Chapter 04
Interest Point Detection and Matching

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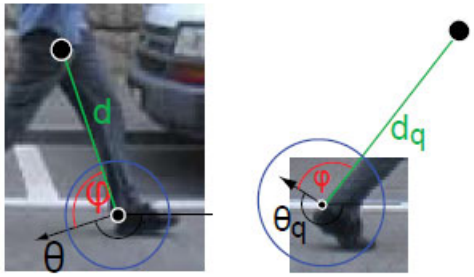
Content

- Local Invariant Features
- Harris Corner Detector
- Scale Invariant Point Detection
- Scale Invariant Feature Transform (SIFT)
- Interest Point Matching

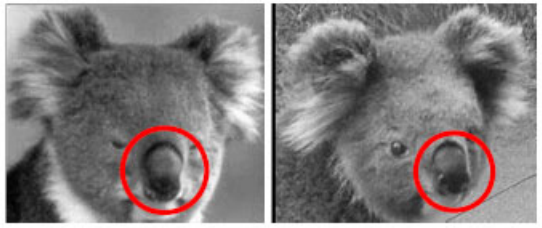
Motivation

- Global representations have major limitations
- Instead, describe and match only local regions
- Increased robustness to
  - Occlusions

  - Articulation



  - Intra-category variations



Motivation

Application: Image Matching

by Diva Sian

by swashford

Motivation

Application: Image Matching

Harder Case

by Diva Sian

by scgbt

Steve Seitz

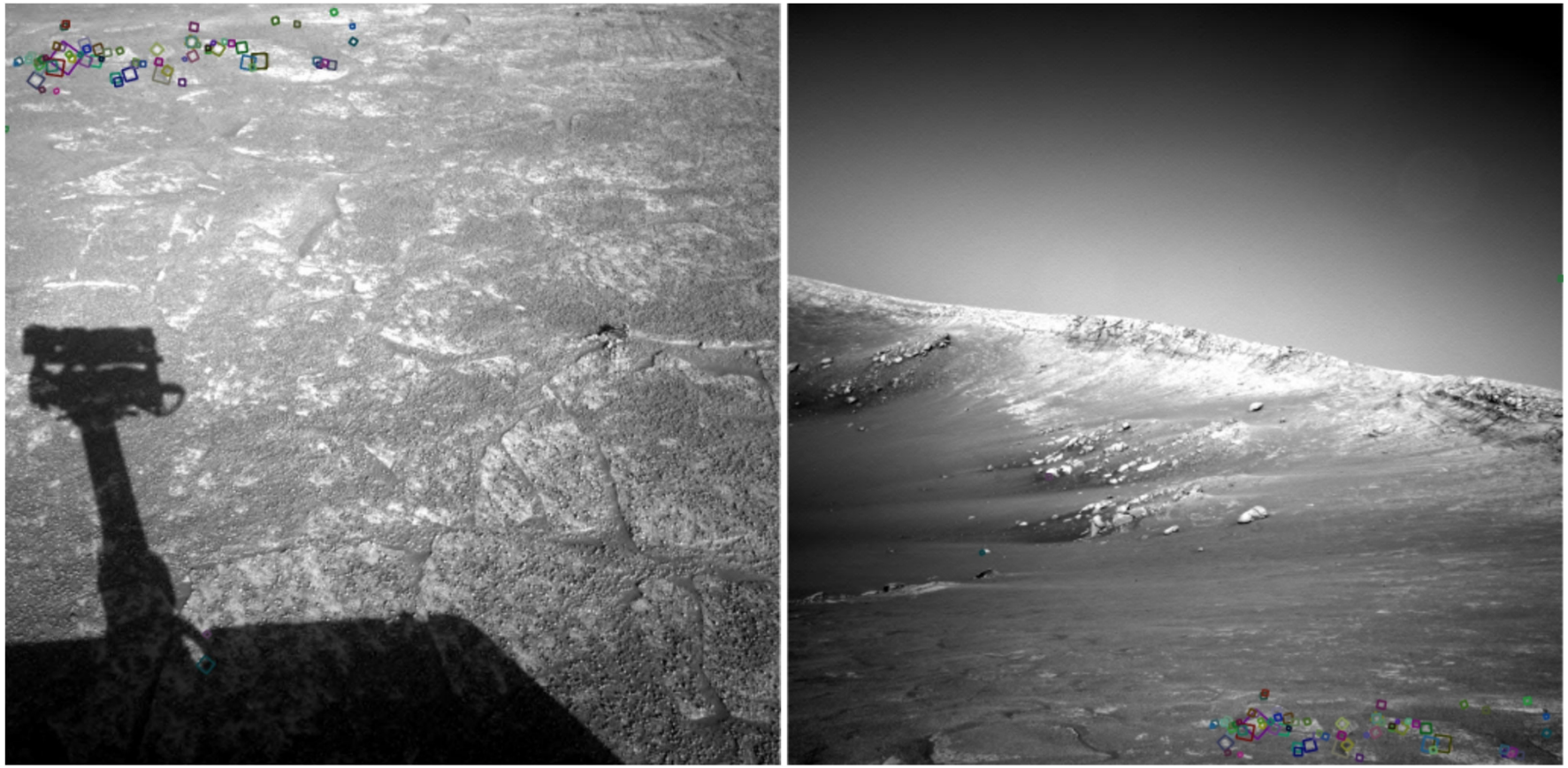
Motivation

Application: Image Matching

NASA Mars Rover Images

Motivation

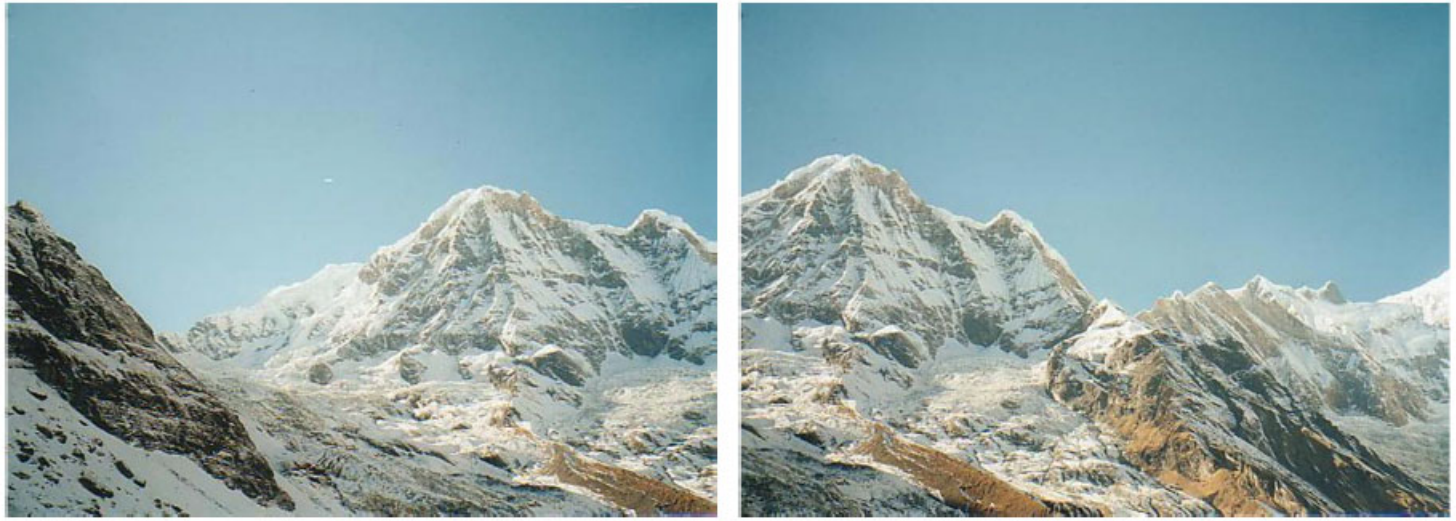
Application: Image Matching (Look for tiny colored squares)



NASA Mars Rover images with SIFT matches

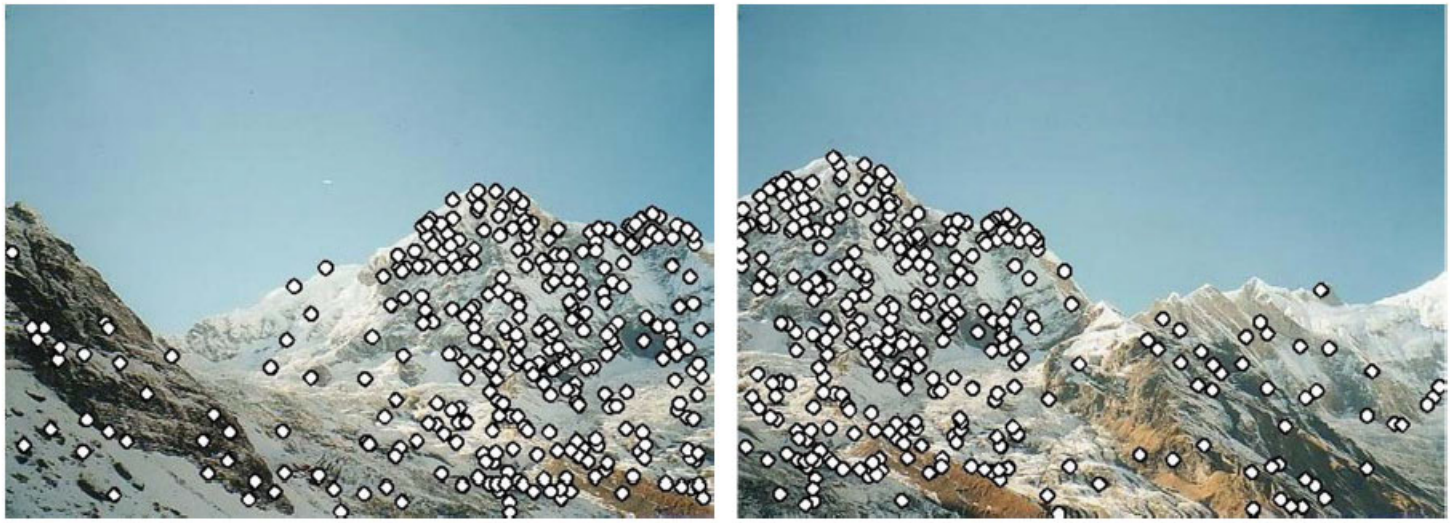
Motivation

- Panorama stitching
  - We have two images – how do we combine them?



Motivation

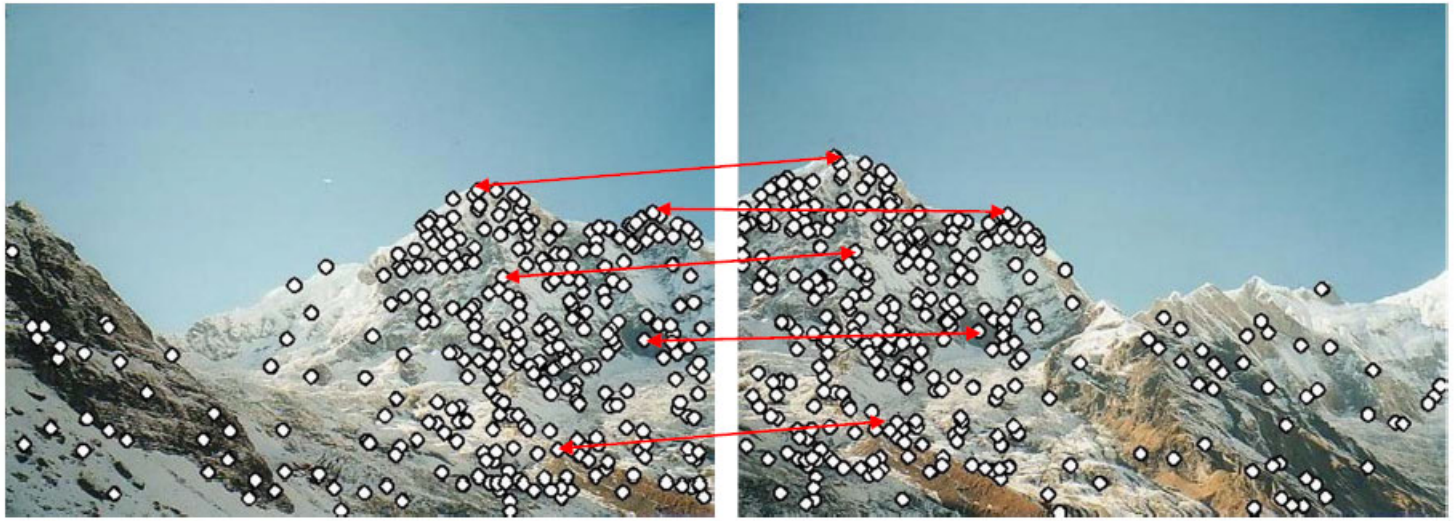
- Panorama stitching
  - We have two images – how do we combine them?



- Procedure:
  - Detect feature points in both images

Motivation

- Panorama stitching
  - We have two images – how do we combine them?



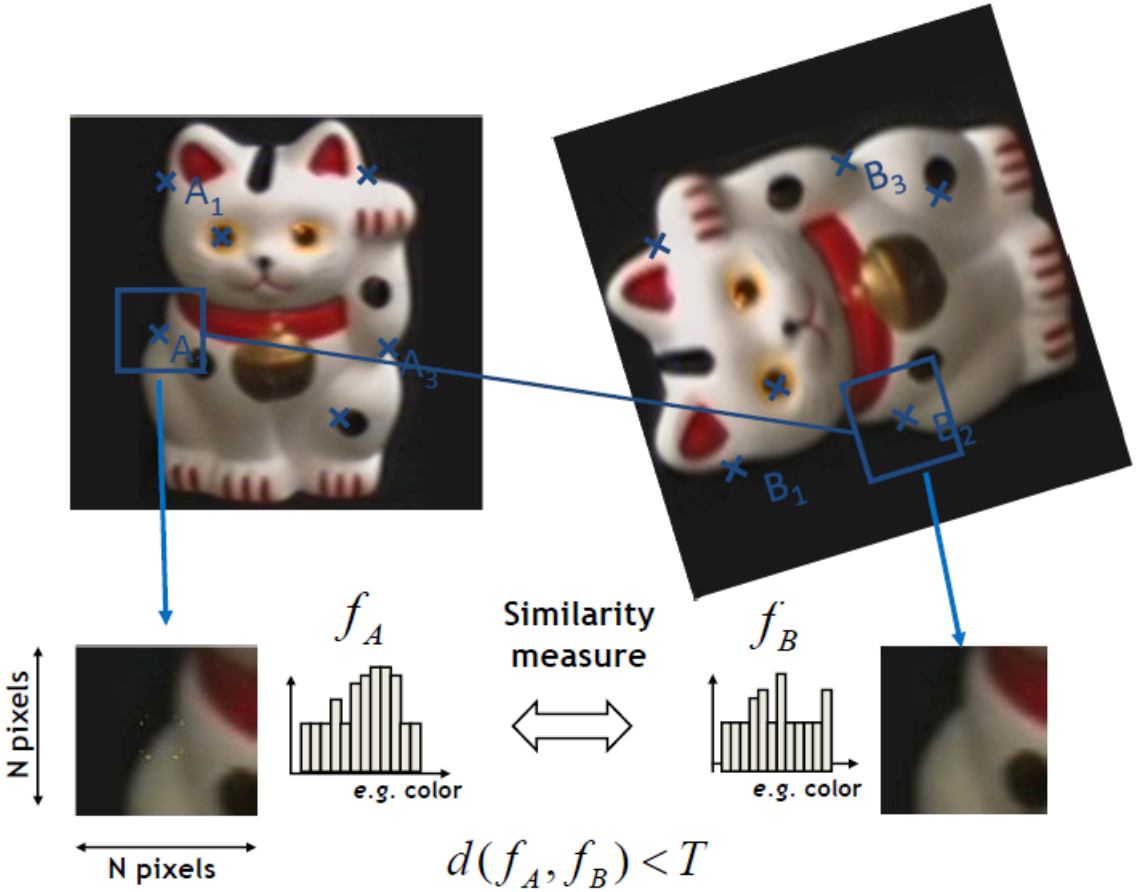
- Procedure:
  - Detect feature points in both images
  - Find corresponding pairs

Motivation

- Panorama stitching
  - We have two images – how do we combine them?

- Procedure:
  - Detect feature points in both images
  - Find corresponding pairs
  - Use these pairs to align the images

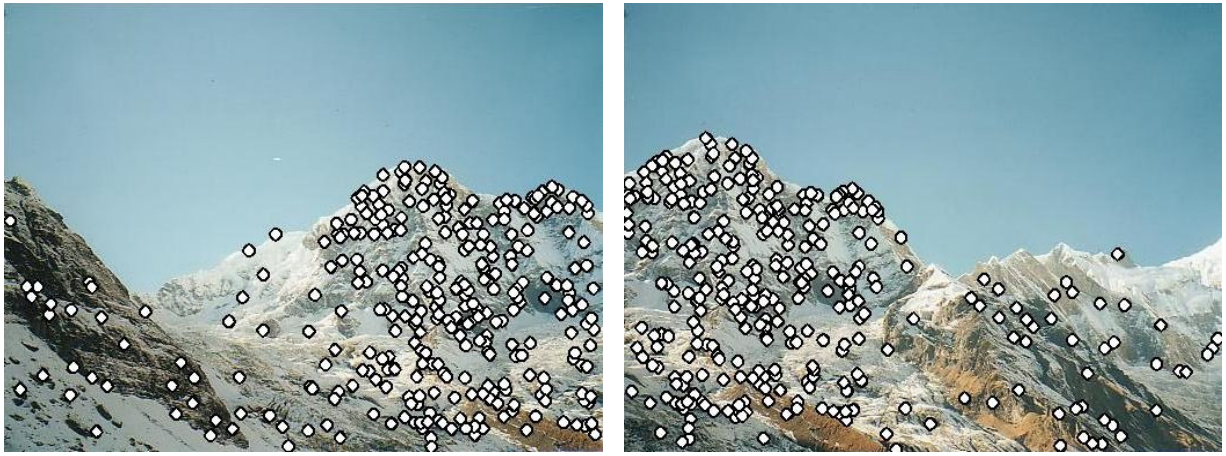
General Approach for Image Matching



1. Find a set of distinctive key-points
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

Source: B. Leibe

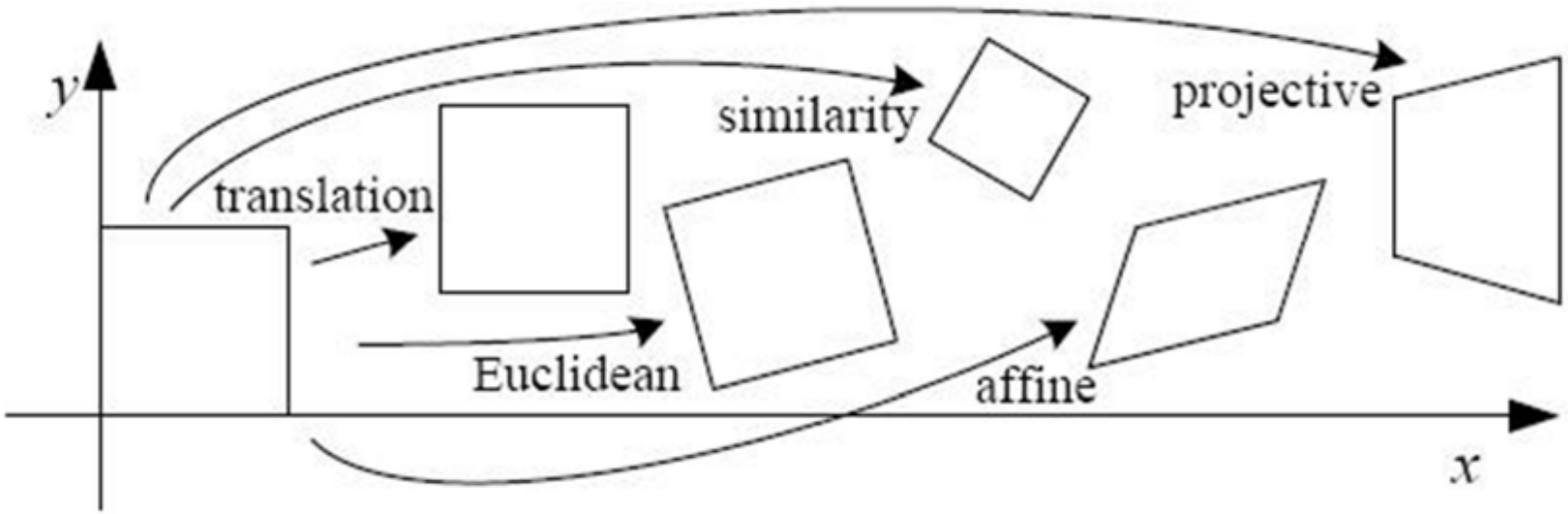
Characteristics of Good Features



- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature has a distinctive description
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

Invariance: Geometric Transformations

Level of Geometric Invariance



Invariance: Photometric Transformations

- Often modeled as a linear transformation:
  - Scaling + Offset

Applications

Feature points are used for:

- Motion tracking
- Image alignment
- 3D reconstruction
- Object recognition
- Indexing and database retrieval
- Robot navigation

Content

- Local Invariant Features
- Harris Corner Detector
- Scale Invariant Point Detection
- Scale Invariant Feature Transform (SIFT)
- Interest Point Matching

Finding Corners

My office,

5:30PM, Sep. 18, 2011

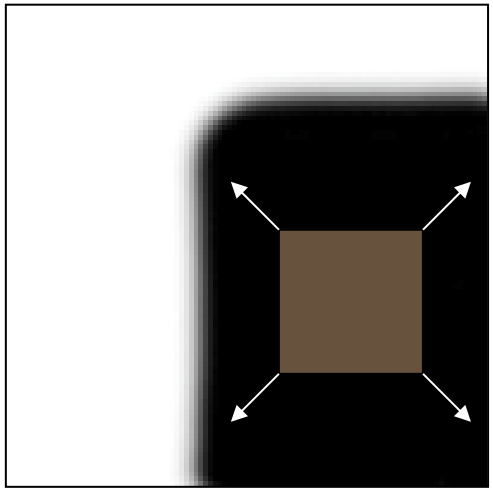
Finding Corners

- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

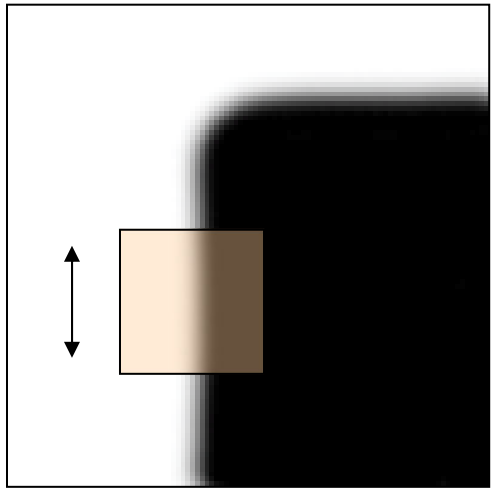
C. Harris and M. Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Corner Detection: Basic Idea

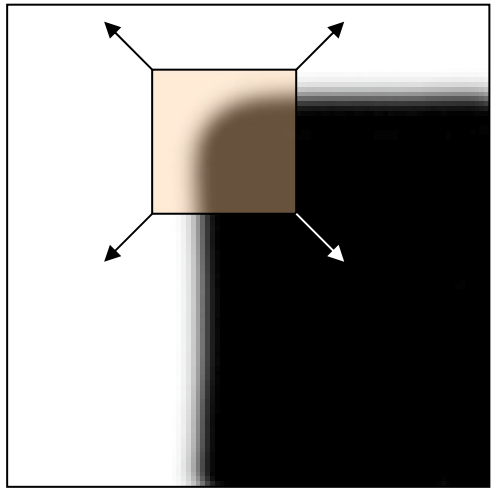
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in all directions

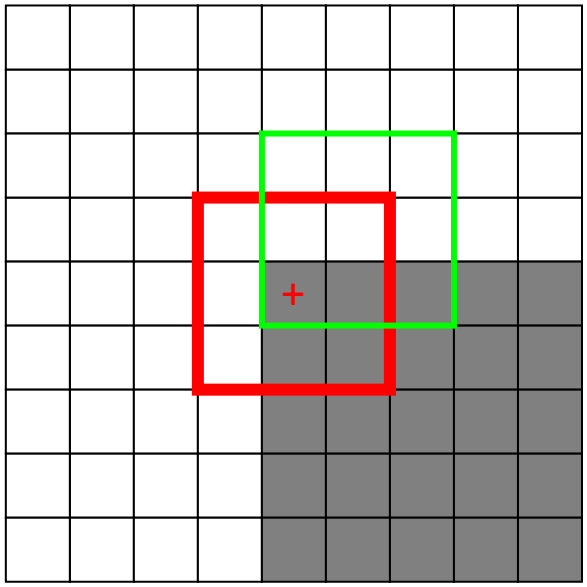


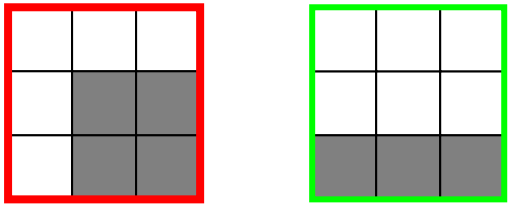
"edge":
no change along the edge direction



"corner":
significant change in all directions

Harris Detector: Basic Idea



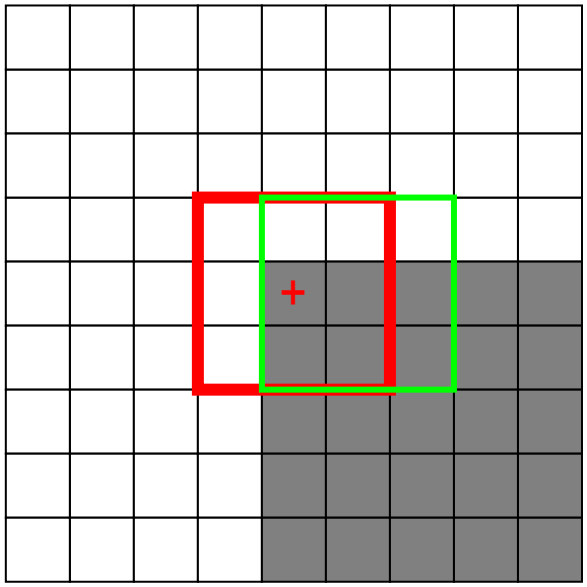


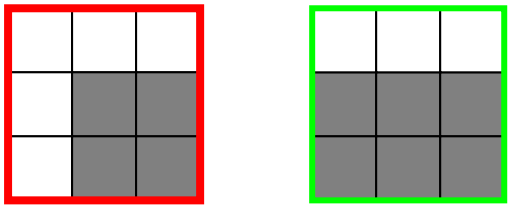
Difference = 3

Demo of a point + with well distinguished neighborhood.

Moving the window in any direction will result in a large intensity change.

Harris Detector: Basic Idea



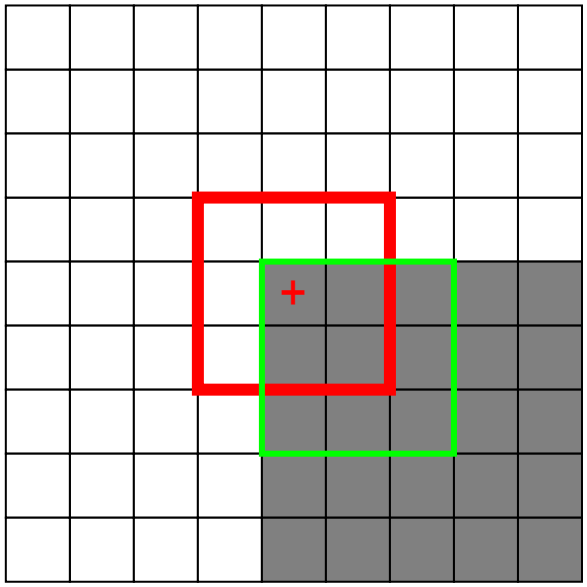


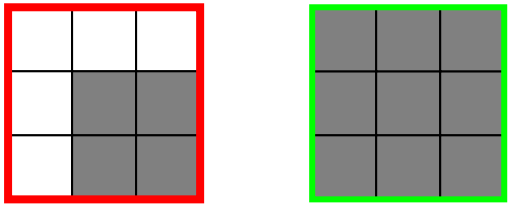
Difference = 2

Demo of a point + with well distinguished neighborhood.

Moving the window in any direction will result in a large intensity change.

Harris Detector: Basic Idea



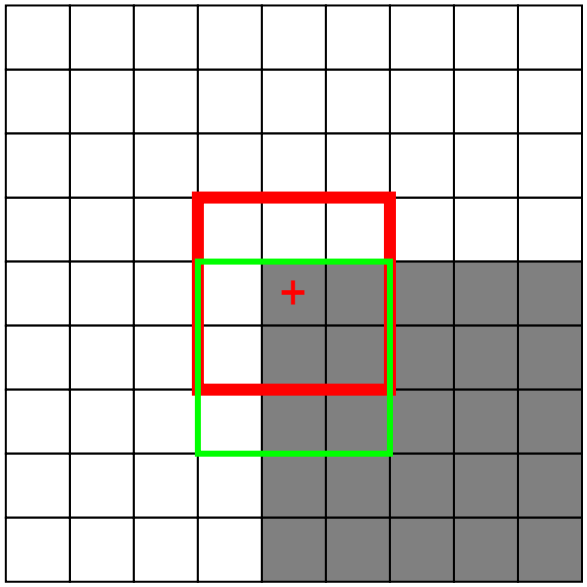


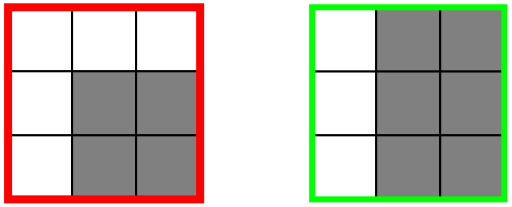
Difference = 5

Demo of a point + with well distinguished neighborhood.

Moving the window in any direction will result in a large intensity change.

Harris Detector: Basic Idea



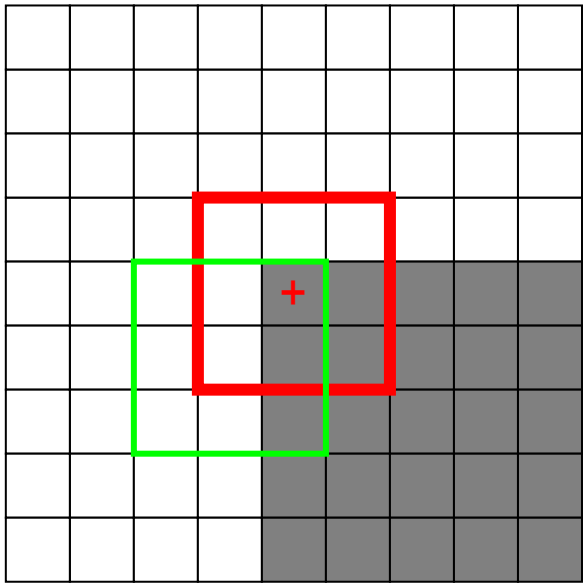


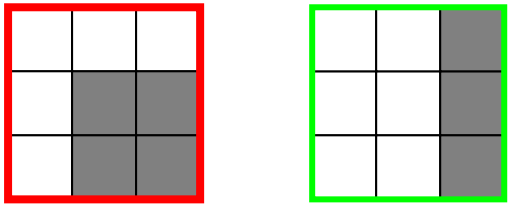
Difference = 2

Demo of a point + with well distinguished neighborhood.

Moving the window in any direction will result in a large intensity change.

Harris Detector: Basic Idea



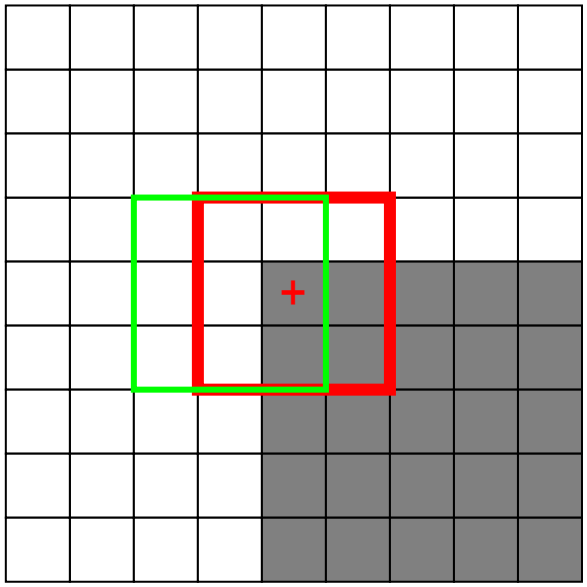


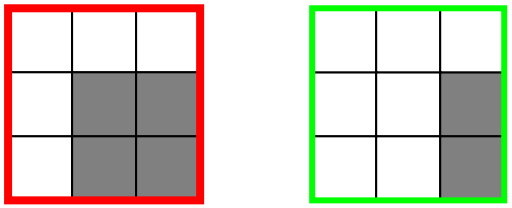
Difference = 3

Demo of a point + with well distinguished neighborhood.

Moving the window in any direction will result in a large intensity change.

Harris Detector: Basic Idea



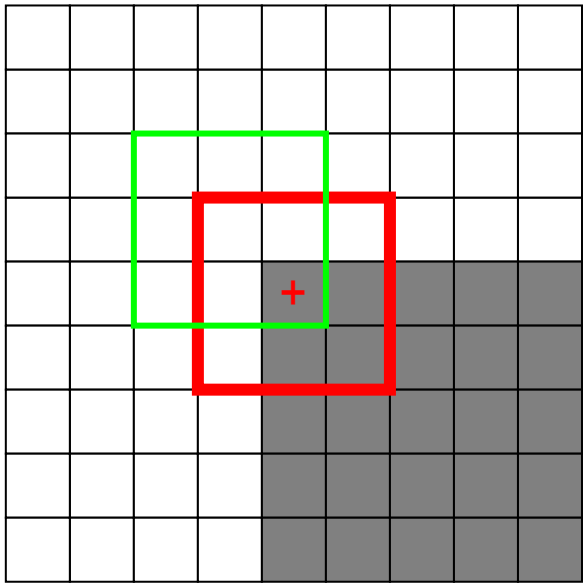


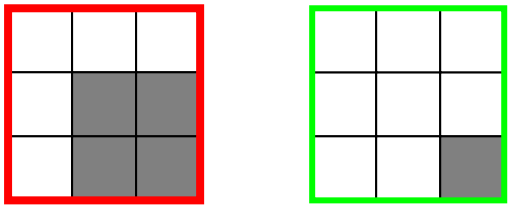
Difference = 2

Demo of a point + with well distinguished neighborhood.

Moving the window in any direction will result in a large intensity change.

Harris Detector: Basic Idea





Difference = 3

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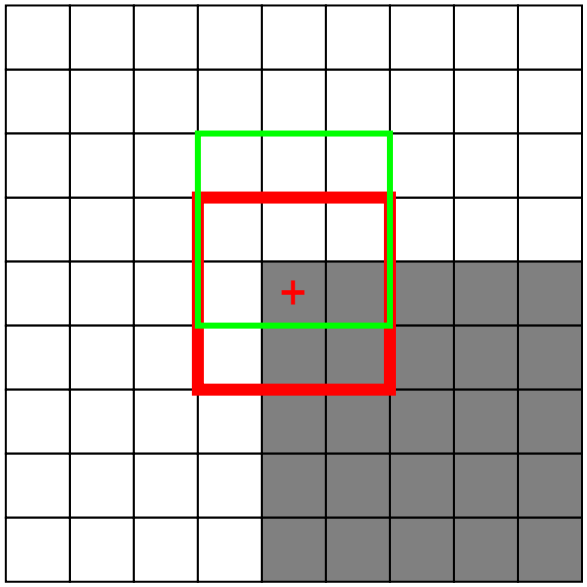
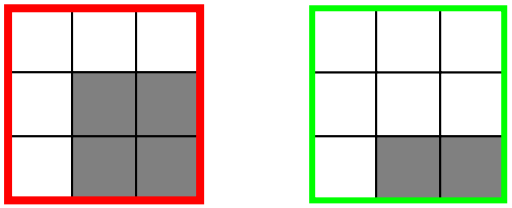
Harris Detector: Basic Idea





Difference = 2

Demo of a point + with well distinguished neighborhood.

Moving the window in any direction will result in a large intensity change.

Harris Corner Detection: Mathematics

Change in appearance of a local patch (defined by a window w) centered at p for the shift $(\Delta x, \Delta y)$:

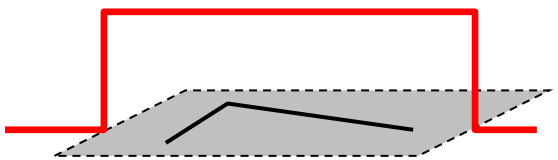
$$S_w(\Delta x, \Delta y) = \sum_{(x_i, y_i) \in w} (f(x_i, y_i) - f(x_i + \Delta x, y_i + \Delta y))^2$$

Window function

Intensity

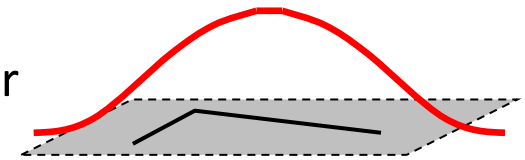
Shifted intensity

Window function w=



1 in window, 0 outside

or



Gaussian

Harris Corner Detection: Mathematics

$$S_w(\Delta x, \Delta y) = \sum_{(x_i, y_i) \in w} (f(x_i, y_i) - f(x_i + \Delta x, y_i + \Delta y))^2 \qquad (1)$$

$$\approx \sum_{(x_i, y_i) \in w} \left(f(x_i, y_i) - f(x_i, y_i) - \left[\frac{\partial f(x_i, y_i)}{\partial x}, \frac{\partial f(x_i, y_i)}{\partial y} \right] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^2 \qquad (2)$$

$$= \sum_{(x_i, y_i) \in w} \left(\left[\frac{\partial f(x_i, y_i)}{\partial x}, \frac{\partial f(x_i, y_i)}{\partial y} \right] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^2 \qquad \text{(Due to } |\mathbf{u}|^2 = \mathbf{u}^T \mathbf{u} \text{)}$$

$$= [\Delta x, \Delta y] \left(\sum_{(x_i, y_i) \in w} \begin{bmatrix} \frac{\partial f(x_i, y_i)}{\partial x} \\ \frac{\partial f(x_i, y_i)}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial f(x_i, y_i)}{\partial x} & \frac{\partial f(x_i, y_i)}{\partial y} \end{bmatrix} \right) \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$= [\Delta x \; \Delta y] \boldsymbol{M} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

Harris Corner Detection

$$\boldsymbol{M} = \begin{bmatrix} \sum_{(x_i, y_i) \in w} \left(\frac{\partial f(x_i, y_i)}{\partial x} \right)^2 & \sum_{(x_i, y_i) \in w} \left(\frac{\partial f(x_i, y_i)}{\partial x} \cdot \frac{\partial f(x_i, y_i)}{\partial y} \right) \\ \sum_{(x_i, y_i) \in w} \left(\frac{\partial f(x_i, y_i)}{\partial x} \cdot \frac{\partial f(x_i, y_i)}{\partial y} \right) & \sum_{(x_i, y_i) \in w} \left(\frac{\partial f(x_i, y_i)}{\partial y} \right)^2 \end{bmatrix}$$

Harris Corner Detection

$$S(\triangle x,\triangle y) \cong [\triangle x,\triangle y]\boldsymbol{M}\begin{bmatrix}\triangle x\\ \triangle y\end{bmatrix}$$

where, $\boldsymbol{M} = \begin{bmatrix} \sum_{(x_i,y_i)\in w}(I_x)^2 & \sum_{(x_i,y_i)\in w}(I_x I_y) \\ \sum_{(x_i,y_i)\in w}(I_x I_y) & \sum_{(x_i,y_i)\in w}(I_y)^2 \end{bmatrix}$

$S(\triangle x,\triangle y) = 1$ actually is the ellipse equation.

The shape of the ellipse is determined by $\boldsymbol{M}$.

Assignment

$\boldsymbol{M}$ can be proved to be a positive semi-definite matrix

In practice, M is positive definite nearly for sure, then $[\Delta x\ \Delta y]\boldsymbol{M}\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix} = 1$ represents an ellipse

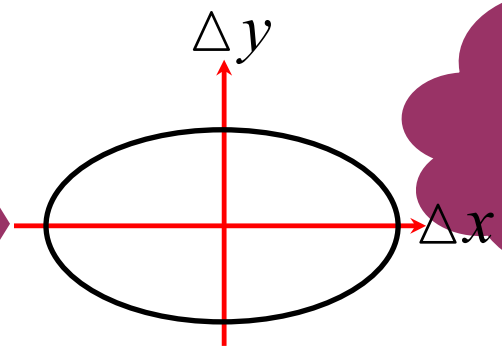
Harris Corner Detection

The "cornerness" of the window w is reflected in $\boldsymbol{M}$

Suppose there are two local windows w_1 and w_2; consider the cases when the moving of the two windows leads to the intensity change equals to 1. The moving vector $[\Delta x, \Delta y]$ of each window satisfies the ellipse equation. Thus,

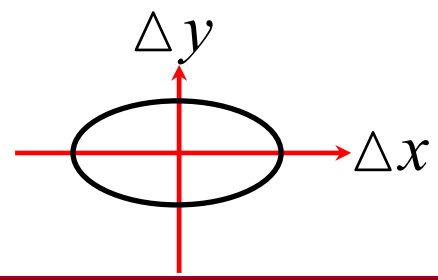
For w_1,

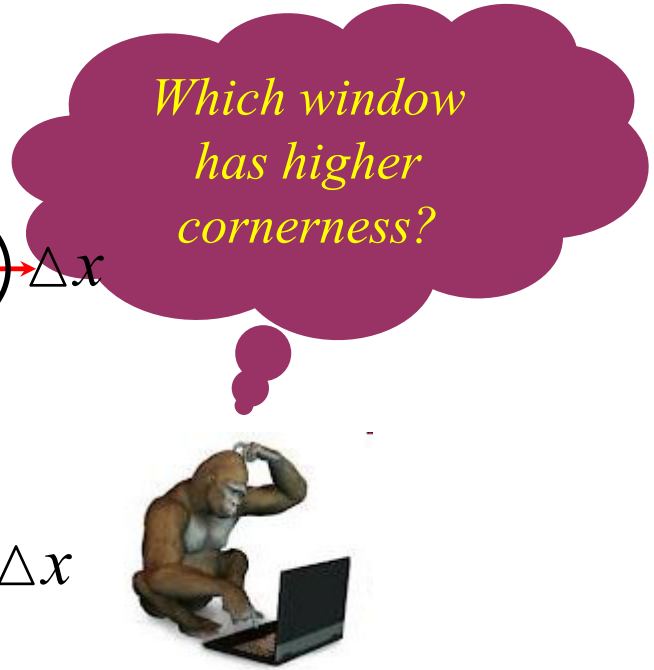
$$[\Delta x, \Delta y]\boldsymbol{M}_1\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}=1$$



For w_2,

$$[\Delta x, \Delta y]\boldsymbol{M}_2\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}=1$$

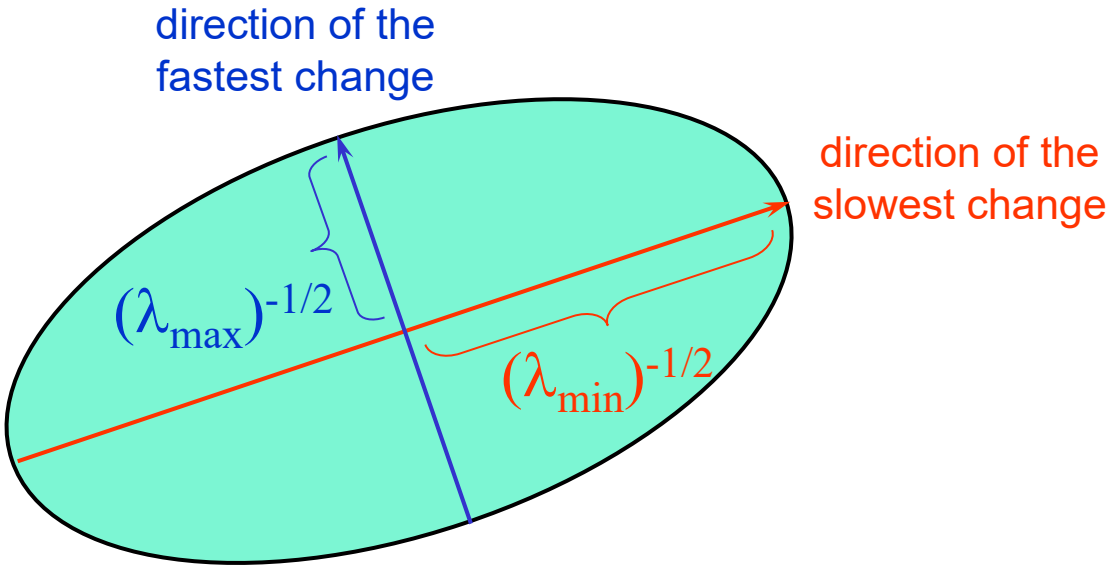




Harris Corner Detection

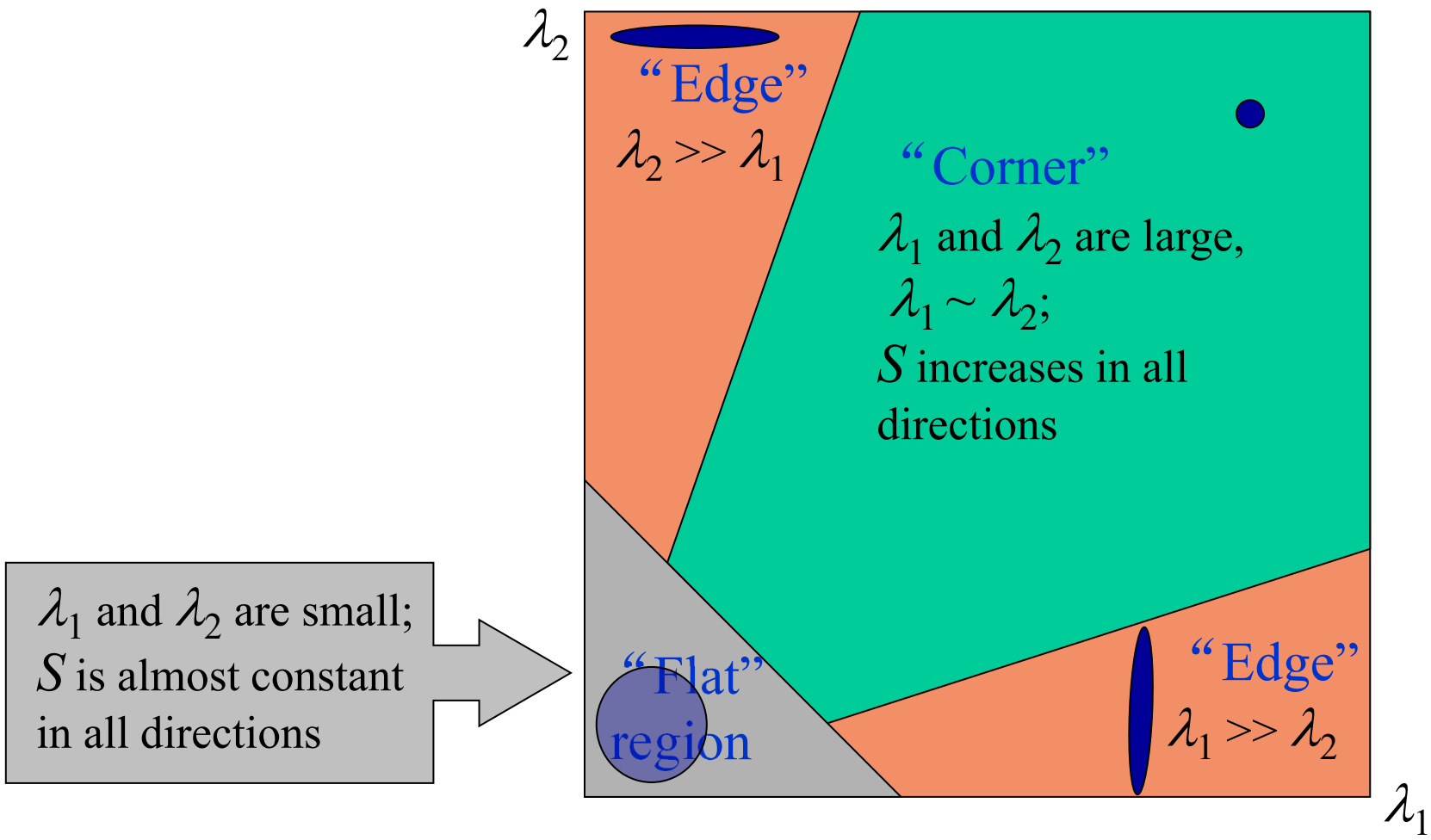
Diagonalization of $\boldsymbol{M}$: $$\boldsymbol{M} = \boldsymbol{R}^{-1}\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}\boldsymbol{R}$$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by $\boldsymbol{R}$



Interpreting the eigenvalues

Classification of image points using eigenvalues of $\boldsymbol{M}$:



Corner response function

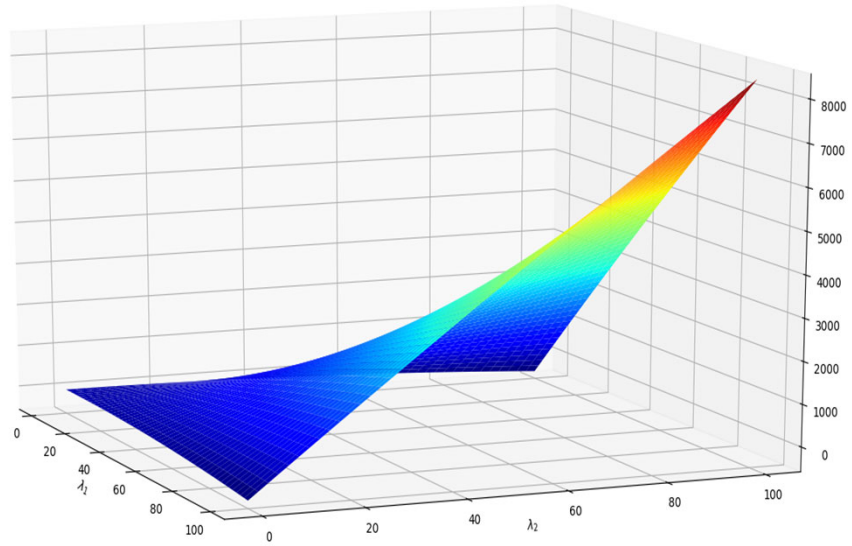
Measure of corner response:

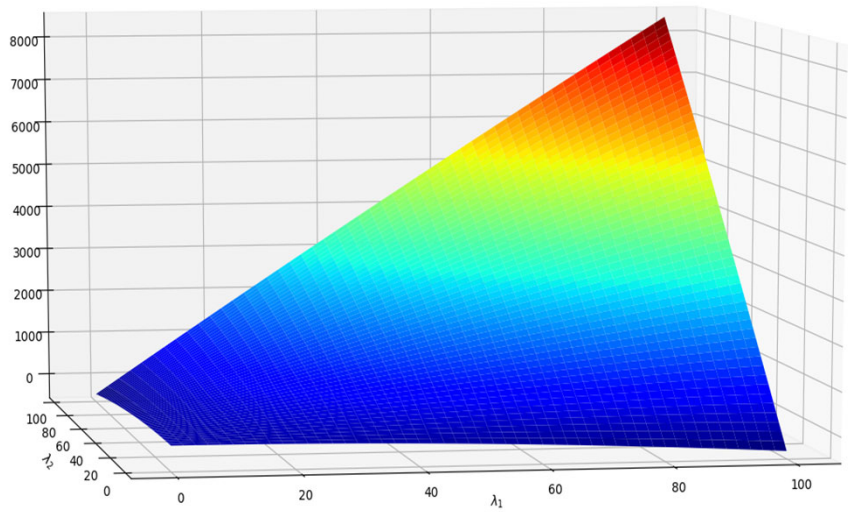
$$R = \det \boldsymbol{M} - \alpha \left(\operatorname{trace} \boldsymbol{M} \right)^2$$

$$\det\left(\boldsymbol{M}\right) = \lambda_1 \lambda_2$$

$$\operatorname{trace}\left(\boldsymbol{M}\right) = \lambda_1 + \lambda_2$$

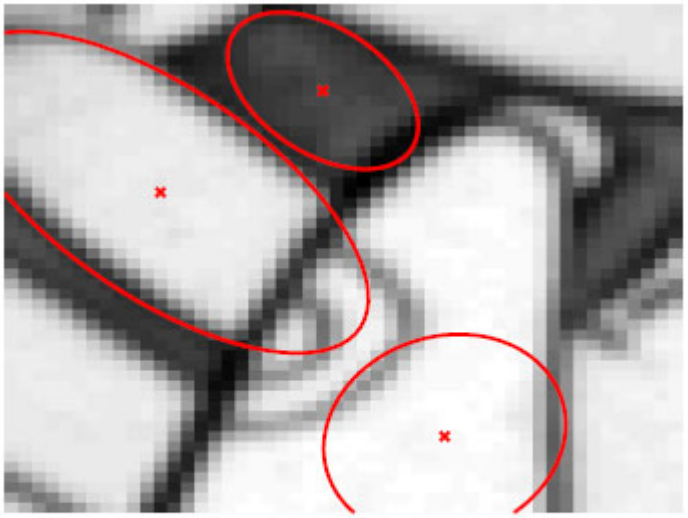
(α – empirical constant, α= 0.04-0.06)



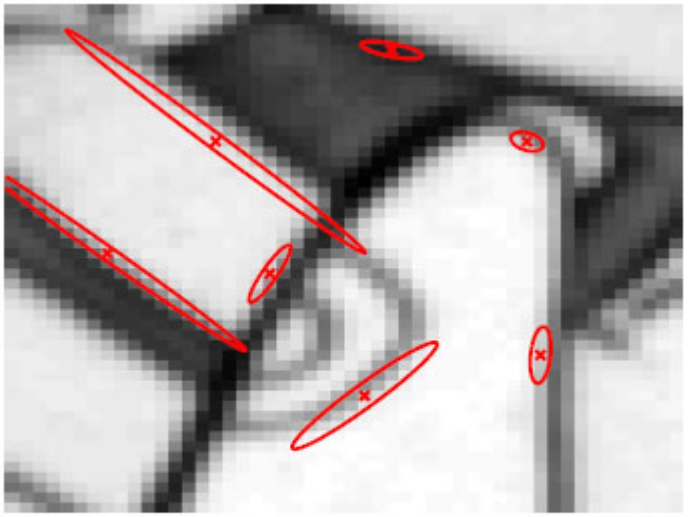


Harris corner detector--illustration

Ellipse with equation : $[\triangle x, \triangle y]\boldsymbol{M}\begin{bmatrix}\triangle x\\ \triangle y\end{bmatrix}=1$



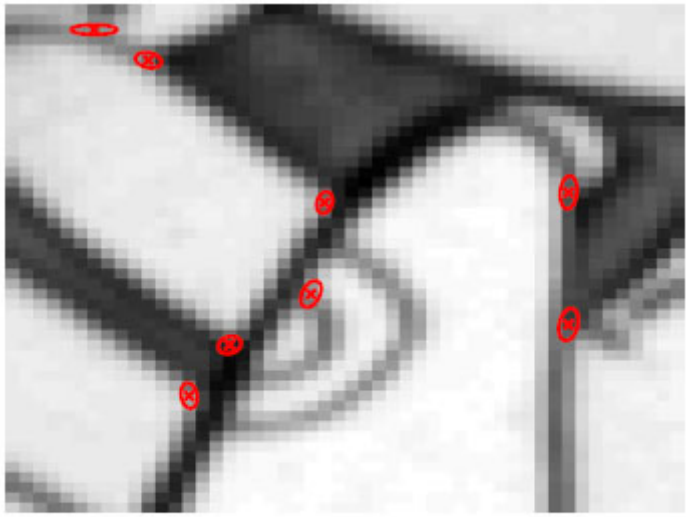
flat region
both eigenvalues small



edge
one small, one large

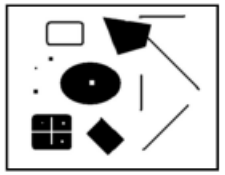
Harris corner detector--illustration

Ellipse with equation : $\left[\triangle x, \triangle y\right] \boldsymbol{M} \begin{bmatrix} \triangle x \\ \triangle y \end{bmatrix} = 1$



corner
both eigenvalues large

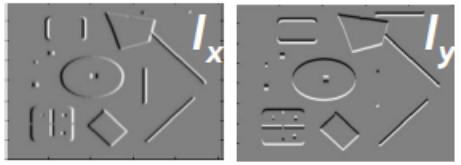
Harris corner detector-Algorithm



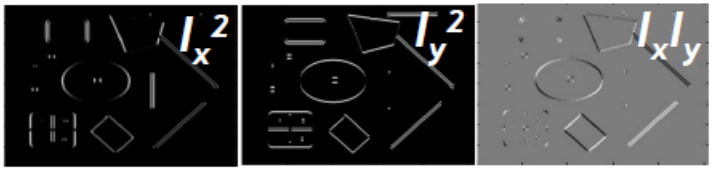
- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I,\sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

1. Image derivatives



2. Square of derivatives



3. Gaussian filter $g(\sigma_I)$

4. Cornerness function - two strong eigenvalues

$$R = \det[M(\sigma_I,\sigma_D)] - \alpha[\mathrm{trace}(M(\sigma_I,\sigma_D))]^2$$
$$= g(I_x^2)g(I_y^2) - [g(I_xI_y)]^2 - \alpha[g(I_x^2)+g(I_y^2)]^2$$

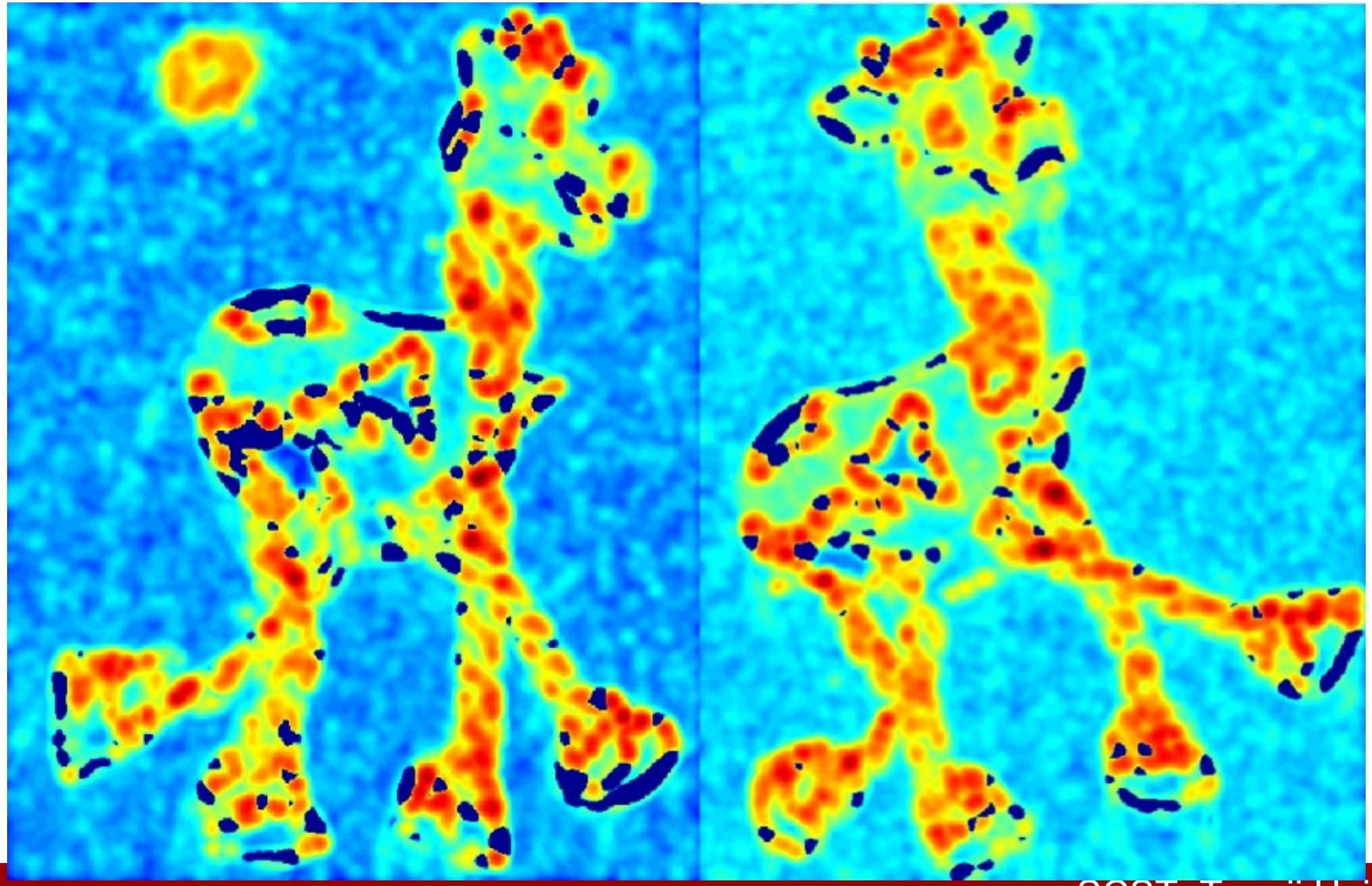
5. Perform non-maximum suppression

Slide credit: Krystian Mikolajczyk

Harris Detector: Steps

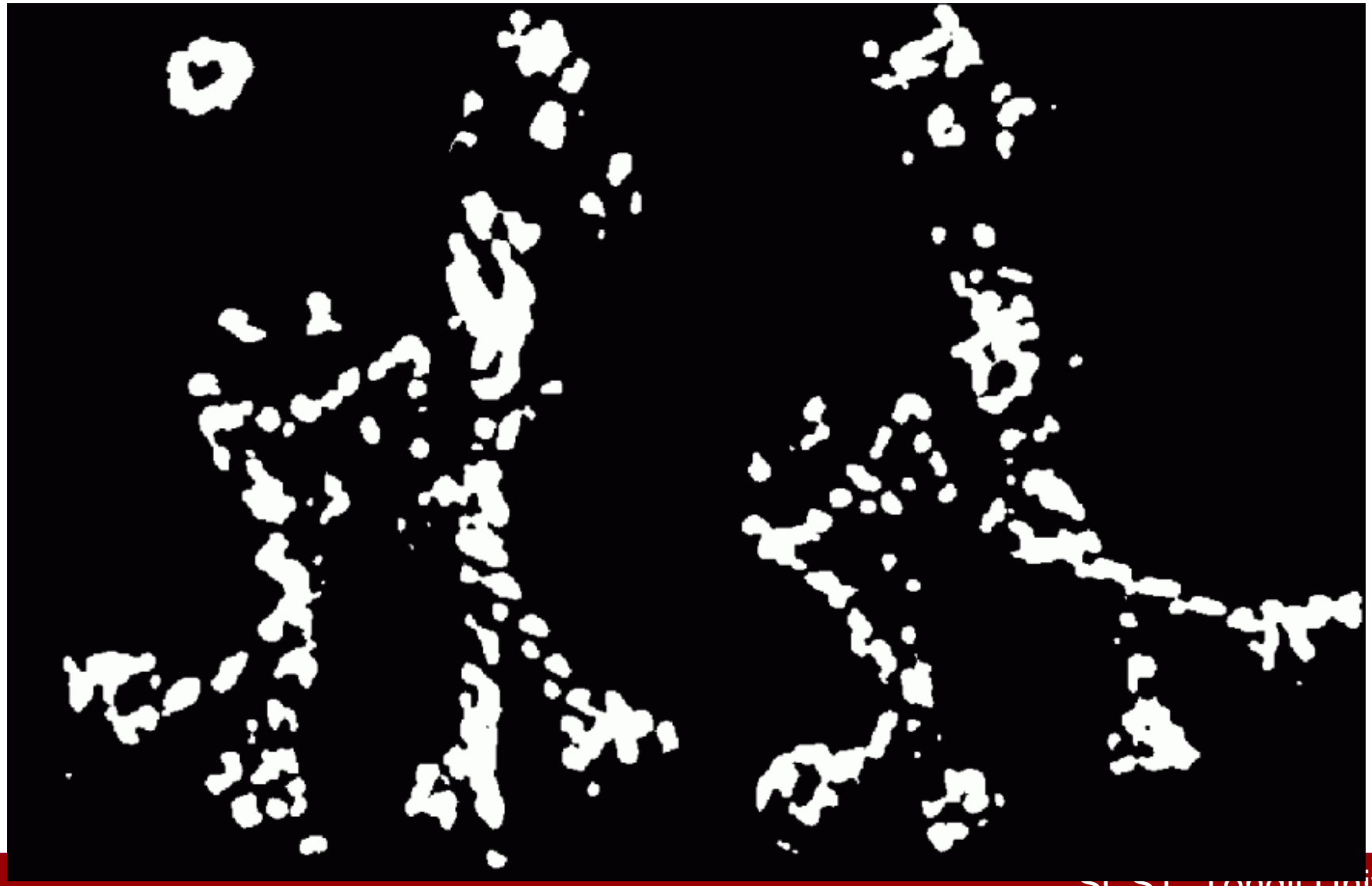
Harris Detector: Steps

Compute corner response R



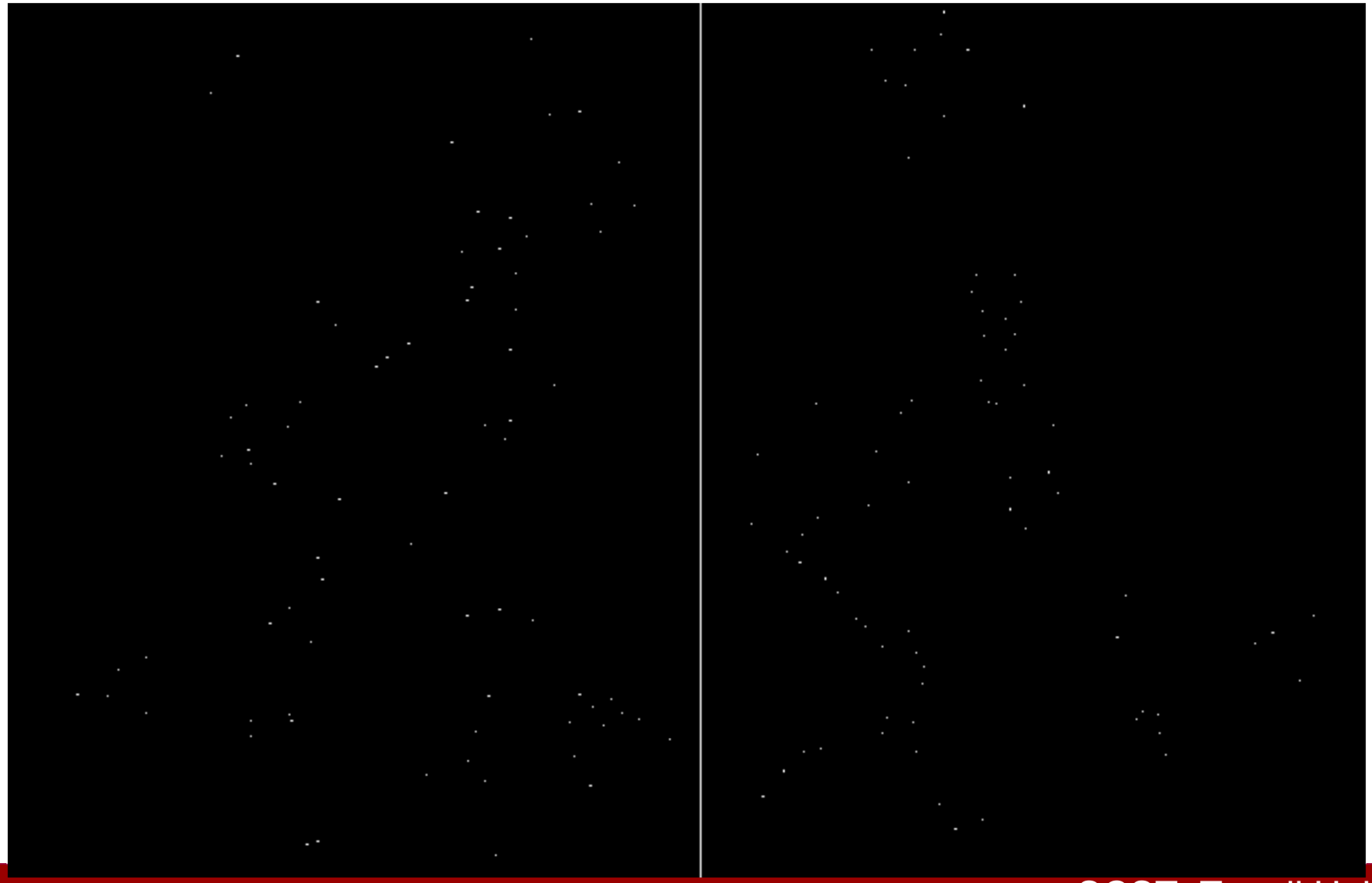
Harris Detector: Steps

Find points with large corner response: R>threshold



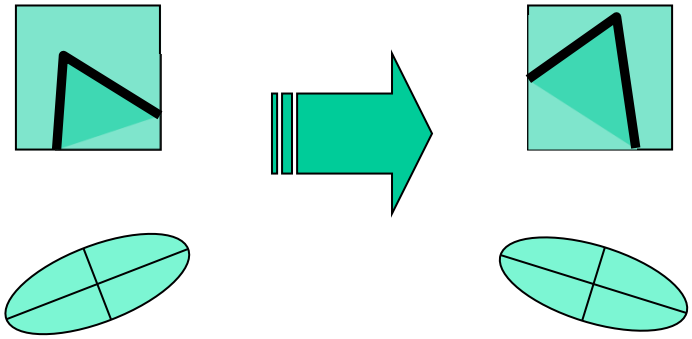
Harris Detector: Steps

Take only the points of local maxima of *R*



Harris Detector: Some Properties

Rotation invariance

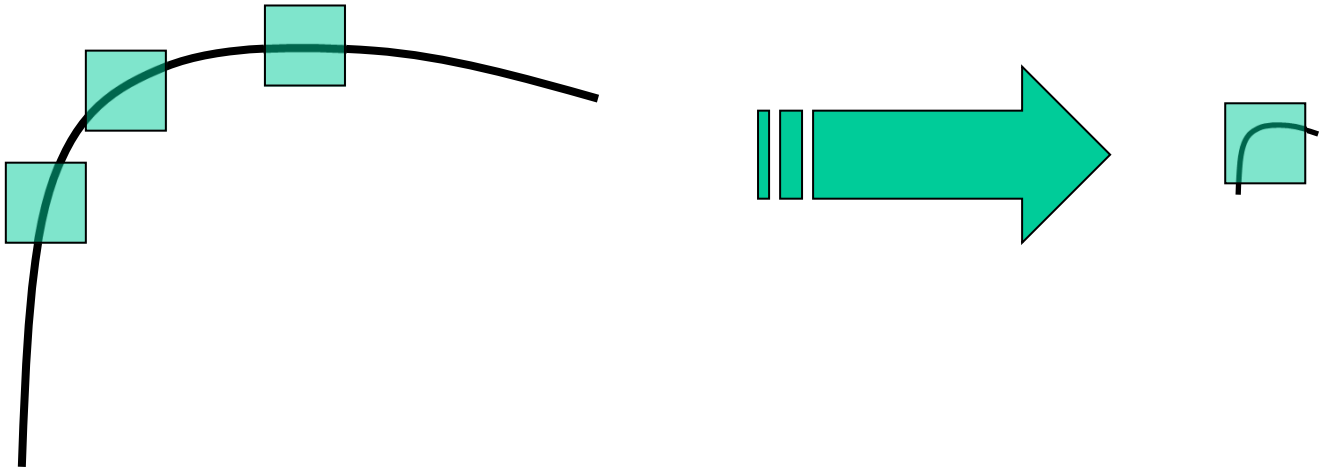


Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response R is invariant to image rotation

Harris Detector: Some Properties

Not invariant to *image scale*!



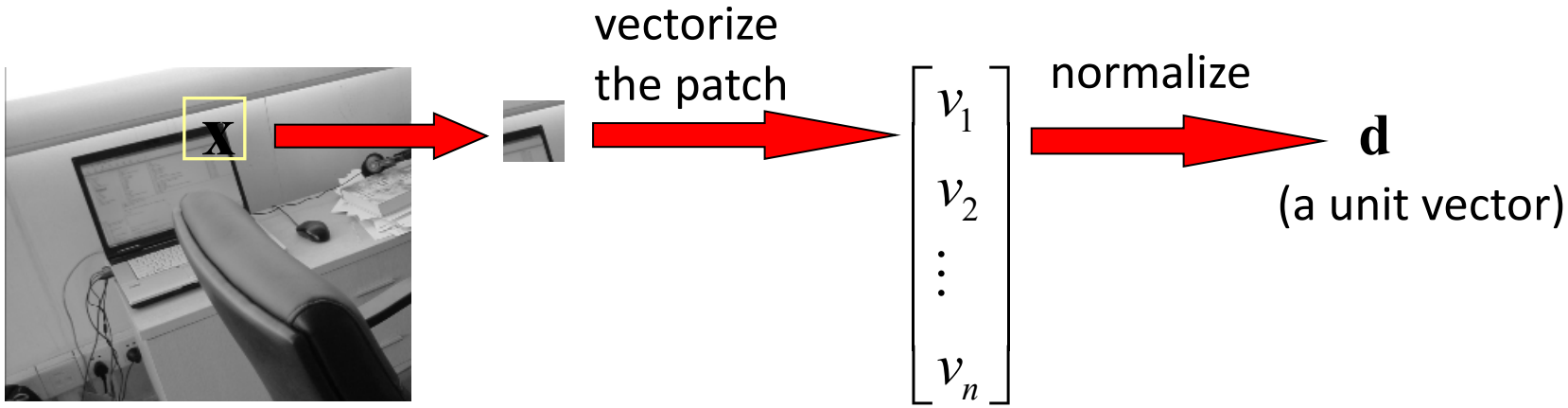
All points will be classified as edges

Corner !

The underlying reason is that Harris corner detection scheme does not provide an automatic and appropriate window size selection method!

“block” descriptor (usually used for Harris corner)

- “Block” descriptor for a Harris corner $\mathbf{x}$
  - Take a region with a fixed size around $\mathbf{x}$
  - Stack the region into a vector and normalize it as $\mathbf{d}$
  - This vector $\mathbf{d}$ serves as the descriptor for $\mathbf{x}$



Re-investigate the “block” descriptor

- “Block” descriptor for a Harris corner $\mathbf{x}$
  - Take a region with a fixed size around $\mathbf{x}$
  - Stack the region into a vector and normalize it as $\mathbf{d}$
  - This vector $\mathbf{d}$ serves as the descriptor for $\mathbf{x}$
- Deficiencies of such simple descriptors
  - Not rotation invariant
  - Not scale invariant

Re-investigate the “block” descriptor

- “Block” descriptor for a Harris corner $\mathbf{x}$
  - Take a region with a fixed size around $\mathbf{x}$
  - Stack the region into a vector and normalize it as $\mathbf{d}$
  - This vector $\mathbf{d}$ serves as the descriptor for $\mathbf{x}$
- We want:
  - Rotation and scale invariant feature points
  - Rotation and scale invariant feature descriptors

Content

- Local Invariant Features
- Harris Corner Detector
- Scale Invariant Point Detection
- Scale Invariant Feature Transform (SIFT)
- Interest Point Matching

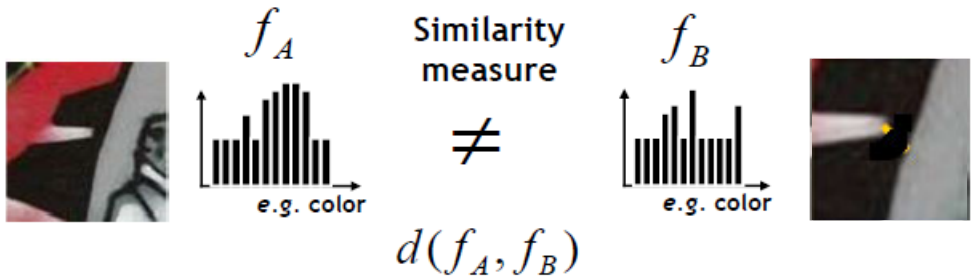
From Points to Regions

- The Harris corner detector defines interest points
  - Precise localization
  - High repeatability

- In order to match those points, we need to compute a descriptor over a region
  - How can we define such a region in a scale invariant manner?
  - That is how can we detect sale invariant regions?

Naïve Approach: Exhaustive Search

- Multi-scale procedure
  - Compare descriptors while varying the patch size



Slide credit: Krystian Mikolajczyk

Naïve Approach: Exhaustive Search

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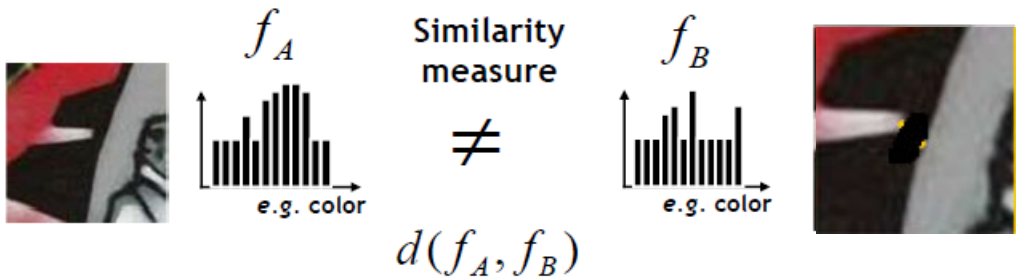

Slide credit: Krystian Mikolajczyk

Naïve Approach: Exhaustive Search

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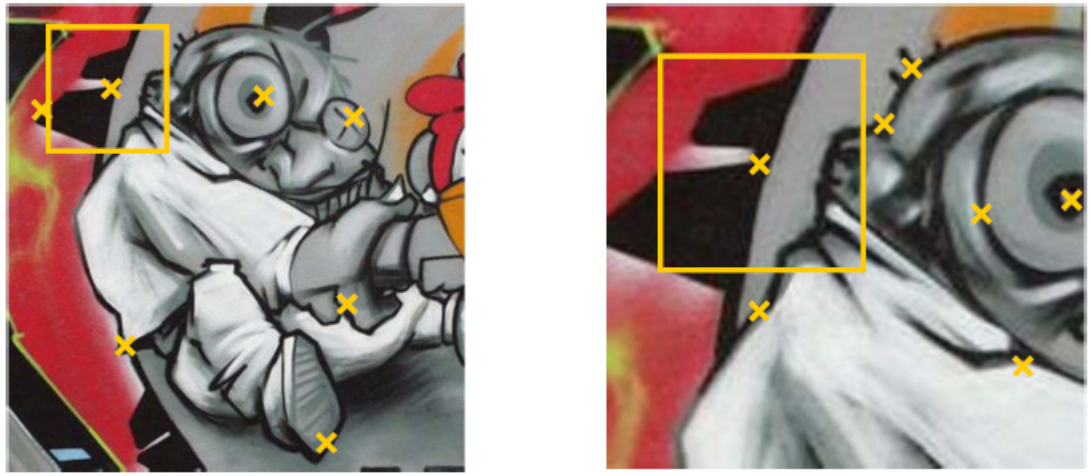
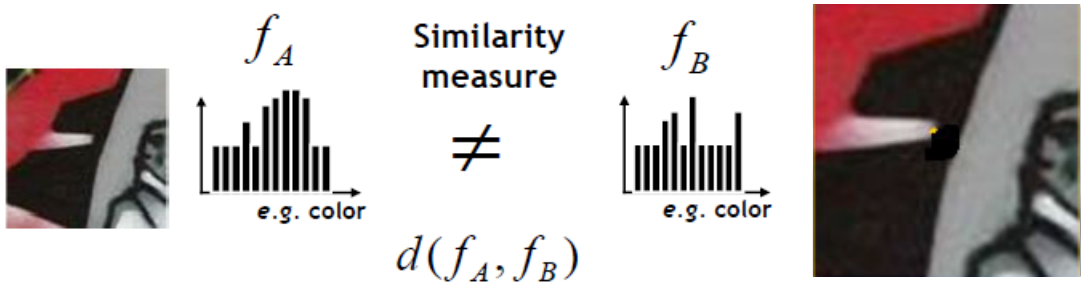
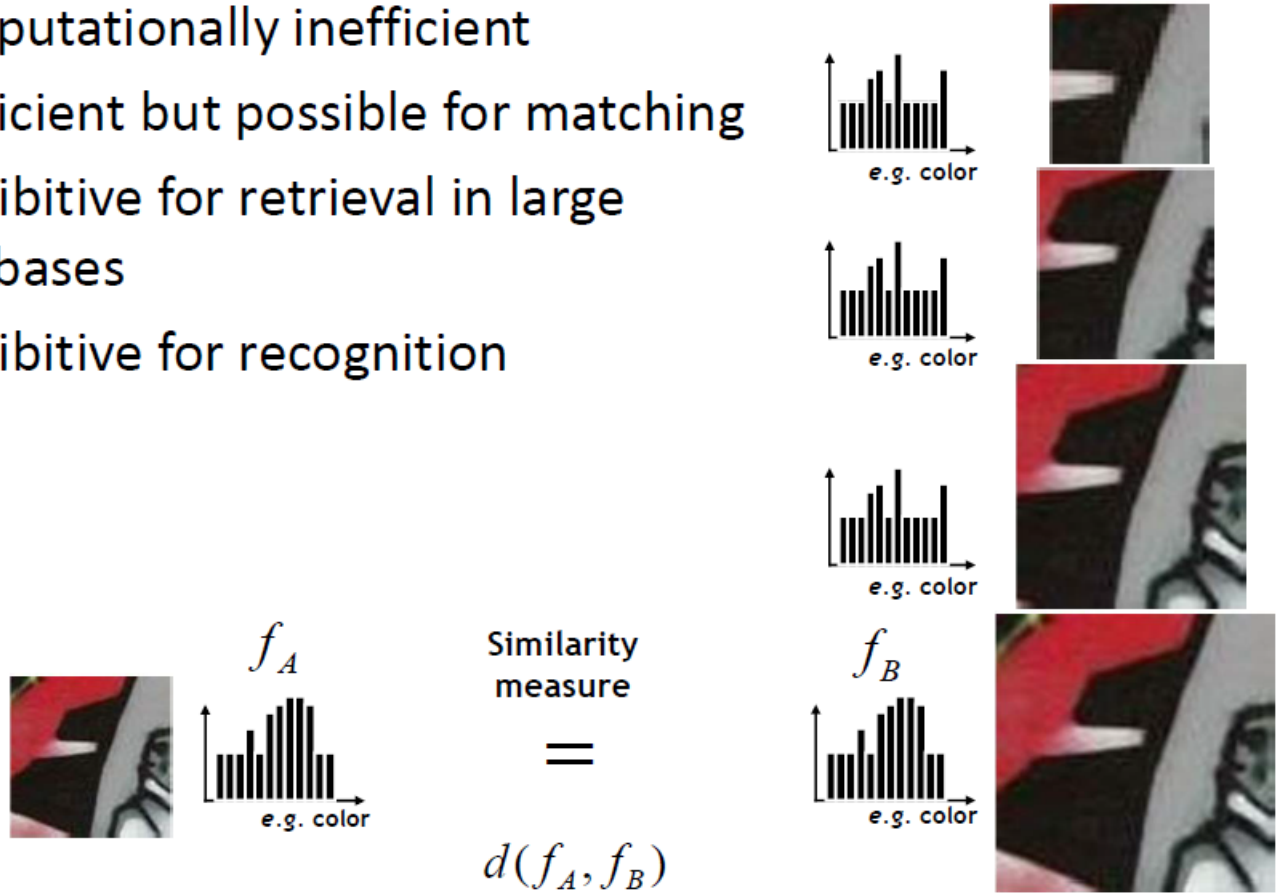




Slide credit: Krystian Mikolajczyk

Naïve Approach: Exhaustive Search

- Comparing descriptors while varying the patch size
  - Computationally inefficient
  - Inefficient but possible for matching
  - Prohibitive for retrieval in large databases
  - Prohibitive for recognition



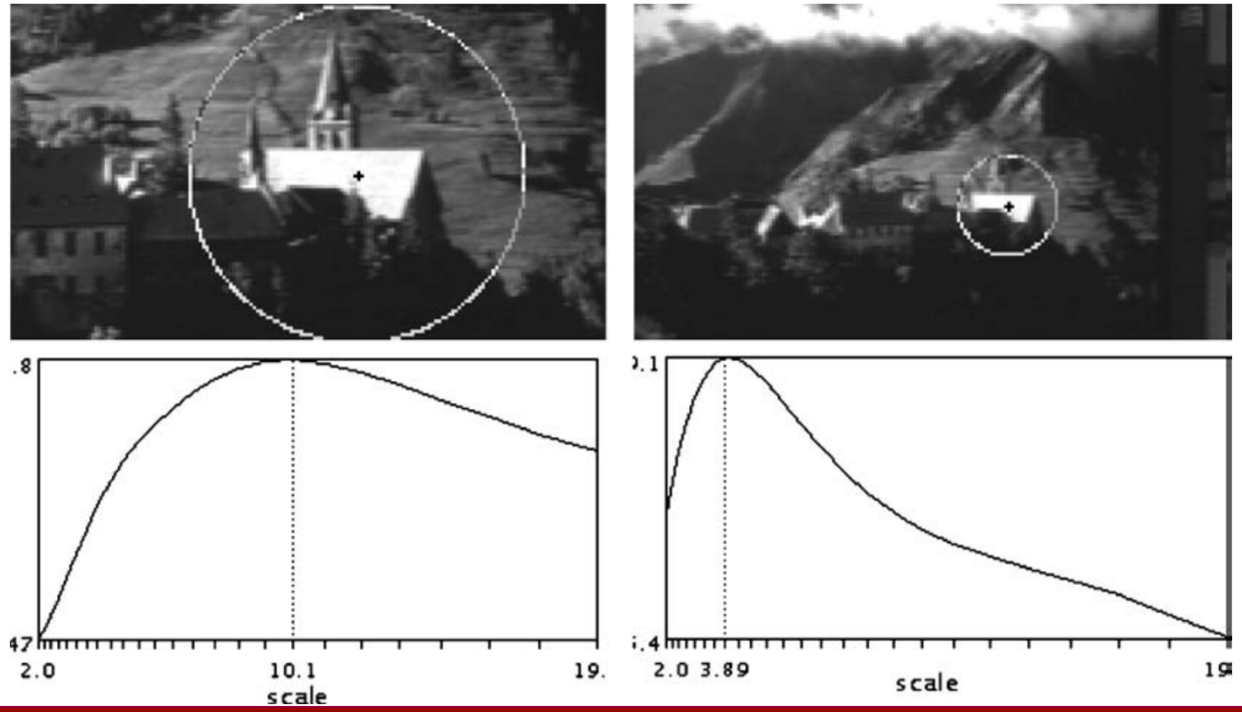
Slide credit: Krystian Mikolajczyk

What do we want to do next?

- Naïve approach for scale invariant local description is not efficient (Detect Harris corners first, and then exhaustively searching for regions with appropriate sizes)
- Now we want to:
  - Find scale invariant points in the image (location)
  - At the same time, we want to know their **characteristic scales** (used to determine the neighborhood for local description)

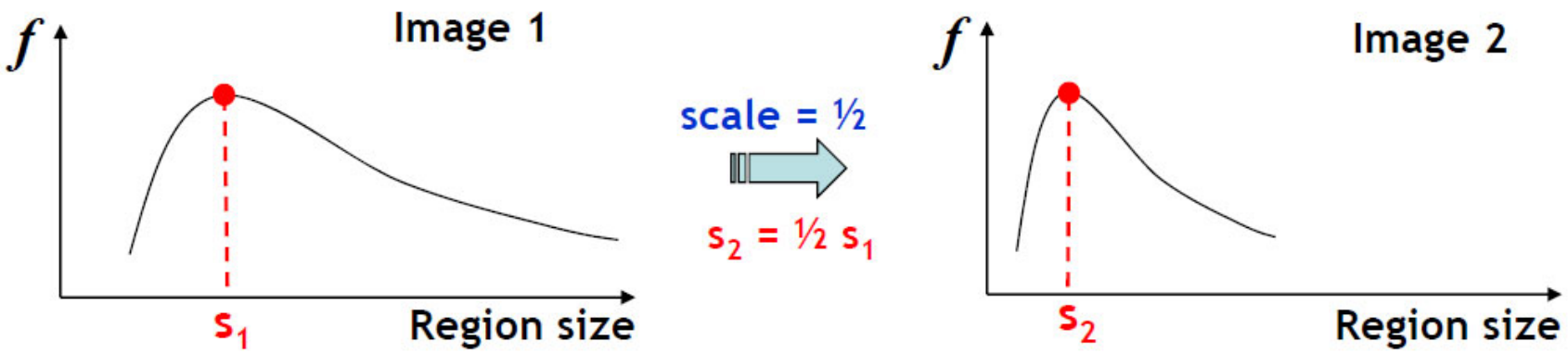
Achieving scale covariance

- Goal: independently detect corresponding regions in scaled versions of the same image
- Need *scale selection* mechanism for finding characteristic region size that is *covariant* with the image transformation



Automatic Scale Selection

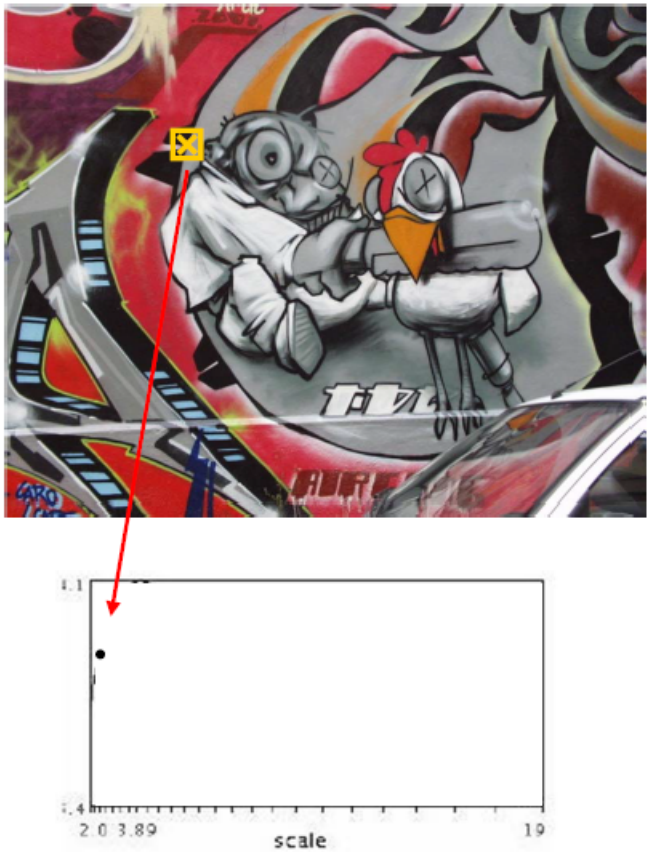
- Common approach
  - Take a local extremum of this function
  - Observation: region size for which the extremum is achieved should be covariant to image scale; this scale covariant region size is found in each image independently



Slide credit: Kristen Grauman

Automatic Scale Selection

- Function responses for increasing scale (scale signature)



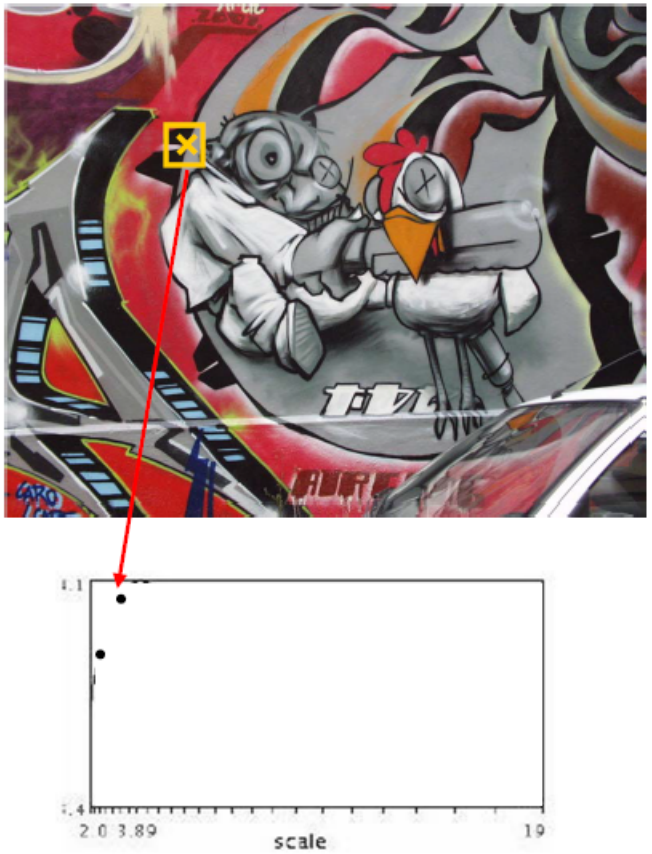
$f(I_{i_1 \ldots i_m}(x, \sigma))$

$f(I_{i_1 \ldots i_m}(x', \sigma))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

- Function responses for increasing scale (scale signature)



$f(I_{i_1 \ldots i_m}(x, \sigma))$

$f(I_{i_1 \ldots i_m}(x', \sigma))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

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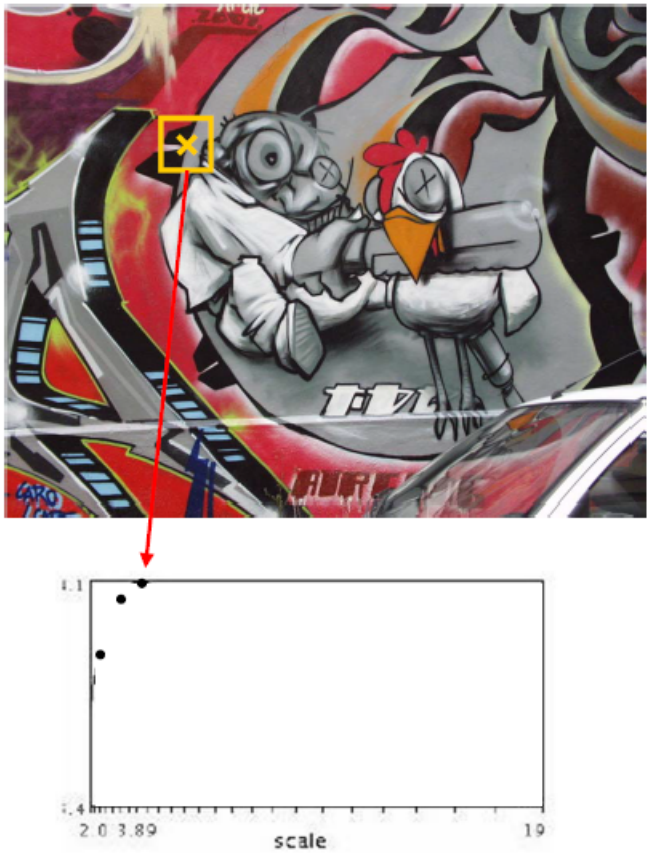

$f(I_{i_1 \ldots i_m}(x, \sigma))$

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Slide credit: Krystian Mikolajczyk

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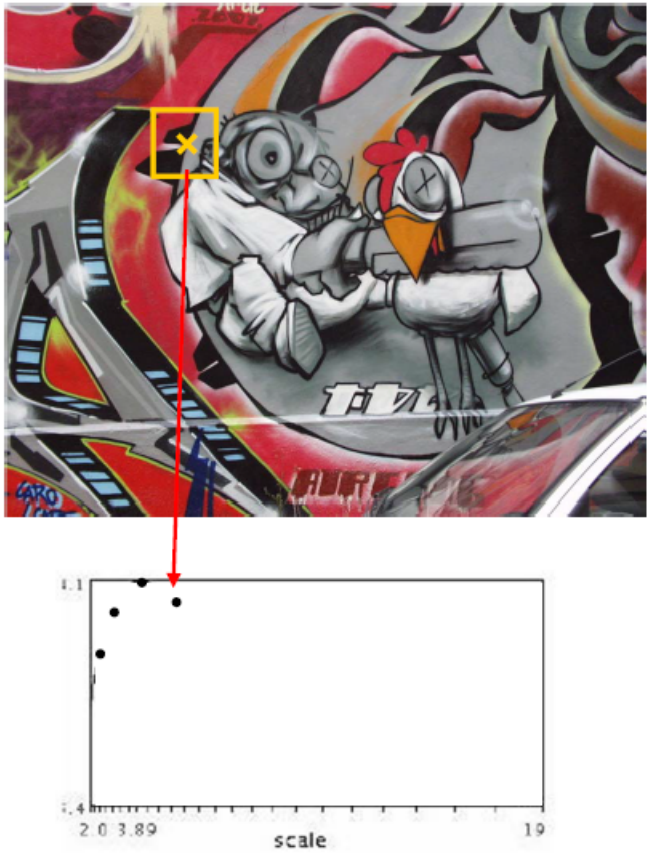
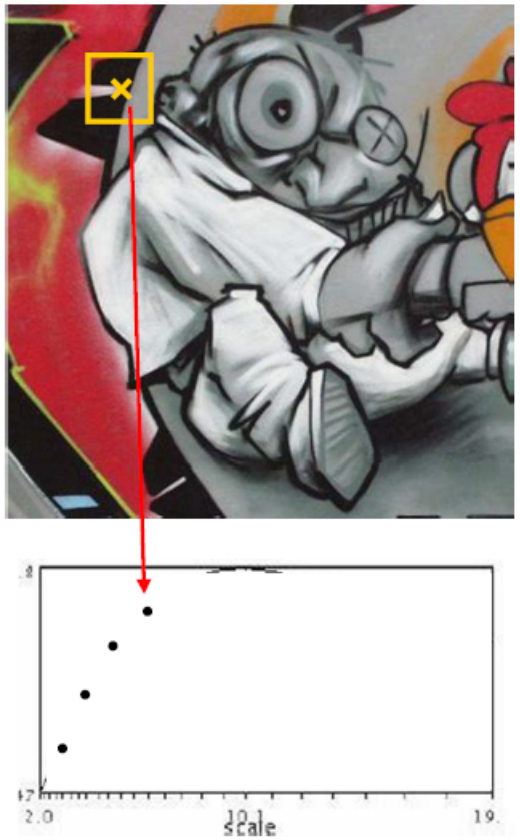


$f(I_{i_1 \ldots i_m}(x, \sigma))$

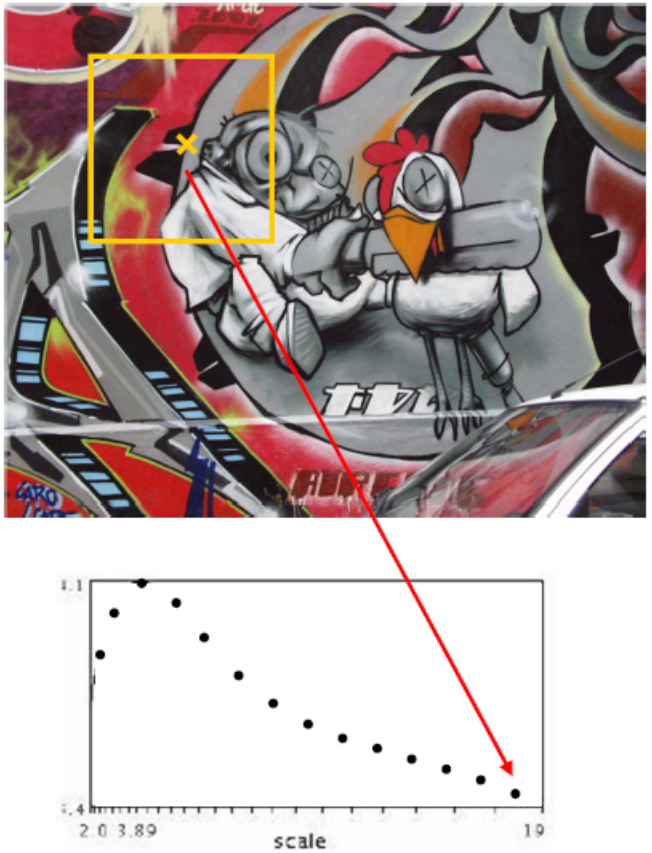


$f(I_{i_1 \ldots i_m}(x', \sigma))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

- Function responses for increasing scale (scale signature)



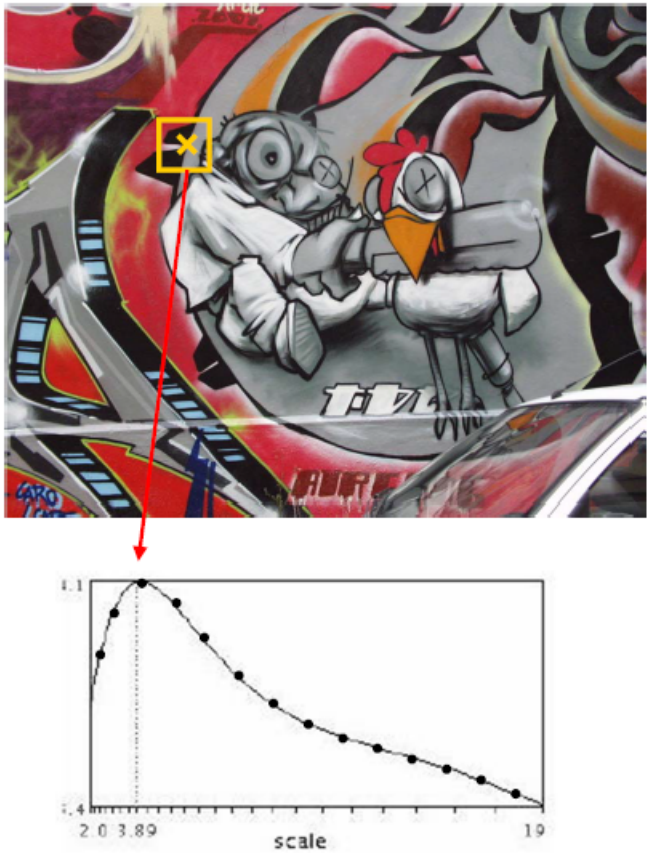
$f(I_{i_1 \ldots i_m}(x, \sigma))$

$f(I_{i_1 \ldots i_m}(x', \sigma))$

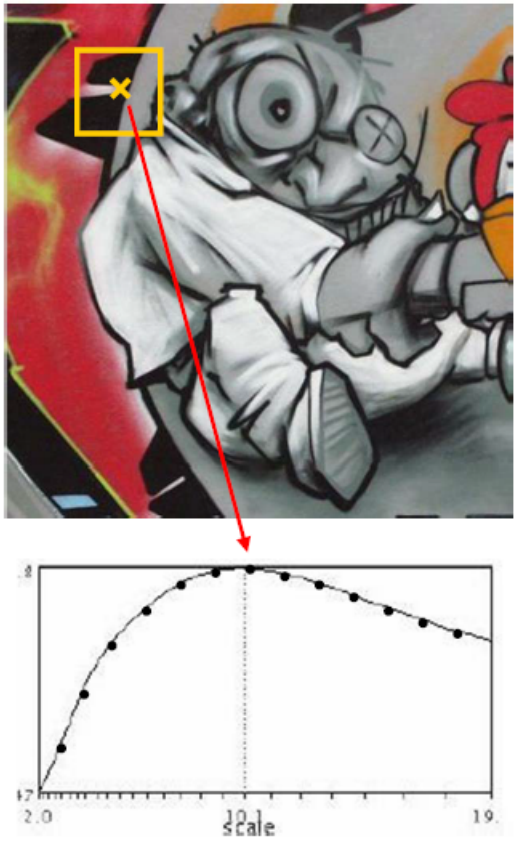
Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

- Function responses for increasing scale (scale signature)



$f(I_{i_1 \ldots i_m}(x,\sigma))$

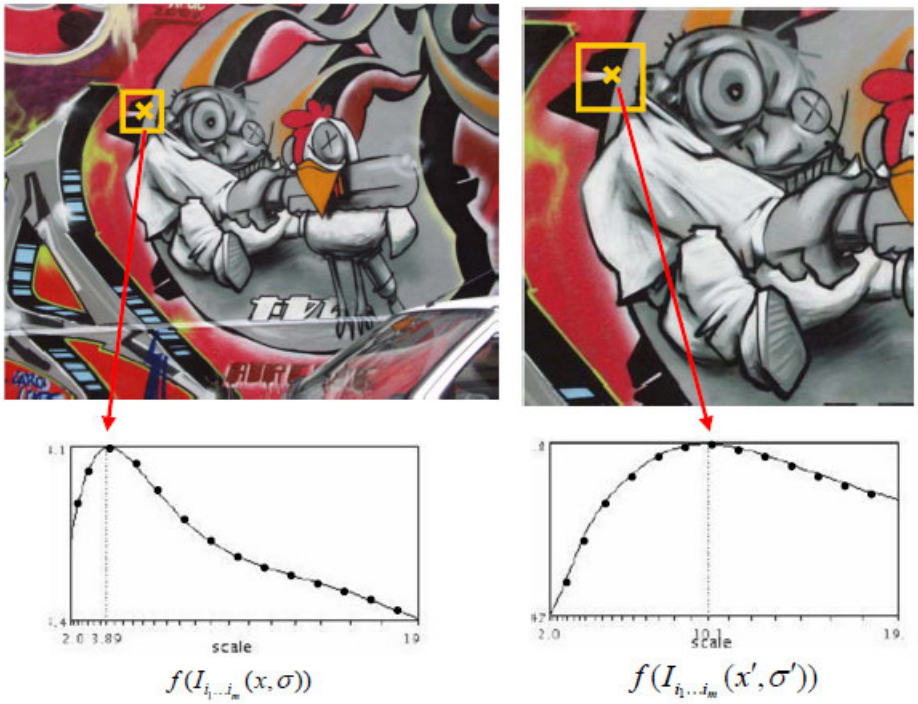


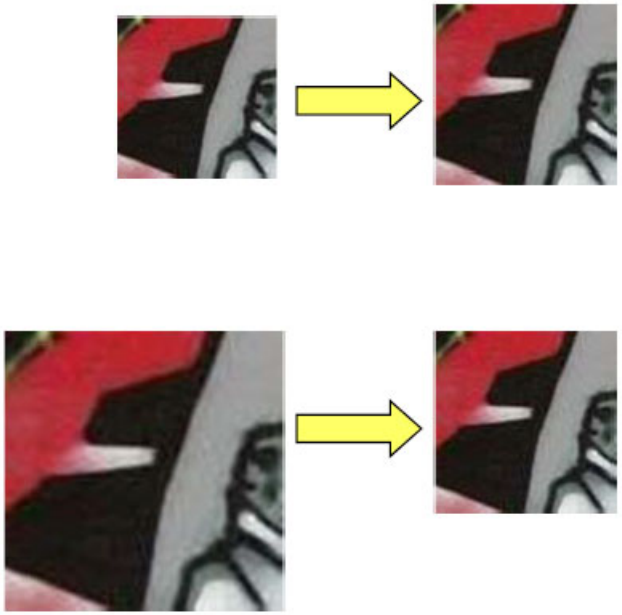
$f(I_{i_1 \ldots i_m}(x',\sigma'))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

- Normalize: Rescale to fixed size

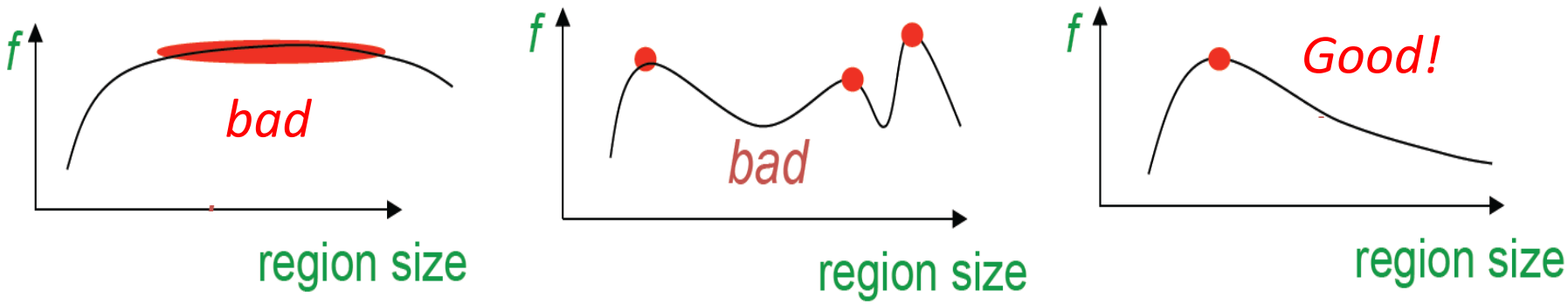




Slide credit: Tinne Tuytelaars

Automatic Scale Selection

- A good function for scale selection
  - It should have one stable sharp peak response to region size



The answer is scale-normalized Laplacian of Gaussian!

Scale-normalized LoG

2D isotropic Gaussian function,

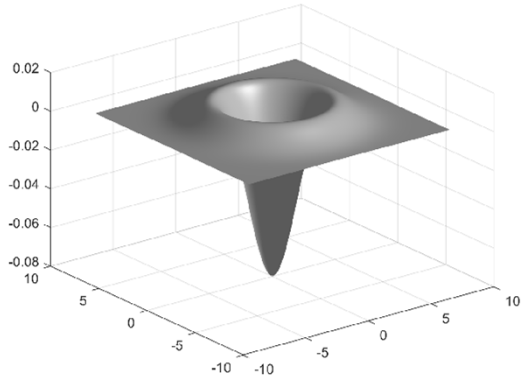
$$g(x,y)=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

LoG,

$$\nabla^2 g=\frac{\partial^2 g}{\partial x^2}+\frac{\partial^2 g}{\partial y^2}=\frac{x^2+y^2-2\sigma^2}{2\pi\sigma^6}e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Scale-normalized LoG,

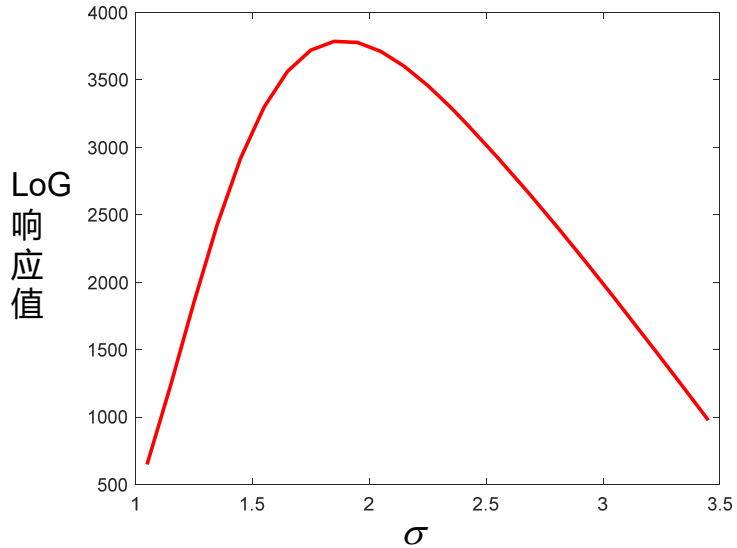
$$\sigma^2\nabla^2 g=\frac{x^2+y^2-2\sigma^2}{2\pi\sigma^4}e^{-\frac{x^2+y^2}{2\sigma^2}}$$



It can be used for blob structure detection

Blob detection using scale-normalized LoG

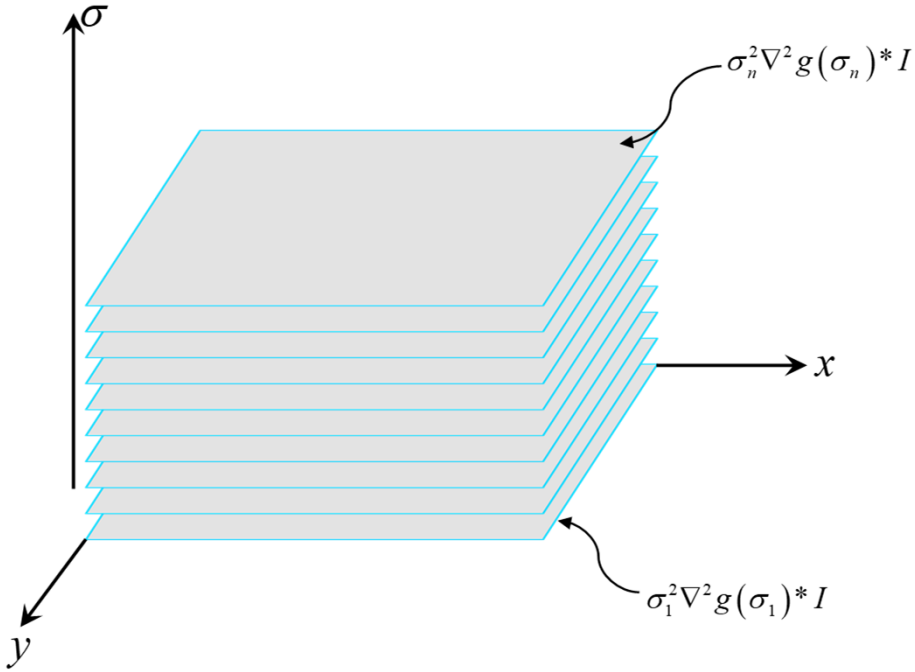
- For blob structures detection on image I
  - We need to know their centers and their spatial extensions (blob sizes)
  - For a blob point, its responses to scale-normalized LoGs with various scales has a unique peak (valley) , and thus we can use the “peak scale” to determine its size



Responses of a blob point to a series of scale-normalized LoGs with various scales

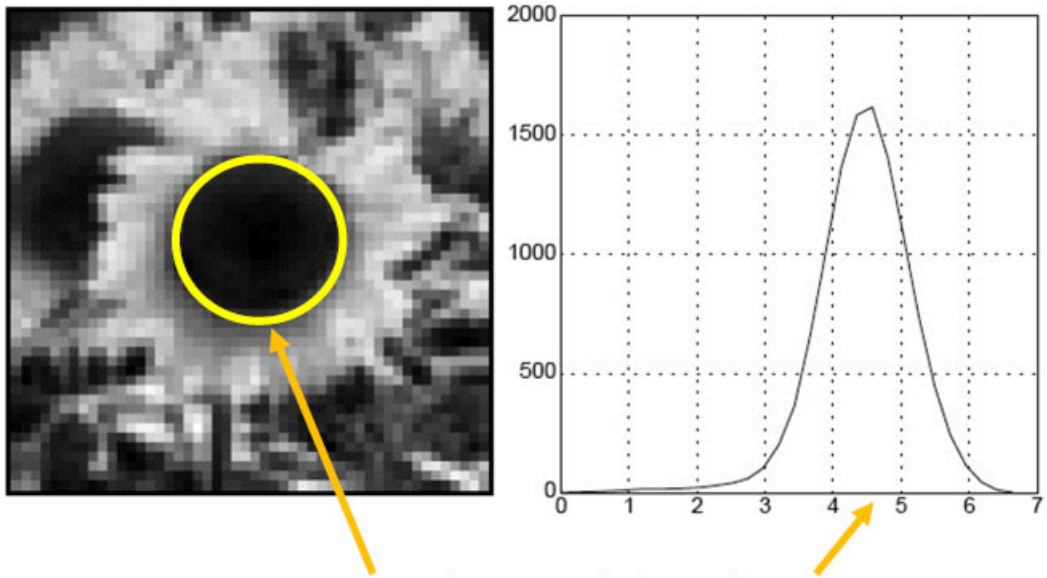
Blob detection using scale-normalized LoG

- For blob structures detection on image I
  - We need to know their centers and their spatial extensions (blob sizes)
  - For a blob point, its responses to scale-normalized LoGs with various scales has a unique peak (valley) , and thus we can use the "peak scale" to determine its size
  - Thus, we can use I's responses to a set of scale-normalized LoGs with various scales to find blobs



Characteristic scale

- We define the characteristic scale as the scale that produces peak (or valley) of scale-normalized Laplacian-of-Gaussian response



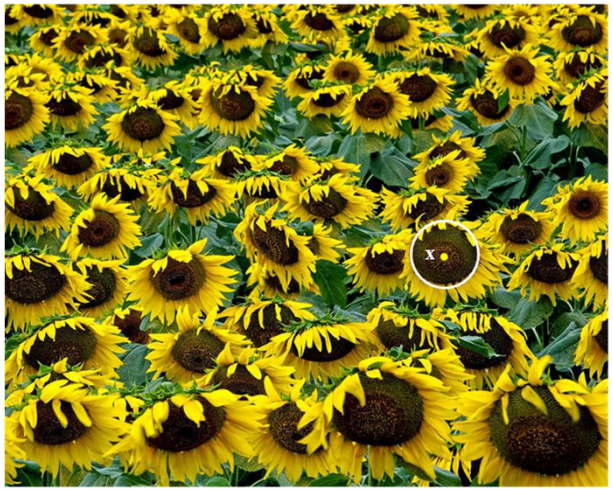
Characteristic scale

T. Lindeberg (1998). "Feature detection with automatic scale selection." *International Journal of Computer Vision* 30 (2): pp 77--116.

Slide credit: Svetlana Lazebnik

Characteristic scale

An example



$\times 2$

x's characteristic scale obtained by scale-normalized LoGs is 1.5

y's characteristic scale obtained by scale-normalized LoGs is 3.0

scale-normalized LoG satisfy our requirement of automatic scale selection

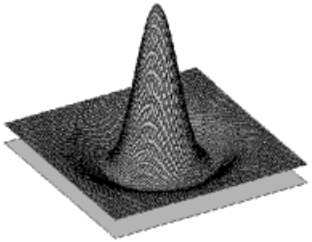
Another Fact

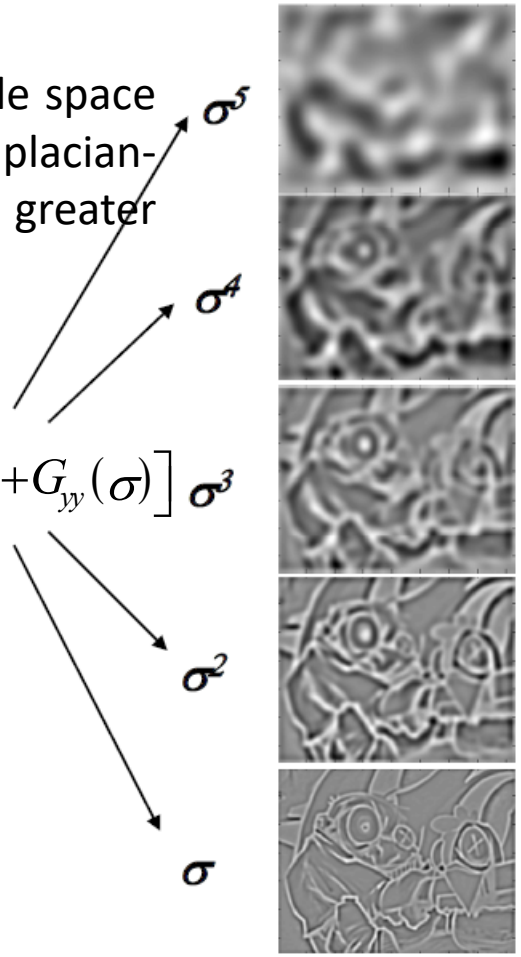
Spatial selection: the magnitude of the scale-normalized Laplacian-of-Gaussian response will achieve an extremum at the center of the blob, provided that its scale is “matched” to the scale of the blob

Scale-Invariant Point Detection

- Interest points:
  - Local extremum in scale space of scale-normalized Laplacian-of-Gaussian and be greater than a threshold

$$\sigma^2\left[G_{xx}(\sigma)+G_{yy}(\sigma)\right]$$





Slide adapted from Krystian Mikolajczyk

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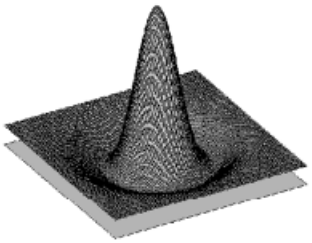

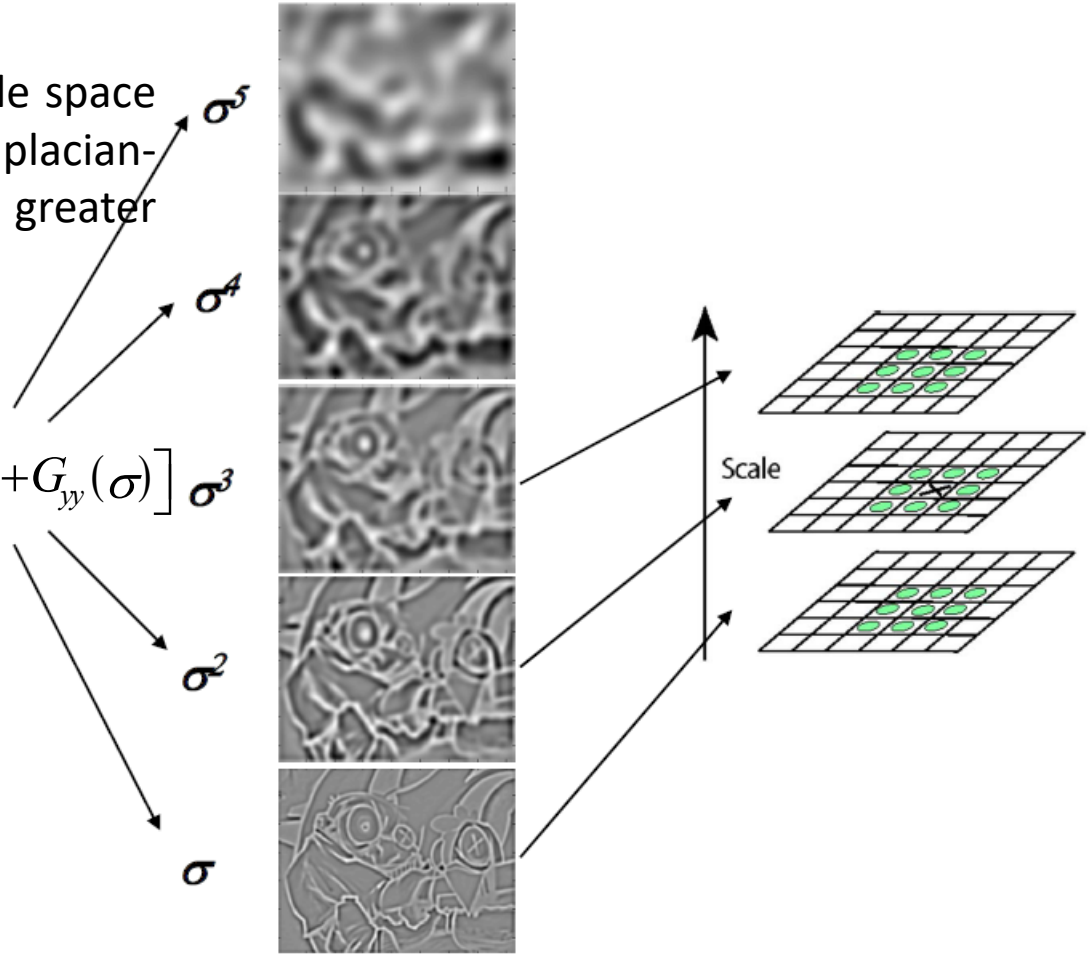

Scale-Invariant Point Detection

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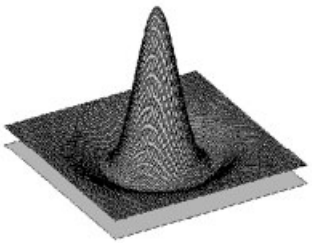
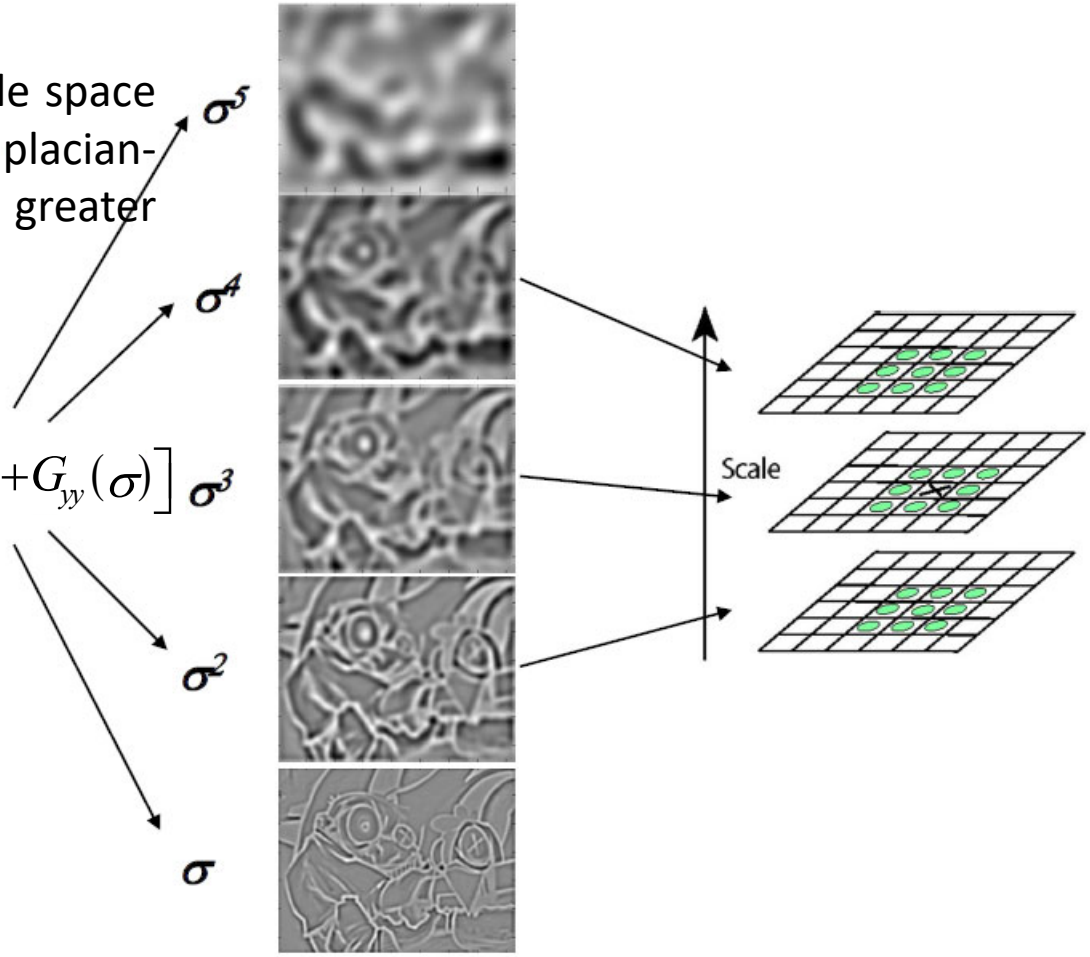
  - Local extremum in scale space of scale-normalized Laplacian-of-Gaussian and be greater than a threshold

$$\sigma^2\left[G_{xx}(\sigma)+G_{yy}(\sigma)\right]$$

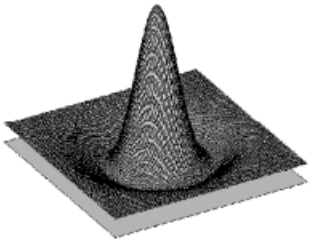


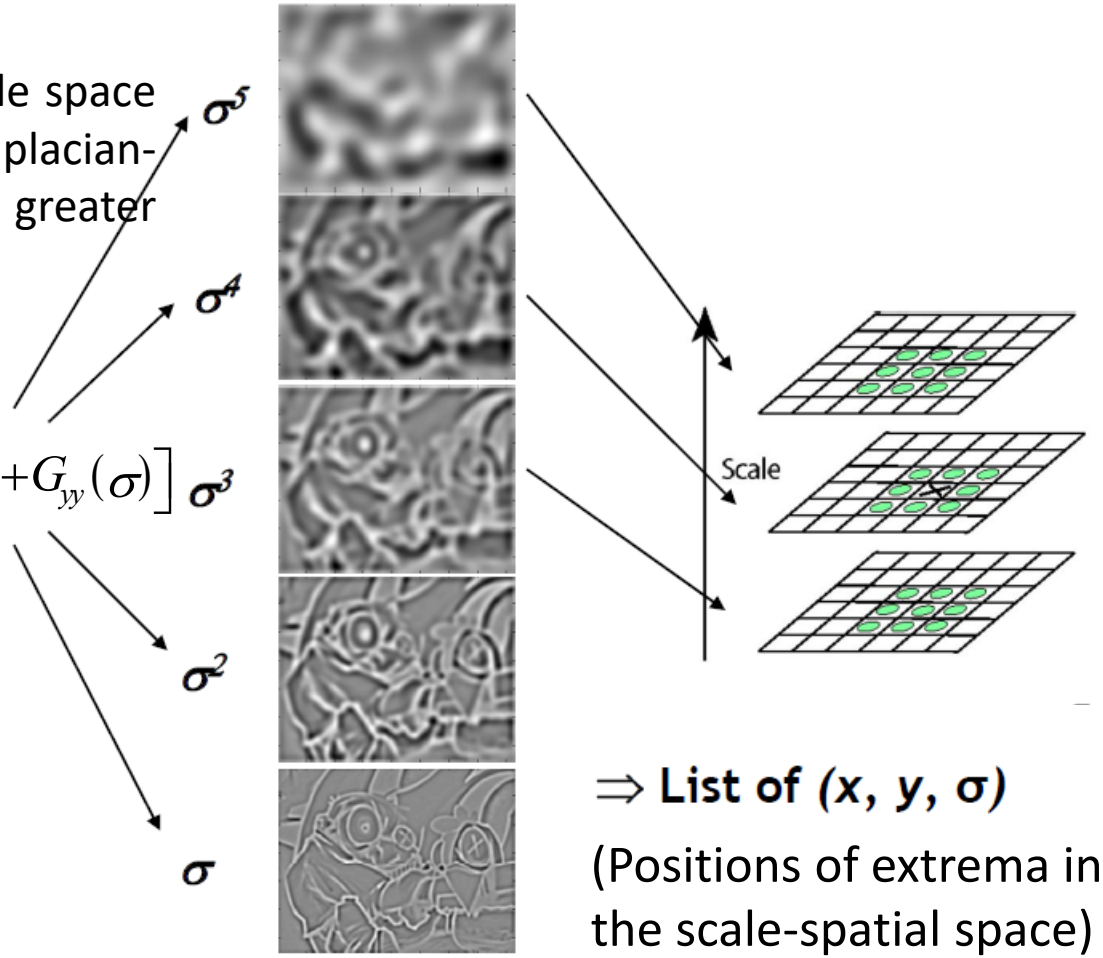


Scale-Invariant Point Detection

- Interest points:
  - Local extremum in scale space of scale-normalized Laplacian-of-Gaussian and be greater than a threshold

$$\sigma^2\left[G_{xx}(\sigma)+G_{yy}(\sigma)\right]$$





⇒ **List of (x, y, σ)**

(Positions of extrema in the scale-spatial space)

We have got want we want!

Note: local extrema is obtained as key points by comparing the examined location with all the other 26 points around it in the scale-space

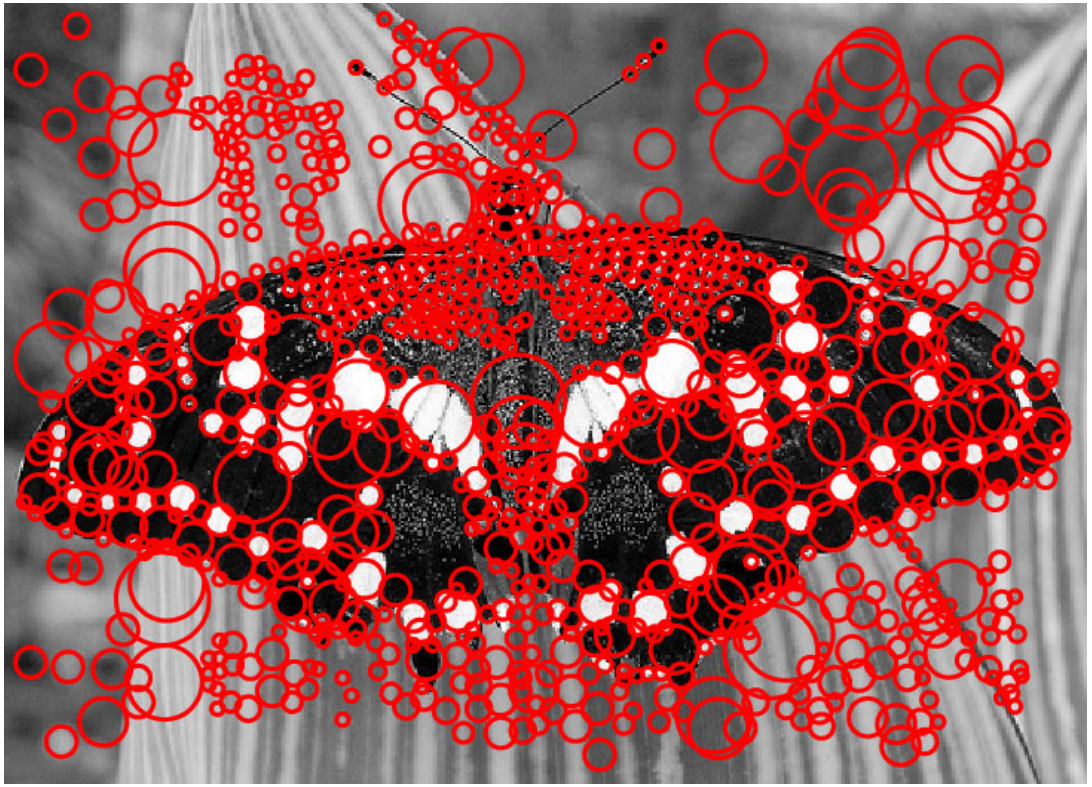
If the local extrema of scale-normalized LoG is achieved at $\mathbf{p}$, two things of $\mathbf{p}$ can be determined: its spatial location and characteristic scale!

Scale-Invariant Point Detection: Example

Scale-Invariant Point Detection: Example

sigma = 11.9912

Scale-Invariant Point Detection: Example



Efficient implementation

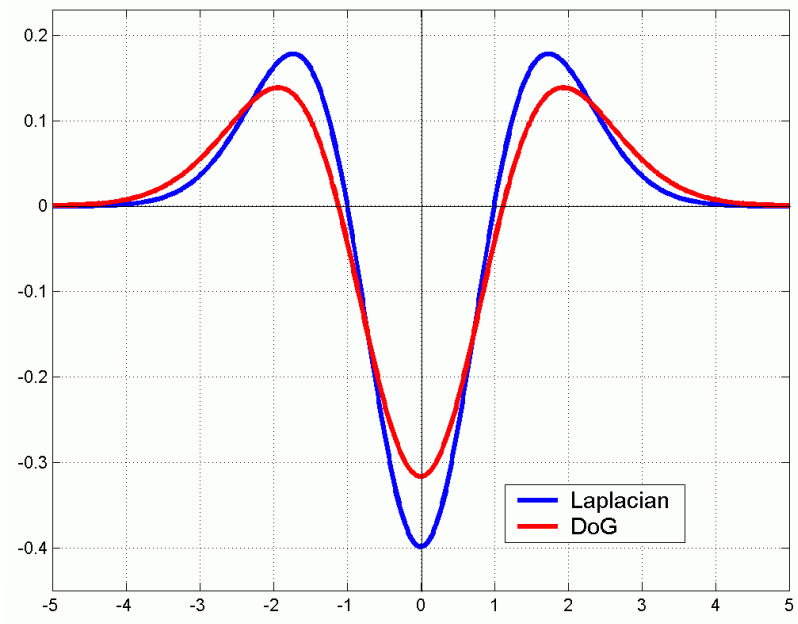
Approximating the scale-normalized LoG with a difference of Gaussians:

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

(scale-normalized LoG)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)

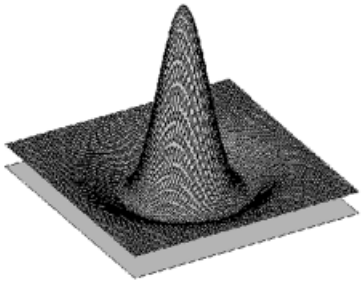


where Gaussian is

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2} \right)$$

DoG

- Difference of Gaussians as approximation of scale-normalized LoG
  - This is used e.g. in Lowe's SIFT pipeline for feature detection.
- Advantages
  - No need to compute 2nd derivatives
  - Gaussians are computed anyway, e.g. in a Gaussian pyramid.



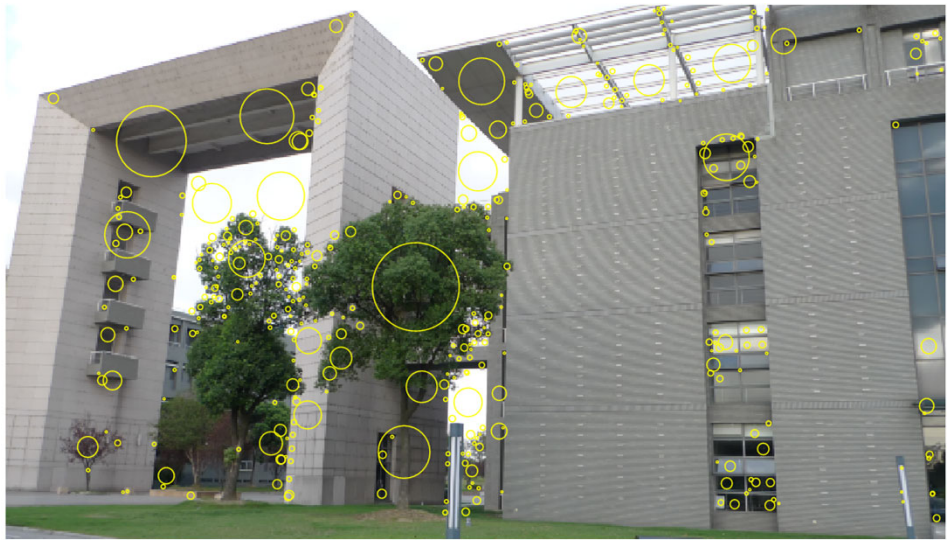
Slide credit: Bastian Leibe

Scale-Invariant Point Detection

- Given: Two images of the same scene with a large *scale difference* between them.
- Goal: Find *the same* interest points *independently* in each image.
- Solution: Search for *maxima* of suitable functions in *scale* and in *space* (over the image).

- Two strategies
  - scale-normalized LoG
  - Difference-of-Gaussian (DoG) as a fast approximation
  - *These can be used either on their own, or in combinations with single-scale keypoint detectors (Harris, Hessian).*

Examples



Content

- Local Invariant Features
- Harris Corner Detector
- Scale Invariant Point Detection
- Scale Invariant Feature Transform (SIFT)
- Interest Point Matching

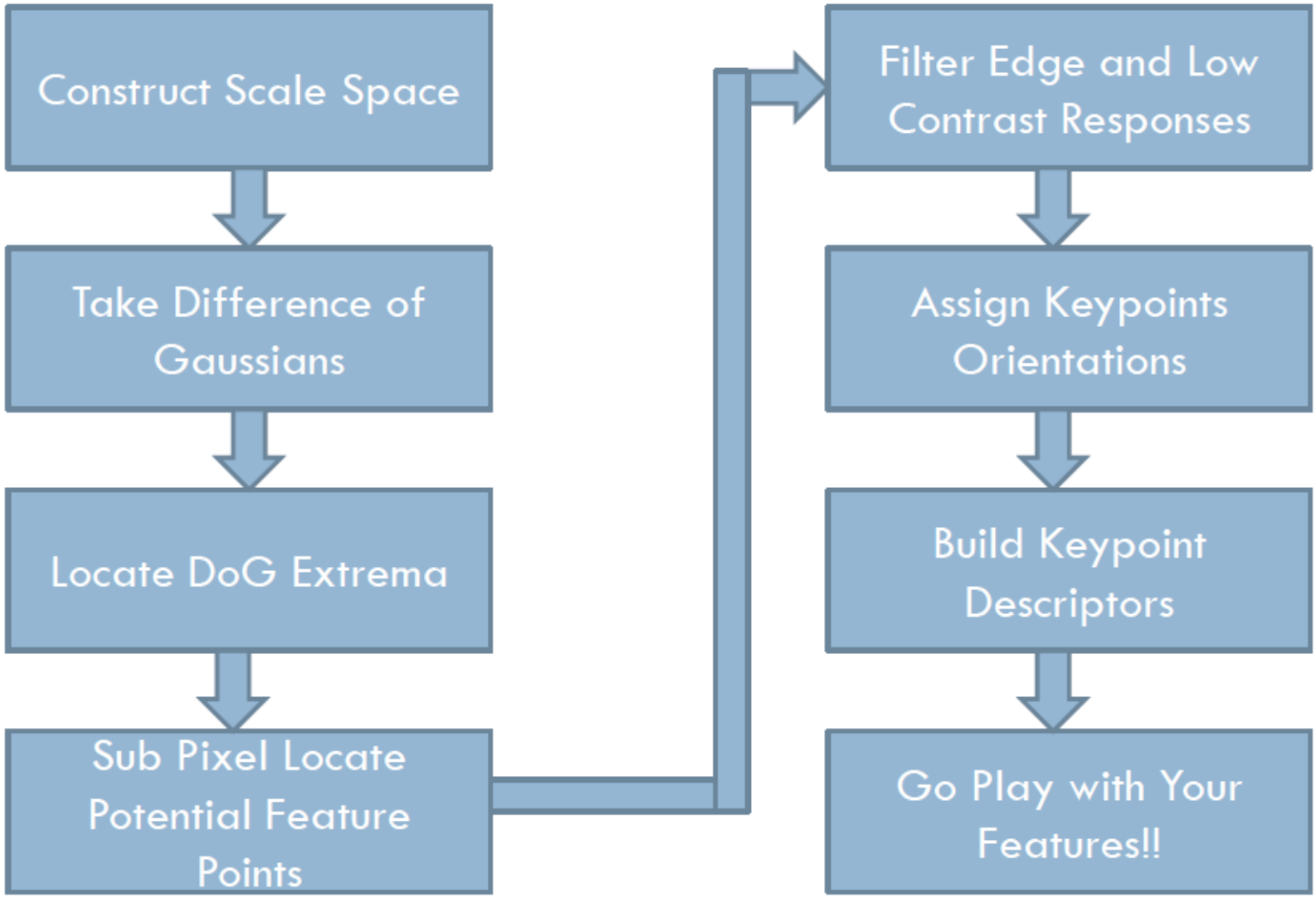
SIFT

- Scale Invariant Feature Transform
  - Proposed in [1]
  - It uses extrema of DoG to detect key points and the associated characteristic scales
  - It uses SIFT to describe a key point

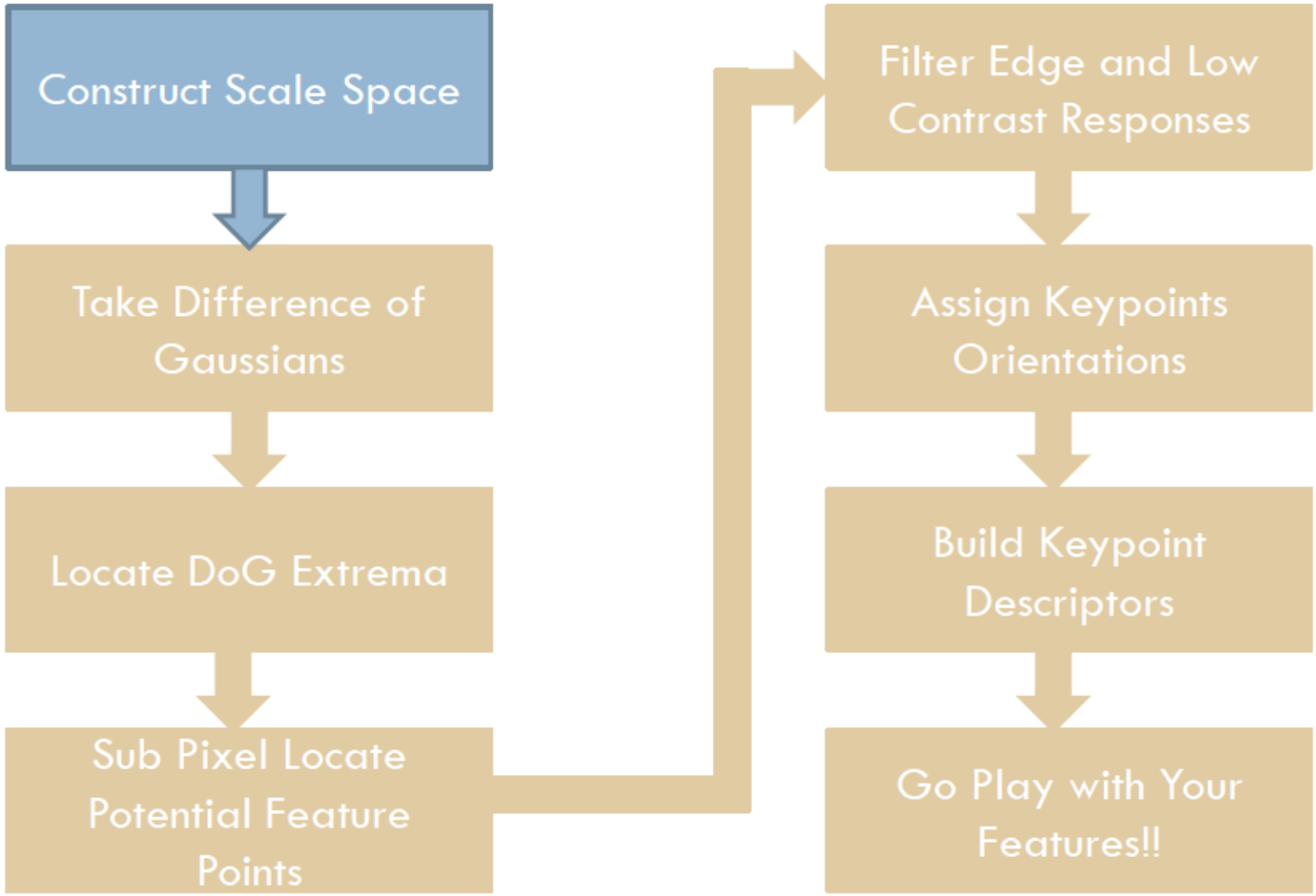
[1] D.G. Lowe, Distinctive image features from scale-invariant keypoints, *IJCV* 60 (2), pp. 91-110, 2004

Prof. David Lowe
University of British Columbia

SIFT


Construct Scale Space
Take Difference of Gaussians
Locate DoG Extrema
Sub Pixel Locate Potential Feature Points
Filter Edge and Low Contrast Responses
Assign Keypoints Orientations
Build Keypoint Descriptors
Go Play with Your Features!!

SIFT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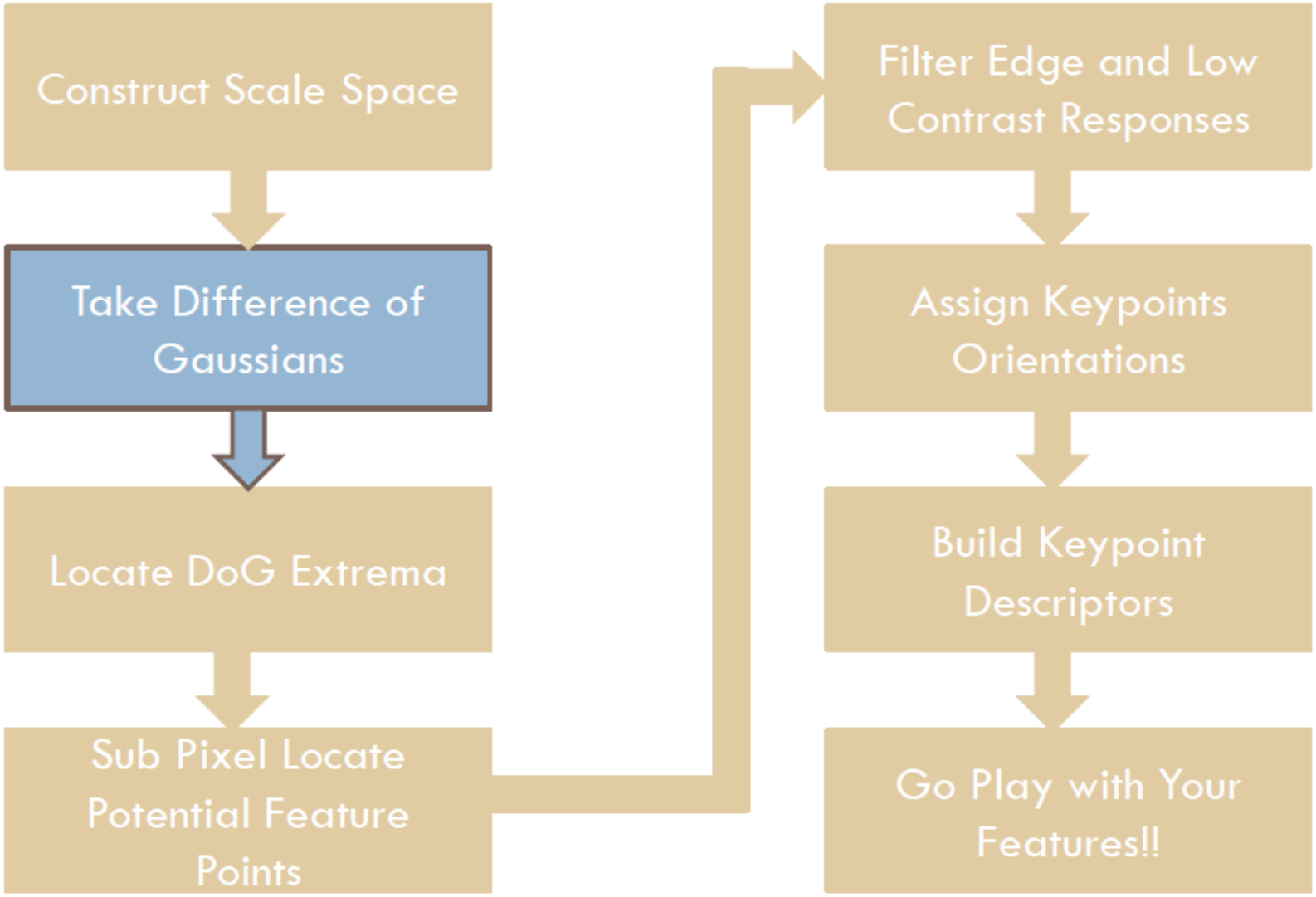
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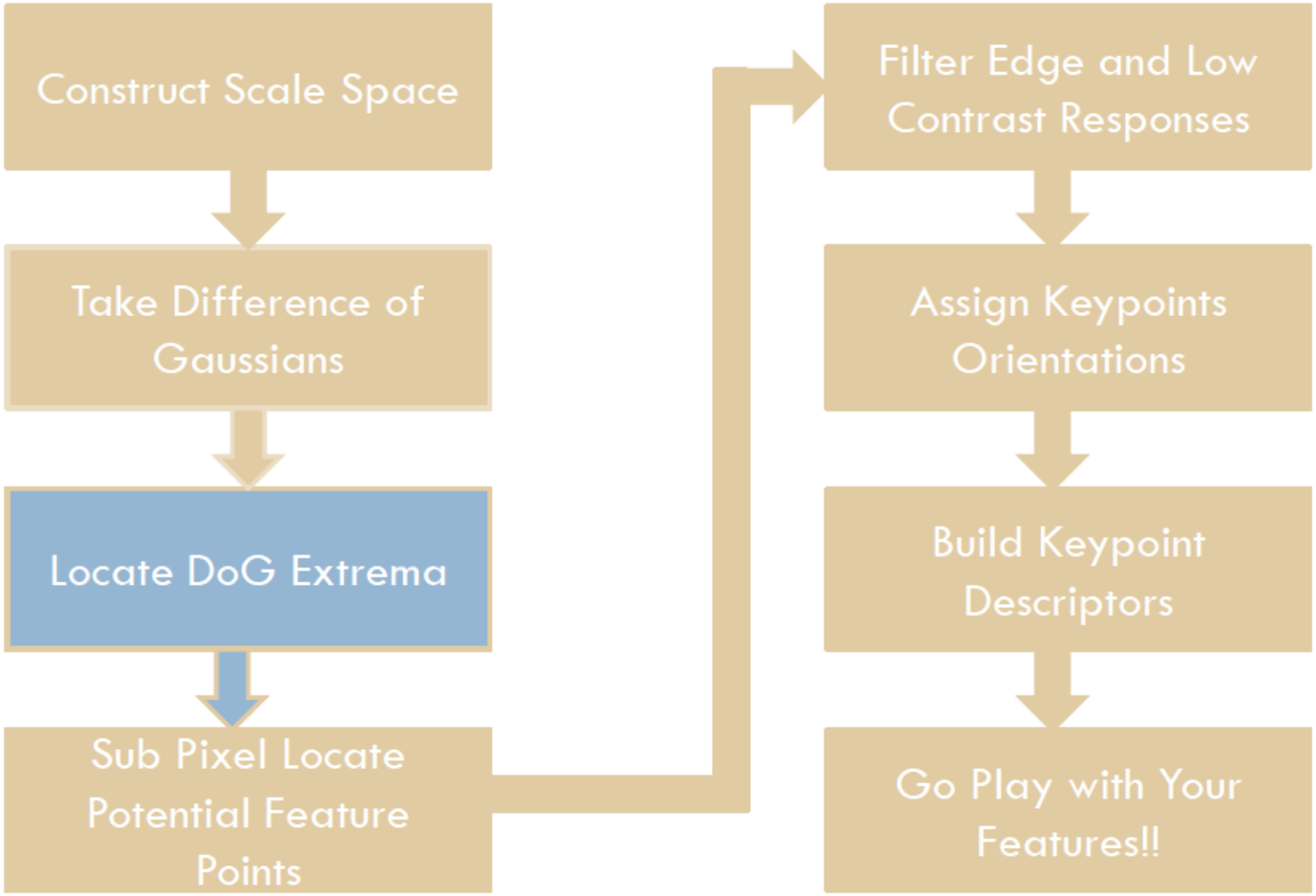
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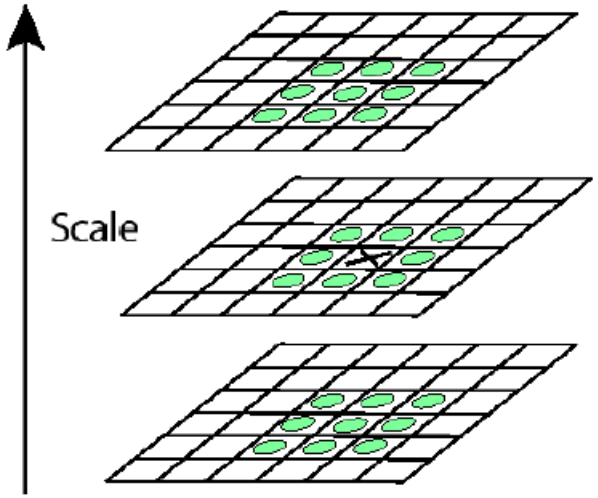
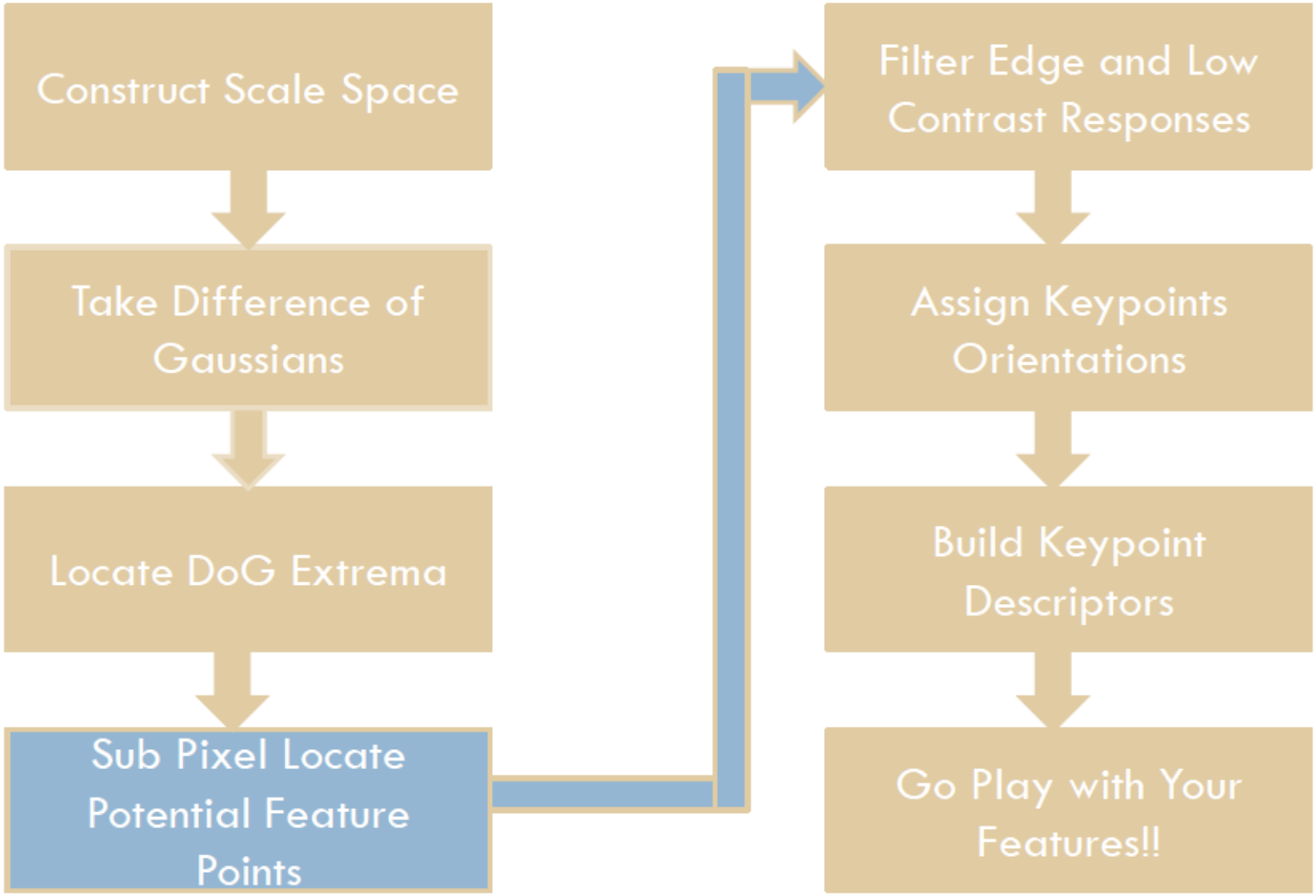
SIFT

- Scan each DOG image
  - Look at all neighboring points (including scale)
  - Identify Min and Max
    - 26 Comparisons



SIFT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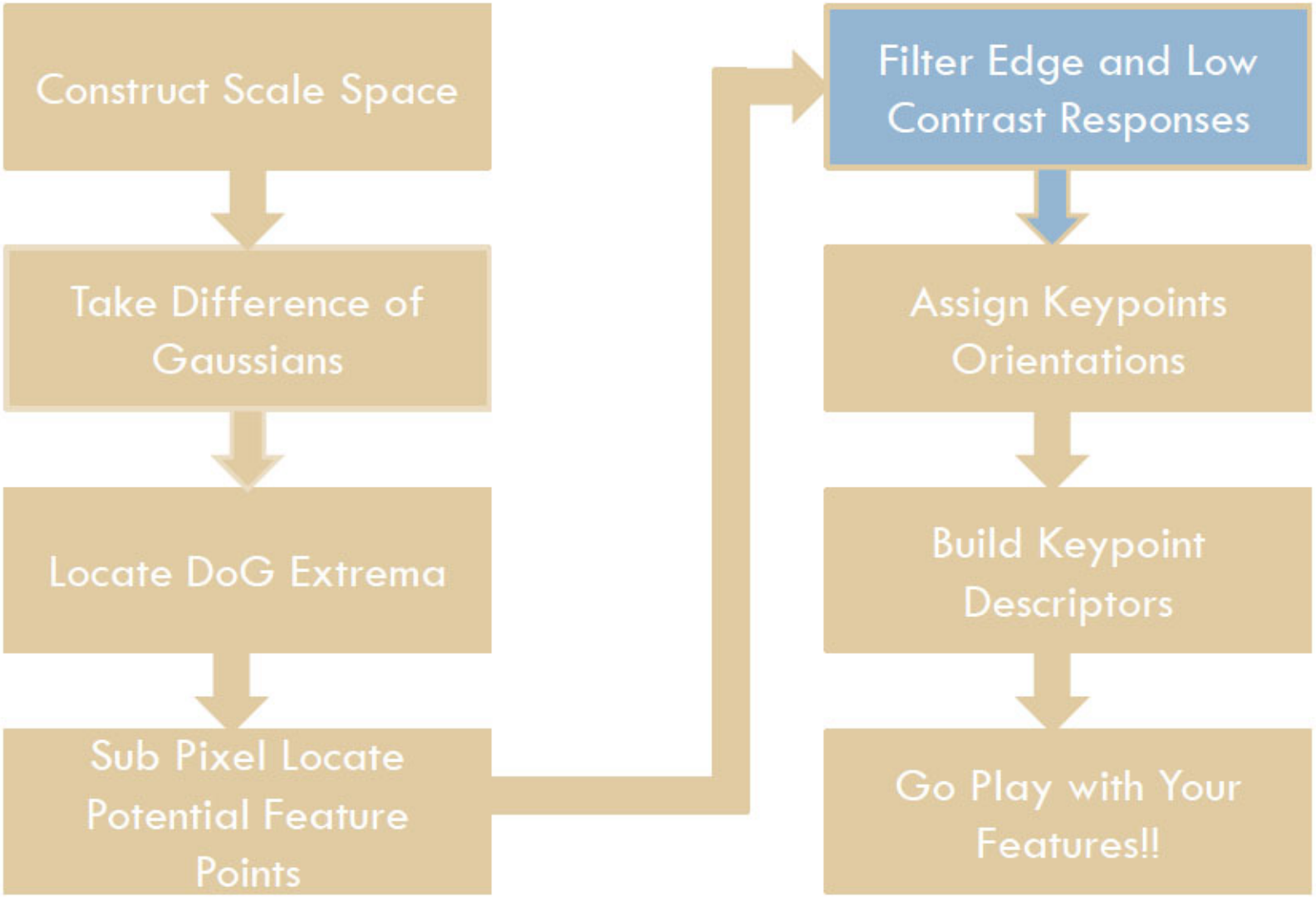
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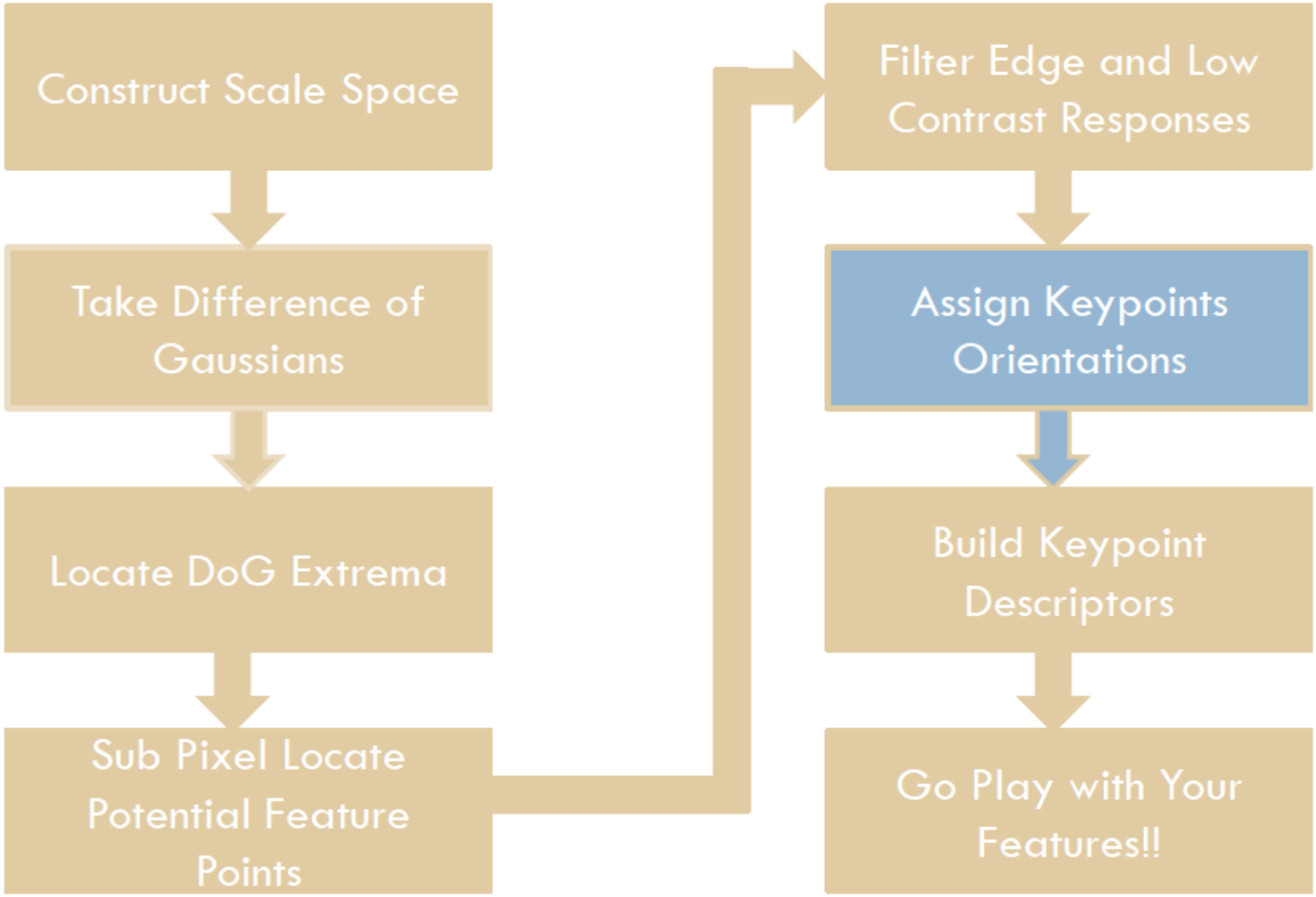
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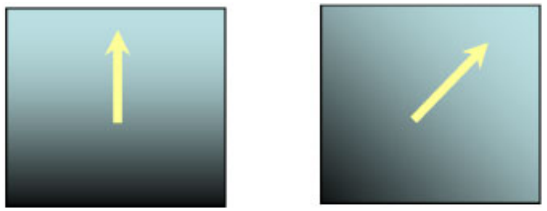
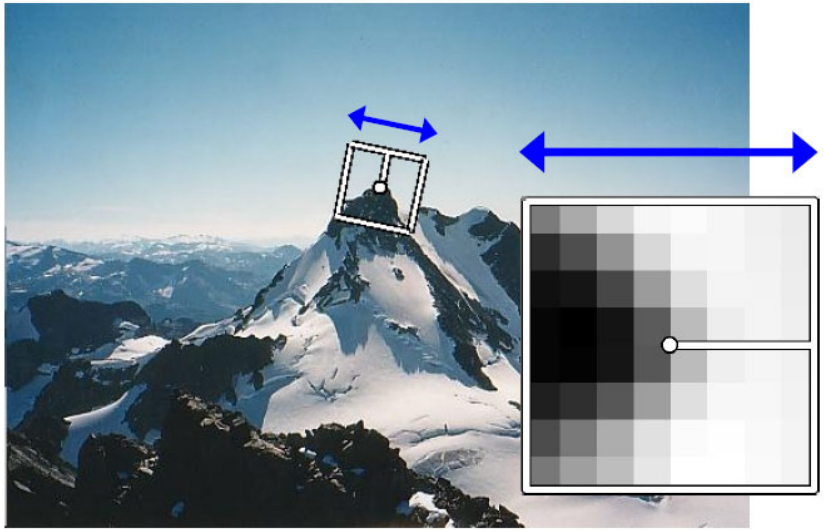
Assign Keypoints Orientations

- Assign orientation to the keypoint
  - Find local orientation: dominant orientation of gradient for the image patch (its size is determined by the characteristic scale)

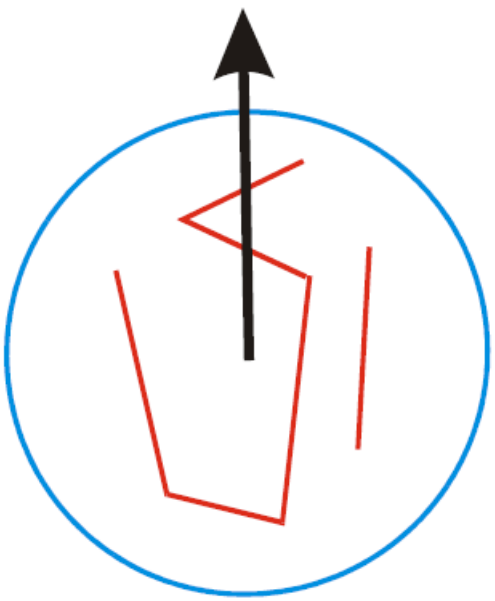


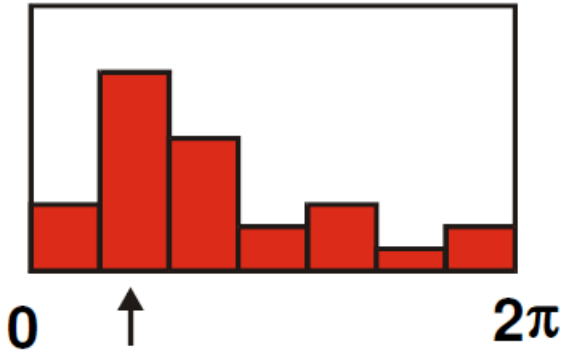
  - Rotate the patch according to this angle; this can achieve rotation invariance description



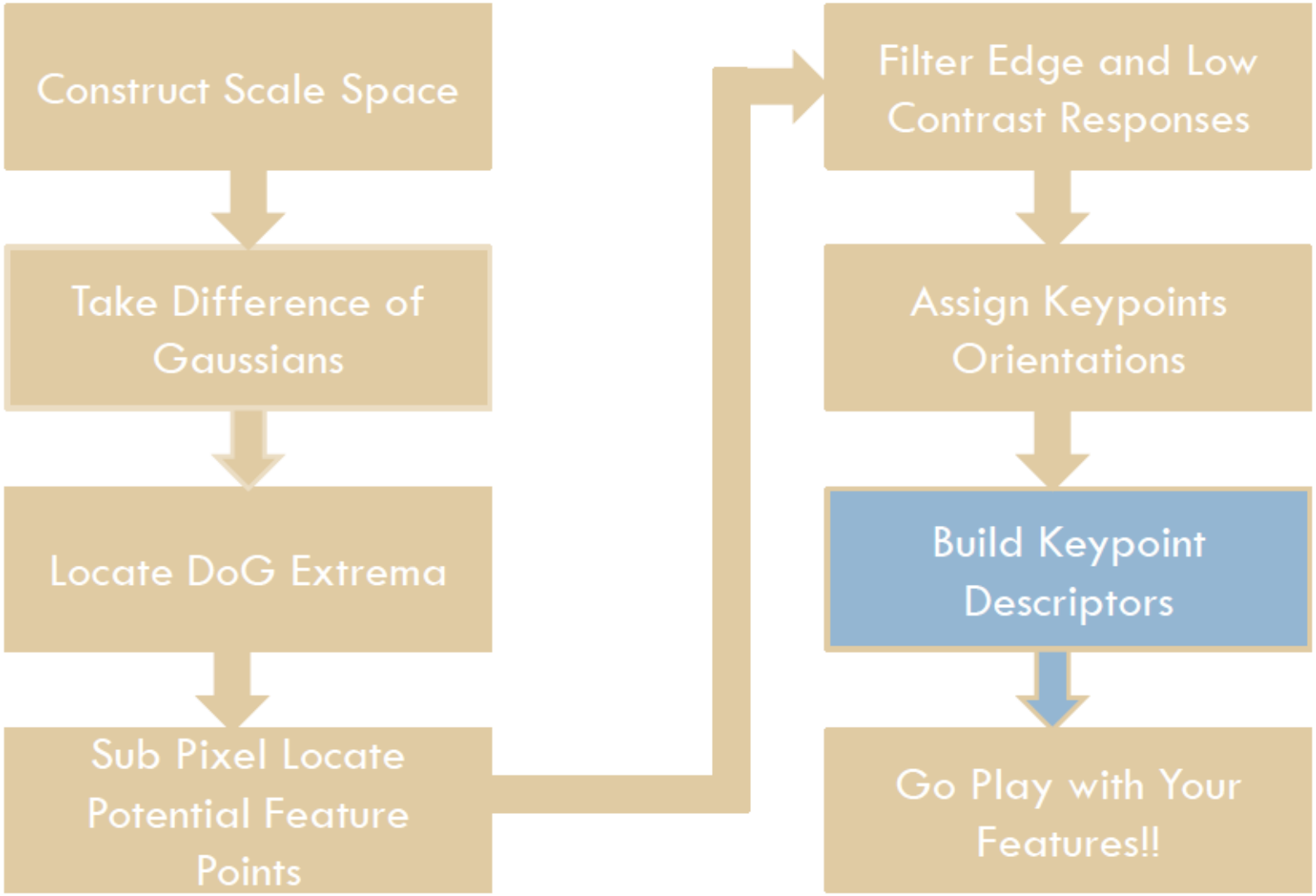
Assign Keypoints Orientations

- Orientation normalization
  - Compute orientation histogram
  - Select dominant orientation
  - Normalization: rotate the patch to the selected orientation





SIFT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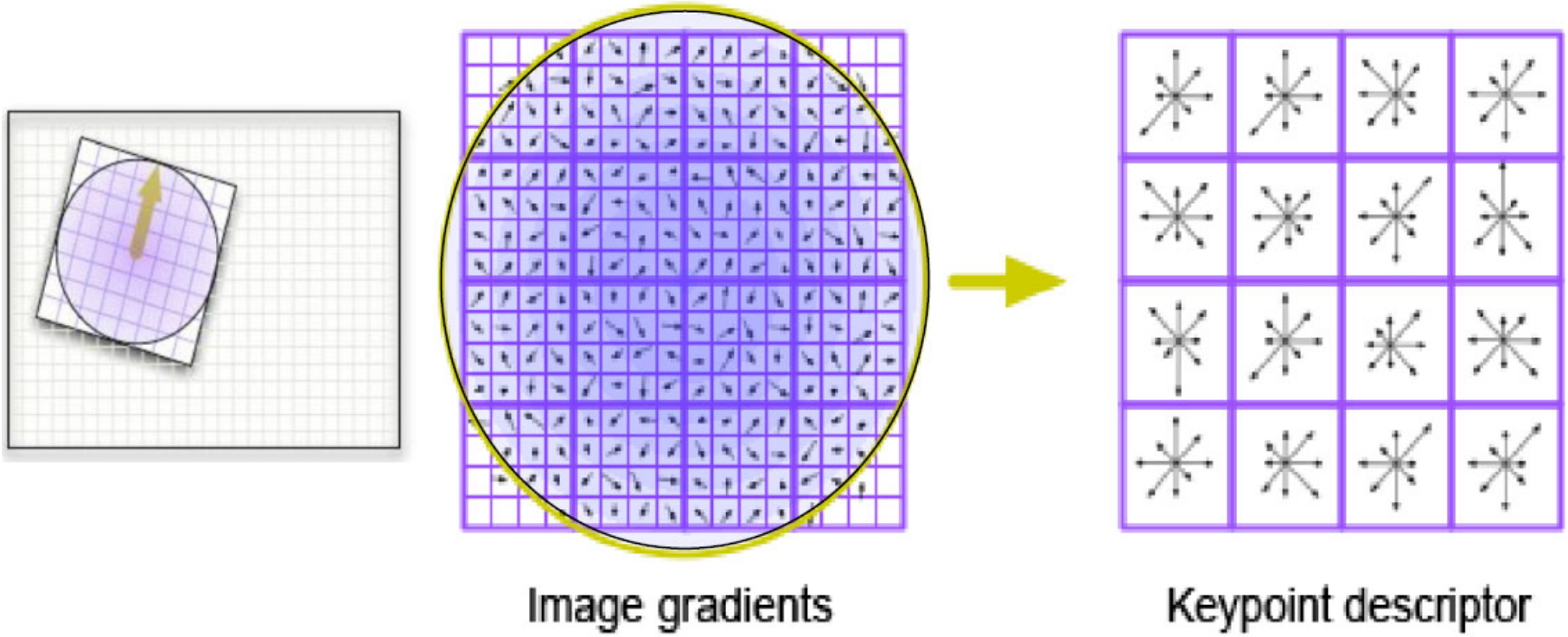
SIFT

- Building the descriptor
  - Sample the points around the keypoint
  - Rotate the gradients and coordinates by the previously computed orientation
  - Separate the region in to 4×4 sub-regions
  - Create gradient-orientation histogram for each sub-region with 8 bins (In real implementation, each sample point is weighted by a Gaussian)

SIFT

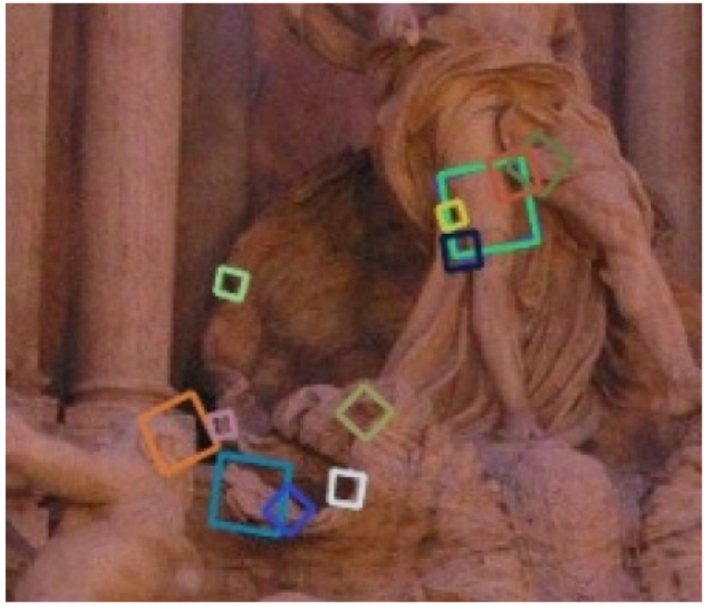
- Building the descriptor



- Actual implementation uses 4*4 sub regions which lead to a 4*4*8 = 128 element vector

SIFT

- One image yields:
  - n 128-dimensional descriptors: each one is a histogram of the gradient orientations within a patch
    - [n x 128 matrix]
  - n scale parameters specifying the size of each patch
    - [n x 1 vector]
  - n orientation parameters specifying the angle of the patch
    - [n x 1 vector]
  - n 2D points giving positions of the patches
    - [n x 2 matrix]



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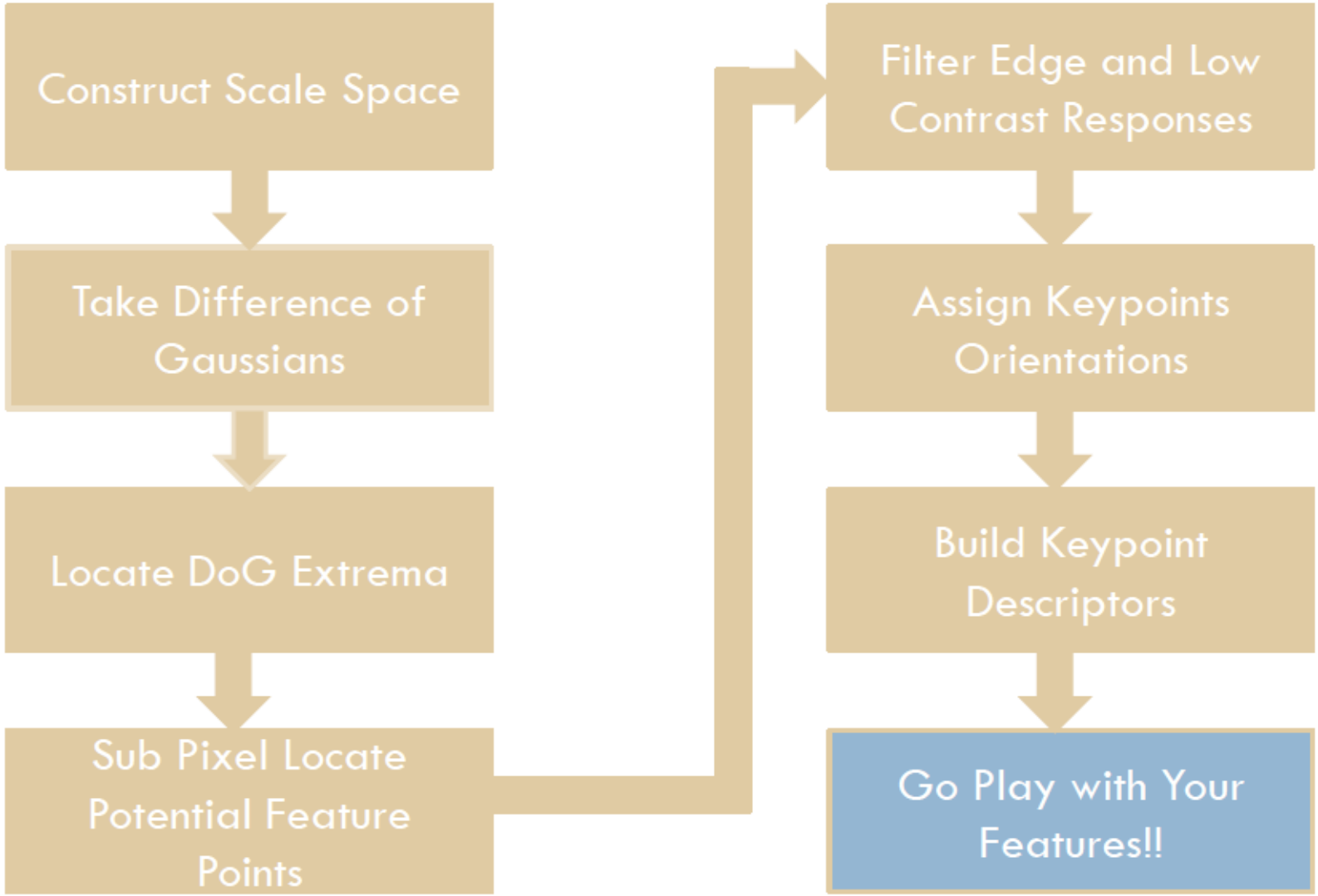
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Applications of SIFT

- Object recognition
- Robot localization and mapping
- Panorama stitching
- 3D scene modeling, recognition and tracking
- Analyzing the human brain in 3D magnetic resonance images

Content

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Distance between two descriptors (not limited to block descriptors)

Given two descriptors $\mathbf{d}_1, \mathbf{d}_2 \in \mathbb{R}^n$, their distance can be computed in different ways

Sum of squared differences (SSD):

$$SSD_{dist}\left(\mathbf{d}_1, \mathbf{d}_2\right) = \left\|\mathbf{d}_1 - \mathbf{d}_2\right\|_2^2 = \sum_{i=1}^{n}\left(d_1^i - d_2^i\right)^2$$

Or sum of absolute differences (SAD):

$$SAD_{dist}\left(\mathbf{d}_1, \mathbf{d}_2\right) = \sum_{i=1}^{n}\left|d_1^i - d_2^i\right|$$

Or normalized cross correlation (NCC):

$$NCC_{dist}\left(\mathbf{d}_1, \mathbf{d}_2\right) = 1 - \frac{1}{n}\frac{\left(\mathbf{d}_1 - \mu\left(\mathbf{d}_1\right)\right)\cdot\left(\mathbf{d}_2 - \mu\left(\mathbf{d}_2\right)\right)}{std\left(\mathbf{d}_1\right)std\left(\mathbf{d}_2\right)}$$

or

$$NCC_{dist}\left(\mathbf{d}_1, \mathbf{d}_2\right) = \arccos\left(\frac{1}{n}\frac{\left(\mathbf{d}_1 - \mu\left(\mathbf{d}_1\right)\right)\cdot\left(\mathbf{d}_2 - \mu\left(\mathbf{d}_2\right)\right)}{std\left(\mathbf{d}_1\right)std\left(\mathbf{d}_2\right)}\right)$$

where $\mu\left(\mathbf{d}_1\right)$ returns the mean value of $\mathbf{d}_1$ and $std(\mathbf{d}_1)$ returns the standard deviation of $\mathbf{d}_1$

Key points matching based on their descriptors

Suppose I_1 and I_2 are two images

I_1's key points and the associated descriptors are $\{\mathbf{x}_i\}_{i=1}^{m}$ and $\mathcal{P}=\{\mathbf{d}_i\}_{i=1}^{m}$

I_2's key points and the associated descriptors are $\{\mathbf{y}_j\}_{j=1}^{n}$ and $\mathcal{Q}=\{\mathbf{e}_j\}_{j=1}^{n}$

If and only if $\mathbf{x}_i$' and $\mathbf{y}_j$'s descriptors $\mathbf{d}_i$ and $\mathbf{e}_j$ satisfy the following conditions, we say the key points $\mathbf{x}_i$ and $\mathbf{y}_j$ match and form a correspondence pair,

1) $dist(\mathbf{d}_i, \mathbf{e}_j)<t_1$, where t_1 is a predefined threshold

2) $\mathbf{d}_i$ and $\mathbf{e}_j$ satisify the "two-direction" confirmation criteria, i.e.,

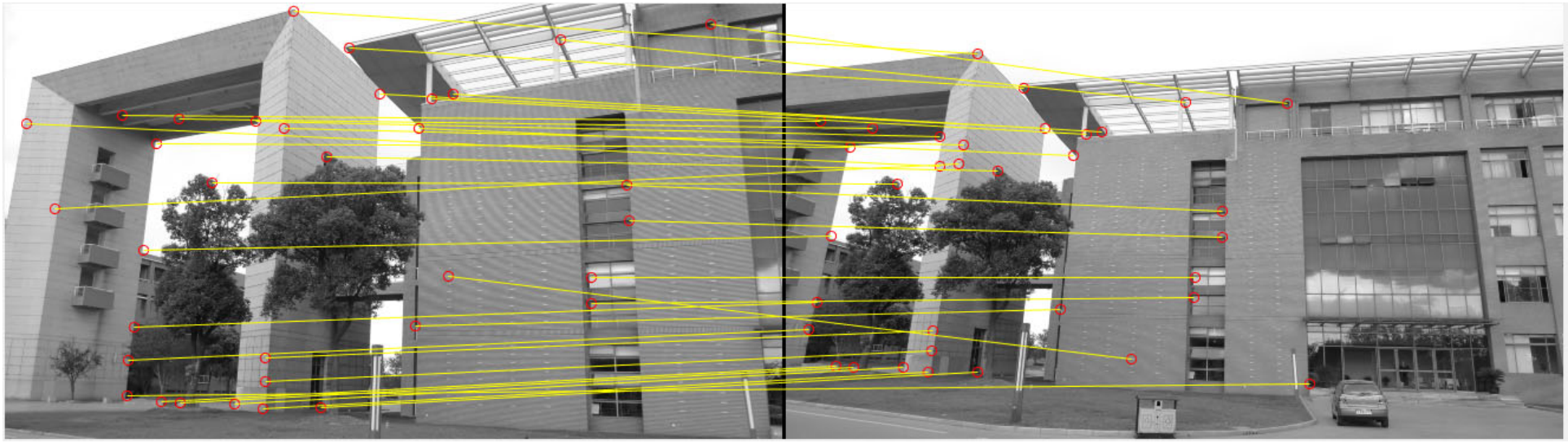
$$\forall \mathbf{v} \in \mathcal{P}, \mathbf{v} \neq \mathbf{d}_i, dist\left(\mathbf{v}, \mathbf{e}_j\right) > dist\left(\mathbf{d}_i, \mathbf{e}_j\right)$$

$$\forall \mathbf{v} \in \mathcal{Q}, \mathbf{v} \neq \mathbf{e}_j, dist\left(\mathbf{d}_i, \mathbf{v}\right) > dist\left(\mathbf{d}_i, \mathbf{e}_j\right)$$

3) $\mathbf{d}_i$ and $\mathbf{e}_j$'s matching is unambiguous

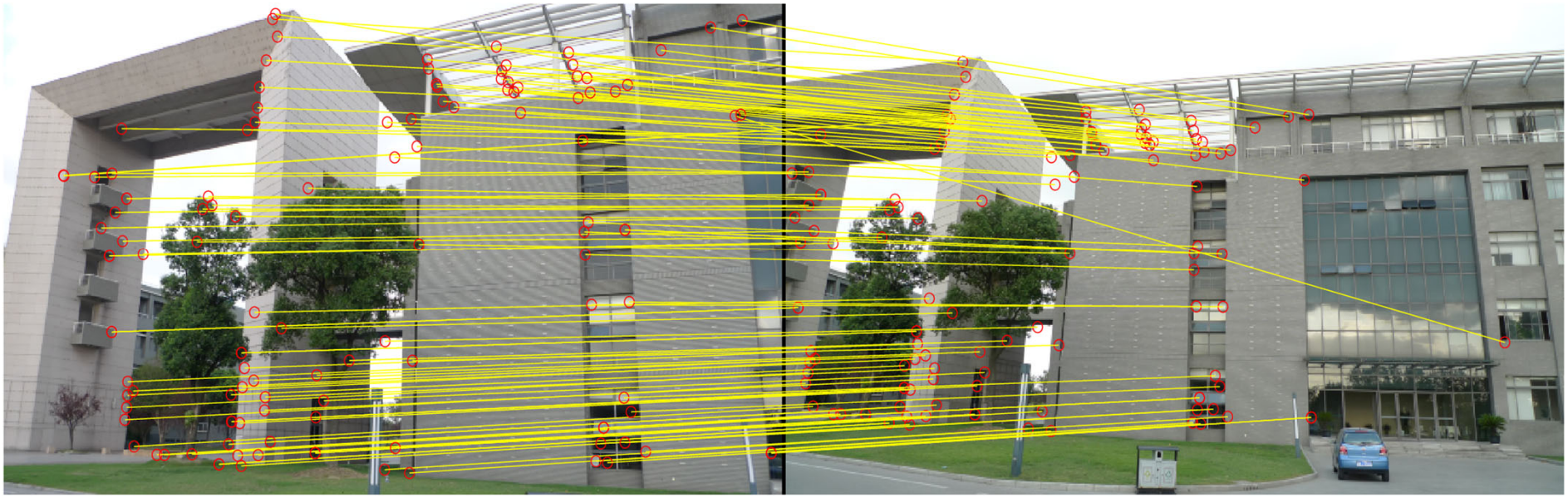
Let $d_1 = dist\left(\mathbf{d}_i, \mathbf{e}_j\right)$. Suppose $\mathbf{e}_k$ is the second best matching descriptor to $\mathbf{d}_i$ and let $d_2 = dist\left(\mathbf{d}_i, \mathbf{e}_k\right)$. Then, $d_1 / d_2 < t_2$, where t_2 is another predefined threshold

Key points matching based on their descriptors



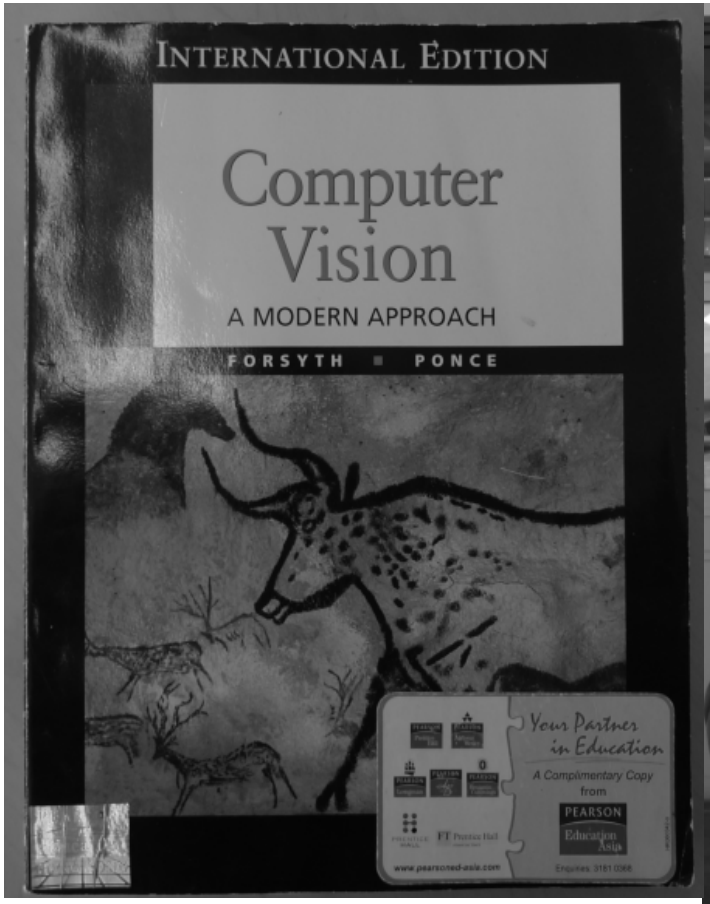
Matching Harris corners using block descriptors

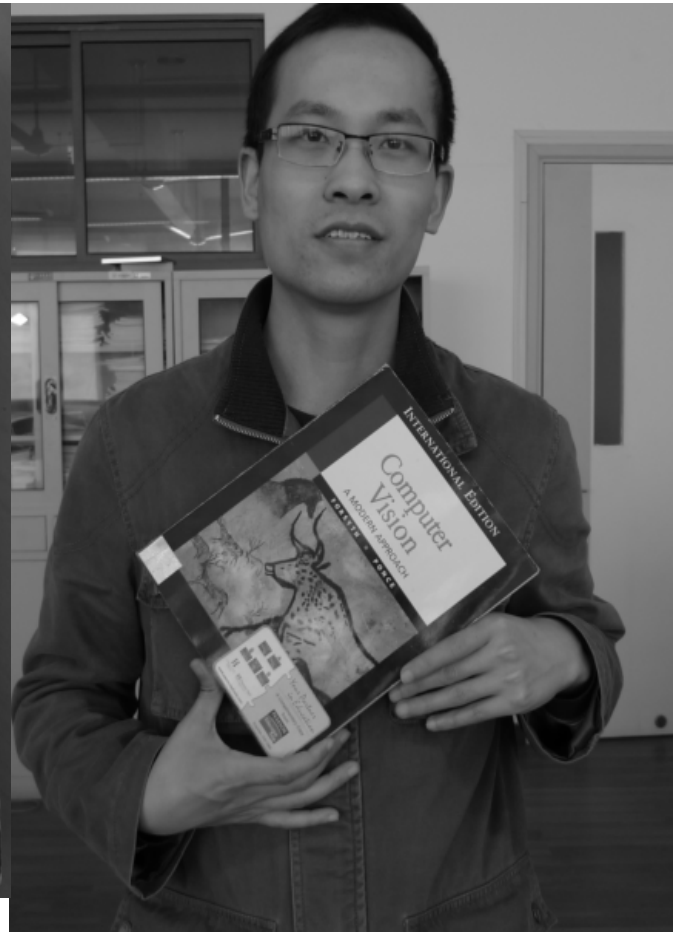
Key points matching based on their descriptors



Matching scale-invariant points using SIFT descriptors

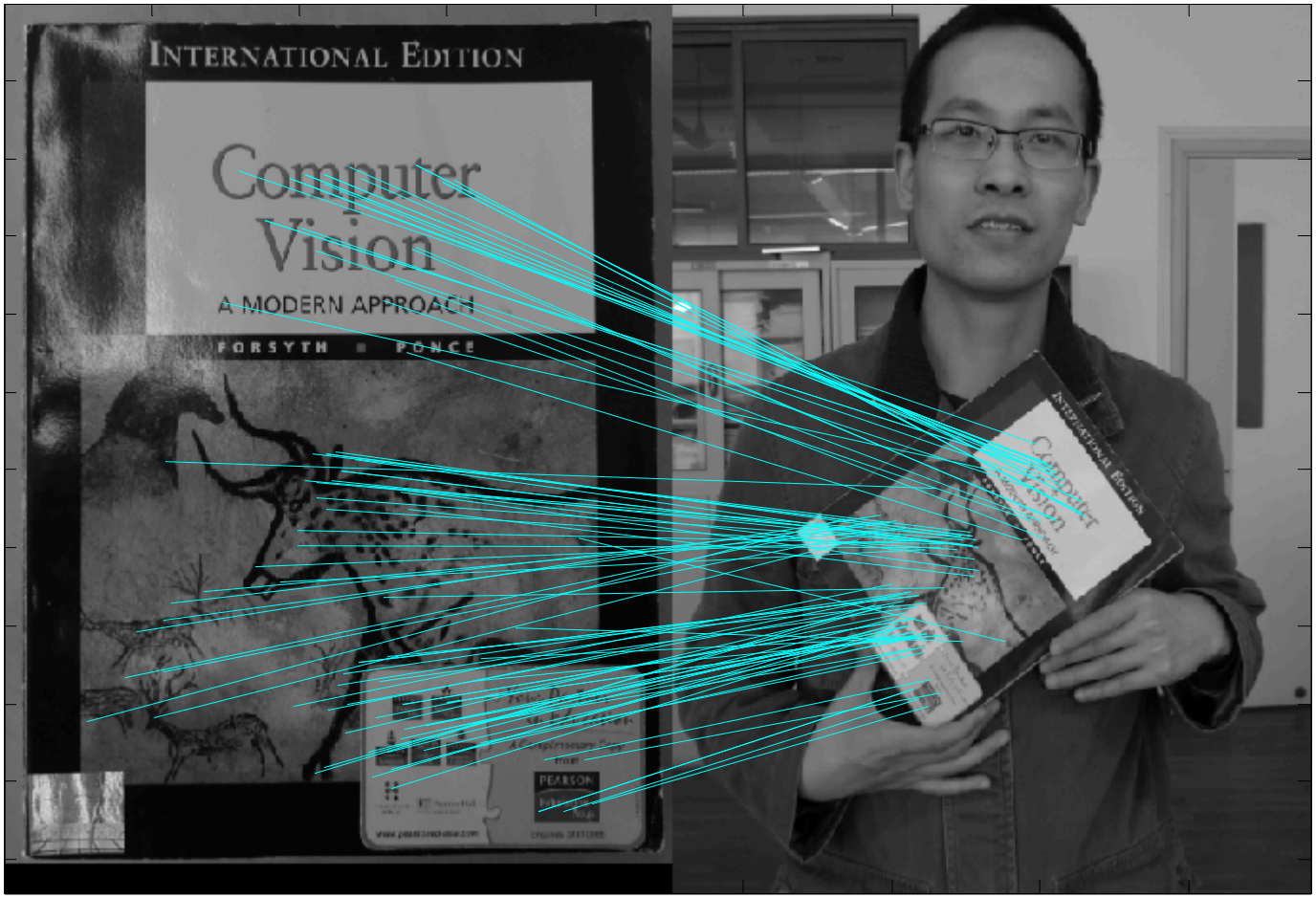
Key points matching based on their descriptors





Key points matching based on their descriptors

Matching scale-invariant points using SIFT descriptors



Q+A