Chapter 06
Robust Estimation of Homography

Lin ZHANG
School of Computer Science and Technology
Tongji University

Outline

- RANSAC—A Framework for Parametric Model Estimation
- Image Interpolation
- Practice for Panorama Stitching

RANSAC—A framework for parametric model estimation

Let's review Chapter 5

Homography estimation—Modeled as a homogeneous linear LS problem

Suppose that by matching keypoints on $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$, we get $\mathcal{S} = \{\boldsymbol{x}_i \leftrightarrow \boldsymbol{x}_i^{'}\}_{i=1}^{p}$

where $\boldsymbol{x}_i \leftrightarrow \boldsymbol{x}_i^{'}$ means that the point $\boldsymbol{x}_i$ from $\boldsymbol{I}_1$ and the point $\boldsymbol{x}_i^{'}$ from $\boldsymbol{I}_2$ matches, and p is the number of correspondence pairs

temporarily suppose there is no outliers in $\mathcal{S}$

All the pairs can be linked via a universal 2D linear geometric transformation $\boldsymbol{H}_{3\times3}$

If there is no prior knowledge about the geometric transformation between $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$

$\boldsymbol{H}$ can take the most general form, the 2D projective transformation,

$$\forall \boldsymbol{x}_i \leftrightarrow \boldsymbol{x}_i^{'} \in \mathcal{S}, \quad c_i \boldsymbol{x}_i^{'} = \boldsymbol{H}_{3\times3}\boldsymbol{x}_i$$

Since all the keypoints are normal points (instead of infinity points), we can suppose $\boldsymbol{x}_i$ and $\boldsymbol{x}_i^{'}$ are of the form "normalized homogeneous coordinate"

$$c_i \begin{bmatrix} x_i^{'} \\ y_i^{'} \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

However, this condition does not hold in real case; that is, in real applications, the observation set $\mathcal{S}$ contains outliers nearly for sure

If all the data in $\mathcal{S}$ are fed into the LS program to solve $\boldsymbol{H}$, the resultant $\boldsymbol{H}$ will not be accurate

How to get rid of outliers when solving $\boldsymbol{H}$? In other words, we expect that only the **inliers** are involved to solve $\boldsymbol{H}$ while the outliers are excluded

RANSAC

RANdom SAmple Consensus (RANSAC) is a framework to estimate a parametric model from observations with outliers

RANSAC—A framework for parametric model estimation

算法6-1：RANSAC模型拟合算法

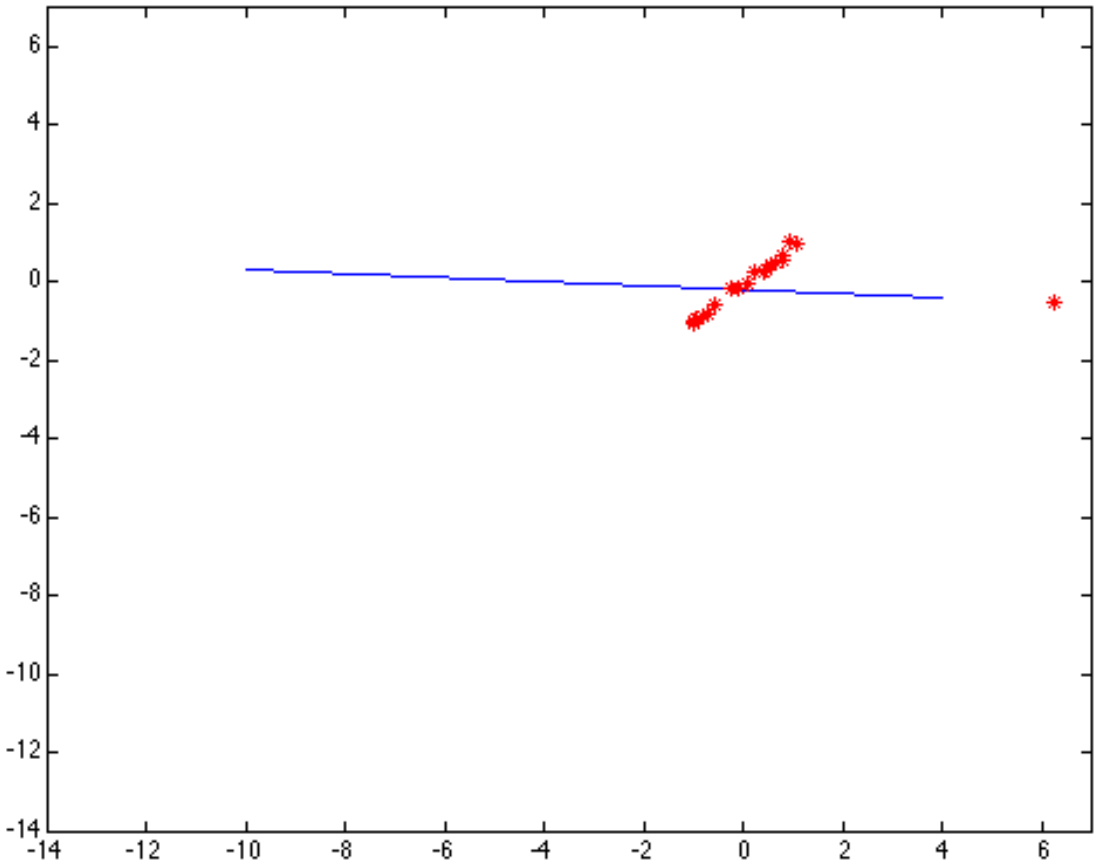
```
输入：
   data //观测数据集
   n //拟合模型所需要的最少的数据点个数
   k //最大允许迭代次数
   t //阈值，若数据点带入模型所得误差小于t，则认为该数据点属于该模型的一致集
   d //阈值，若当前模型的一致集中数据点的个数多于d，则认为该一致集已经足够好

输出：bestFit //拟合出来的模型参数，若为空则表明拟合失败

iterations = 0
bestFit = null
bestErr = something really large

while iterations < k do
   maybeInliers := n randomly selected values from data
   maybeModel := model parameters fitted to maybeInliers
   alsoInliers := empty set
   for every point in data not in maybeInliers do //计算maybeModel的一致集
      if point fits maybeModel with an error smaller than t
         add point to alsoInliers
      end if
   end for
   if the number of elements in alsoInliers is > d then
      // 这意味着我们可能已经找到了一个很好的模型
      // 把该模型从当前一致集中拟合出来
      betterModel := model parameters fitted to all points in maybeInliers and alsoInliers
      thisErr := a measure of how well betterModel fits these points
      if thisErr < bestErr then //完成输出模型及其误差更新
         bestFit := betterModel
         bestErr := thisErr
      end if
   end if
   increment iterations
end while
return bestFit
```

Note: For RANSAC, there is no universal “correct” implementation. If necessary, its implementation can be adjusted to fit the characteristics of specific applications

RANSAC

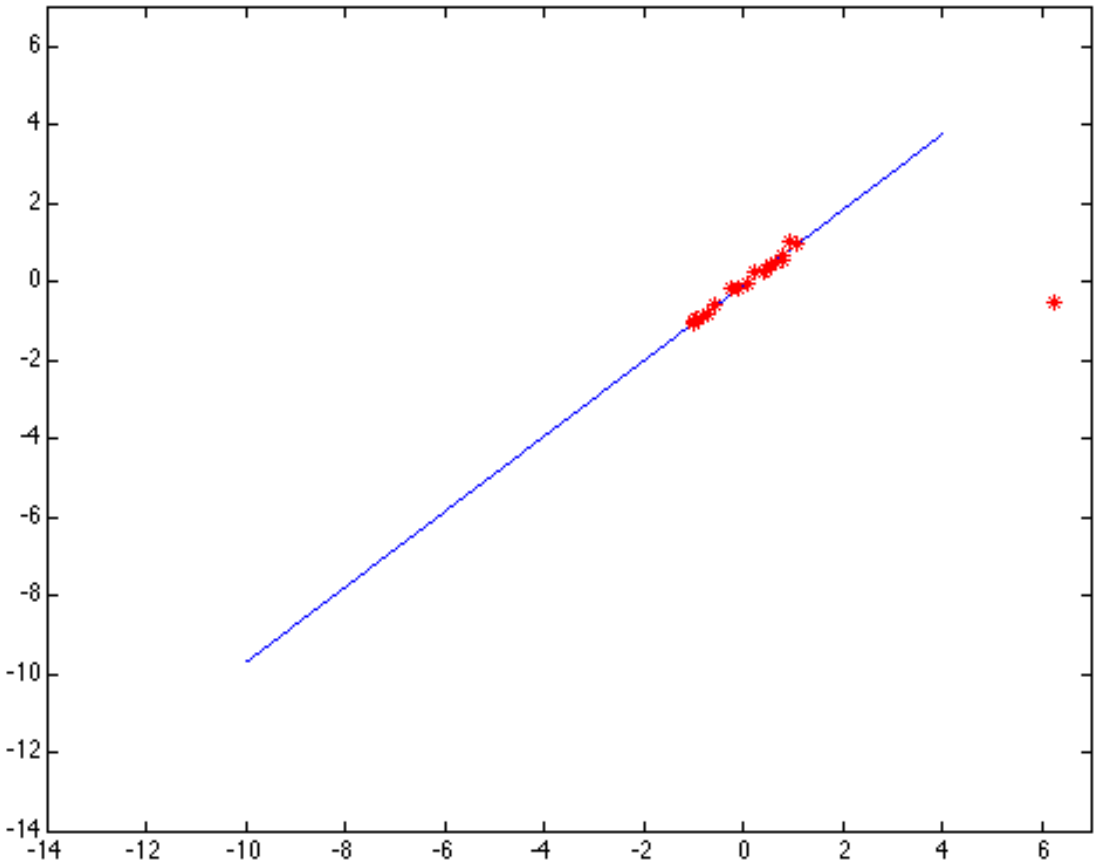
FISCHLER M A, BOLLES R C. Random sample consensus: A paradigm for model fitting with applications to image analysis and automated cartography[J]. Communications of the ACM, 1981, 24(6): 381-395.

RANSAC—Illustration by line-fitting



The result of line fitting using all the data points

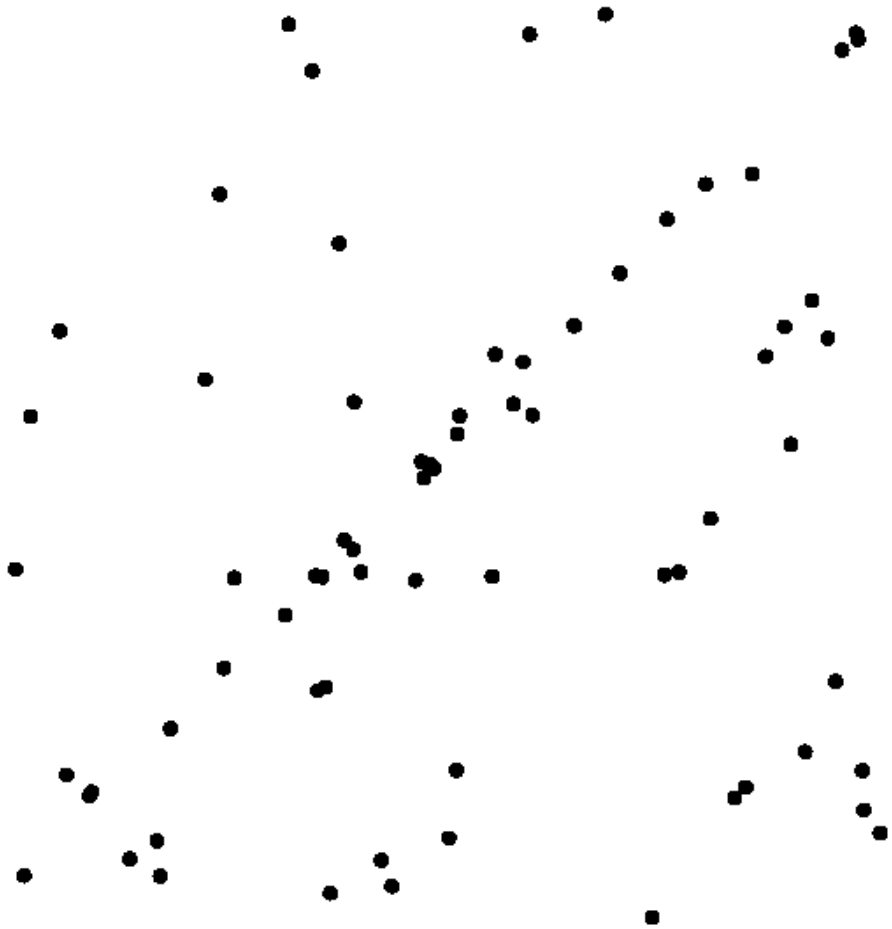
RANSAC—Illustration by line-fitting



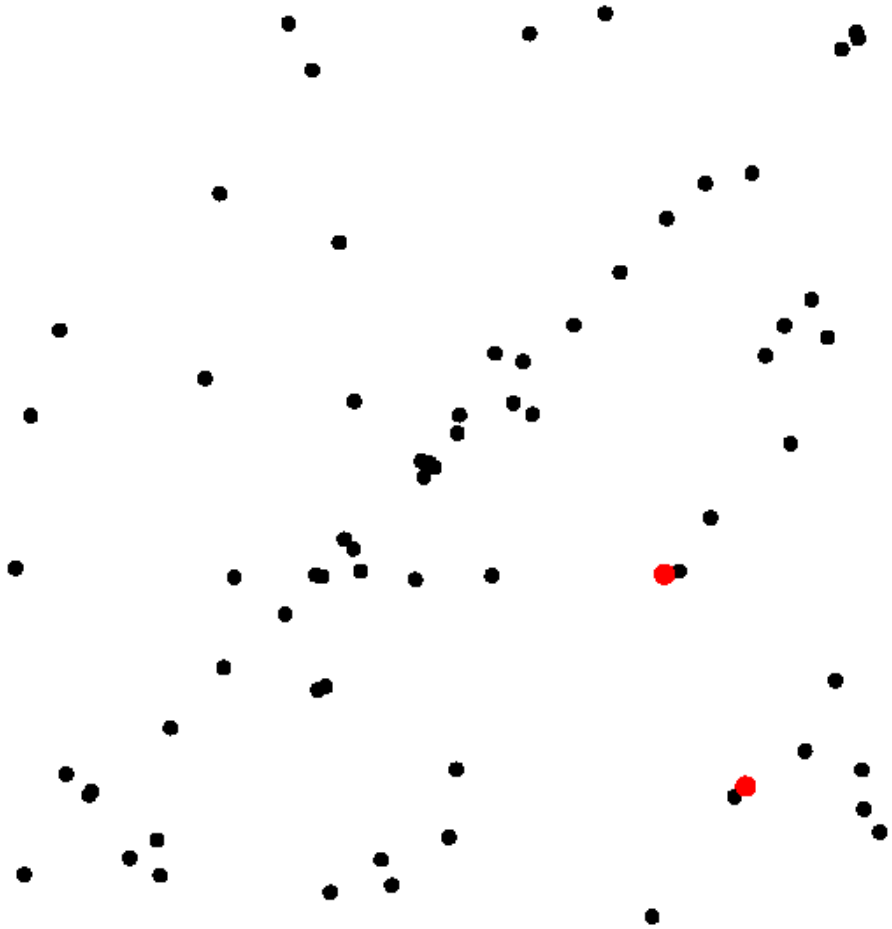
The result of line fitting using RANSAC. With RANSAC, the outlier actually is not involved in line fitting

RANSAC—Illustration by line-fitting

Line fitting by RANSAC



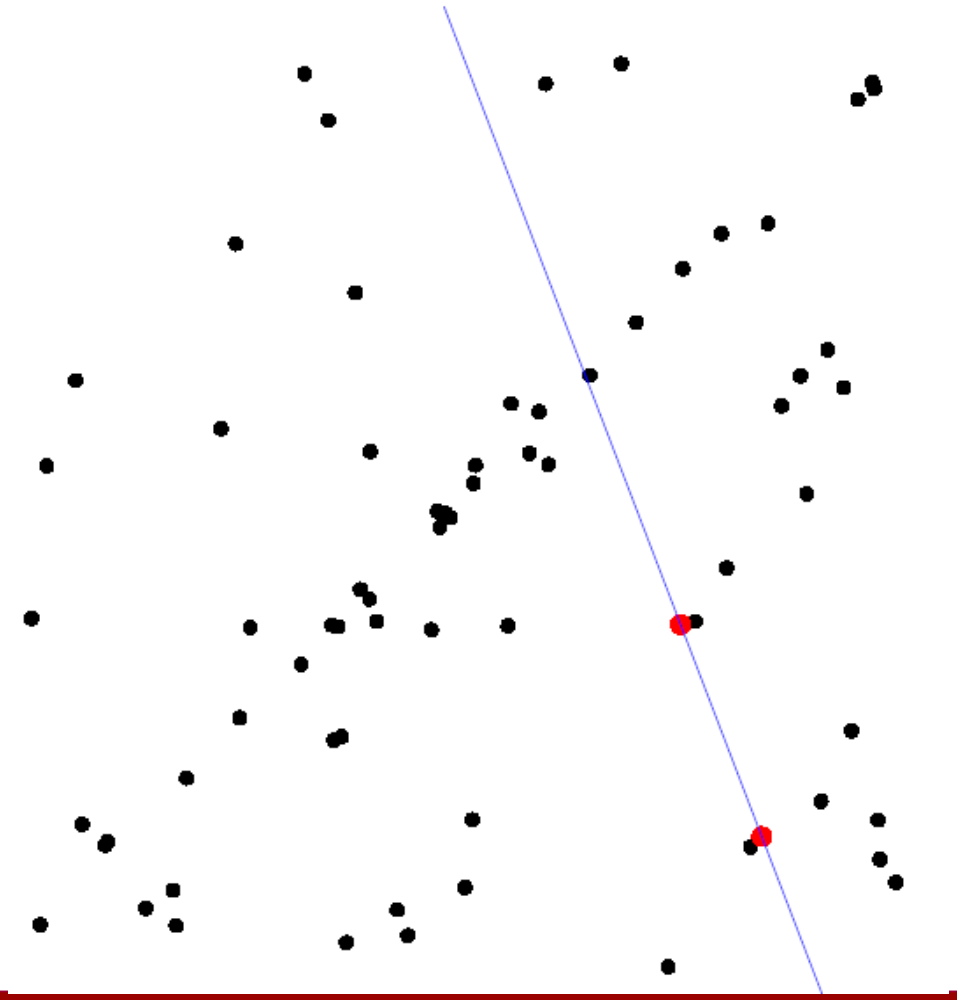
RANSAC—Illustration by line-fitting



Line fitting by RANSAC

- Randomly select two points

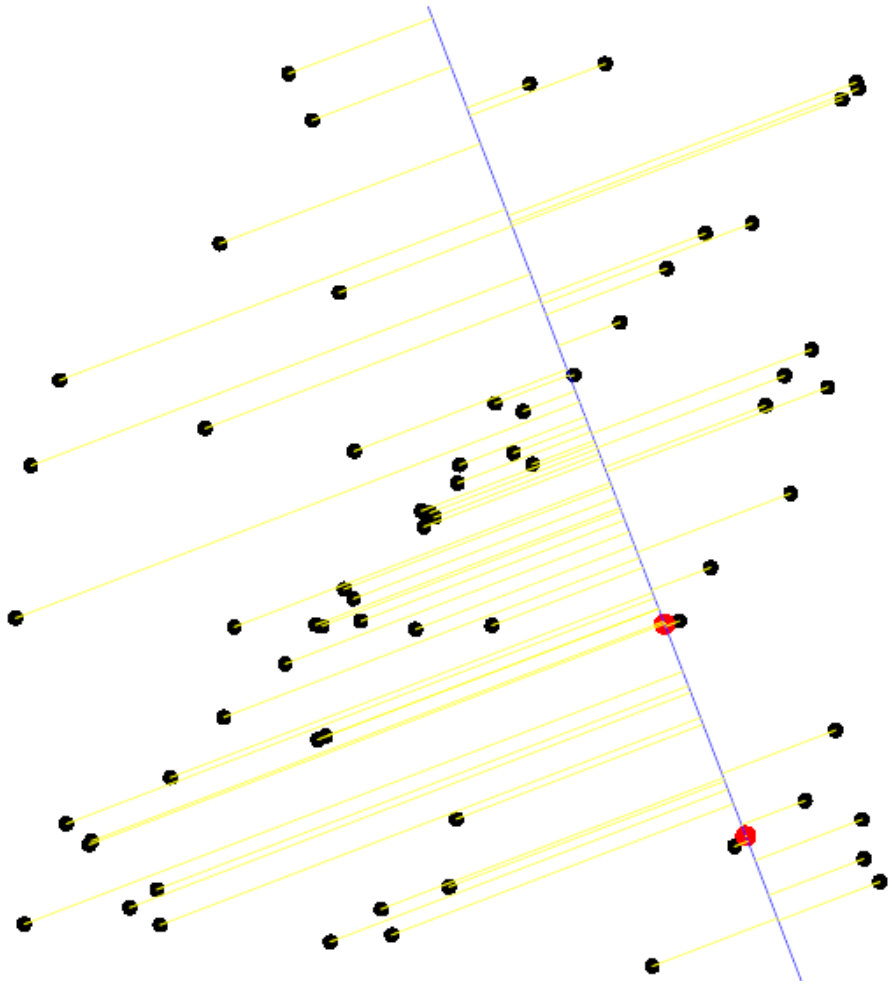
RANSAC—Illustration by line-fitting



Line fitting by RANSAC

- Randomly select two points
- The hypothesized model is the line passing through the two points

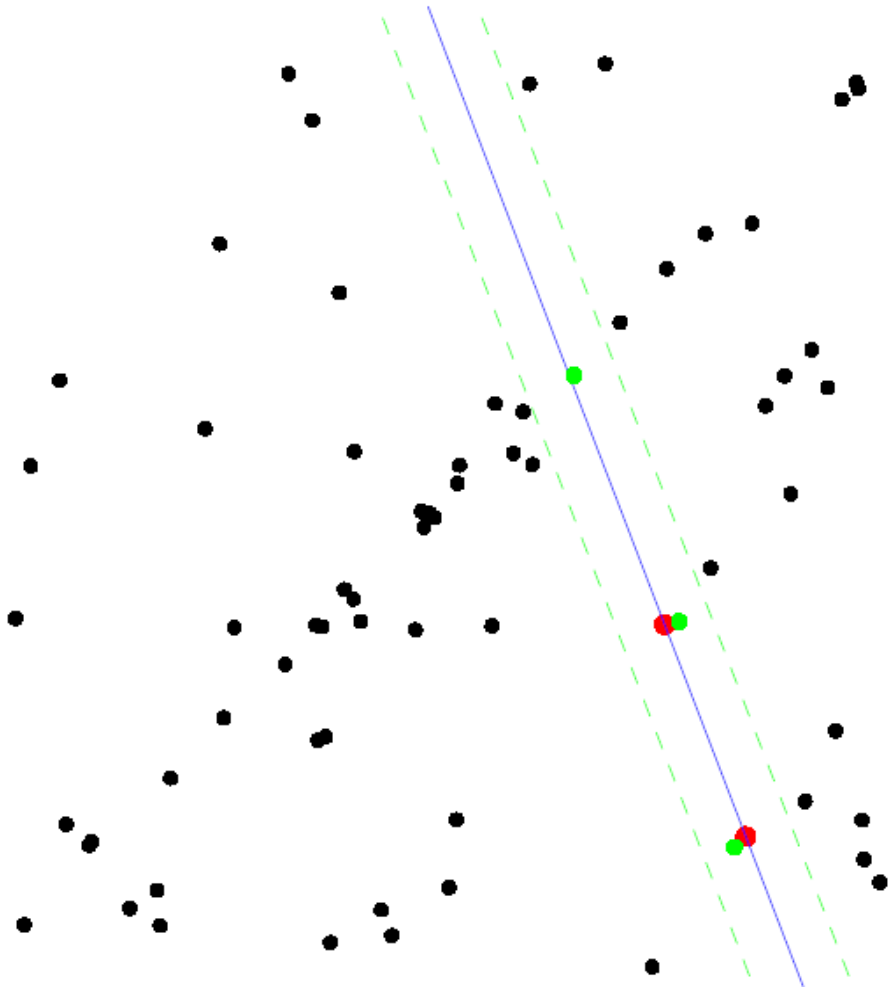
RANSAC—Illustration by line-fitting



Line fitting by RANSAC

- Randomly select two points
- The hypothesized model is the line passing through the two points
- Compute the “distance” of each data point to the hypothesized model

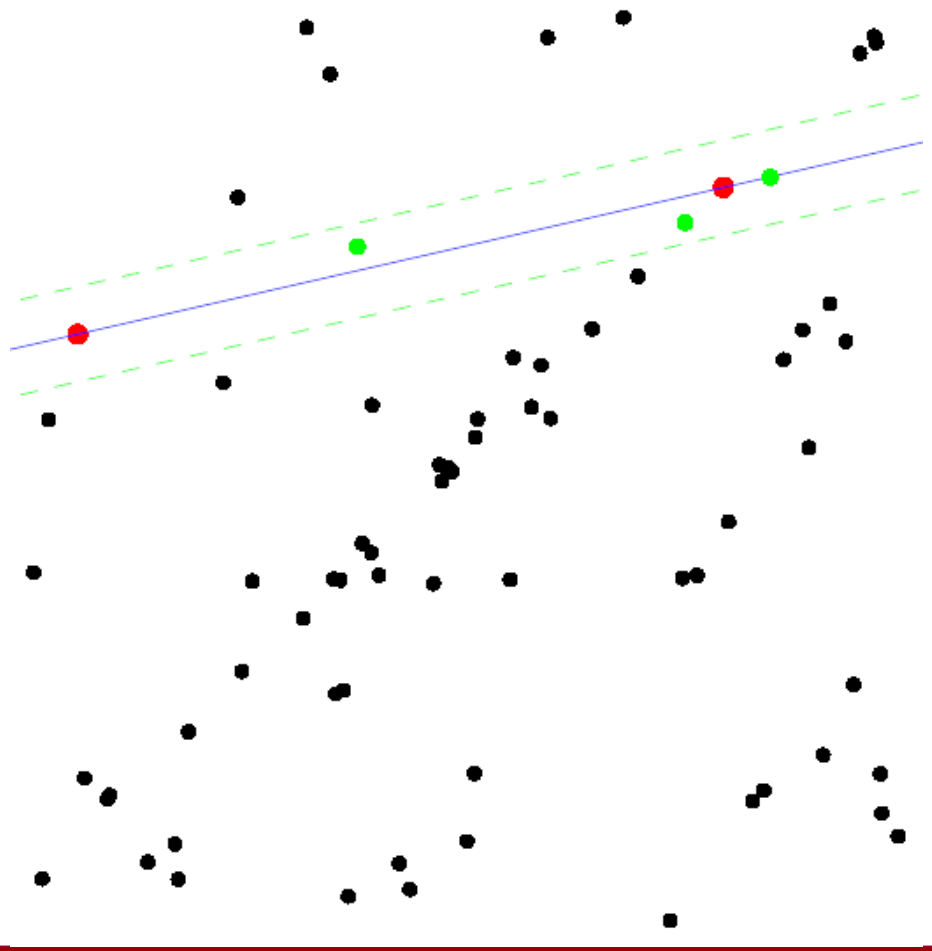
RANSAC—Illustration by line-fitting



Line fitting by RANSAC

- Randomly select two points
- The hypothesized model is the line passing through the two points
- Compute the “distance” of each data point to the hypothesized model
- Determine the consensus set of the current hypothesized model
  - ✓ If it is too small, try another 2 points

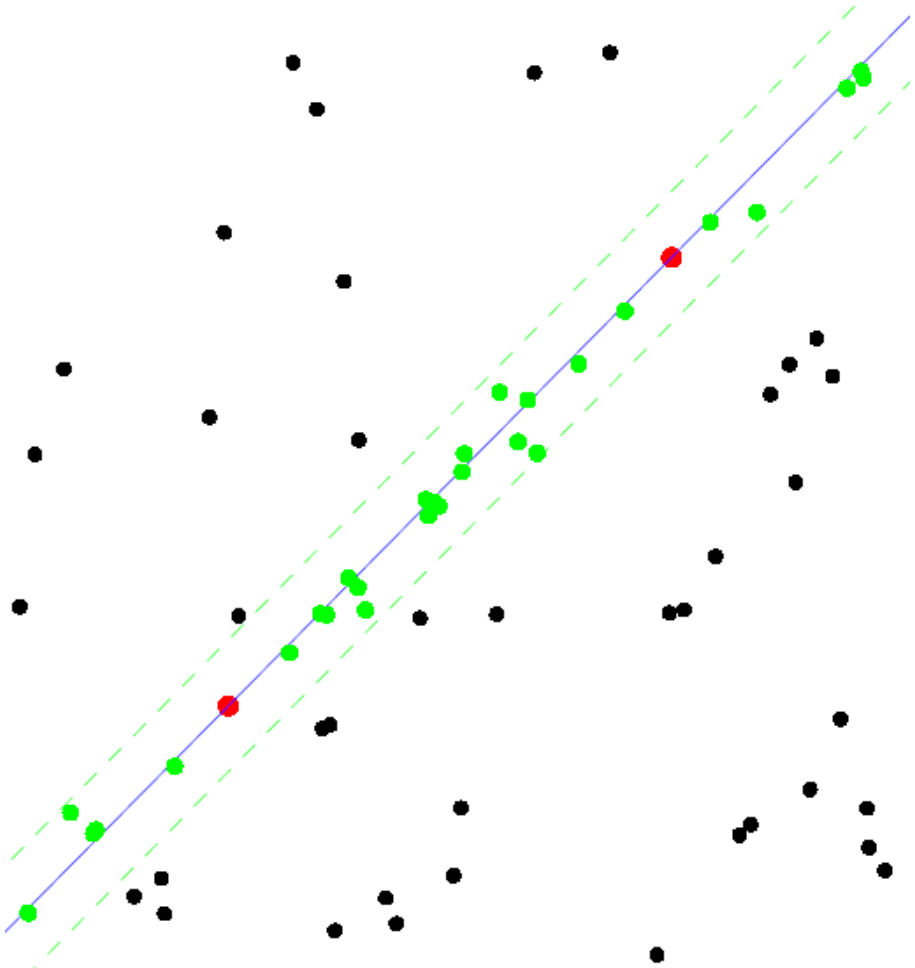
RANSAC—Illustration by line-fitting



Line fitting by RANSAC

- Test another two points

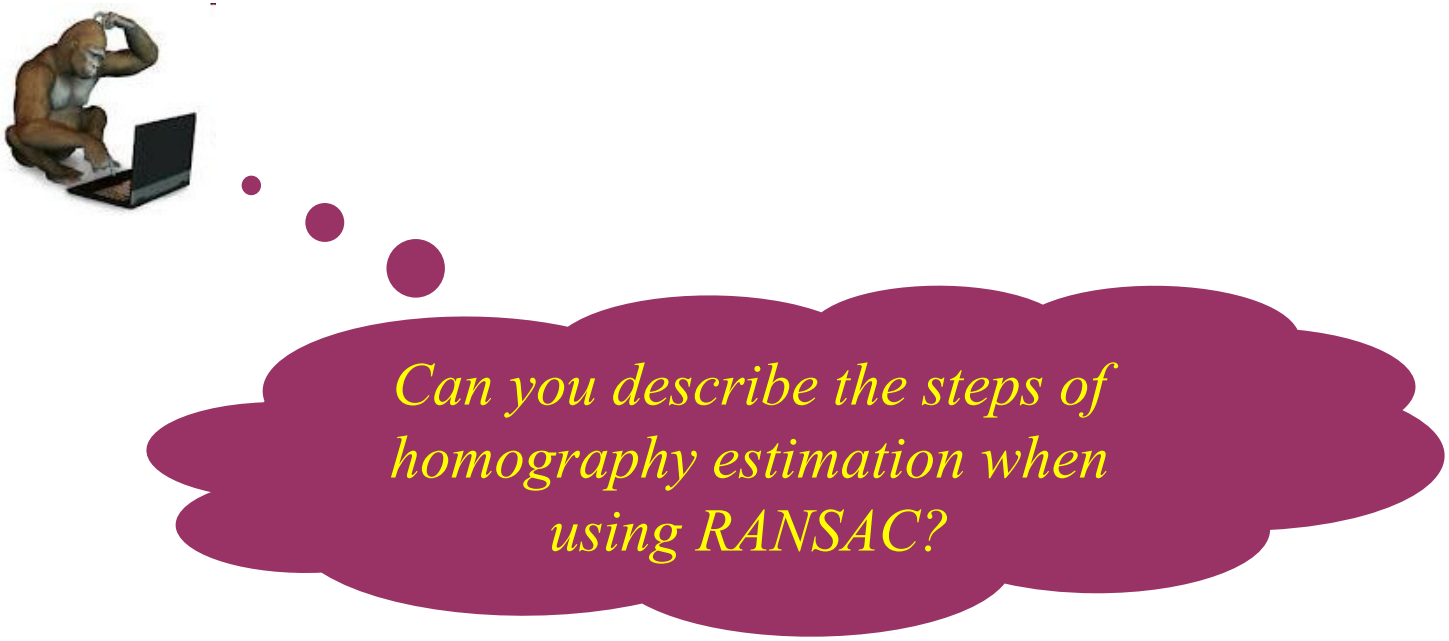
RANSAC—Illustration by line-fitting



Line fitting by RANSAC

- The final fitting result

RANSAC—A framework for parametric model estimation



Outline

- RANSAC—A Framework for Parametric Model Estimation
- Image Interpolation
- Practice for Panorama Stitching

Image interpolation

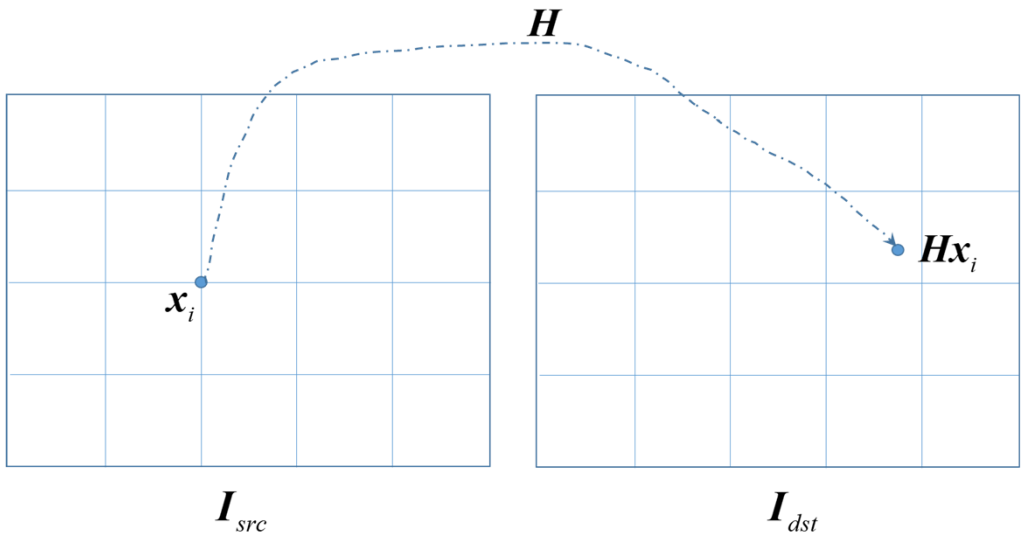
Suppose the geometric transformation between $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ is $\boldsymbol{H}$. To stitch $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$, we need to transform the point $\boldsymbol{x}_i$ (homogeneous coordinate) on $\boldsymbol{I}_1$ to a new position $\boldsymbol{Hx}_i$. How to implement?

In other words, how to implement the geometric transformation to an image?

Denote the original image by $\boldsymbol{I}_{src}$ and the transformed image by $\boldsymbol{I}_{dst}$.

Idea 1:

$$\boldsymbol{I}_{dst}\left(\boldsymbol{Hx}_i\right) \leftarrow \boldsymbol{I}_{src}\left(\boldsymbol{x}_i\right) \qquad \textit{Is it possible in implementation?}$$



NO! Since $\boldsymbol{Hx}_i$ is not an integer coordinate, the pixel assignment operation "$\boldsymbol{I}_{dst}(\boldsymbol{Hx}_i) = \boldsymbol{I}_{src}(\boldsymbol{x}_i)$" cannot actually be completed

Image interpolation

Idea 2:

$$\forall \boldsymbol{y}_i \in \boldsymbol{I}_{dst}\,,\ \boldsymbol{I}_{dst}\left(\boldsymbol{y}_i\right) \leftarrow \boldsymbol{I}_{src}\left(\boldsymbol{H}^{-1}\boldsymbol{y}_i\right)$$

is not an integer coordinate

The pixel value $\boldsymbol{I}_{src}(\boldsymbol{H}^{-1}\boldsymbol{y}_i)$ can be interpolated from its 4 neighbors

Bilinear image interpolation technique

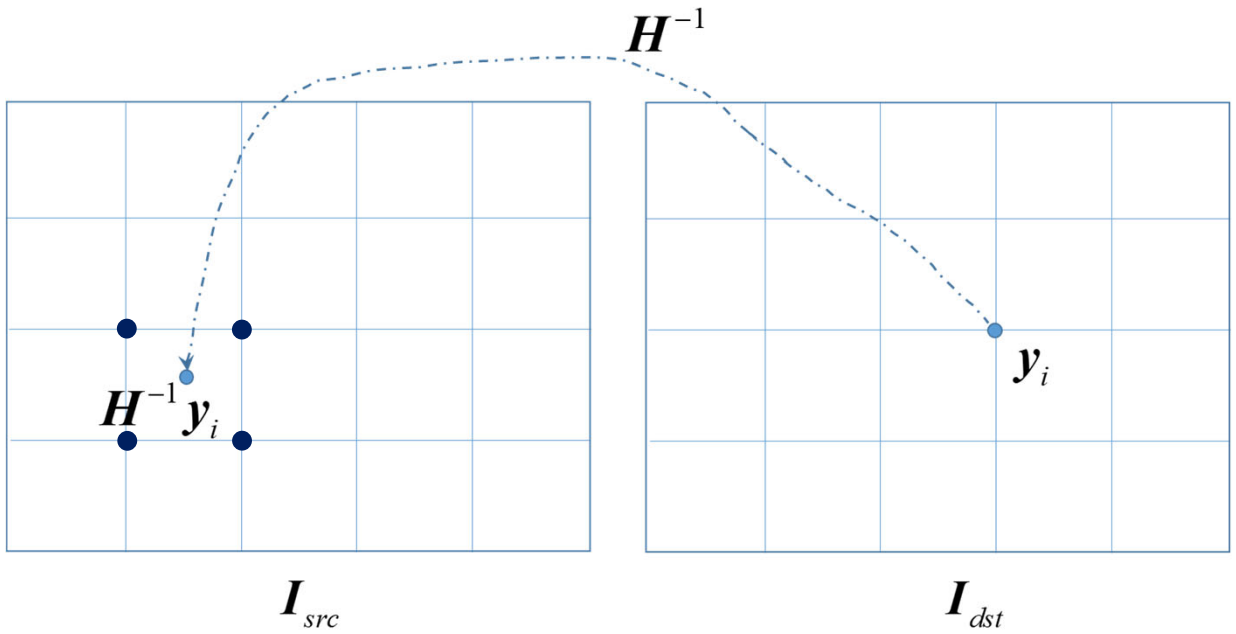
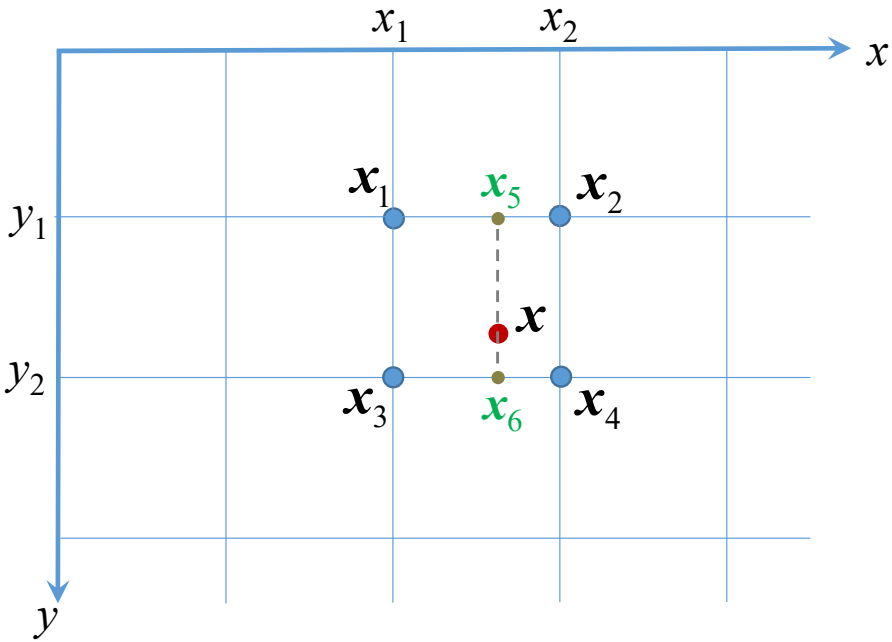


Image interpolation

Bilinear image interpolation technique



Given the digital image $f(\boldsymbol{x}):\mathbb{R}^2 \to \mathbb{R}$, the bilinear interpolation technique is used to calculate the pixel intensity $f(\boldsymbol{x})$ at a non-integer position $\boldsymbol{x}=(x, y)$ based on its four neighbors $\boldsymbol{x}_1(x_1, y_1)$, $\boldsymbol{x}_2(x_2, y_1)$, $\boldsymbol{x}_3(x_1, y_2)$ and $\boldsymbol{x}_4(x_2, y_2)$ by,

$$f(\boldsymbol{x}_5)=\frac{x_2-x}{x_2-x_1}f(\boldsymbol{x}_1)+\frac{x-x_1}{x_2-x_1}f(\boldsymbol{x}_2)$$

$$f(\boldsymbol{x}_6)=\frac{x_2-x}{x_2-x_1}f(\boldsymbol{x}_3)+\frac{x-x_1}{x_2-x_1}f(\boldsymbol{x}_4)$$

$$f(\boldsymbol{x})=\frac{y_2-y}{y_2-y_1}f(\boldsymbol{x}_5)+\frac{y-y_1}{y_2-y_1}f(\boldsymbol{x}_6)$$

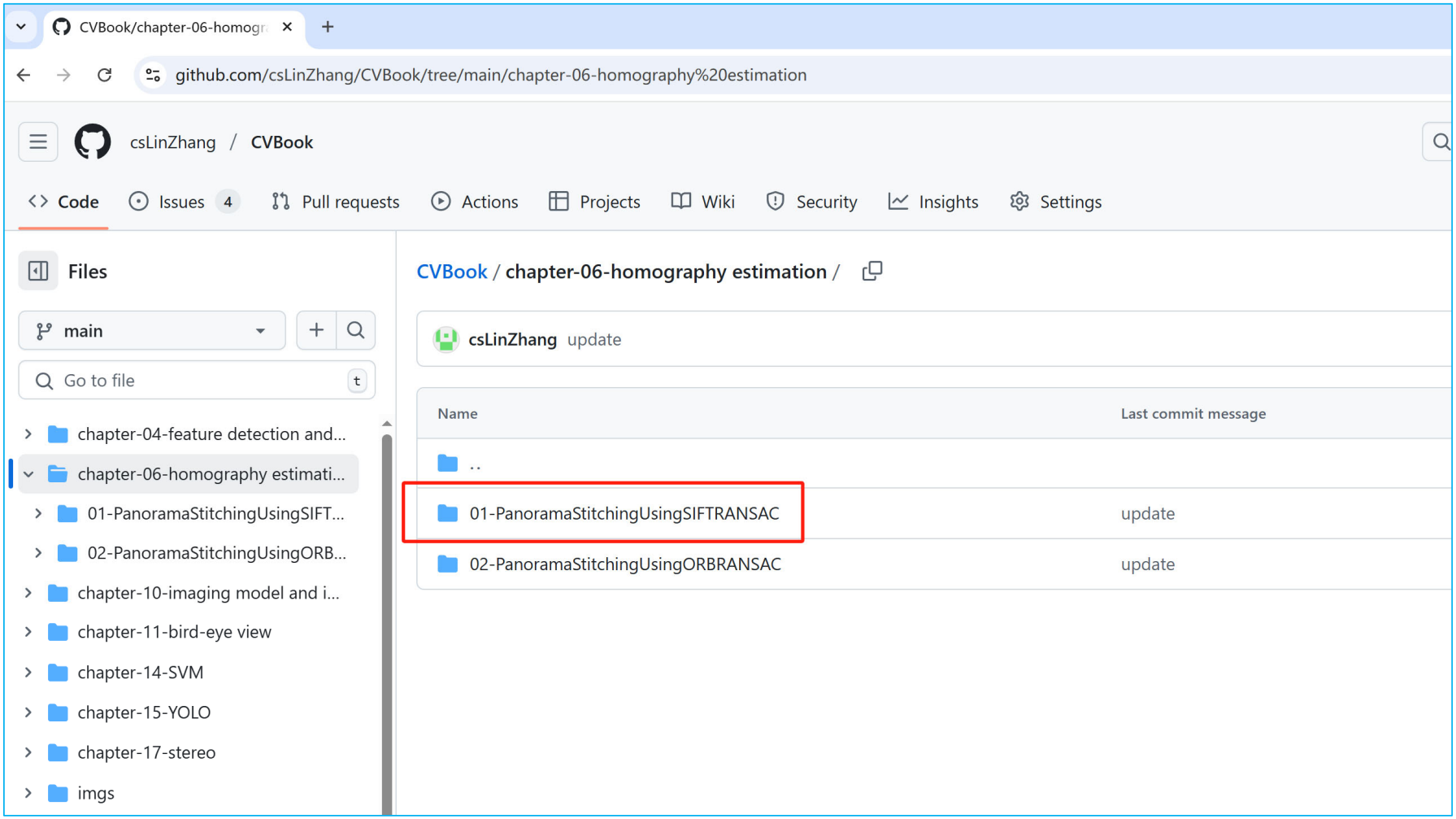
since x_2-x_1=1, y_2-y_1=1

$$f(\boldsymbol{x})=(y_2-y)(x_2-x)f(\boldsymbol{x}_1)+(y_2-y)(x-x_1)f(\boldsymbol{x}_2)+(y-y_1)(x_2-x)f(\boldsymbol{x}_3)+(y-y_1)(x-x_1)f(\boldsymbol{x}_4)$$

Outline

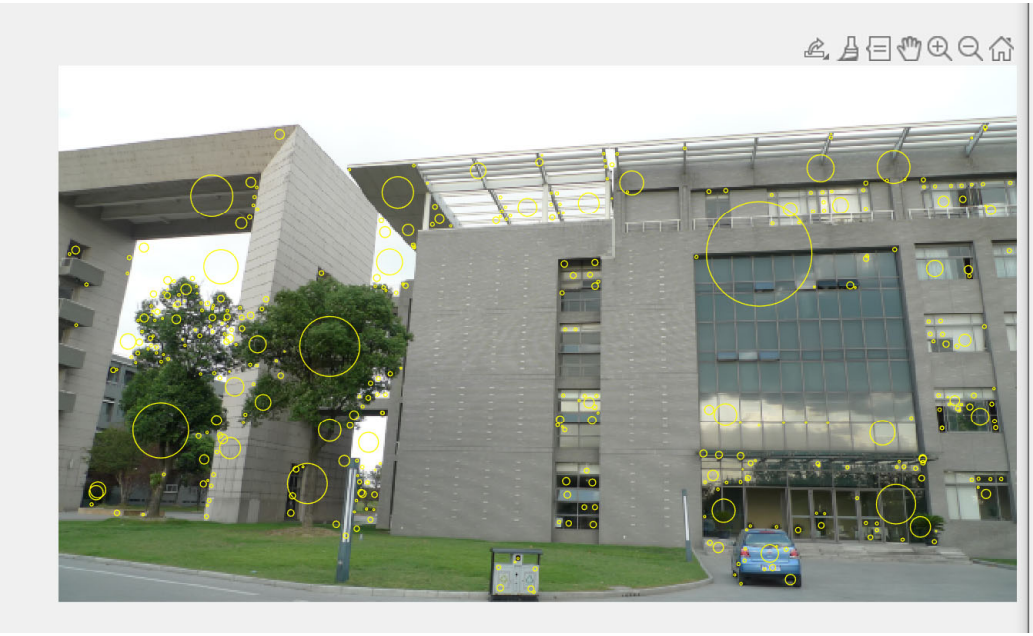
- RANSAC—A Framework for Parametric Model Estimation
- Image Interpolation
- Practice for Panorama Stitching

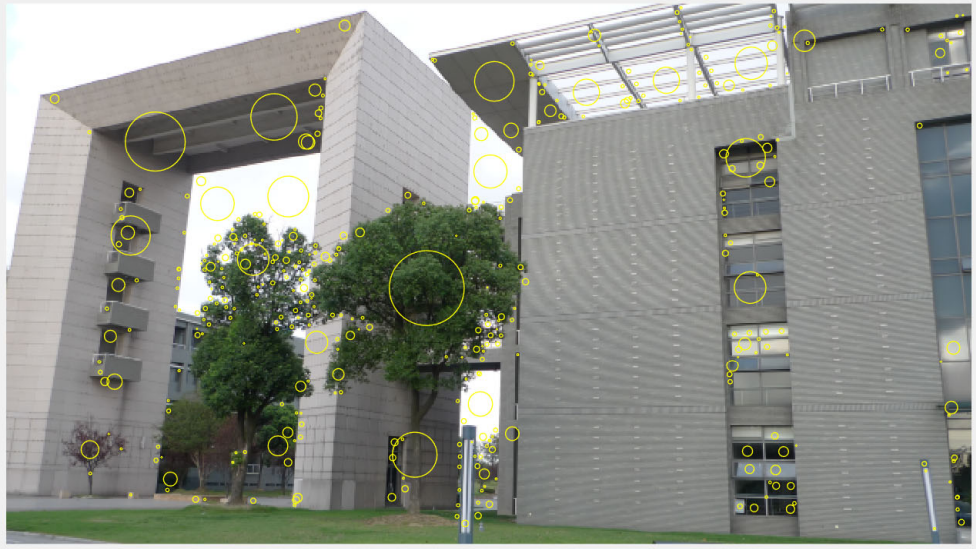
Practice for panorama stitching—A Matlab implementation



Practice for panorama stitching—A Matlab implementation

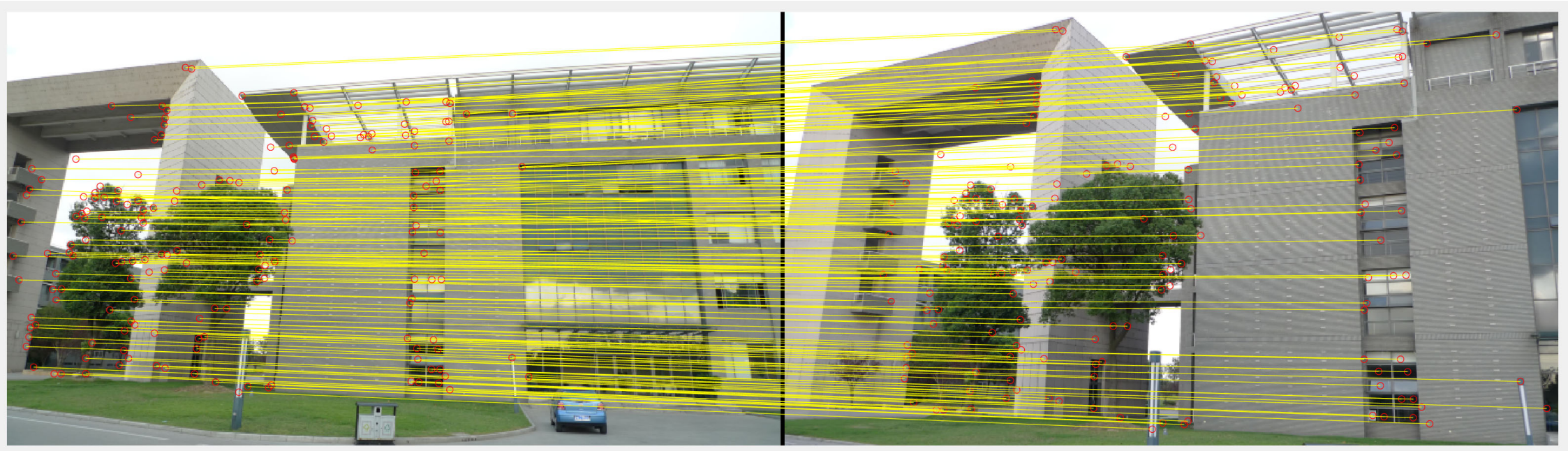
Practice for panorama stitching—A Matlab implementation





SIFT feature point detection

Practice for panorama stitching—A Matlab implementation



SIFT feature point matching using RANSAC

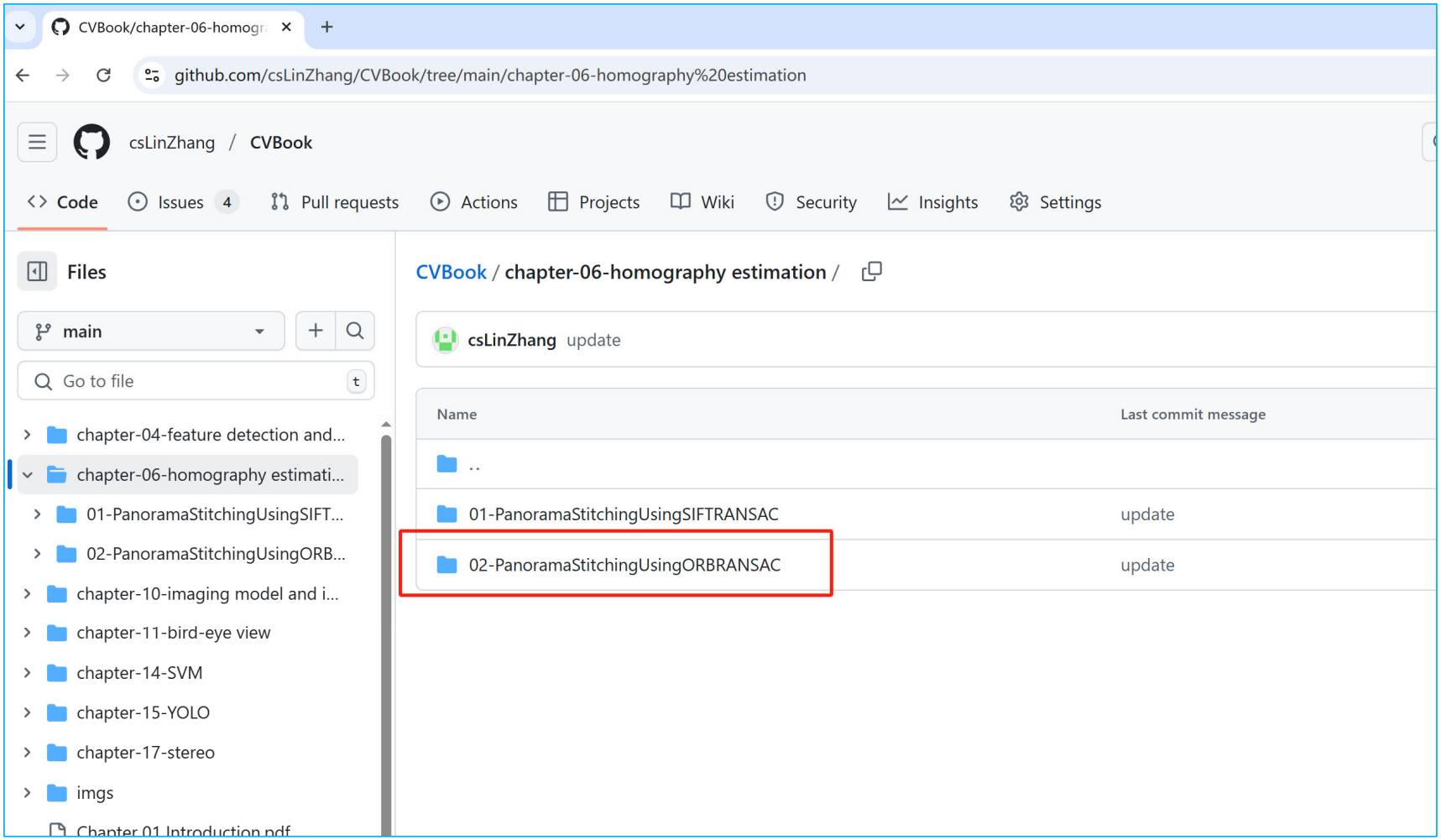
Practice for panorama stitching—A Matlab implementation

Transformed image1 using the estimated homography matrix

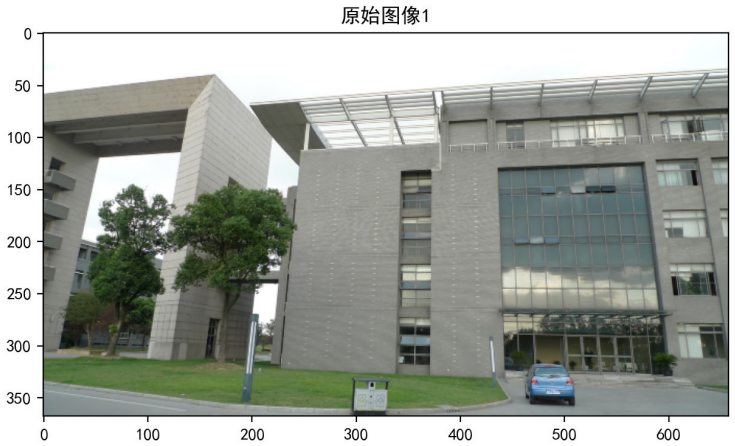
Practice for panorama stitching—A Matlab implementation

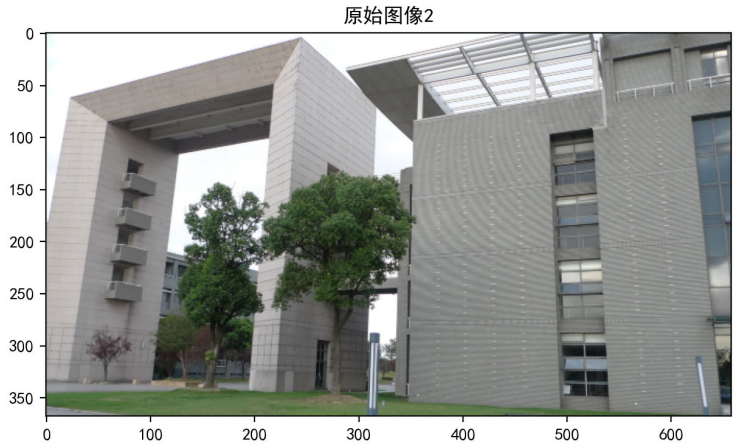
The final stitching result

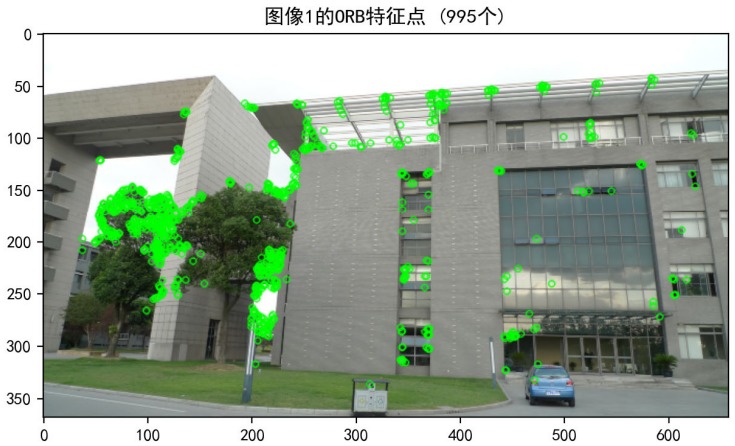
Practice for panorama stitching—A Python implementation

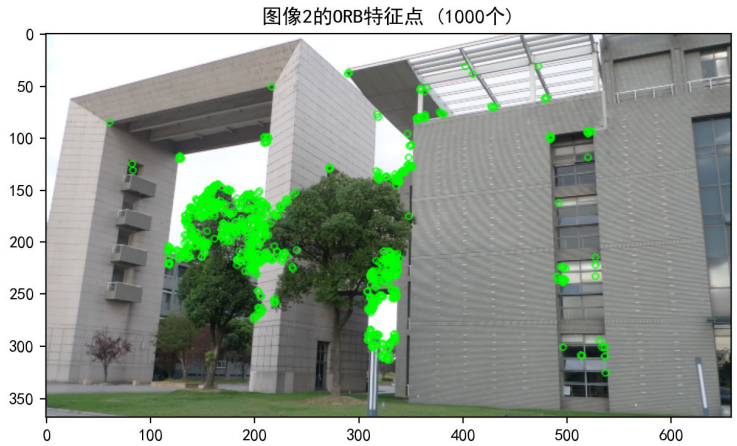


Practice for panorama stitching—A Python implementation

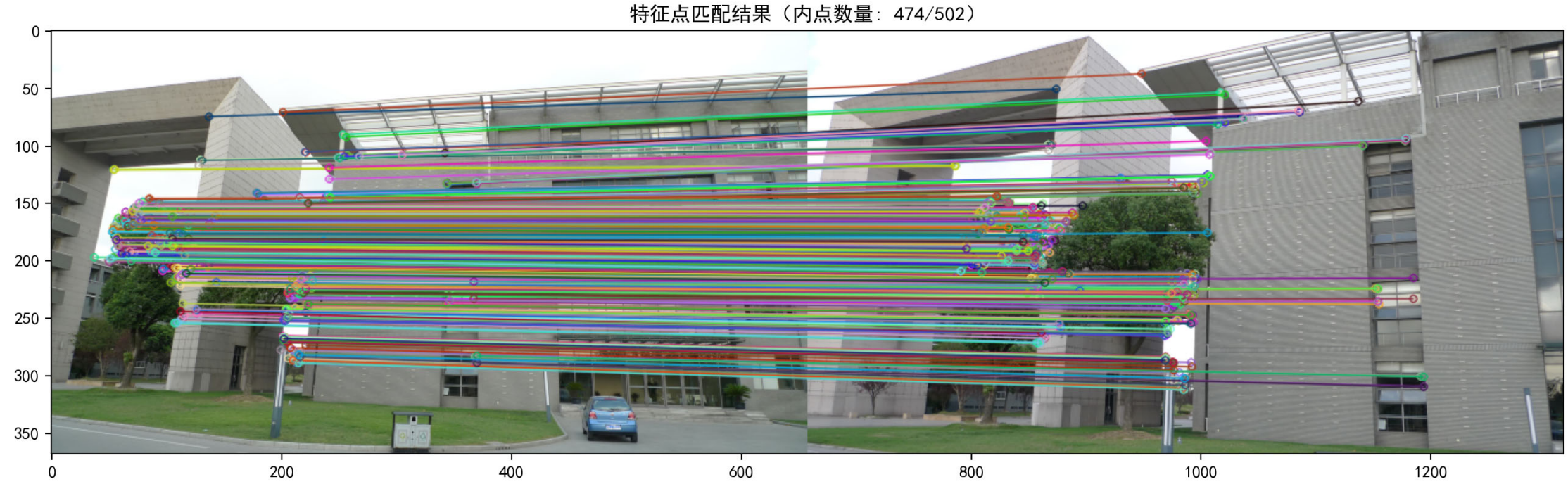




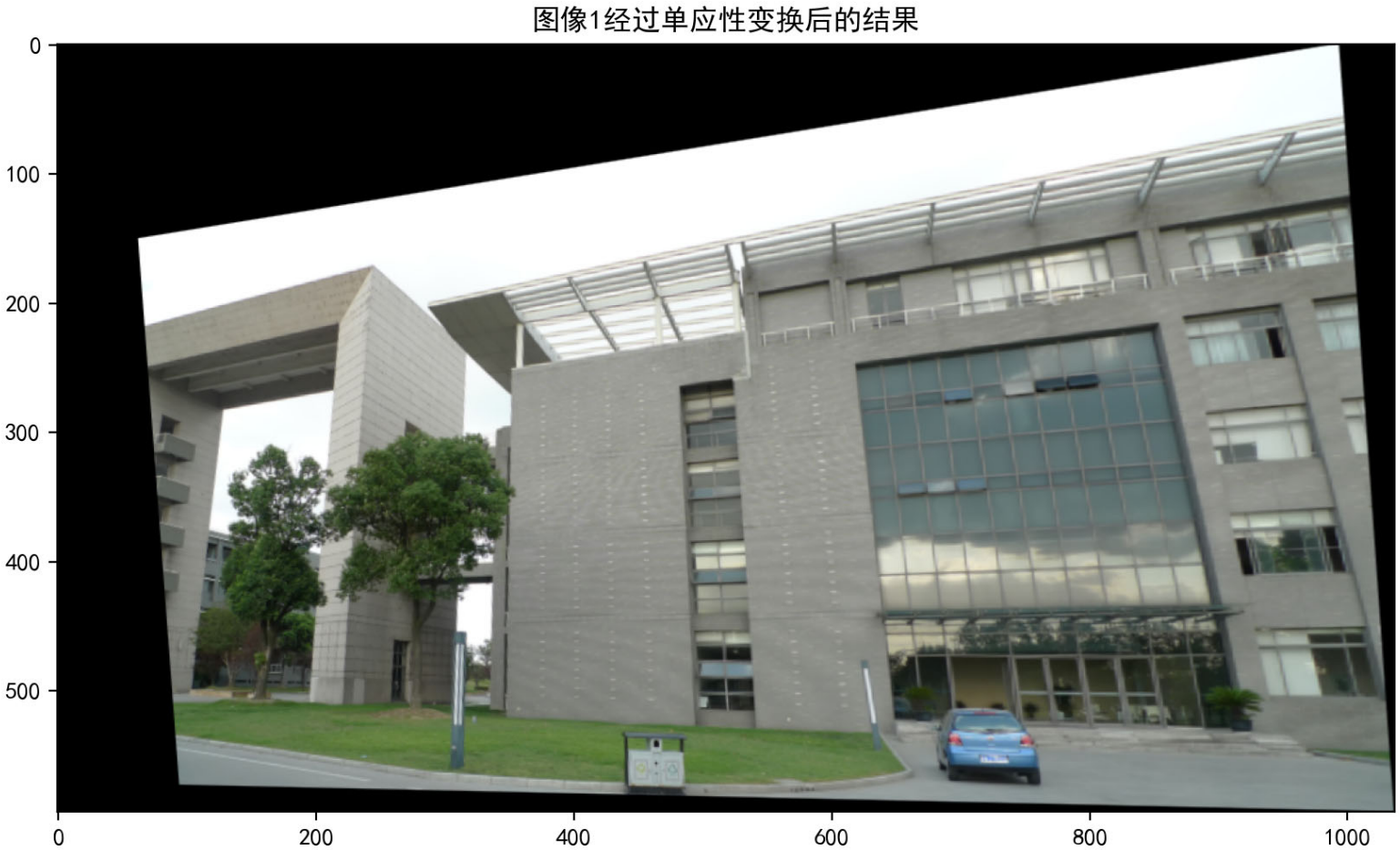




Practice for panorama stitching—A Python implementation



Practice for panorama stitching—A Python implementation



Practice for panorama stitching—A Python implementation

Project products of students from 2009 Media&Arts

Q+A