Ensuring Non-Opacity in Discrete Event Systems

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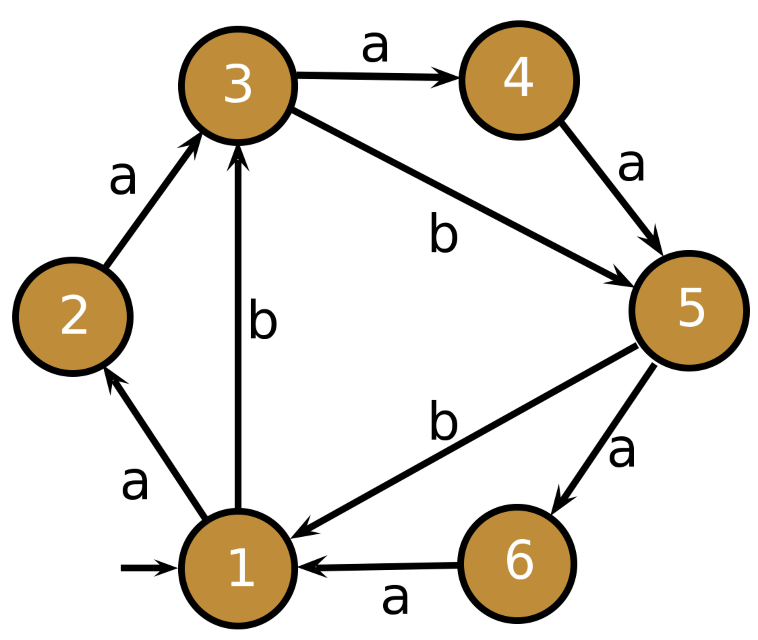
Introduction

In order to model systems where there are a finite number of states and changes in states after discrete events, the field of discrete event systems uses finite automata, written as a 4-tuple (Q, Σ, δ, q_o) with a corresponding directed graph representation:



A Discrete Event System

- $G = (Q, \Sigma, \delta, q_o)$
- $Q = \{1,2,3,4,5,6\}$, $q_o = 1$
- $\Sigma = \{a, b\}$, $\Sigma^* = \{ab, aabba, ...\}$
- $\delta : Q \times \Sigma^* \to Q$
  e.g. $\delta(1,a) = 2$, $\delta(5, ba) = 2$
- $\delta = \{(1,a,2),(2,a,3),(1,b,3),...\}$ is also the set of directed edges in the graph

- $\delta(q, st) = \delta(\delta(q,s), t)$
- Agent: ability to observe $\delta' \subseteq \delta$, ability to send information according to a policy $com : L(G) \to 2^{\Sigma}$.
- We are interested in an agent's ability to distinguish certain states (opacity) in a system with two agents who are communicating to each other.

Problem Statement

Given a plant $G = (Q, \Sigma, \delta, q_o)$, two agents who can observe $\delta_1, \delta_2 \subseteq \delta$ respectively, the set of secret states Q_L, and the set of non-secret states Q_K. To find a set of observer-based (plant-based) policy implementations $(G_1, G_2, \varphi_1, \varphi_2)$ that are minimal and make the system non-opaque to at least one of the agents with respect to Q_K and Q_L.

Definitions/Theorems

Observation under Communication: Given agents who observe $\delta_1, \delta_2 \subseteq \delta$ and each have policy com_{21}/com_{12}.
Then $\mathcal{C}(com_{21}, com_{12}) := (\theta_{21}, \theta_{12})$ defined as follows:

$$\theta_{ij}(\epsilon) = \epsilon$$

$\forall s \in \Sigma^*, \forall \sigma \in \Sigma,$

$$\theta_{ij}(s\sigma) = \begin{cases} \theta_{ij}(s)\sigma & \text{if } (\delta(q_o,s),\sigma,\delta(q_o,s\sigma)) \in \delta_j \\ & \vee((\delta(q_o,s),\sigma,\delta(q_o,s\sigma)) \in \delta_i \wedge \sigma \in com_{ij}(s)) \\ \theta_{ij}(s) & \text{otherwise} \end{cases}$$

State Estimation $SE^{\theta} : \theta(L(G)) \to Q$ is an agent's estimation of the system's state. Formally,

$$SE^{\theta}(s) = \{q \in Q : \exists t \in L(G), \delta(q_o, t) = q \wedge \theta(t) = s\}$$

Opacity: Given two sets of states $Q_K, Q_L \subseteq Q$, the system is opaque under θ with respect to Q_K and Q_L if

$$\exists s \in L(G), (Q_K \cap SE^{\theta}(s) \neq \emptyset) \wedge (Q_L \cap SE^{\theta}(s) \neq \emptyset),$$

i.e., states in Q_L cannot be distinguished form states in Q_K.

Observer: Given an agent whose observation is characterized by some θ, one can create an *observer* by relabeling all unobservable transitions as ϵ and doing an NFA-DFA transformation.

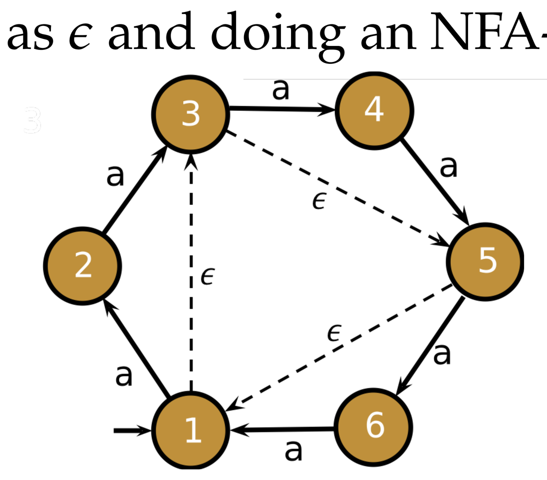


Figure 1: NFA: $(Q, \Sigma, \delta_{\epsilon}, q_o)$
$\delta_1 = \{\text{all transitions by event a}\}$
$\forall s \in L(G), com_{21}(s) = \emptyset$

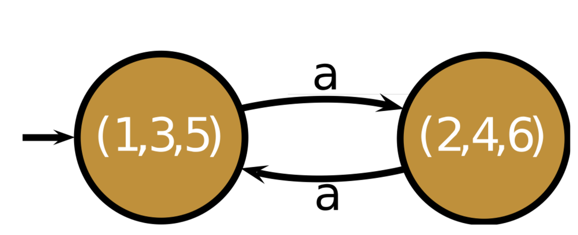


Figure 2: DFA: Observer (X, Σ, ξ, x_o)
$SE^{\theta}(aaa) = \xi(x_o, aaa) = \{2,4,6\}$

Theorem: A state in the observer automaton represents the agent's estimation of the system's current state. Formally,

$$SE^{\theta}(s) = \xi(x_o, s),$$

where ξ is the transition function of the observer.
As a result, a system is opaque under θ iff

$$\exists s \in \theta(L(G)), \xi(x_o, s) \cap Q_L \neq \emptyset \wedge \xi(x_o, s) \cap Q_K \neq \emptyset$$

Constraints

Implementable: A communication policy $com : L(G) \to 2^{\Sigma}$ under θ is implementable if there exists (H, φ) where $H = (Y, \Sigma, \eta, y_o)$ so that $L(H) = \theta(L(G))$, $\varphi : Y \to 2^{\Sigma}$, and for any $s \in L(G)$,

$$com(s) = \varphi(\eta(y_o, \theta(s))).$$

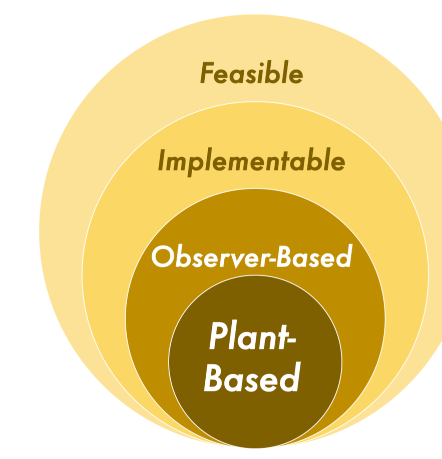


Figure 5: Hierarchy of Different Constraints

Feasibility: A policy $com : L(G) \to 2^{\Sigma}$ is feasible under θ if

$$\forall s, t \in L(G), \theta(s) = \theta(t) \Rightarrow com(s) = com(t).$$

Observer-Based: A pair of communication policies with the finite implementation $(H_1, H_2, \varphi_1, \varphi_2)$ is observer-based if

$$\eta_i(y_{io}, s) = SE^{\theta_{ji}}(s).$$

Plant-Based: A policy (H_i, φ_i) is plant-based if

$$\forall Q \in X_i, \forall q \in Q, (\delta(q,\sigma)! \wedge \sigma \in \varphi_i(Q)) \Rightarrow (q \in Q' \in X_i \Rightarrow \sigma \in \varphi_i(Q'))$$

Challenges

Circular Dependency: What Agent 1 sends to Agent 2 affects what Agent 2 can send. *How can you tell that two policies are feasible with respect to each other?*

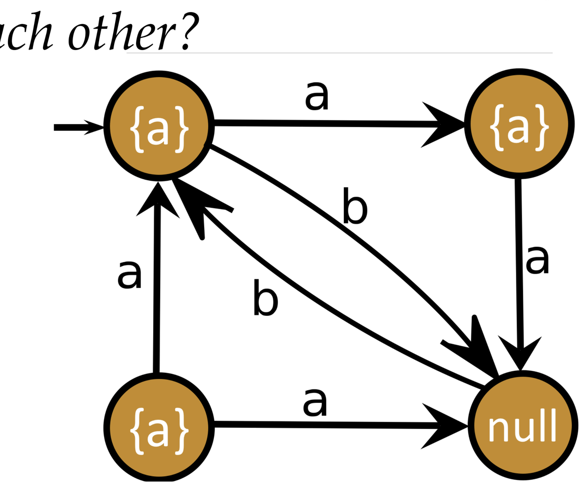


Figure 3: Potential Policy for Agent 1

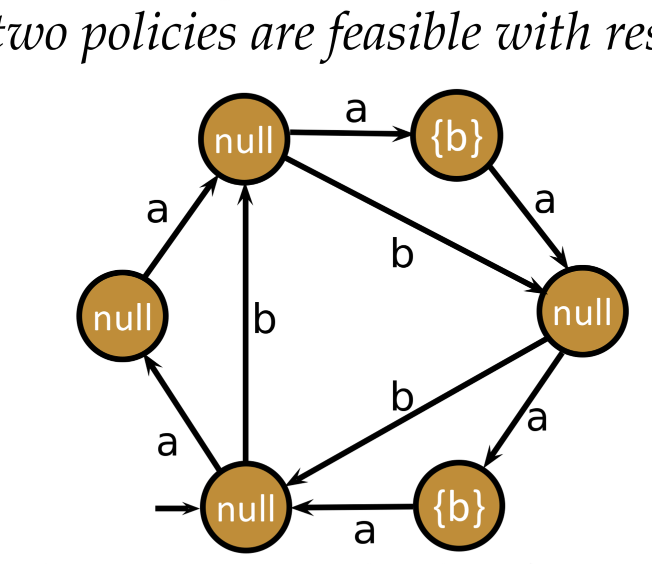


Figure 4: Potential Policy for Agent 2

Results and Conclusions

- Checking Feasibility:
  $\mathcal{R}(H_1, H_2, \varphi_1, \varphi_2) := (X_R, \Sigma, \xi_R, r_o)$ defined as follows:
  $r_o := (q_o, x_{1o}, x_{2o})$, and $\forall c \in \Sigma$ such that $\delta(q,c)!$,

$$\xi_R(r,c) = \begin{cases} (\delta(q,c), \eta_1(Q_a,c), \eta_2(Q_b,c)) & \text{if } c \in o_1(r) \wedge c \in o_2(r) \\ (\delta(q,c), \eta_1(Q_a,c), Q_b) & \text{if } c \in o_1(r) \wedge c \notin o_2(r) \\ (\delta(q,c), Q_a, \eta_2(Q_b,c)) & \text{if } c \notin o_1(r) \wedge c \in o_2(r) \\ (\delta(q,c), Q_a, Q_b) & \text{if } c \notin o_1(r) \wedge c \notin o_2(r) \end{cases}$$

  where

$$r = (q, Q_a, Q_b)$$
$$o_1(q, Q_a, Q_b) := \{c \in \Sigma : (q,c) \in \delta_1 \vee ((q,c) \in \delta_2 \wedge c \in \varphi_2(Q_b))\}$$
$$o_2(q, Q_a, Q_b) := \{c \in \Sigma : (q,c) \in \delta_2 \vee ((q,c) \in \delta_1 \wedge c \in \varphi_1(Q_a))\}$$

  A pair of policies is feasible iff $\delta(q,c)! \Rightarrow \xi_R(r,c)!$ everywhere, i.e. $c \in o_i(r) \Rightarrow \eta_i(Q_x, c)$, in which case we call the "run-through" successful.
- Checking Opacity: Epsilonize ξ_R according to o_1 and o_2 is equivalent to creating the observer
- Algorithm for Finding Plant-Based Solutions: complete and sound, $O(2^{|\delta|}(|Q|^2 \cdot |\Sigma| + |Q|^3))$
- Minimality: guaranteed by enumeration

References

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More Information

Email flin@g.hmc.edu
Download the PDF for this poster at https://github.com/feiyanglin/FUSRP-OpacityProjectPoster/releases/latest

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