Chapter 06
Robust Estimation of Homography

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Outline

- Problem of Homography Estimation
- RANSAC-based Homography Estimation

Problem of Homography Estimation

Problem definition:

On two projective planes π_1 and π_2, there is a set of corresponding points $\left\{\mathbf{x}_i, \mathbf{x}_i^{'}\right\}_{i=1}^{n}$, and we suppose that there is a homography matrix linking the two planes,

$$c_i \mathbf{x}_i^{'} = H\mathbf{x}_i, i = 1, 2, ..., n$$

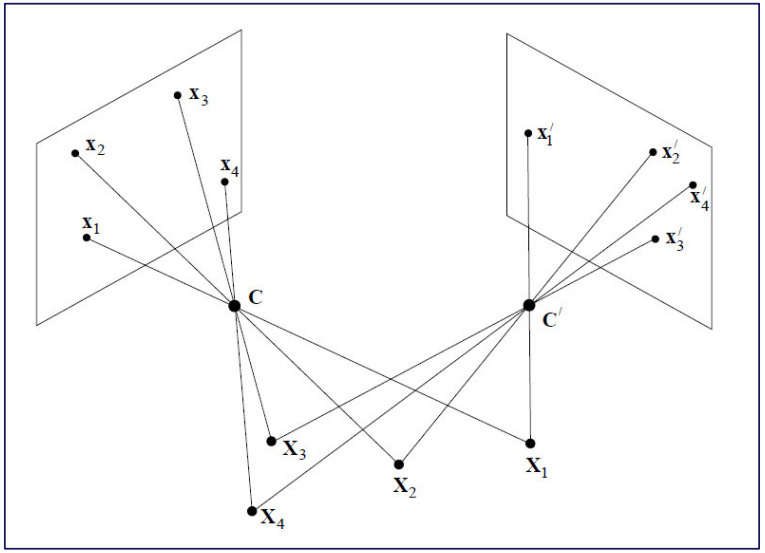
Coordinates of $\left\{\mathbf{x}_i\right\}_i^n$ and $\left\{\mathbf{x}_i^{'}\right\}_{i=1}^{n}$ are known, we need to find H

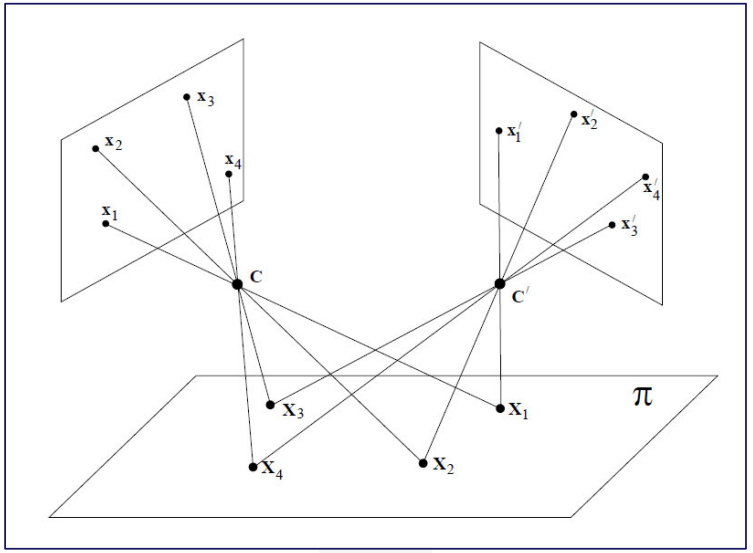
$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

Note: H is defined up to a scale factor. In other words, it has 8 DOFs

Problem of Homography Estimation

Note: Theoretically speaking, homography can only be estimated between two planes, i.e., when you use such a technique to stitch two images, image contents should be roughly on the same plane





Problem of Homography Estimation

4 point-correspondence pairs can uniquely determine a homography matrix since each correspondence pair solves two degrees of freedom

$$c\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \rightarrow \begin{cases} h_{11}u + h_{12}v + h_{13} = cx \\ h_{21}u + h_{22}v + h_{23} = cy \\ h_{31}u + h_{32}v + h_{33} = c \end{cases}$$

$$\rightarrow \begin{cases} \dfrac{h_{11}u + h_{12}v + h_{13}}{h_{31}u + h_{32}v + h_{33}} = x \\ \dfrac{h_{21}u + h_{22}v + h_{23}}{h_{31}u + h_{32}v + h_{33}} = y \end{cases}$$

Note: here we assume that the matching points are all finite points (no points at infinity)

Problem of Homography Estimation

4 point-correspondence pairs can uniquely determine a homography matrix since each correspondence pair solves two degrees of freedom

$$\begin{pmatrix} u & v & 1 & 0 & 0 & 0 & -ux & -vx & -x \\ 0 & 0 & 0 & u & v & 1 & -uy & -vy & -y \end{pmatrix} \begin{pmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{pmatrix} = \mathbf{0}$$

Thus, four correspondence pairs generate 8 equations

Problem of Homography Estimation

4 point-correspondence pairs can uniquely determine a homography matrix since each correspondence pair solves two degrees of freedom

$$A\mathbf{h} = \mathbf{0} \quad (1)$$

8×9 $\quad$ 9×1

Normally, $Rank(A) = 8$; thus (1) has 1 (9-8) solution vector (linear independant) in its solution space

In our case, since we have n>4 point pairs, we get

$$\mathbf{A}_{2n\times 9}\mathbf{h}_{9\times 1} = \mathbf{0}$$

It is an overdetermined homogeneous linear equation system

Problem of Homography Estimation

Since only the ratios among the elements of H take effect, in another way we can fix h_{33}=1 (suppose that h_{33}!=0),

$$c\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \Rightarrow \begin{cases} h_{11}u + h_{12}v + h_{13} = cx \\ h_{21}u + h_{22}v + h_{23} = cy \\ h_{31}u + h_{32}v + 1 = c \end{cases} \Rightarrow \begin{cases} \dfrac{h_{11}u + h_{12}v + h_{13}}{h_{31}u + h_{32}v + 1} = x \\ \dfrac{h_{21}u + h_{22}v + h_{23}}{h_{31}u + h_{32}v + 1} = y \end{cases}$$

$$\Rightarrow \begin{pmatrix} u & v & 1 & 0 & 0 & 0 & -ux & -vx \\ 0 & 0 & 0 & u & v & 1 & -uy & -vy \end{pmatrix} \begin{pmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{pmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow$$

Since we have n>4 point pairs, we get

$$\mathbf{A}_{2n\times 8}\mathbf{h}_{8\times 1} = \mathbf{b}_{2n\times 1}$$

It is an overdetermined inhomogeneous linear equation system

Outline

- Problem of Homography Estimation
- RANSAC-based Homography Estimation

RANSAC-based Homography Estimation

- When there are more than 4 correspondence pairs, is it a proper way to use the LS method to solve the model directly?
  - ➢ NO! Because usually, outliers exist among the correspondence pairs

RANdom SAmple Consensus (RANSAC) is an iterative framework to estimate a parametric model from observations with noisy outliers

RANSAC-based Homography Estimation

Objective

Robust fit a model to a data set S which contains outliers

Algorithm

(1) Randomly select a sample of s data points from S and instantiate the model from this subset

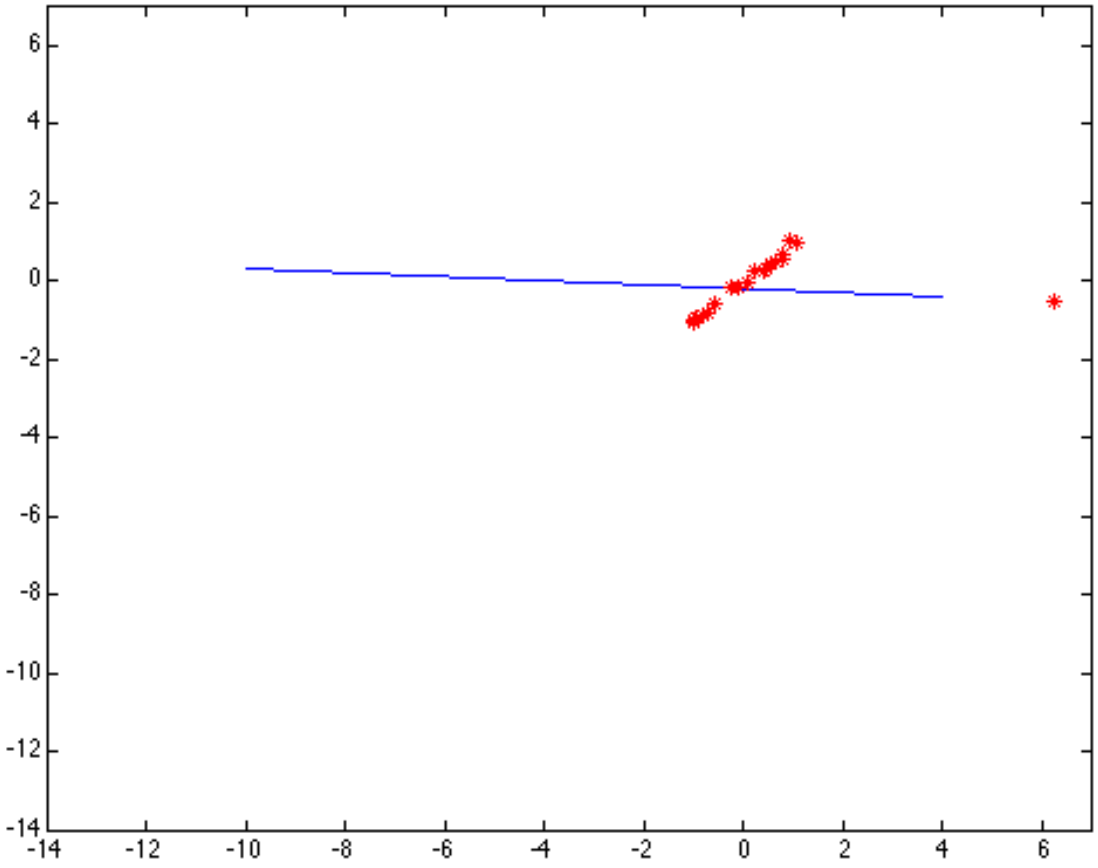
(2) Determine the set of data points S_i which are within a distance threshold t of the model. The set S_i is the consensus set of the sample and defines the inliers of S

(3) If the size of S_i (the number of inliers) is greater than some threshold T, re-estimate the model using all points in S_i and terminate

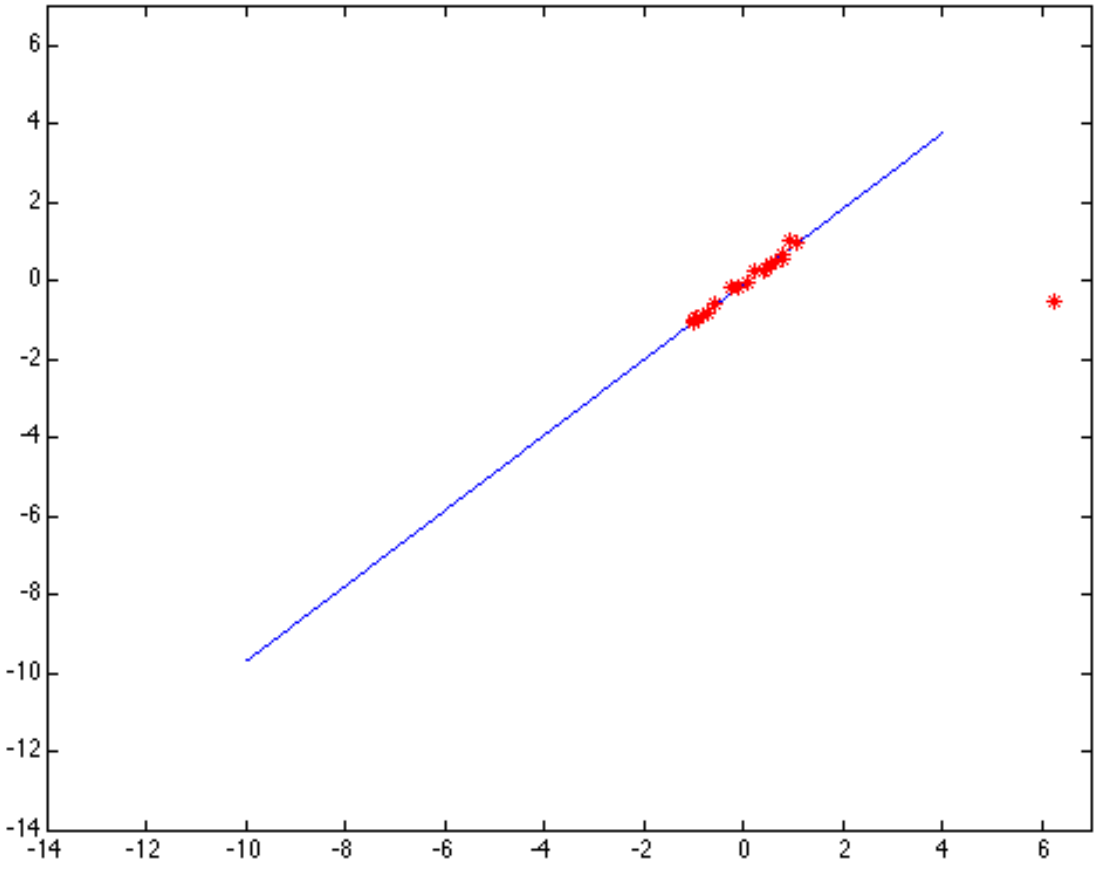
(4) If the size of S_i is less than T, select a new subset and repeat the above

(5) After N trials the largest consensus set S_i is selected, and the model is re-estimated using all points in the subset S_i

RANSAC-based Homography Estimation



RANSAC-based Homography Estimation

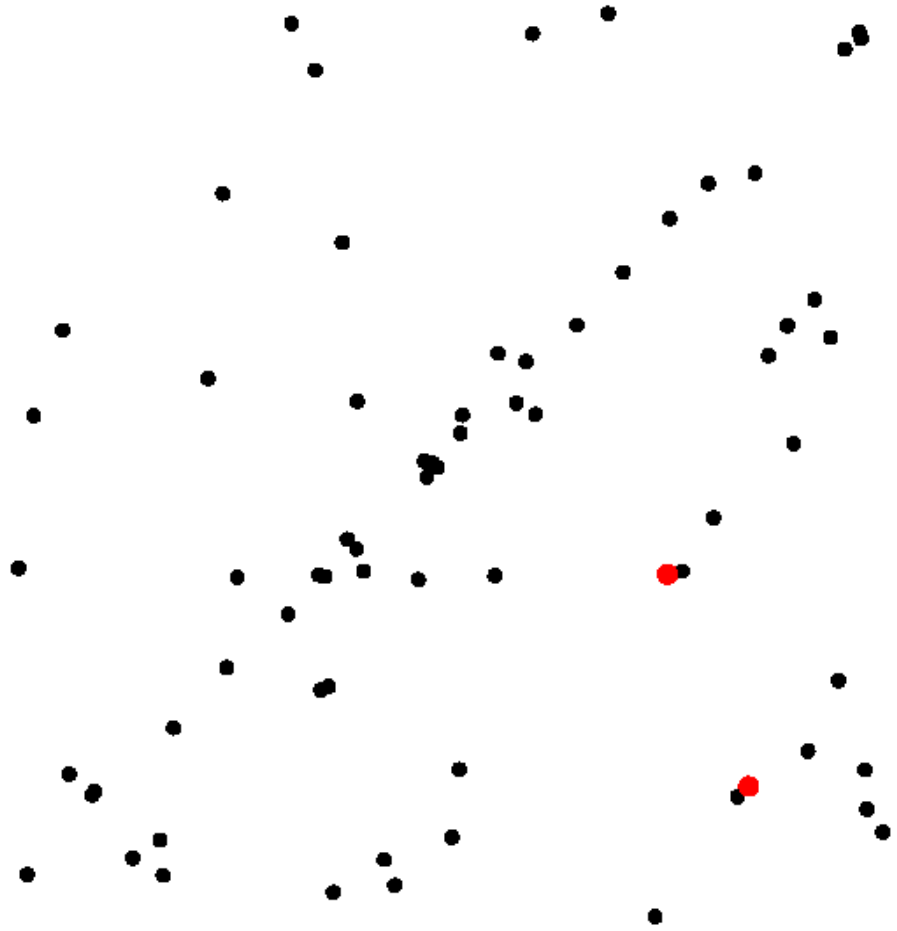


RANSAC-based Homography Estimation

Line fitting by RANSAC

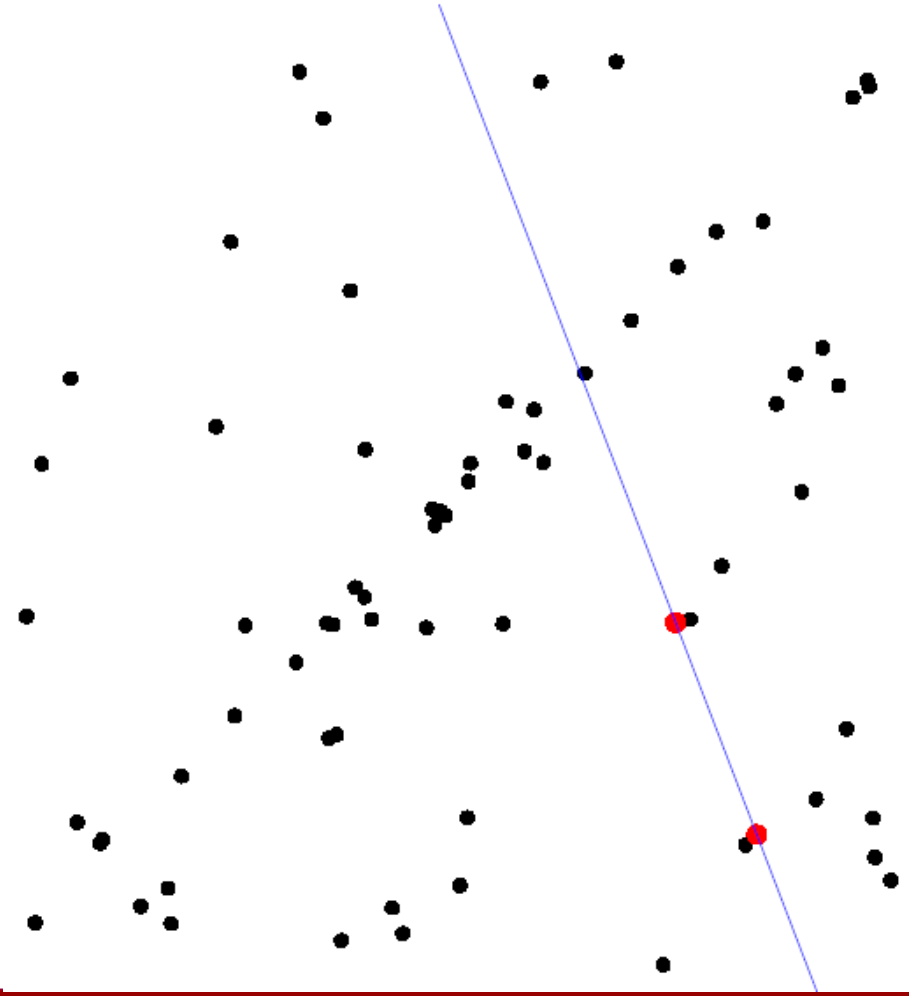
RANSAC-based Homography Estimation

Line fitting by RANSAC



- Randomly select two points

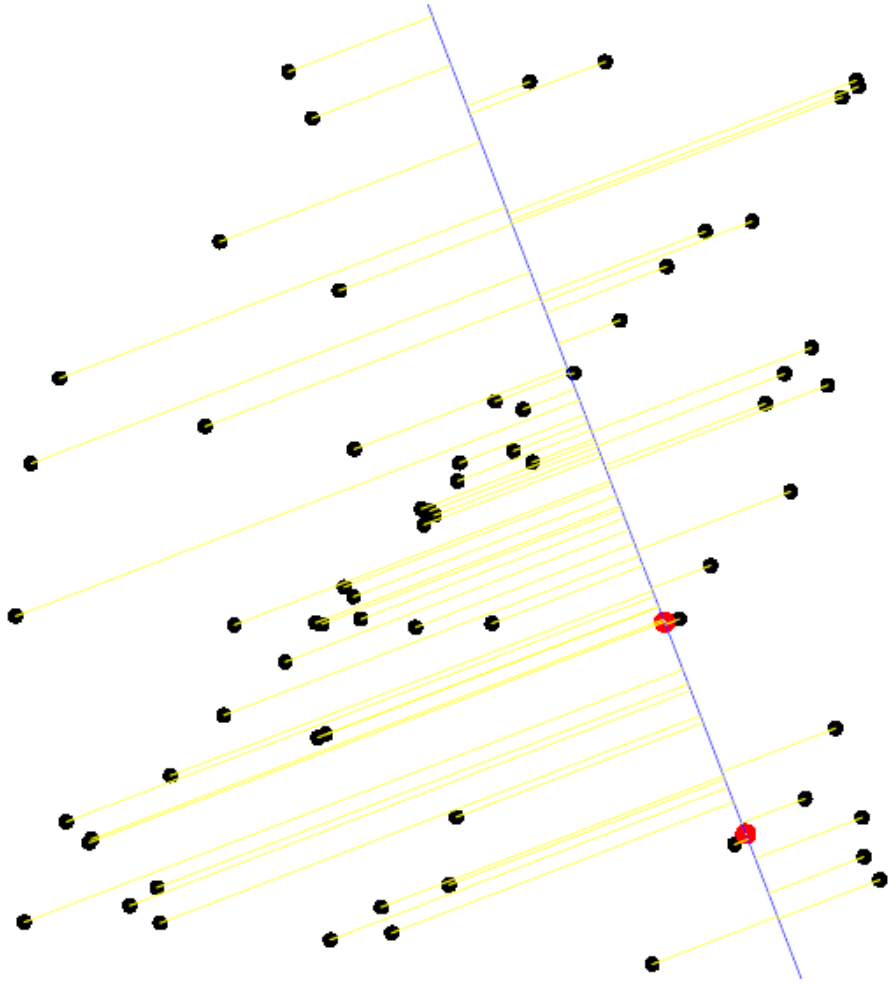
RANSAC-based Homography Estimation



Line fitting by RANSAC

- Randomly select two points
- The hypothesized model is the line passing through the two points

RANSAC-based Homography Estimation



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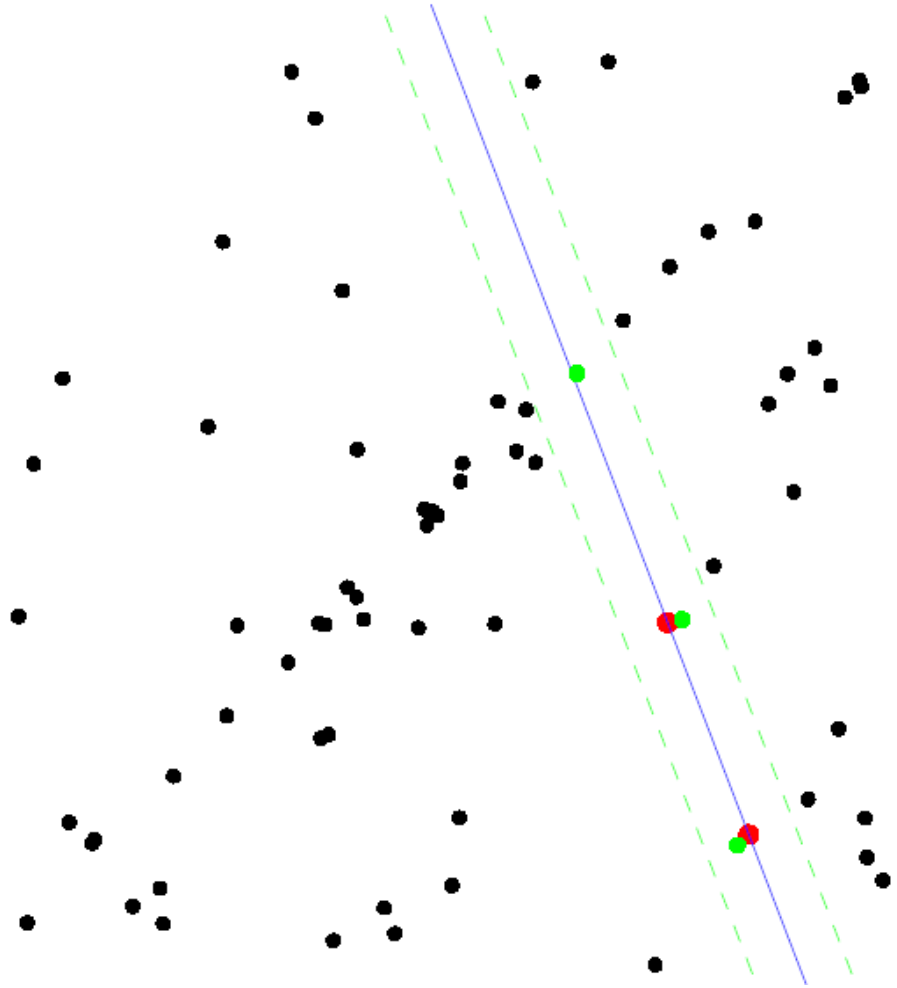
RANSAC-based Homography Estimation



Line fitting by RANSAC

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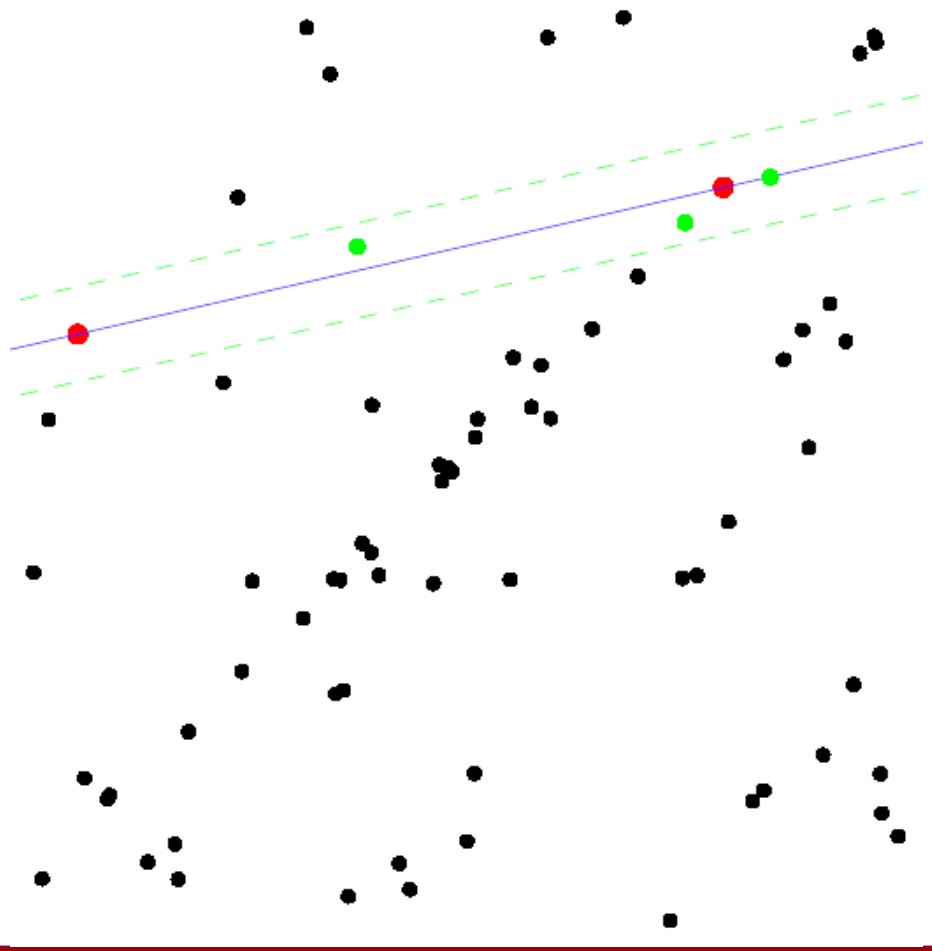
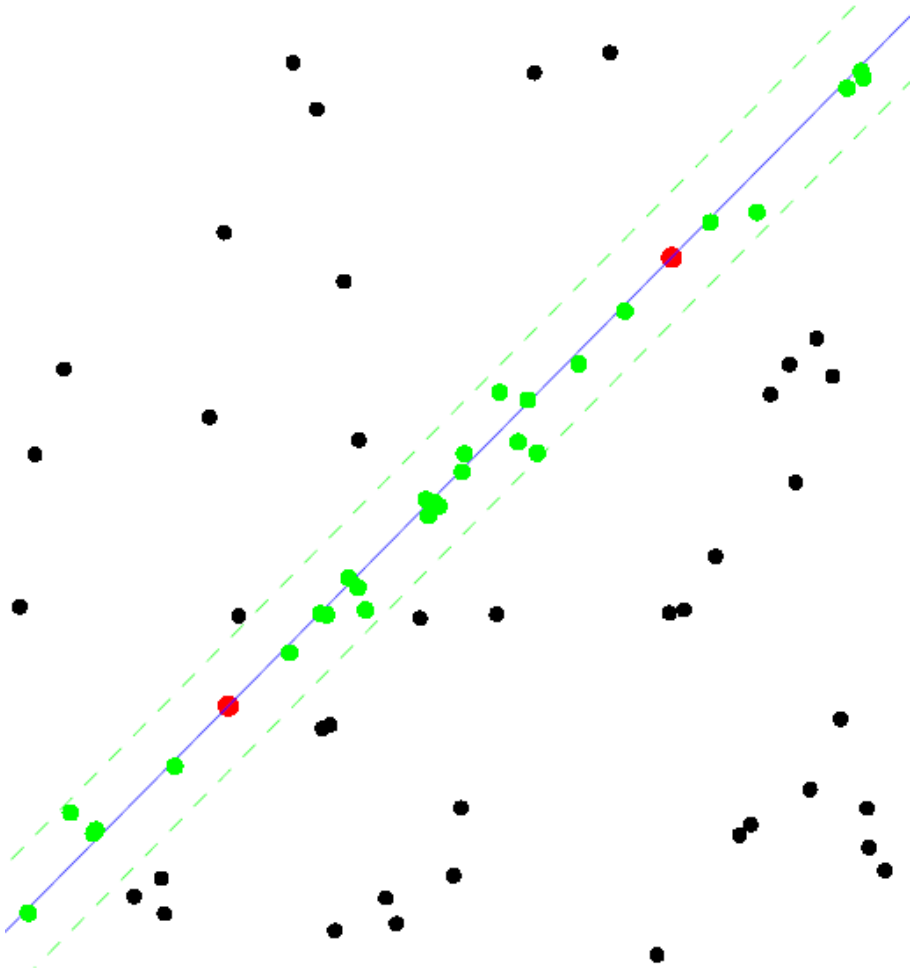
RANSAC-based Homography Estimation



Line fitting by RANSAC

- Test another two points

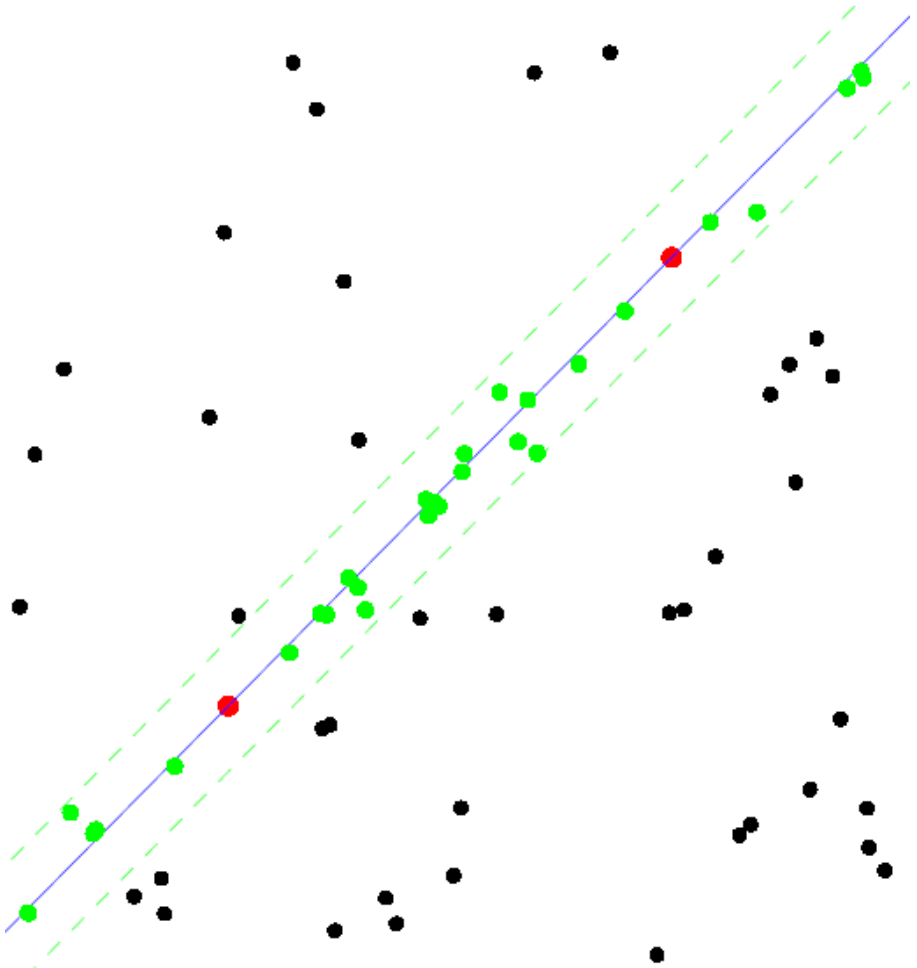
RANSAC-based Homography Estimation



Line fitting by RANSAC

- The final fitting result

RANSAC-based Homography Estimation



Line fitting by RANSAC

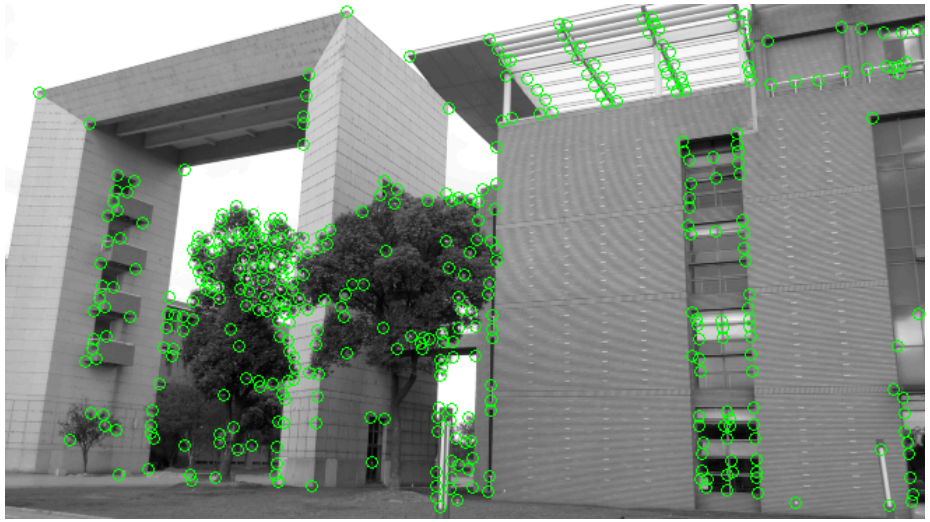
- The final fitting result

RANSAC-based Homography Estimation

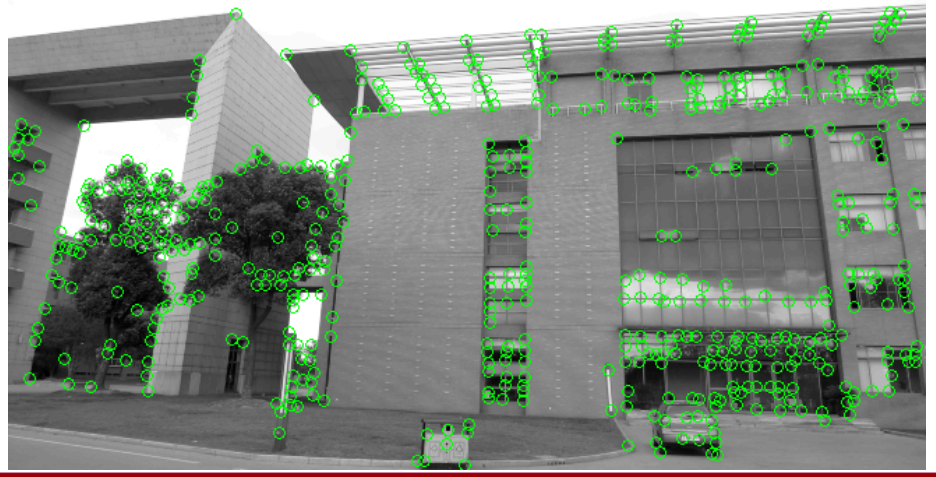
Can you describe the steps of homography estimation when using RANSAC?

Homography Estimation: Example 1

Homography Estimation: Example 1



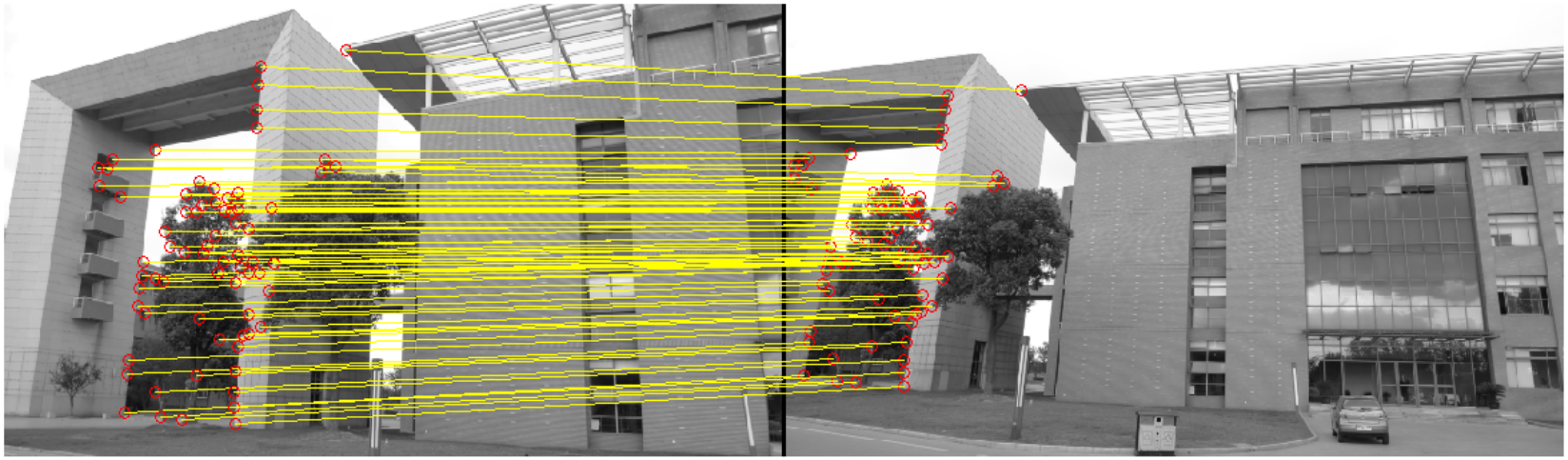
Interest points detection



Homography Estimation: Example 1

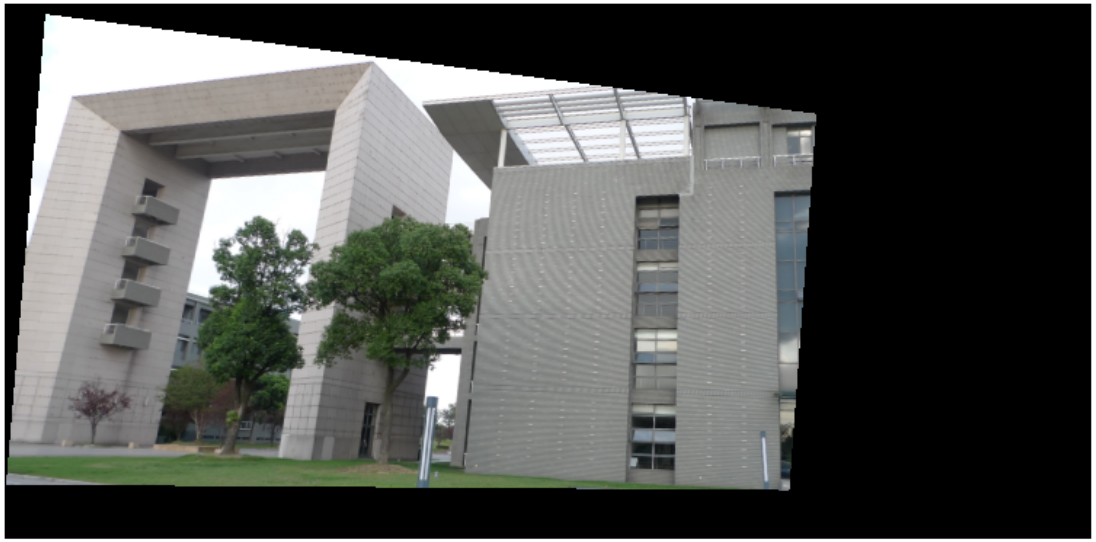
Correspondence estimation

Then, the homography matrix can be estimated by using the correspondence pairs with RANSAC



Homography Estimation: Example 1

Transform image one using the estimated homography matrix



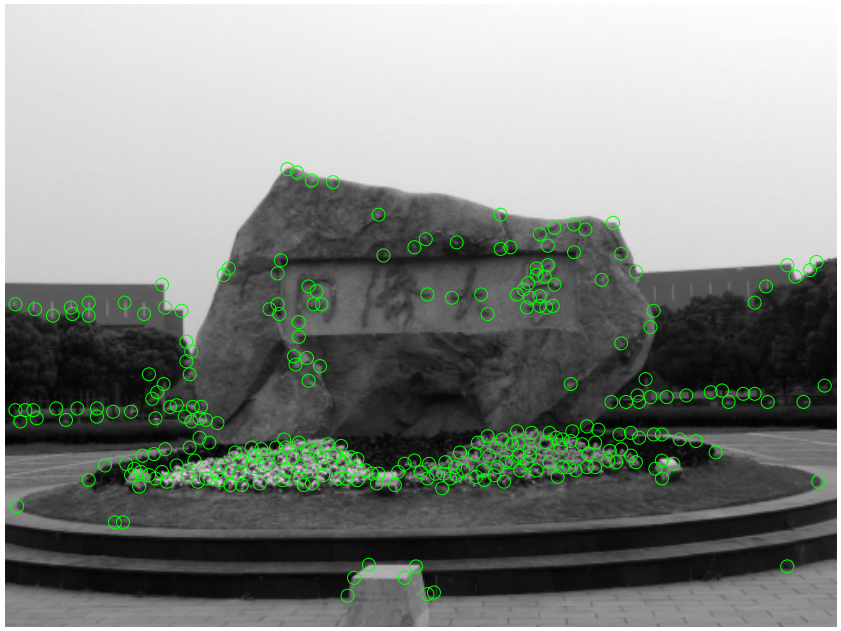
Homography Estimation: Example 1

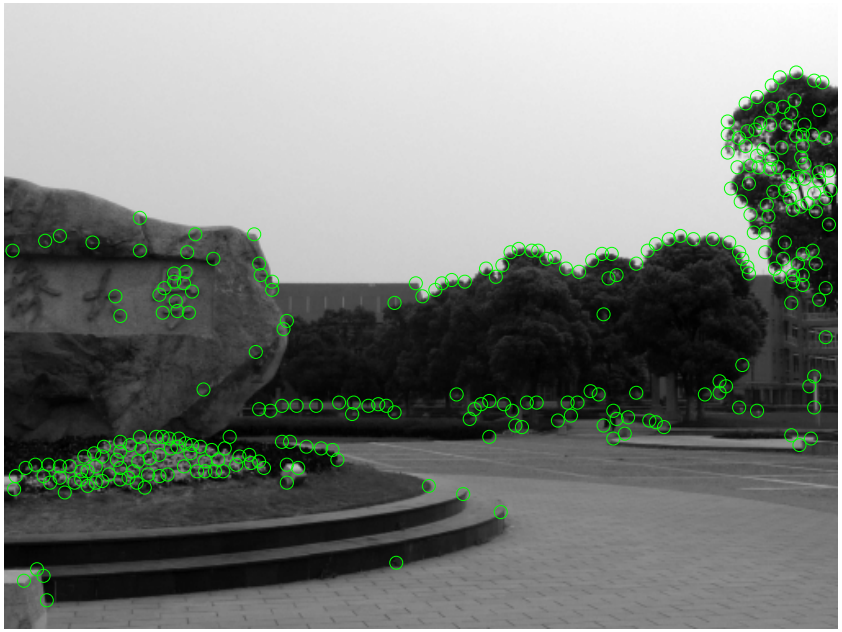
Finally, stitch the transformed image one with image two

Homography Estimation: Example 2

Homography Estimation: Example 2

Interest points detection

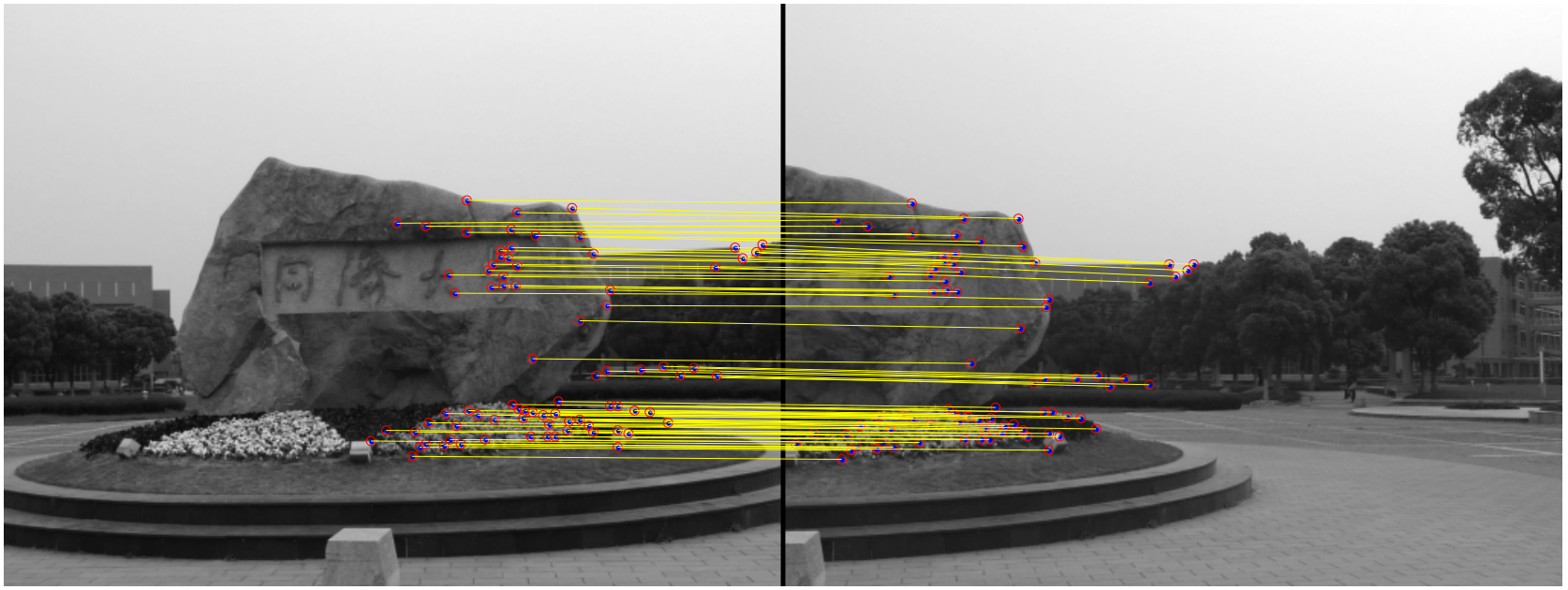




Homography Estimation: Example 2

Correspondence estimation

Then, the homography matrix can be estimated by using the correspondence pairs with RANSAC

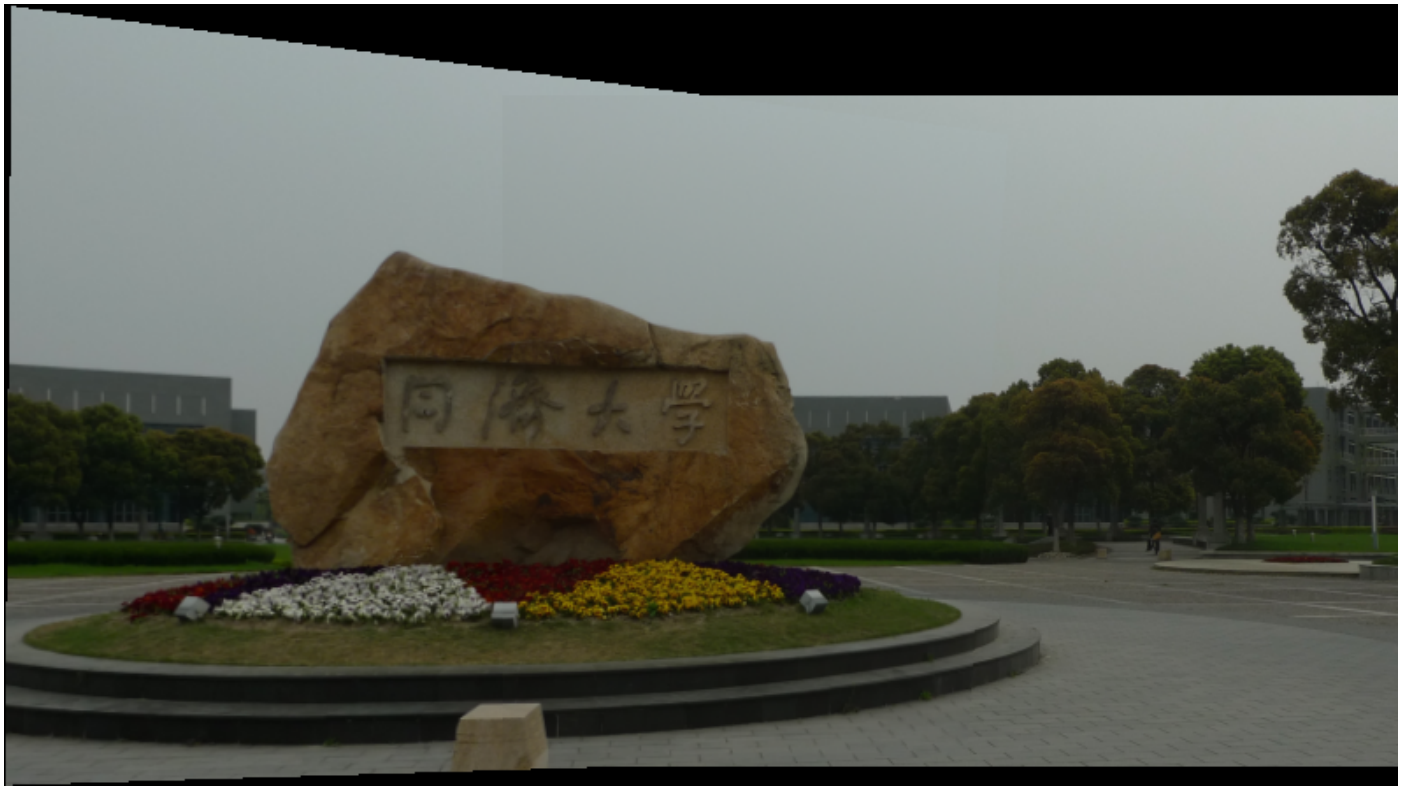


Homography Estimation: Example 2

Transform image one using the estimated homography matrix

Homography Estimation: Example 2

Finally, stitch the transformed image one with image two



Homography Estimation: Example 3

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Q+A