HW2

April 25, 2023

STAT 207 HW2

Due May 5th

Your Name

All homeworks should be completed independently; make your answers and codes as concise as possible; avoid excessive outputs; submit BOTH your source code and output file to Canvas.

1. NAS Problem 6.7.1

Verify the vector norm inequalities for p and q chosen from $\{1, 2, \infty\}$.

$$\|x\|_q \leq \|x\|_p, \|x\|_p \leq m^{1/p-1/q}\|x\|_q$$

2. NAS Problem 6.7.8

Prove that $1 \leq \|I\|$ and $||A||^{-1} \leq ||A^{-1}||$ for any matrix norm.

3. CS (Computational Statistics) Exercise 5.1

Calculate condition numbers $\kappa_1(A), \kappa_2(A)$ and $\kappa_\infty(A)$ of the given example, where κ_p corresponding to the respective ℓ_p induced norm.

4. CS Exercise 5.3

Implementation of Gram-Schmidt orthonormalization.

5. Sweeping

Consider the matrix

$$A = \frac{1}{3}\begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix}.$$

Compute by hand its inverse by sweeping. Determine whether A is positive definite based on the intermediate results of sweeping.

6. NAS Problem 8.5.8

Implement the following algorithm to find the largest and smallest eigenvalues of Ω:

$$\Omega = \begin{bmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{bmatrix}$$

Suppose the $m \times m$ symmetric matrix A has eigenvalues $\lambda_1 < \lambda_2 < \cdots < \lambda_{m-1} < \lambda_m$. The iterative scheme $x_{n+1} = (A - \eta_n I)x_n$ can be used to approximate either λ_1 or λ_m. Consider the criterion

$$\sigma_n = \frac{x_{n+1}^T A x_{n+1}}{x_{n+1}^T x_{n+1}}.$$

Choosing η_n to maximize σ_n causes $\lim_{n\to\infty} \sigma_n = \lambda_m$, while choosing η_n to minimize σ_n causes $\lim_{n\to\infty} \sigma_n = \lambda_1$. If $\tau_k = x_n^T A^k x_n$, then the extrema of σ_n as a function of η can be obtained as the roots of the quadratic equation

$$0 = \det \begin{bmatrix} 1 & \eta & \eta^2 \\ \tau_0 & \tau_1 & \tau_2 \\ \tau_1 & \tau_2 & \tau_3 \end{bmatrix}$$

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