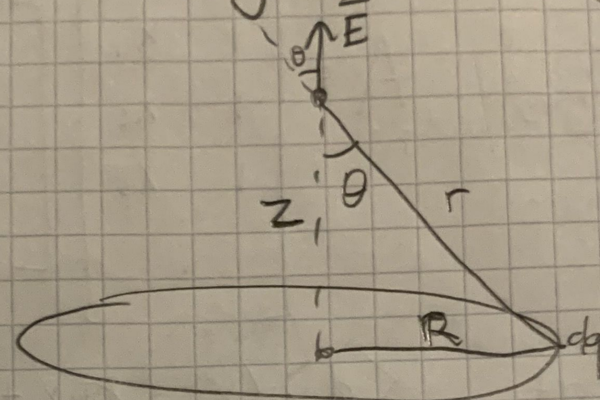
PS2

Problem 1

Take a ring of charge with charge q:



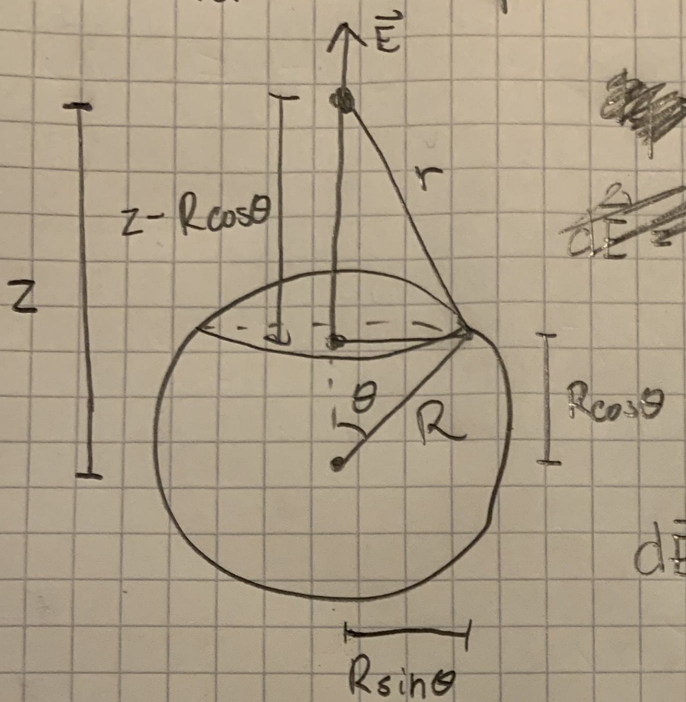
$$\Rightarrow dE_z = \frac{dq}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$r^2 = z^2 + R^2, \quad \cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2+R^2}}$$

$$\Rightarrow dE_z = \underbrace{\frac{1}{4\pi\varepsilon_0}\cdot\frac{z}{(z^2+R^2)^{3/2}}}_{\text{all of this is constant}} dq$$

$$\Rightarrow \vec{E} = \frac{zq}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}$$

Now for the sphere:



$$d\vec{E} = \hat{r}\,\frac{(z - R\cos\theta)\,dq}{4\pi\varepsilon_0\left((z-R\cos\theta)^2 + R^2\sin^2\theta\right)^{3/2}}$$

$$d\vec{E} = \hat{r}\,\frac{(z - R\cos\theta)\,dq}{4\pi\varepsilon_0\left(z^2 - 2Rz\cos\theta + R^2\cos^2\theta + R^2\sin^2\theta\right)^{3/2}}$$

$$d\vec{E} = \hat{r}\,\frac{(z - R\cos\theta)\,dq}{4\pi\varepsilon_0\left(z^2 - 2Rz\cos\theta + R^2\right)^{3/2}}$$

$$dq = \sigma\, dA = \frac{Q}{4\pi R^2}\, dA = \frac{Q}{4\pi R^2} R^2 \sin\theta\, d\theta\, d\phi$$

$$\Rightarrow \int d\phi = 2\pi$$

$$dq = \frac{1}{2} Q \sin\theta\, d\theta$$

$$\Rightarrow d\vec{E} = \hat{r}\,\frac{Q}{8\pi\varepsilon_0}\left[\frac{(z - R\cos\theta)\sin\theta}{\left(z^2 - 2Rz\cos\theta + R^2\right)^{3/2}}\right] d\theta$$

then we integrate with $0 \leq \theta \leq \pi$.