 用三要素法可求

得:

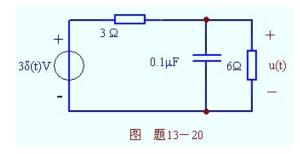
$$u_0(0^+) = u_c(0^+) = u_c(0^-) = 0.5V$$

$$u_0(\infty) = u_c(\infty) = 3V,$$

$$\tau = RC = \frac{1}{12}S$$

得
$$u_0(t) = 3 - (3 - 0.5) = (3 - 2.5e^{-12t})U(t)V$$

13-20 图题 13-20 所示电路, 求 u(t)。

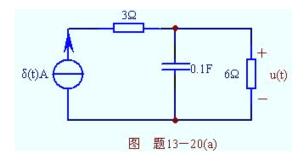


答案

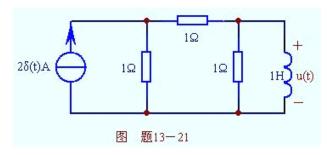
解: 求 u(t)的等效电路如图题 13-10(a) 所示。

故
$$u(0^+) = \frac{1}{C} = 10V$$
, $u(\infty) = 0$, $\tau = \frac{3 \times 6}{3 + 6} \times 0.1 = 0.2S$

故得
$$u(t) = u(0^+)e^{-\frac{1}{\tau}t} = 10e^{-5t}U(t)V_{\circ}$$



13-21 图题 13-21 所示电路。求 u(t)。



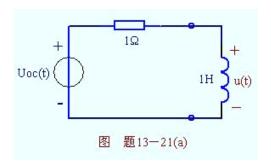
答案

解: 求 u(t)、 i(t)的等效电压源电路如图题 13–21(a)所示。其中 $u_{oc}(t)=\delta(t)V_{,}~~R_0=1\Omega_{~o.}$ 故有

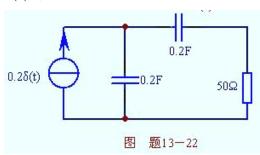
$$i(0^{+}) = \frac{1}{L} = 1A, \quad i(\infty) = 0, \quad \tau = \frac{L}{R_0} = 1S$$

$$i(t) = i(0^{+})e^{-\frac{1}{\tau'}} = 1e^{-t}U(t)A$$

$$u(t) = \frac{di(t)}{dt} = \delta(t) - e^{-t}U(t)V$$
故



13-22 图题 13-22 所示电路,求 u(t)。



答案

解: t>0 时的等效电路如图题 13-22(a) 所示。

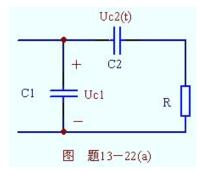
$$u_{C1}(0^+) = 0.2 \frac{1}{C_1} = 1V$$

$$u_{C2}(0^+) = u_{C2}(0^-) = 0$$

$$u(0^+) = u_{c1}(0^+) - u_{c2}(0^+) = 1V$$

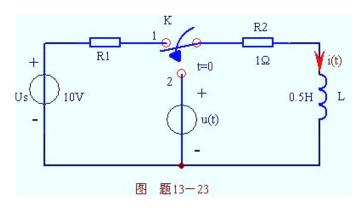
$$u(\infty) = 0$$
, $\tau = R \frac{C_1 C_2}{C_1 + C_2} = 6S$

故
$$u(t) = 1e^{-\frac{1}{6}t}U(t)V$$
。



13-23 图题 13-23 所示电路,t<0 时,K 接在 " 1 ",电路已达稳定。今于 t=0 时将 K 接道 " 2 "。今欲使 t>0 时电路中的电流只存在正弦稳态响应,

问应选多达数值? 已知 $u(t) = 10\cos 2tV$ 。



答案

m: t<0时 K 在 1 ,电路已达稳定,故有

$$i(0^-) = \frac{1}{R_1 + 1} A$$

▷0 时 K 在 2 。 先求零输入响应 $i_x(t)$:

$$i_x(0^+) = i(0^-) = \frac{1}{R_1 + 1} A$$

$$i_x(\infty) = 0, \quad \tau = \frac{L}{R_2} = 0.5 S$$

故得
$$i_x(t) = \frac{1}{R_1 + 1} e^{-2t} U(t) A$$
。

再求零状态响应 $i_f(t)$:

$$\dot{U} = \frac{10}{\sqrt{2}}V$$

$$Z = R_2 + j\omega L = 1 + j1 = \sqrt{2} \angle 45^{\circ} \Omega$$

$$\dot{I} = \frac{\dot{U}}{Z} = 5 \angle -45^{\circ} A$$

故得正弦稳态响应为 $i_s(t) = 5\sqrt{2}\cos(2t - 45^\circ)A_o$

其瞬态响应为 $i_{t}(t) = Be^{-\frac{1}{\tau}t} = Be^{2t}U(t)A_{\circ}$

故零状态响应为 $i_r(t) = i_s(t) + i_r(t) = 5\sqrt{2}\cos(2t - 45^\circ) + Be^{-2t}$

$$i_f(0^+) = 0 = 5\sqrt{2}\cos(-45^\circ) + B$$
,

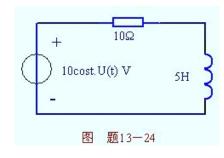
故得 B=-5

故
$$i_f(t) = 5\sqrt{2}\cos(2t - 45^\circ) - 5e^{-2t}U(t)A$$

 $i(t) = i_x(t) + i_f(t) = \frac{1}{R_1 + 1} e^{-2t} + 5\sqrt{2}\cos(2t - 45^\circ) - 5e^{-2t}$ 故全响应为

可见,欲使
$$i(t)=5\sqrt{2}\cos(2t-45^{\circ})A$$
,则必须有 R_1+1 -5=0,故得 $R_0=1\Omega$ 。

13-24 图题 13-24 所示电路,为使全响应电路 i(t)中的瞬态响应分量为零。求的 $i(0^-)$ 。



<u>答案</u>

解: 零输入响应为
$$i_x(t) = i_x(0^+)e^{-\frac{1}{\tau}t} = i_x(0^-)e^{-2t}U(t)A$$
,

其中
$$\tau = \frac{L}{R} = \frac{5}{10} = 0.5S$$
。

零状态响应为 $i_f(t)$:

$$\dot{U}_s = \frac{10}{\sqrt{2}}V$$

$$Z = 10 + j\omega L = 11.18 \angle 26.57^{\circ} \Omega$$

$$\dot{I} = \frac{\dot{U}_s}{Z} = 0.633 \angle -26.57^{\circ} A$$

故得正弦稳态响应为 $i_s(t) = 0.633\sqrt{2}\cos(t - 26.57^\circ)A$

自由响应为
$$i_f(t) = Be^{-2t}U(t)A$$

故得零状态响应为

$$i_f(t) = i_t(t) + i_s(t) = Be^{-2t} + 0.633\sqrt{2}\cos(t - 26.57^\circ)A$$

$$\inf_{t \mid t} i_f(0^+) = 0 = B + 0.633\sqrt{2}\cos(-26.57^\circ)$$

故得
$$B = -0.797$$

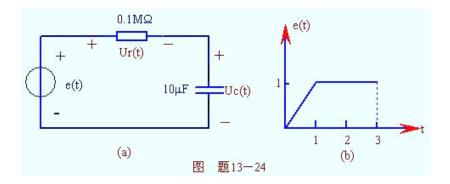
故得
$$i_f(t) = -0.797 e^{-2t} + 0.633\sqrt{2}\cos(t - 26.57^\circ)A$$

故得全响应为

$$i(t) = i_x(t) + i_x(t) = i(0^-)e^{-2t} - 0.797 + 0.633\sqrt{2}\cos(t - 26.57^\circ)A$$

故欲使
$$i(t) = 0.633\sqrt{2}\cos(t-26.57^{\circ})A$$
,则必须有 $i(0^{-}) = 0.797$ A。

13-25 图(a)所示电路,e(t)的波形如图(b)所示。求零状态响应 $u_c(t)$ 和 $u_R(t)$ 。



答案

解:单位冲激响应

$$h(t) = e^{-t}U(t)$$

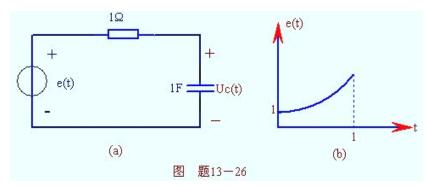
$$\pm u_C(t) = h(t) * e(t)$$

$$= \begin{cases} 0, t < 0 \\ -1 + t + e^{-t}, 0 \le t \le 1 \\ 1 + e^{-1} - e^{-(t-1)}, 1 \le t \le 3 \\ e^{-t} - e^{-(t-1)} - e^{-(t-3)}, t > 3 \end{cases}$$

$$u_R(t) = e(t) - u_C(t)$$

$$= \begin{cases} 0, t < 0 \\ 1 - e^{-t}, 0 \le t \le 1 \\ e^{-(t-1)} - e^{-t}, 1 \le t \le 3 \\ e^{-(t-1)} - e^{-t} + e^{-(t-3)}, t > 3 \end{cases}$$

13-26 图(a)所示电路, $e(t) = e^t[U(t) - U(t-1)]$ 其波形图如图所示。求零状态响应 $u_c(t)$ 。



答案

解:
$$h(t) = e^{-t}U(t)$$

$$u_C(t) = h(t) * e(t)$$

$$= \begin{cases} \frac{1}{2}(e^{t} - e^{-t}), 0 \le t \le 1\\ \frac{1}{2}e^{-(t-2)} - \frac{1}{2}e^{-t} = 3.2e^{-t}, \quad t \ge 1 \end{cases}$$