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Discrete logistics network design model under interval hierarchical OD demand based on interval genetic algorithm

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Abstract: Aimed at the uncertain characteristics of discrete logistics network design, an interval hierarchical triangular uncertain OD demand model based on interval demand and network flow is presented. Under consideration of the system profit, the uncertain demand of logistics network is measured by interval variables and interval parameters, and an interval planning model of discrete logistics network is established. The risk coefficient and maximum constrained deviation are defined to realize the certain transformation of the model. By integrating interval algorithm and genetic algorithm, an interval hierarchical optimal genetic algorithm is proposed to solve the model. It is shown by a tested example that in the same scenario condition an interval solution [3 275.3, 3 603.7] can be obtained by the model and algorithm which is obviously better than the single precise optimal solution by stochastic or fuzzy algorithm, so it can be reflected that the model and algorithm have more stronger operability and the solution result has superiority to scenario decision.

Key words: uncertainty; interval planning; hierarchical OD; logistics network design; genetic algorithm

1 Introduction

Discrete logistics network design is a system optimization problem. Its purpose is to solve optimal allocation and layout of logistics nodes and transport routes on logistics network within a certain range to ensure operational efficiency and benefit of the whole logistics system. So, this problem is defined as a facility location problem, and as a research hotspot on the academia it also has received wide attention. Currently, the research is mainly focused on the model improved and algorithm designed in logistics network design under different scenario statuses [1−5]. In recent years, with the rapid development of uncertain theory, logistics network under uncertain demand is received more and more attention. From the view of stochastic and fuzzy programming, some uncertain optimal algorithms are designed to solve this kind of problems [6−9]. But the solving process depends on some stochastic or fuzzy selections, and the solving result is also a single precise optimal solution. Actually, the direct influence of uncertainty is that the solving result is variable, while the single precise optimal solution can not reflect the selection behavior of decision-maker to decision scheme.

Interval planning and interval algorithm were proposed by MOORE [10], which has been proven to be a better approach to solve uncertain problem. The largest advantage of interval planning is that an interval solution can be obtained which can express the superiority of scenario decision. Interval planning has been an important medium and mode to research uncertain problem, and interval genetic algorithm has also been widely used to solve this uncertain optimal problem. TAKAO et al [11] formulated an optimal design about system reliability as a nonlinear integer programming problem with interval coefficients, and solved it directly with keeping nonlinearity of the objective function by using genetic algorithms. TAKEAKI et al [12] researched reliability optimal design problem with interval constraint based on hybrid genetic algorithms. GUPTA et al [13] applied interval variables to constrain uncertain inventory problem, established a planning model based on sales, inventory and demand, and

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adopted genetic algorithm to solve. SOARES et al [14] constructed an interval planning model for uncertain multi-objective optimal problem, and designed an interval robust multi-objective optimal solving algorithm. JENG et al [15] proposed an interval competitive agglomeration clustering algorithm to solve uncertain interval planning problem. BHUNIA et al [16] proposed an interval constraint model to reliability stochastic optimization problem, and componed interval genetic algorithm to solve. LIN and TSAN [17] proposed a robust multi-market newsvendor model with interval demand data, and designed solving algorithm under different standards. LAXMINARAYAN et al [18] researched multi-objective reliability optimal problem in interval environment, and designed genetic algorithm to solve. LINET and ONUR [19] researched a hierarchical clustering and routing procedure for large scale disaster relief logistics planning. SHEU and LIN [20] researched a hierarchical facility network planning model for global logistics network configurations.

As can be seen, the researches about uncertain discrete logistics network design are mainly focused on the following aspects: 1) Stochastic and fuzzy programming is used as the currently main measure to solve uncertain logistics network design, but the solving result is only a single precise optimal solution, so the actual decision-making basis is not strong; 2) The researches about discrete logistics network under complex environment and multi objectives have received wide attention, but it moreover lacks research of this problem from the perspective of hierarchical OD demand; 3) Interval uncertain model and interval algorithm are widely applied to solving general mathematical programming problem, but it seldom can be used to research logistics network design problem.

In this work, the uncertain demand characteristic of discrete logistics network and network flow is considered, and the hierarchical triangular uncertain OD demand is proposed. Based on this, constrained uncertain demand with interval variables and interval parameters can be used to establish an interval discrete logistics network planning model. Genetic algorithm and interval algorithm are combined to design the solving mode.

2 Problem description

2.1 Demand characteristic of uncertain discrete logistics network

With the rapid development of logistics industry and society demand, the tendency of multi-commodity flows, fast switching frequency and cycle time of goods become more and more conspicuous, and large-scale commodities circulation induces large-scale logistics. Logistics network presents the characteristic with multi-commodity, multi-manufacturer, multi-node of distribution and multi-demand consumers. Due to the uncertain demand, and the variation of current costs of the system, the complexity of logistics network becomes more prominent. The demand structure and characteristics are shown in Fig. 1.

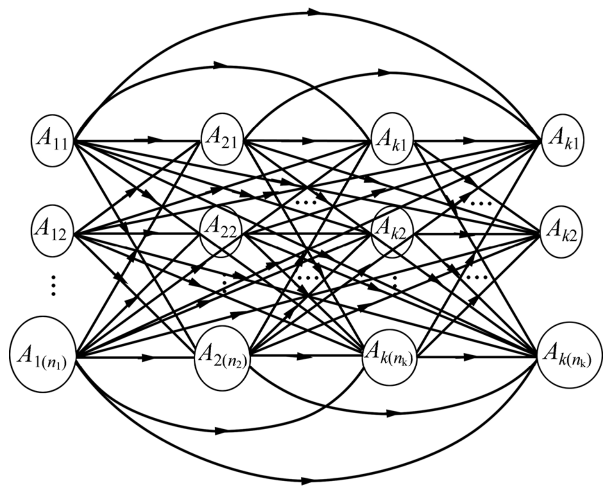

Fig. 1 Hierarchical OD demand structure of logistics network

1) Logistics network has some characteristics of multi-layer, multi-node, multi-commodity flows. In Fig. 1, layer A_1 and layer A_k are the first layer and the bottom layer of logistics network, respectively, and there are many other middle currency layers which have a great quantity predetermined nodes.

2) Hierarchical triangular OD demand relation in logistics network. Demand of traditional logistics network is often considered that the logistics only flow between two adjacent layers, but in fact, demand under network flows should exist between any nodes in any layer of downstream and any node in any layer of upstream. It expresses a hierarchical triangular OD demand relation.

3) There is interval uncertain demand of logistics network. Discrete logistics network design requires a predetermined and selected scheme assembly, and each node included in this assembly is a feasible solution. The purpose of decision is to find out the optimal scheme to guarantee a maximum efficiency of the whole system, and this scheme is the final decision scheme. Under different constrained conditions, different results may be obtained to solve the same problem. Because logistics network has uncertainty, different constraints may also bring out different influences on the total object of system. Therefore, the uncertain design of logistics network should be reflected on the interval uncertainty.

That is to say, it must consider uncertain factors, and demarcate all uncertain variables and parameters by interval numbers. Only with a reasonable understanding about interval variation range of design variables and parameters, it can more accurately catch the uncertain influence on the solving problem, and service to the scientific and reasonable decision.

2.2 Interval logistics network planning

Interval logistics network planning is to measure the uncertain variables in the form of interval numbers, while interval operational criteria are applied to realize the certain transformation of logistics network design model, and interval optimal algorithm is combined to solve. Interval numbers can be expressed as

$$X^{\mathrm{I}}=[x^{\mathrm{L}},x^{\mathrm{R}}]=\{x\in\mathbf{R}\,|\,x^{\mathrm{L}}\le x\le x^{\mathrm{R}}\} \tag{1}$$

where X^{I} is an interval in which interval variable x is included; x^{L} is interval lower limit; x^{R} is interval upper limit; x is an interval variable.

During the process of uncertain logistics network design, all uncertain variables and parameters can be considered varying in some interval ranges, and it also can be constrained by interval numbers.

3 Model

3.1 Basic assumption

Interval uncertain discrete logistics network design under hierarchical OD demand is a complex system. In this work, there are some basic assumptions to establish planning model as follows:

1) The planning model is a 0−1 mixed integer programming and discrete nodes-decision problem, and each layer and each node can meet service demand of system.

2) It can use a measurable interval form to express each uncertain variable and parameter in the model.

3) It can form a stable supply chain structure among corresponding layers, and the hierarchical OD demand characteristics are very clear.

4) Each hierarchical node's benefit and cost functions are linear and can be computed.

5) It does not consider dynamic variation of interval benefit and cost functions.

3.2 Planning model

According to network structure of Fig. 1, an interval logistics network demand system can be defined which includes network layers assembly $A=\{A_k|k=1, 2, \cdots, K]\}$, and network nodes assembly $A=\{A_{k(n_k)}|k=1, 2, \cdots, K]\}$. All nodes in the first layer A_1 and the bottom layer $\boldsymbol{A}_{K(n_K)}$ are fixed and necessary, so this work is to select some nodes during the middle (K−2) layers based on the maximum whole benefits of objective function of logistics network.

1) Basic variables and parameters are as follows. m is commodity category ($1\le m\le M$); n_k is the nodes number in the k-th layer, because the first and the bottom layers are fixed ($2\le k\le K-1$); $(C^{ij}_{kk'm})^{\mathrm{I}}$ is the interval unit transportation rate of commodity m from node i in layer k to node j in layer k', according to the hierarchical OD demand of logistics network, $1\le k\le K-1$, $k+1\le k'\le K$, $1\le i\le n_k$, $1\le j\le n_k$; $(X^{ij}_{kk'm})^{\mathrm{I}}$ is interval logistics volume of commodity m from node i in layer k to node j in layer k'; $(\partial^{ij}_{kk'm})^{\mathrm{I}}$ is the interval inflow increment coefficient of commodity m from node i in layer k to node j in layer k'; $(\beta^{ij}_{kk'm})^{\mathrm{I}}$ is the interval special benefit coefficient; $(\lambda^{ij}_{kk'm})^{\mathrm{I}}$ is the interval outflow increment coefficient; $(E^{i}_{km})^{\mathrm{I}}$ is the interval operational rate of commodity m on node i in layer k($1\le k\le K-1$, $1\le i\le n_k$); $(X^{i}_{km})^{\mathrm{I}}$ is interval logistics volume of commodity m through node i in layer k; Y^i_k is a 0−1 decision variable, if $Y^i_k=1$, it shows that node i in layer k is selected, otherwise it is not selected; $(F^i_k)^{\mathrm{I}}$ is interval investment cost about node i in layer k; $(V^i_{km})^{\mathrm{I}}$ is interval passing or storage capability of commodity m on node i in layer k; M_k is the most allowed and selected nodes numbers in layer k ($1\le M_k\le n_k$); $(a^i_m)^{\mathrm{I}}$ is interval demand volume of commodity m on node i in the bottom layer; D^{I} is an large positive number interval.

2) Cost function

Through the above definitions, in the structure of hierarchical OD demand, the whole interval cost of logistics system includes three aspects: interval network transportation cost C_1^{I}, interval nodes consumed cost C_2^{I} and interval nodes investment cost C_3^{I}, as follows:

$$C_1^{\mathrm{I}}=\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}(C^{ij}_{kk'm})^{\mathrm{I}}(X^{ij}_{kk'm})^{\mathrm{I}}Y^i_k \tag{2}$$

$$C_2^{\mathrm{I}}=\sum_{k=2}^{K-1}\sum_{i=1}^{n_k}\sum_{m=1}^{M}(E^{i}_{km})^{\mathrm{I}}(X^{i}_{km})^{\mathrm{I}}Y^i_k \tag{3}$$

$$C_3^{\mathrm{I}}=\sum_{k=2}^{K-1}\sum_{i=1}^{n_k}(F^{i}_{k})^{\mathrm{I}}Y^i_k \tag{4}$$

So, the total interval cost function is

$$\begin{aligned}C^{\mathrm{I}}&=C_1^{\mathrm{I}}+C_2^{\mathrm{I}}+C_3^{\mathrm{I}}=\\&\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}(C^{ij}_{kk'm})^{\mathrm{I}}(X^{ij}_{kk'm})^{\mathrm{I}}Y^i_k+\\&\sum_{k=2}^{K-1}\sum_{i=1}^{n_k}\sum_{m=1}^{M}(E^{i}_{km})^{\mathrm{I}}(X^{i}_{km})^{\mathrm{I}}Y^i_k+\sum_{k=2}^{K-1}\sum_{i=1}^{n_k}(F^{i}_{k})^{\mathrm{I}}Y^i_k\end{aligned} \tag{5}$$

3) Benefit function

In the whole logistics network chain, because the commodity flows from upstream layer to downstream layer, some incomes exist, such as: interval inflow increment income P_1^{I} (commodity income after handled, manufactured, transported, storaged), interval special income P_2^{I} (commodity saled, promoted), interval outflow increment income P_3^{I} (commodity income during the process of outflow, such as loading and unloading, transportation, distribution, packaging and sorting). The interval incomes function can be expressed as

$$P_1^{\mathrm{I}}=\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}(\partial_{kk'm}^{ij})^{\mathrm{I}}(X_{kk'm}^{ij})^{\mathrm{I}}Y_k^j \tag{6}$$

$$P_2^{\mathrm{I}}=\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}(\beta_{kk'm}^{ij})^{\mathrm{I}}(X_{kk'm}^{ij})^{\mathrm{I}}Y_k^j \tag{7}$$

$$P_3^{\mathrm{I}}=\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}(\lambda_{kk'm}^{ij})^{\mathrm{I}}(X_{kk'm}^{ij})^{\mathrm{I}}Y_k^i \tag{8}$$

So, the total interval income function is

$$\begin{aligned}P^{\mathrm{I}}&=P_1^{\mathrm{I}}+P_2^{\mathrm{I}}+P_3^{\mathrm{I}}=\\&\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}(Y_k^j(\partial_{kk'm}^{ij})^{\mathrm{I}}+\\&Y_k^j(\beta_{kk'm}^{ij})^{\mathrm{I}}+Y_k^i(\lambda_{kk'm}^{ij})^{\mathrm{I}})(X_{kk'm}^{ij})^{\mathrm{I}}\end{aligned} \tag{9}$$

The three interval income coefficients $(\partial_{kk'm}^{ij})^{\mathrm{I}}$, $(\beta_{kk'm}^{ij})^{\mathrm{I}}$ and $(\lambda_{kk'm}^{ij})^{\mathrm{I}}$ can be expressed by a comprehensive income coefficient $(\mu_{kk'm}^{ij})^{\mathrm{I}}$ as

$$(\mu_{kk'm}^{ij})^{\mathrm{I}}=(\partial_{kk'm}^{ij})^{\mathrm{I}}+(\beta_{kk'm}^{ij})^{\mathrm{I}}+(\lambda_{kk'm}^{ij})^{\mathrm{I}} \tag{10}$$

Equation (9) can be transformed to

$$P^{\mathrm{I}}=\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}Y_k^j(\mu_{kk'm}^{ij})^{\mathrm{I}}(X_{kk'm}^{ij})^{\mathrm{I}} \tag{11}$$

4) Total model

Define Z^{I} as the total interval system benefit, and an interval planning model through income−cost analysis can be established as

$$\begin{aligned}\max Z^{\mathrm{I}}=&\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}Y_k^j(\mu_{kk'm}^{ij})^{\mathrm{I}}(X_{kk'm}^{ij})^{\mathrm{I}}-\\&\sum_{k=2}^{K-1}\sum_{i=1}^{n_k}(F_k^i)^{\mathrm{I}}Y_k^i-\\&\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}(C_{kk'm}^{ij})^{\mathrm{I}}(X_{kk'm}^{ij})^{\mathrm{I}}Y_k^i-\\&\sum_{k=2}^{K-1}\sum_{i=1}^{n_k}\sum_{m=1}^{M}(E_{km}^i)^{\mathrm{I}}(X_{km}^i)^{\mathrm{I}}Y_k^i\end{aligned} \tag{12}$$

$$\begin{aligned}\text{s.t.}\quad&\sum_{m=1}^{M}(X_{km}^i)^{\mathrm{I}}\le(V_{km}^i)^{\mathrm{I}}\\&(X_{km}^i)^{\mathrm{I}}=\sum_{k'=k+1}^{K}\sum_{j=1}^{n_{k'}}(X_{kk'm}^{ij})^{\mathrm{I}}\\&\sum_{k=1}^{K-1}(X_{km}^i)^{\mathrm{I}}\le(a_m^i)^{\mathrm{I}}\\&\sum_{i=1}^{n_k}Y_k^i\le M_k,\quad 2\le k\le K-1\\&(X_{kk'm}^{ij})^{\mathrm{I}}\le Y_k^i\cdot D^{\mathrm{I}}\\&(X_{kk'm}^{ij})^{\mathrm{I}},(X_{km}^i)^{\mathrm{I}}\ge 0\end{aligned} \tag{13}$$

where $\sum_{m=1}^{M}(X_{km}^i)^{\mathrm{I}}\le(V_{km}^i)^{\mathrm{I}}$ meets interval acceptance constraint in each node; $\sum_{k=1}^{K-1}(X_{km}^i)^{\mathrm{I}}\le(a_m^i)^{\mathrm{I}}$ meets total interval supply and demand; $\sum_{i=1}^{n_k}Y_k^i\le M_k$ meets nodes selected numbers constraint; $(X_{km}^i)^{\mathrm{I}}=\sum_{k'=k+1}^{K}\sum_{j=1}^{n_{k'}}(X_{kk'm}^{ij})^{\mathrm{I}}$ conforms interval volume conservation in each node.

4 Solution and algorithm

4.1 Interval operation

Interval numbers meet the general operational criterion about real numbers, but logistics network is a practically applied problem and is different to the general mathematical programming problem, and the uncertain parameters and variables of logistics network are non negative constraints. In this work, it only involves interval multiplication, interval addition and interval subtraction operations, which can be defined as

$$\begin{aligned}&X^{\mathrm{I}}+Y^{\mathrm{I}}=[x^{\mathrm{L}},x^{\mathrm{R}}]+[y^{\mathrm{L}},y^{\mathrm{R}}]=[x^{\mathrm{L}}+y^{\mathrm{L}},x^{\mathrm{R}}+y^{\mathrm{R}}]\\&X^{\mathrm{I}}+R=[x^{\mathrm{L}},x^{\mathrm{R}}]+R=[x^{\mathrm{L}}+R,x^{\mathrm{R}}+R]\end{aligned} \tag{14}$$

$$\begin{aligned}&X^{\mathrm{I}}-Y^{\mathrm{I}}=[x^{\mathrm{L}},x^{\mathrm{R}}]-[y^{\mathrm{L}},y^{\mathrm{R}}]=[x^{\mathrm{L}}-y^{\mathrm{R}},x^{\mathrm{R}}-y^{\mathrm{L}}]\\&X^{\mathrm{I}}-R=[x^{\mathrm{L}},x^{\mathrm{R}}]-R=[x^{\mathrm{L}}-R,x^{\mathrm{R}}-R]\end{aligned} \tag{15}$$

$$\begin{aligned}&(C_{kk'm}^{ij})^{\mathrm{I}}(X_{kk'm}^{ij})^{\mathrm{I}}Y_k^i=\\&\qquad[(C_{kk'm}^{ij})^{\mathrm{L}}(X_{kk'm}^{ij})^{\mathrm{L}}Y_k^i,(C_{kk'm}^{ij})^{\mathrm{R}}(X_{kk'm}^{ij})^{\mathrm{R}}Y_k^i]\\&(E_{km}^i)^{\mathrm{I}}(X_{km}^i)^{\mathrm{I}}Y_k^i=[(E_{km}^i)^{\mathrm{L}}(X_{km}^i)^{\mathrm{L}}Y_k^i,(E_{km}^i)^{\mathrm{R}}(X_{km}^i)^{\mathrm{R}}Y_k^i]\\&(F_k^i)^{\mathrm{I}}Y_k^i=[(F_k^i)^{\mathrm{L}}Y_k^i,(F_k^i)^{\mathrm{R}}Y_k^i]\\&(\mu_{kk'm}^{ij})^{\mathrm{I}}(X_{kk'm}^{ij})^{\mathrm{I}}Y_k^j=\\&\qquad[(\mu_{kk'm}^{ij})^{\mathrm{L}}(X_{kk'm}^{ij})^{\mathrm{L}}Y_k^j,(\mu_{kk'm}^{ij})^{\mathrm{R}}(X_{kk'm}^{ij})^{\mathrm{R}}Y_k^j]\\&(X_{km}^i)^{\mathrm{I}}=[(X_{km}^i)^{\mathrm{L}},(X_{km}^i)^{\mathrm{R}}]\\&(V_{km}^i)^{\mathrm{I}}=[(V_{km}^i)^{\mathrm{L}},(V_{km}^i)^{\mathrm{R}}]\end{aligned}$$

$$(a_m^i)^{\mathrm{I}}=[(a_m^i)^{\mathrm{L}},(a_m^i)^{\mathrm{R}}] \tag{16}$$

Equations (14) and (15) are interval addition and interval subtraction operational criterion about interval variables, and Eq. (16) is interval multiplication operational results involved in this work. Obviously, the bound of new interval variable after interval operation may become bigger.

4.2 Certain transformation of model

According to Eq. (16), it can transform Eq. (12) to

$$\max Z^{\mathrm{L}}=\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}Y_k^j(\mu_{kk'm}^{ij})^{\mathrm{L}}(X_{kk'm}^{ij})^{\mathrm{L}}-\sum_{k=2}^{K-1}\sum_{i=1}^{n_k}(\boldsymbol{F}_k^i)^{\mathrm{L}}Y_k^i-\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}(C_{kk'm}^{ij})^{\mathrm{L}}(X_{kk'm}^{ij})^{\mathrm{L}}Y_k^i-\sum_{k=2}^{K-1}\sum_{i=1}^{n_k}\sum_{m=1}^{M}(E_{km}^i)^{\mathrm{L}}(X_{km}^i)^{\mathrm{L}}Y_k^i \tag{17}$$

$$\max Z^{\mathrm{R}}=\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}Y_k^j(\mu_{kk'm}^{ij})^{\mathrm{R}}(X_{kk'm}^{ij})^{\mathrm{R}}-\sum_{k=2}^{K-1}\sum_{i=1}^{n_k}(F_k^i)^{R}Y_k^i-\sum_{k=1}^{K-1}\sum_{k'=k+1}^{K}\sum_{i=1}^{n_k}\sum_{j=1}^{n_{k'}}\sum_{m=1}^{M}(C_{kk'm}^{ij})^{\mathrm{R}}(X_{kk'm}^{ij})^{\mathrm{R}}Y_k^i-\sum_{k=2}^{K-1}\sum_{i=1}^{n_k}\sum_{m=1}^{M}(E_{km}^i)^{\mathrm{R}}(X_{km}^i)^{\mathrm{R}}Y_k^i \tag{18}$$

And there is $Z^{\mathrm{I}}=[Z^{\mathrm{L}}, Z^{\mathrm{R}}]$ (19)

Define $Z'(Z^{\mathrm{L}}\le Z'\le Z^{\mathrm{R}})$ which is included in interval number Z^{I}, and give a risk coefficient θ:

$$\theta=\frac{Z'-Z^{\mathrm{L}}}{Z^{\mathrm{R}}-Z^{\mathrm{L}}} \tag{20}$$

θ is a risk factor which can make the solving result of objective function greater than or equal to Z' under the uncertainty of decision variables $(X_{kk'm}^{ij})^{\mathrm{I}},(X_{km}^i)^{\mathrm{I}}$, and it meets $0\le\theta\le1$. There is

$$Z'=\theta Z^{\mathrm{R}}+(1-\theta)Z^{\mathrm{L}} \tag{21}$$

So objective function Eq. (12) can be transformed to

$$\max Z^{\mathrm{I}}=\theta Z^{\mathrm{R}}+(1-\theta)Z^{\mathrm{L}} \tag{22}$$

In addition, because there have some uncertain parameters, such as, $(C_{kk'm}^{ij})^{\mathrm{I}},(E_{km}^i)^{\mathrm{I}}$ and $(F_k^i)^{\mathrm{I}}$ in the model which can bring out deviation for the solution of decision variables, it also should define the corresponding decision deviation constraints as

$$d(Z)=Z^{\mathrm{R}}-Z^{\mathrm{L}}\le d_{\max} \tag{23}$$

where $d_{\max}$ expresses the allowed objective function interval solution by decision-maker, and it must be prior given by decision-maker.

Similarly, the constraint conditions can be transformed to

$$\begin{aligned}\text{s.t.}\quad &(V_{km}^i)^{\mathrm{L}}\le\sum_{m=1}^{M}(X_{km}^i)^{\mathrm{L}}\le\sum_{m=1}^{M}(X_{km}^i)^{\mathrm{R}}\le(V_{km}^i)^{\mathrm{R}}\\ &(X_{km}^i)^{\mathrm{L}}=\sum_{k'=k+1}^{K}\sum_{j=1}^{n_{k'}}(X_{kk'm}^{ij})^{\mathrm{L}}\\ &(X_{km}^i)^{\mathrm{R}}=\sum_{k'=k+1}^{K}\sum_{j=1}^{n_{k'}}(X_{kk'm}^{ij})^{\mathrm{R}}\\ &(a_m^i)^{\mathrm{L}}\le\sum_{k=1}^{K-1}(X_{km}^i)^{\mathrm{L}}\le\sum_{k=1}^{K-1}(X_{km}^i)^{\mathrm{R}}\le(a_m^i)^{\mathrm{R}}\\ &\sum_{i=1}^{n_k}Y_k^i\le M_k,\quad 2\le k\le K-1\\ &Y_k^i\cdot D^{\mathrm{L}}\le(X_{kk'm}^{ij})^{\mathrm{L}}\le(X_{kk'm}^{ij})^{\mathrm{R}}\le Y_k^i\cdot D^{\mathrm{R}}\\ &(X_{km}^i)^{\mathrm{L}},(X_{km}^i)^{\mathrm{R}},(X_{kk'm}^{ij})^{\mathrm{L}},(X_{kk'm}^{ij})^{\mathrm{R}}\ge0\end{aligned} \tag{24}$$

In this model, it is shown that to a given logistics network with multi layers and multi nodes, in the uncertain demand, each basic cost parameter, benefit parameter and demand variable exist in the form of "interval", so it can be expressed by the operation with "interval number" in the model. The model is based on the total maximum benefit. The objective function is composed of three parts: interval benefit function, interval transportation cost function and interval nodes cost function, and the constrained condition can meet the interval supply-demand equilibrium in each OD pair, interval produce-attract equilibrium of the total system, interval capacity limitation of each node and number of constraints about decided nodes. So, this model is an NP-hard problem which includes interval uncertain demand.

4.3 Judgement of interval solution

Discrete logistics network design is a NP problem, which is often solved by optimal algorithm techniques, so judgement, comparison and elimination can be used during the whole search process. In the mode of interval constraint, judgement of interval solution had three scenes: interval intersection, interval outer-section and interval inclusion.

Assume that there are two interval solutions of objective function in the process of solution, such as

$(Z_0)^{\mathrm{I}}=[(Z_0)^{\mathrm{L}},(Z_0)^{\mathrm{R}}]$ and $(Z_1)^{\mathrm{I}}=[(Z_1)^{\mathrm{L}},(Z_1)^{\mathrm{R}}]$.

1) If $(Z_0)^{\mathrm{L}}\ge(Z_1)^{\mathrm{R}}$, it can express the form of interval outer-section, so $(Z_0)^{\mathrm{I}}\ge(Z_1)^{\mathrm{I}}$, $(Z_1)^{\mathrm{I}}$ is the optimal interval solution.

2) If $(Z_0)^{\mathrm{L}}\le(Z_1)^{\mathrm{L}}\le(Z_1)^{\mathrm{R}}\le(Z_0)^{\mathrm{R}}$, it can express the form of interval inclusion, so $(Z_1)^{\mathrm{I}}\subseteq(Z_0)^{\mathrm{I}}$, and $(Z_1)^{\mathrm{I}}$ is the optimal interval solution.

3) If $(Z_0)^{\mathrm{R}}\ge(Z_1)^{\mathrm{L}}$, it can express the form of interval intersection, so an interval possibility function is defined as

$$P=\max\{0,1-\max\left[0,\frac{(Z_0)^{\mathrm{R}}-(Z_1)^{\mathrm{L}}}{((Z_0)^{\mathrm{R}}-(Z_0)^{\mathrm{L}})+((Z_1)^{\mathrm{R}}-(Z_1)^{\mathrm{L}})}\right]\}\tag{25}$$

If $P\ge0.5$, where $(Z_0)^{\mathrm{I}}\ge(Z_1)^{\mathrm{I}}$, or $(Z_0)^{\mathrm{I}}<(Z_1)^{\mathrm{I}}$.

4.4 Interval genetic algorithm

Genetic algorithm can consider overall optimization performance and robustness of solution, which can be thought as a better method to solve NP problem, and is widely used in discrete logistics network design. In this work, interval operation and genetic algorithms are combined to solve the discrete interval logistics network planning model. An interval hierarchical optimization genetic algorithm is designed whose basic idea is: Firstly, define a two-layer interval optimal decision process, and give an interval initial solution which meets the upper layer decision problem. In this environment, the lower layer decision can option own object in a possible range on its preference; Then, it may feedback this result to the upper layer problem, and on the base of the best reaction between the upper layer and the lower layer, explore an interval optimal solution about the whole problem. The solving process is as follows.

Step 1: Code the interval variables and parameters. Use the mode of interval codes, where each code expresses an interval value assembly.

Step 2: Define an interval slack variable $(s_{kk'm}^{ij})^{\mathrm{I}}$ for 0−1 decision variable and unknown interval variable $(X_{kk'm}^{ij})^{\mathrm{I}}$, which may meet

$$(X_{kk'm}^{ij})^{\mathrm{I}}=Y_k^i\cdot D^{\mathrm{I}}-(s_{kk'm}^{ij})^{\mathrm{I}}\tag{26}$$

Step 3: Generate interval initial population. With the stochastic simulation technique of computer, it can generate an initial solution assembly U, $(Y_k^i)_t\in U$, and $(Y_k^i)_t$ is the $t(0\le t\le u)$ solution in U. In the state of U_0, N_{gen}=0. Set the population evolution generation N.

Step 4: Set decision parameters. Define risk factor as θ and maximum decision deviation $d_{\max}$.

Step 5: Construct interval fitness function. Because the hierarchical OD discrete logistics network seeks the total maximum benefit as its system object under interval constraint, the fitness function is a reciprocal of objective function, that is

$$F_{\mathrm{fit}}((X_{kk'm}^{ij})^{\mathrm{I}},Y_k^i)=\frac{1}{Z^{\mathrm{I}}}=\frac{1}{\theta Z^{\mathrm{R}}+(1-\theta)Z^{\mathrm{L}}}\tag{27}$$

Step 6: Distribute all $(Y_k^i)_t$ in population U to the lower layer calculation unit.

Step 7: Feedback the operational result to the upper layer calculation unit, and operate Eq. (28) after the operational result is received to candidate solution Z_t:

$$\begin{aligned}(Z_t)^{\mathrm{L}}&=\min Z_t((X_{kk'm}^{ij})^{\mathrm{I}},Y_k^i)\\(Z_t)^{\mathrm{R}}&=\max Z_t((X_{kk'm}^{ij})^{\mathrm{I}},Y_k^i)\end{aligned}\tag{28}$$

Step 8: Define penalty factor G and calculate comprehensive objective function:

$$\max Z_t=\theta(Z_t)^{\mathrm{R}}+(1-\theta)(Z_t)^{\mathrm{L}}-G\max(0,d(Z_t)-d_{\max})\tag{29}$$

Step 9: According to the approach of Section 4.3, judge and compare the property of interval solutions, and reserve it if it reaches an optimal solution.

Step 10: If evolution generation has been finished, turn to Step 12 or to be continued.

Step 11: Define the probabilities of interval selection, interval intersect and interval variation, proceed interval genetic operation for the current generation optimal solution, and generate new populations, turn to Step 3.

Step 12: Output the optimal interval solution.

Step 13: Calculate slack variable $(s_{kk'm}^{ij})^{\mathrm{I}}$, substitute Eq. (26) to objective problem, and obtain a new $(Y_k^i)_{t+1}$.

Step 14: Test of deviation. If $d(Z)=Z^{\mathrm{R}}-Z^{\mathrm{L}}\le d_{\max}$, terminate the operation.

Step 15: Output the result. Proceed generated operation under $N_{\mathrm{gen}}=N_{\mathrm{gen}}+1$, if $N_{\mathrm{gen}}=N$, terminate the operation, and select the corresponding solving result of the first chromosome in the current population as the final solution, and the $1/F_{\mathrm{fit}}$ is the optimal interval solution of objective function; if $N_{\mathrm{gen}}<N$, turn to Step 3 and continue.

5 Analysis of example

There is a discrete complex logistics network, the current commodities M=2, network layers K=4, nodes included in each layer n_k={3, 4, 3, 5}, limited nodes selected in middle layers M_k={M_2, M_3}={3, 2}, the other interval parameters are defined as follows.

$$(\boldsymbol{C}_{kk'1}^{ij})^{\mathrm{I}}=\begin{bmatrix} OD & A_{11} & A_{12} & A_{13} & A_{21} & A_{22} & A_{23} & A_{24} & A_{31} & A_{32} & A_{33} \\ A_{21} & [1.2,1.6] & [0.8,1.1] & [1.3,1.7] & & & & & & & \\ A_{22} & [1.3,1.5] & [1.0,1.3] & [0.9,1.2] & & & 0 & & & & \\ A_{23} & [0.9,1.1] & [1.1,1.4] & [1.3,1.6] & & & & & & & \\ A_{24} & [1.5,1.8] & [1.4,1.7] & [1.8,2.1] & & & & & & 0 & \\ A_{31} & [1.8,2.1] & [2.1,2.3] & [1.2,1.5] & [1.0,1.3] & [1.1,1.4] & [1.3,1.6] & [1.2,1.4] & & & \\ A_{32} & [2.2,2.4] & [1.6,1.9] & [1.8,2.3] & [0.9,1.2] & [0.8,1.1] & [1.0,1.3] & [1.4,1.6] & & & \\ A_{33} & [1.7,2.0] & [1.8,2.2] & [2.2,2.7] & [1.4,1.7] & [1.3,1.5] & [1.2,1.4] & [1.3,1.7] & & & \\ A_{41} & [2.5,2.9] & [2.4,2.6] & [2.6,2.8] & [1.5,1.8] & [1.8,2.3] & [1.6,1.9] & [1.8,2.1] & [1.4,1.8] & [1.6,1.9] & [1.0,1.4] \\ A_{42} & [2.2,2.6] & [2.7,3.0] & [2.7,3.0] & [1.9,2.3] & [2.0,2.4] & [2.1,2.5] & [2.4,2.6] & [1.0,1.2] & [1.2,1.4] & [1.6,1.8] \\ A_{43} & [2.4,2.8] & [2.4,2.8] & [2.2,2.6] & [2.2,2.6] & [2.5,2.7] & [2.3,2.6] & [2.9,3.2] & [1.4,1.7] & [1.1,1.3] & [1.4,1.7] \\ A_{44} & [1.9,2.3] & [3.2,3.5] & [2.8,3.1] & [2.4,2.7] & [2.0,2.4] & [1.8,2.2] & [2.5,2.8] & [0.9,1.3] & [1.0,1.5] & [1.1,1.3] \\ A_{45} & [2.0,2.4] & [2.8,3.1] & [2.5,2.9] & [1.9,2.3] & [1.7,2.2] & [2.3,2.6] & [2.2,2.6] & [1.5,1.8] & [1.7,1.9] & [1.8,2.1] \end{bmatrix}$$

$$(\boldsymbol{C}_{kk'2}^{ij})^{\mathrm{I}}=\begin{bmatrix} OD & A_{11} & A_{12} & A_{13} & A_{21} & A_{22} & A_{23} & A_{24} & A_{31} & A_{32} & A_{33} \\ A_{21} & [1.3,1.6] & [0.9,1.2] & [1.4,1.7] & & & & & & & \\ A_{22} & [1.4,1.7] & [1.3,1.5] & [0.9,1.3] & & & 0 & & & & \\ A_{23} & [1.0,1.3] & [1.0,1.4] & [1.2,1.5] & & & & & & & \\ A_{24} & [1.1,1.5] & [1.1,1.5] & [1.4,1.8] & & & & & & 0 & \\ A_{31} & [1.8,2.1] & [1.6,1.9] & [1.9,2.2] & [0.8,1.1] & [1.3,1.5] & [1.0,1.3] & [0.9,1.4] & & & \\ A_{32} & [2.3,2.7] & [2.0,2.4] & [2.5,2.8] & [1.1,1.4] & [1.4,1.7] & [1.4,1.7] & [1.5,1.8] & & & \\ A_{33} & [2.0,2.4] & [1.7,1.9] & [2.4,2.7] & [1.3,1.5] & [1.0,1.3] & [1.6,1.9] & [1.0,1.3] & & & \\ A_{41} & [2.7,3.0] & [2.2,2.6] & [2.6,3.0] & [1.8,2.2] & [1.8,2.2] & [1.9,2.3] & [1.7,2.0] & [1.4,1.6] & [1.5,1.8] & [1.3,1.7] \\ A_{42} & [3.1,3.4] & [2.9,3.3] & [2.8,3.2] & [2.3,2.6] & [2.1,2.5] & [2.6,2.8] & [2.3,2.7] & [1.0,1.3] & [1.1,1.4] & [1.2,1.5] \\ A_{43} & [2.6,3.1] & [2.4,2.8] & [1.9,2.5] & [2.2,2.5] & [2.6,2.8] & [3.2,3.5] & [2.6,2.9] & [1.2,1.5] & [1.6,2.0] & [1.8,2.1] \\ A_{44} & [2.8,3.2] & [2.5,2.9] & [2.6,3.1] & [1.9,2.3] & [2.4,2.8] & [2.7,2.9] & [2.4,2.8] & [1.8,2.0] & [1.8,2.3] & [2.3,2.6] \\ A_{45} & [2.5,3.0] & [3.1,3.5] & [3.2,3.6] & [2.5,2.8] & [3.0,3.4] & [3.4,3.8] & [2.9,3.3] & [2.1,2.4] & [2.4,2.6] & [2.7,3.0] \end{bmatrix}$$

$$(\boldsymbol{\mu}_{kk'1}^{ij})^{\mathrm{I}}=\begin{bmatrix} OD & A_{11} & A_{12} & A_{13} & A_{21} & A_{22} & A_{23} & A_{24} & A_{31} & A_{32} & A_{33} \\ A_{21} & [4.4,4.9] & [5.1,5.5] & [5.2,5.5] & & & & & & & \\ A_{22} & [4.8,5.2] & [4.9,5.3] & [4.4,4.7] & & & & & & & \\ A_{23} & [5.3,5.6] & [5.5,5.8] & [5.1,5.6] & & & 0 & & & & \\ A_{24} & [4.7,5.0] & [4.9,5.3] & [5.1,5.4] & & & & & & 0 & \\ A_{31} & [5.6,5.9] & [5.7,6.0] & [6.2,6.5] & [5.1,5.3] & [5.2,5.4] & [4.9,5.3] & [5.0,5.3] & & & \\ A_{32} & [6.1,6.5] & [6.4,6.6] & [6.7,7.0] & [5.0,5.4] & [4.8,5.1] & [4.6,5.1] & [4.6,4.9] & & & \\ A_{33} & [4.9,5.3] & [5.2,5.4] & [4.7,5.0] & [4.9,5.2] & [5.3,5.6] & [5.1,5.3] & [5.2,5.6] & & & \\ A_{41} & [5.2,5.7] & [5.7,6.0] & [5.3,5.7] & [5.4,5.7] & [5.5,5.9] & [5.6,5.8] & [5.7,6.1] & [5.5,5.8] & [5.0,5.4] & [4.9,5.3] \\ A_{42} & [5.0,5.4] & [4.8,5.2] & [4.5,4.8] & [6.0,6.4] & [6.1,6.5] & [6.0,6.4] & [6.3,6.5] & [4.9,5.2] & [5.1,5.3] & [5.2,5.5] \\ A_{43} & [4.7,4.9] & [4.5,5.0] & [4.7,5.0] & [4.7,5.1] & [5.3,5.6] & [5.4,5.7] & [5.2,5.6] & [4.8,5.1] & [4.5,4.8] & [4.8,5.1] \\ A_{44} & [5.2,5.6] & [5.1,5.4] & [5.3,5.7] & [4.8,5.3] & [4.5,4.8] & [4.8,5.1] & [4.9,5.2] & [5.6,5.9] & [5.7,6.0] & [5.8,6.2] \\ A_{45} & [5.8,6.2] & [5.3,5.6] & [5.2,5.4] & [5.5,5.8] & [5.7,6.0] & [5.4,5.8] & [5.9,6.2] & [5.8,6.2] & [5.7,6.2] & [6.0,6.3] \end{bmatrix}$$

$$(\boldsymbol{E}_{2m}^{i})^{\mathrm{I}}=\begin{bmatrix} [1.5,1.8] & [1.6,2.0] & [1.2,1.4] & [1.3,1.5] \\ [1.6,1.8] & [1.5,1.8] & [1.0,1.4] & [1.1,1.4] \end{bmatrix}$$

$$(\boldsymbol{E}_{3m}^{i})^{\mathrm{I}}=\begin{bmatrix} [1.2,1.5] & [1.1,1.4] & [0.9,1.2] \\ [1.0,1.3] & [1.4,1.7] & [1.2,1.5] \end{bmatrix}$$

$$(\boldsymbol{V}_{2m}^{i})^{\mathrm{I}}=\begin{bmatrix} [225,232] & [178,184] & [136,142] & [215,224] \\ [178,185] & [236,245] & [202,213] & [145,157] \end{bmatrix}$$

$$(\boldsymbol{\mu}_{kk'2}^{ij})^{\mathrm{I}}=\begin{bmatrix} OD & A_{11} & A_{12} & A_{13} & A_{21} & A_{22} & A_{23} & A_{24} & A_{31} & A_{32} & A_{33} \\ A_{21} & [5.2,5.5] & [5.3,5.5] & [4.8,5.2] & & & & & & & \\ A_{22} & [4.8,5.1] & [5.0,5.3] & [5.5,5.8] & & & & & & & \\ A_{23} & [5.7,6.0] & [5.5,5.8] & [6.0,6.3] & & & 0 & & & & \\ A_{24} & [5.1,5.4] & [4.8,5.2] & [5.2,5.4] & & & & & & 0 & \\ A_{31} & [5.8,6.2] & [5.9,6.2] & [5.5,5.8] & [5.0,5.3] & [4.9,5.3] & [5.2,5.5] & [4.7,5.2] & & & \\ A_{32} & [4.4,4.8] & [4.8,5.2] & [4.7,5.0] & [5.7,6.0] & [5.5,5.8] & [4.6,4.9] & [4.5,4.8] & & & \\ A_{33} & [5.0,5.3] & [5.3,5.6] & [5.6,6.0] & [4.7,4.9] & [4.4,4.7] & [5.3,5.5] & [5.1,5.5] & & & \\ A_{41} & [6.1,6.4] & [5.9,6.2] & [5.5,5.9] & [4.8,5.0] & [5.3,5.6] & [5.7,6.0] & [4.2,4.5] & [4.8,5.0] & [4.7,4.9] & [4.8,5.2] \\ A_{42} & [5.9,6.2] & [5.4,5.7] & [6.0,6.3] & [4.5,4.8] & [5.2,5.8] & [5.4,5.8] & [5.3,5.6] & [4.5,4.7] & [4.5,4.8] & [5.2,5.4] \\ A_{43} & [4.8,5.1] & [4.6,4.8] & [4.7,4.9] & [5.0,5.3] & [4.7,4.9] & [4.7,4.9] & [5.1,5.3] & [4.1,4.4] & [4.6,4.9] & [4.4,4.6] \\ A_{44} & [4.2,4.5] & [4.7,5.0] & [5.2,5.6] & [4.4,4.7] & [5.6,5.9] & [5.3,5.6] & [4.9,5.2] & [5.3,5.6] & [5.4,5.7] & [4.5,4.8] \\ A_{45} & [5.0,5.4] & [5.4,5.7] & [5.4,5.8] & [4.6,4.8] & [4.8,5.2] & [5.2,5.5] & [5.2,5.4] & [5.0,5.3] & [5.3,5.6] & [4.6,4.9] \end{bmatrix}$$

$$(\boldsymbol{V}_{3m}^{i})^{\mathrm{I}}=\begin{bmatrix} [236,246] & [197,208] & [165,176] \\ [168,178] & [265,274] & [208,215] \end{bmatrix}$$

$$(\boldsymbol{F}_{2}^{i})^{\mathrm{I}}=\begin{bmatrix} [128,132] & [143,149] & [118,123] & [145,152] \end{bmatrix}$$

$$(\boldsymbol{F}_{3}^{i})^{\mathrm{I}}=\begin{bmatrix} [169,174] & [152,159] & [179,185] \end{bmatrix}$$

$$(\boldsymbol{a}_{m}^{i})^{\mathrm{I}}=\begin{bmatrix} [289,295] & [304,311] & [276,285] & [274,280] & [212,220] \\ [305,311] & [297,305] & [295,304] & [265,274] & [228,235] \end{bmatrix}$$

According to the solving strategy in Section 4.4, define population evolution generations and penalty coefficient. In the different decision preference θ and $d_{\max}$ of decision-maker, the simulated operational results are shown in Table 1 and Table 2, respectively.

From Table 1 and Fig. 2 and Fig. 3, if $d_{\max}$ is fixed, with the increase of risk coefficient ζ, moreover the upper and lower bounds of the final interval solution of objective function increase, but the interval width of interval solution becomes more and more narrow. It is shown that the reliability about the final interval solution of objective function becomes more strong with the increase of ζ. Similarly, if ζ is fixed, the upper and lower bounds of the final interval solution of objective function come into corresponding changes with the increase of the maximum constrained deviation $d_{\max}$. The result is that the upper bound of the interval expands and the lower bound reduces. It is shown that the superiority of interval solution becomes more stronger with the reduction of $d_{\max}$. On the other hand, from Fig. 2, it can express that $d_{\max}$ has a limited interval, there is a minimum $d_{\max}$ to solution under each scenario, for example, under the scenario of ζ=0.2 and $d_{\max}$<236.8, there is no solution for the tested example. As well, if $d_{\max}$≥878.5, the final interval solution of objective function maintains in a fixed interval. This demonstrates that if the initially defined $d_{\max}$ is too small, it can not obtain optimal interval solution for the interval planning problem; and if the initially defined $d_{\max}$ is too big, the solution of interval planning has no risk, and the solving strategy has no value.

Table 2 expresses the solving results under different scenarios. Not only the interval solution of objective function but also the decided results of nodes selection vary with the variation of risk coefficient ζ and the maximum constrained deviation $d_{\max}$, for example under the same constraint ζ=0.2, the decision results of nodes selection of $d_{\max}$=350 and $d_{\max}$=750 are A_{21}, A_{22}, A_{23}, A_{32}, A_{33} and A_{21}, A_{22}, A_{23}, A_{31}, A_{33}, and there exists obvious difference.

There have some differences to this example with different solution algorithms in Table 3. Under the same condition (ζ=0.5, $d_{\max}$=350), with random algorithm or fuzzy algorithm to solve this example, it can only obtain a single precise optimal solution which can not completely reflect the variation by uncertainty. But interval algorithm may integrate genetic algorithm and interval operation to solve. It can provide interval optimal solution in different scenarios, and completely reflect the variation by uncertainty in the form of "interval". Also it can give service to decision-maker to realize scenario selection, for example during the two scenarios (ζ=0.5, $d_{\max}$=350) and (ζ=0.2, $d_{\max}$=350), the decided scheme and objective interval value are also different, while ζ and $d_{\max}$ are decided by decision-maker who has a prior decision attitude. Obviously, the method has some superiorities to interval optimal solution, scenario decision and stronger algorithm operability.

Table 1 Operational results under different scenarios

Maximum constrained deviation	Risk coefficient, ζ=0.2			Risk coefficient, ζ=0.5			Risk coefficient, ζ=0.8		
	max Z^L	max Z^R	Interval width	max Z^L	max Z^R	Interval width	max Z^L	max Z^R	Interval width
	3 204.5	3 441.3	236.8	3 378.5	3 604.3	225.8	3 485.6	3 703.1	217.5
250	3 198.7	3 447.3	248.6	3 318.5	3 559.7	241.2	3 476.2	3 712.7	236.5
350	3 165.4	3 498.1	332.7	3 275.3	3 603.7	328.4	3 438.5	3 760.5	322.0
450	3 124.8	3 553.4	428.6	3 211.5	3 631.0	419.5	3 398.6	3 810.4	411.8
550	3 097.5	3 612.9	515.4	3 198.7	3 705.4	506.7	3 354.2	3 854.5	500.3
650	3 054.2	3 664.0	609.8	3 165.4	3 765.7	600.3	3 314.0	3 908.2	594.2
750	3 018.4	3 719.0	700.6	3 122.9	3 817.1	694.2	3 287.8	3 971.0	683.2
850	2 975.1	3 770.7	795.6	3 099.4	3 883.1	783.7	3 245.5	4 015.6	770.1
	2 910.3	3 788.8	878.5	3 076.1	3 941.5	865.4	3 219.2	4 073.6	854.4
Instruction	<236.8 no solution ≥878.5 stable solution			<225.8 no solution ≥865.4 stable solution			<217.5 no solution ≥854.4 stable solution		

Table 2 Decided results under different scenarios

Maximum constrained deviation	Risk coefficient, ζ=0.2		Risk coefficient ζ=0.5		Risk coefficient ζ=0.8	
	Decision of second layer	Decision of third layer	Decision of second layer	Decision of third layer	Decision of second layer	Decision of third layer
250	A_{21}, A_{22}, A_{23}	A_{32}, A_{33}	A_{21}, A_{22}, A_{24}	A_{32}, A_{33}	A_{21}, A_{22}, A_{24}	A_{32}, A_{33}
350	A_{21}, A_{22}, A_{23}	A_{32}, A_{33}	A_{21}, A_{22}, A_{24}	A_{32}, A_{33}	A_{21}, A_{22}, A_{24}	A_{32}, A_{33}
450	A_{21}, A_{22}, A_{23}	A_{32}, A_{33}	A_{21}, A_{22}, A_{24}	A_{32}, A_{33}	A_{21}, A_{22}, A_{24}	A_{32}, A_{33}
550	A_{21}, A_{22}, A_{23}	A_{32}, A_{33}	A_{21}, A_{22}, A_{24}	A_{32}, A_{33}	A_{21}, A_{22}, A_{24}	A_{32}, A_{33}
650	A_{21}, A_{22}, A_{23}	A_{32}, A_{33}	A_{21}, A_{22}, A_{24}	A_{32}, A_{33}	A_{21}, A_{22}, A_{24}	A_{32}, A_{33}
750	A_{21}, A_{22}, A_{23}	A_{31}, A_{33}	A_{21}, A_{22}, A_{24}	A_{31}, A_{33}	A_{21}, A_{22}, A_{24}	A_{31}, A_{33}
850	A_{21}, A_{22}, A_{23}	A_{31}, A_{33}	A_{21}, A_{22}, A_{24}	A_{31}, A_{33}	A_{21}, A_{22}, A_{24}	A_{31}, A_{33}
Instruction	<236.8 no solution ≥878.5 stable solution		<225.8 no solution ≥865.4 stable solution		<217.5 no solution ≥854.4 stable solution	

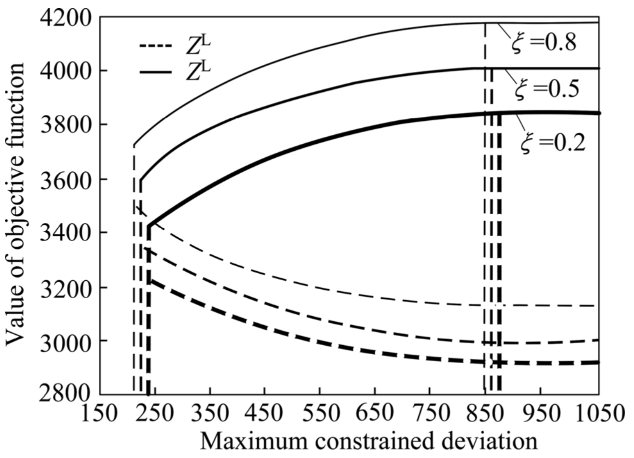

Fig. 2 Simulated results about max Z^I in different ζ

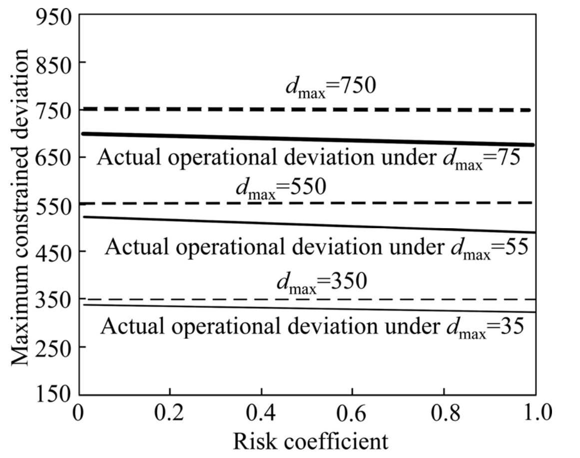

Fig. 3 Actual results of operational deviation under different ζ

Table 3 Comparison of different algorithms

Algorithm	max Z	Decision of second layer	Decision of third layer
Radom algorithm	3 485.2	A_{21}, A_{22}, A_{24}	A_{31}, A_{33}
Fuzzy algorithm	3 526.7	A_{21}, A_{22}, A_{24}	A_{31}, A_{33}
Interval algorithm	[3 275.3, 3 603.7]	A_{21}, A_{22}, A_{24}	A_{32}, A_{33}

6 Conclusions

1) There are some limitations about the design idea of uncertainty to traditional discrete logistics network. From the characteristic of network flow, the uncertain demand of logistics network must be a mode of hierarchical interval triangular OD.

2) Interval logistics network design model with the constraint of interval variables and interval parameters can be transformed to certainty by risk coefficient and the maximum constrained deviation. The result of solution has superiority to interval scenario decision.

3) Interval hierarchical optimal genetic algorithm can consider overall optimization performance and robustness of solution. The interval optimal solution can provide more selected space for the uncertain decision problem, and the operability of the model and algorithm is stronger than that of traditional accurate solution.

4) In some uncertain logistics networks, there may be difficult to quantify some uncertain demand, so how to define more general logistics OD demand interval is still further deepened.

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