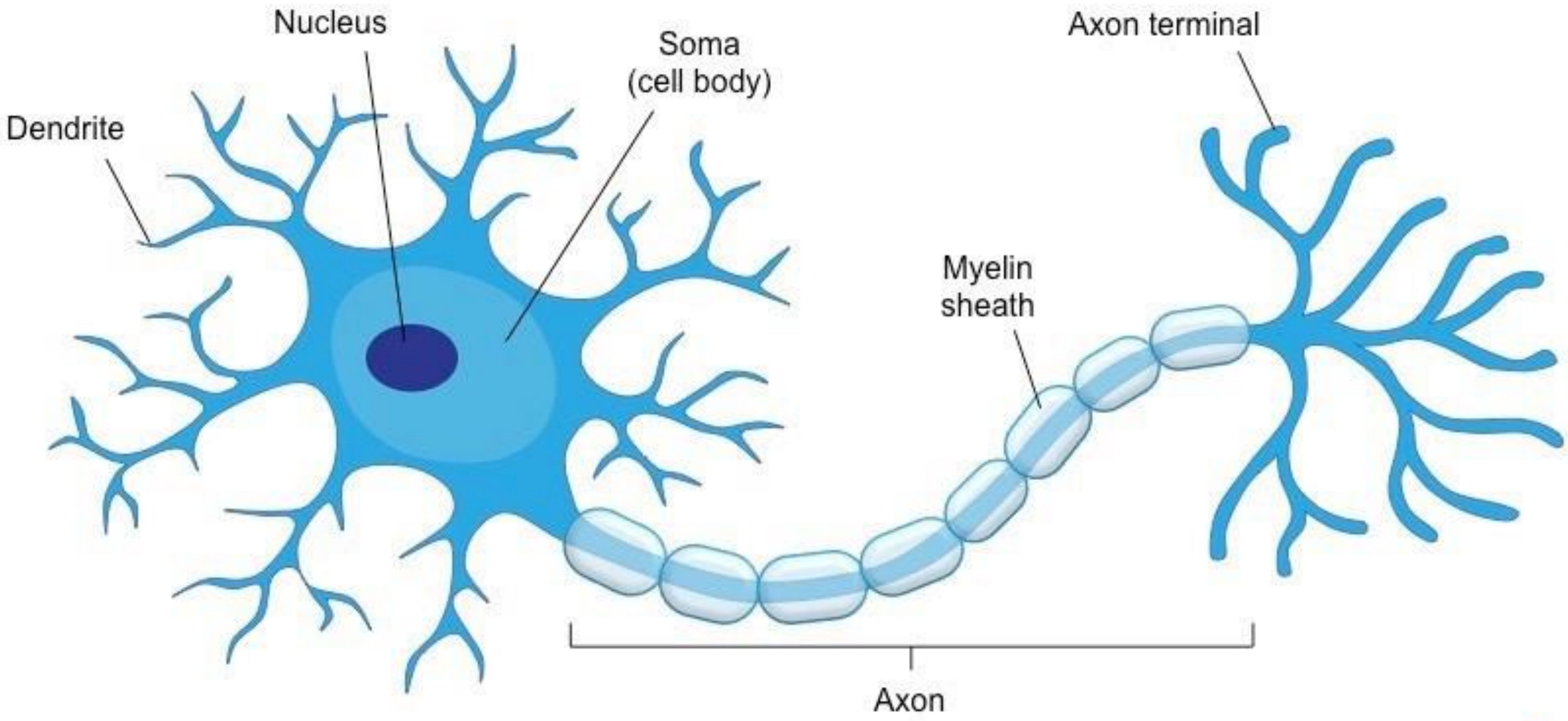
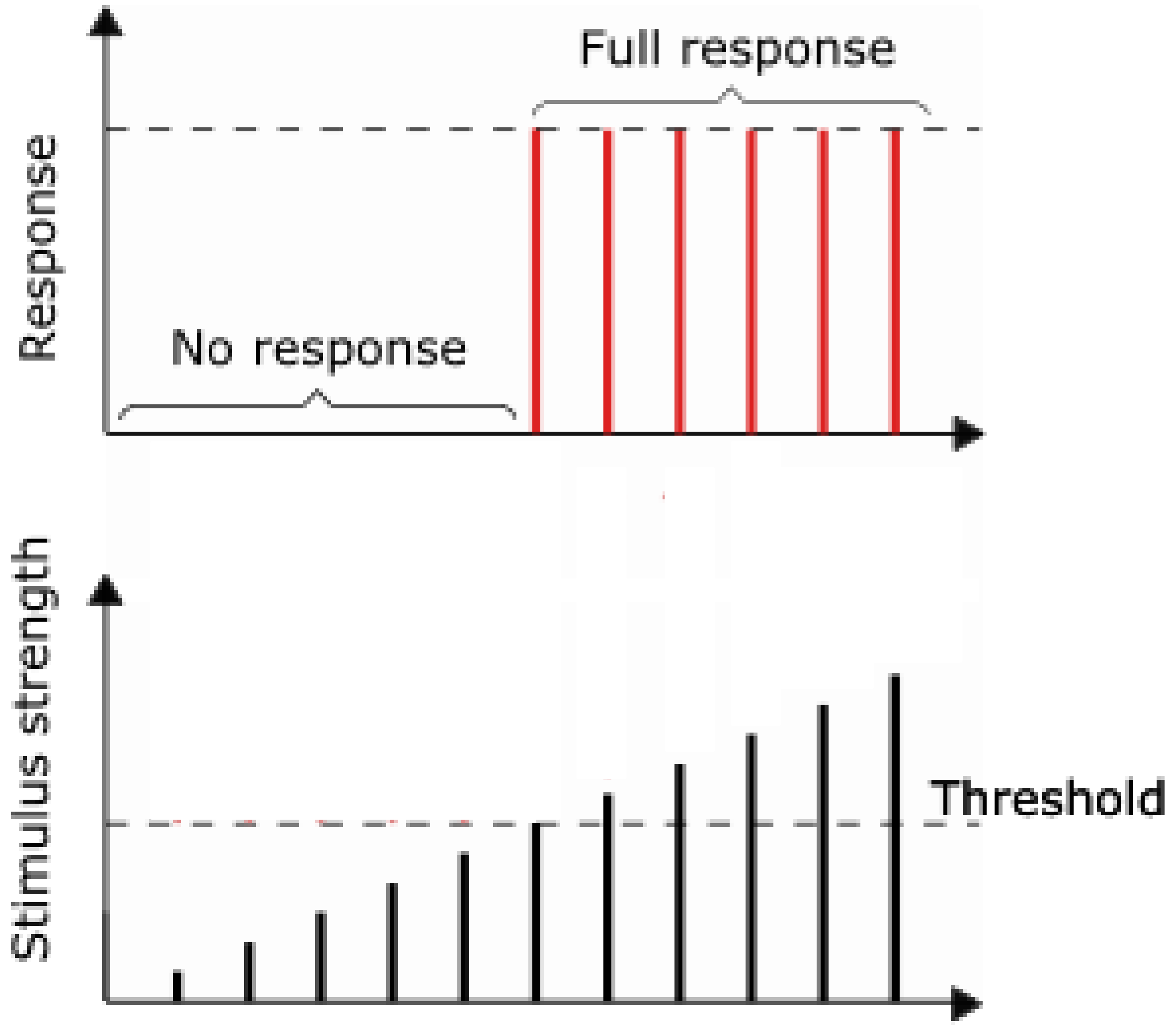
PERCEPTRON

THE DADDY OF NEURAL NETWORKS

FELIPE BUCHBINDER


Nucleus
Soma
(cell body)
Axon terminal
Dendrite
Myelin
sheath
Axon



ALL-OR-NOTHING LAW OF NEURONAL ACTIVATION

(WHY IS IT, THEN, THAT WE SOMETIMES FEEL MORE PAIN OR LESS PAIN?)

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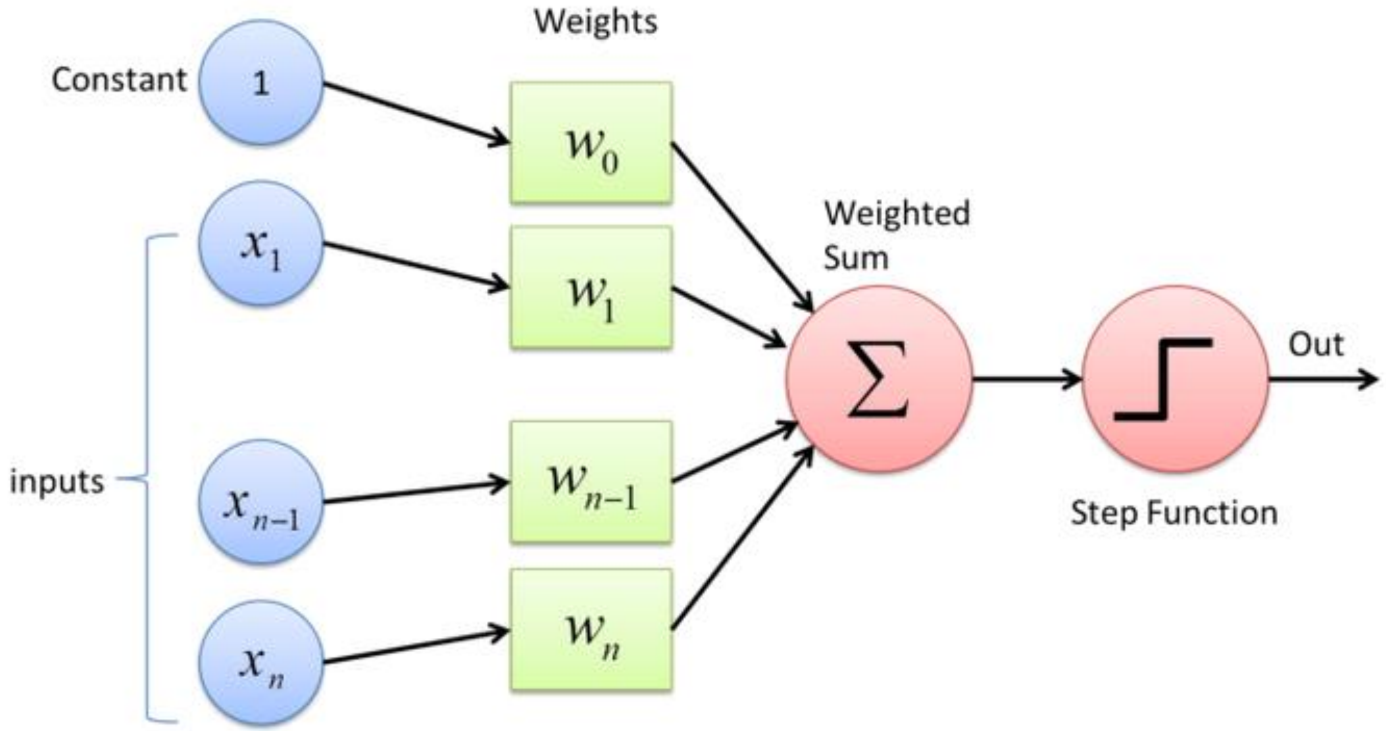
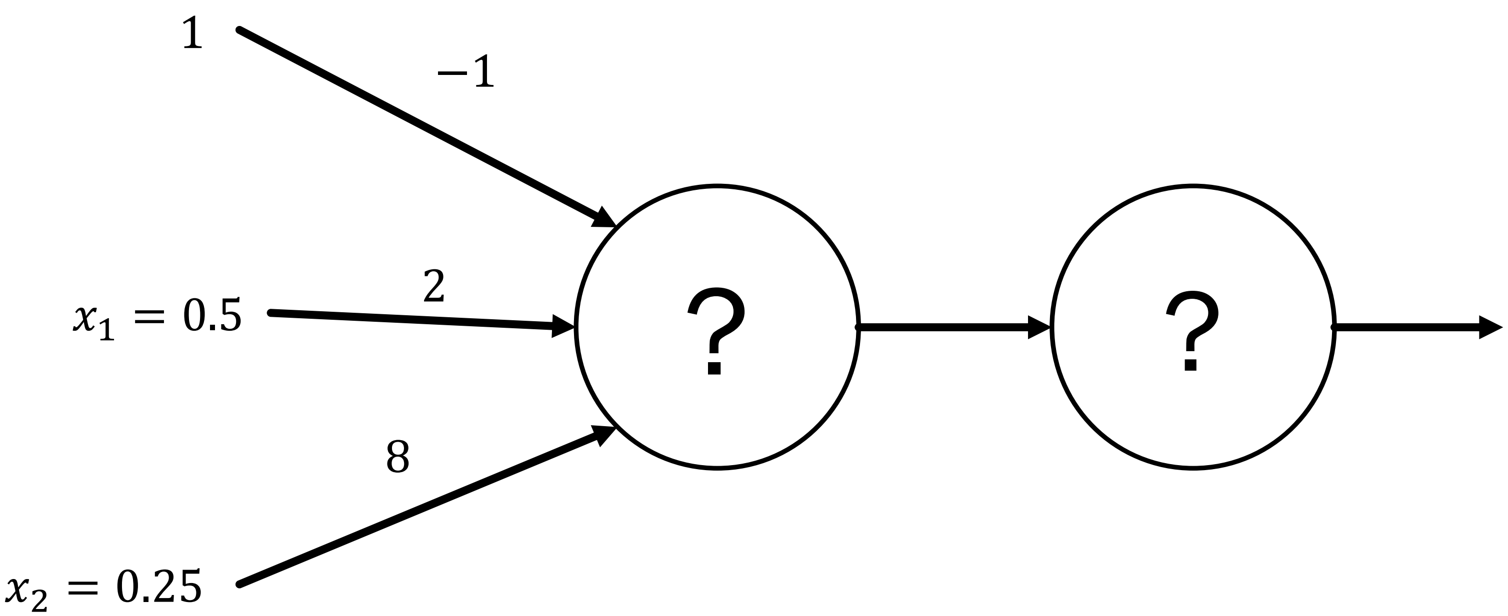
THE PERCEPTRON



SIMPLE EXAMPLE (2 INPUTS)



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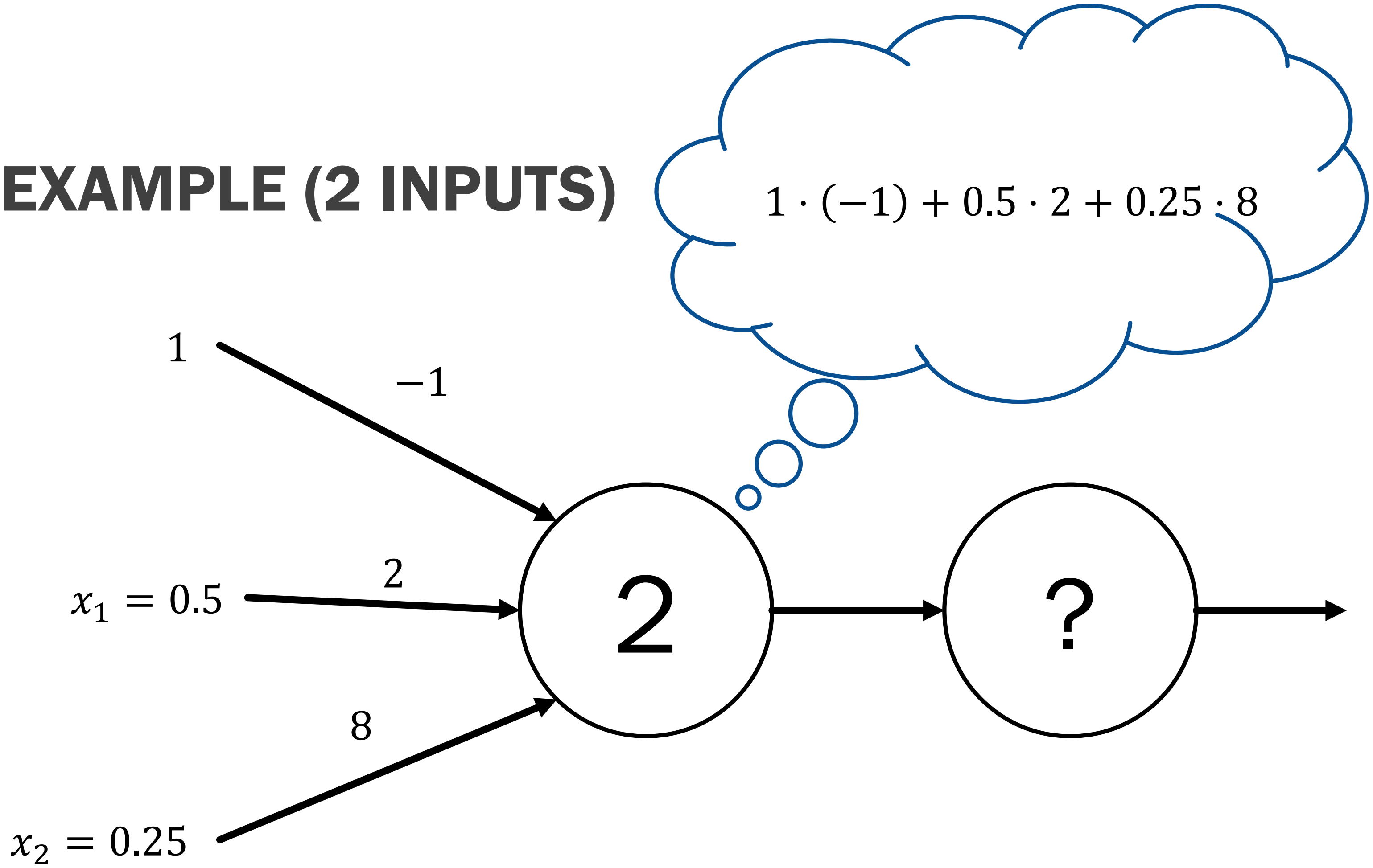

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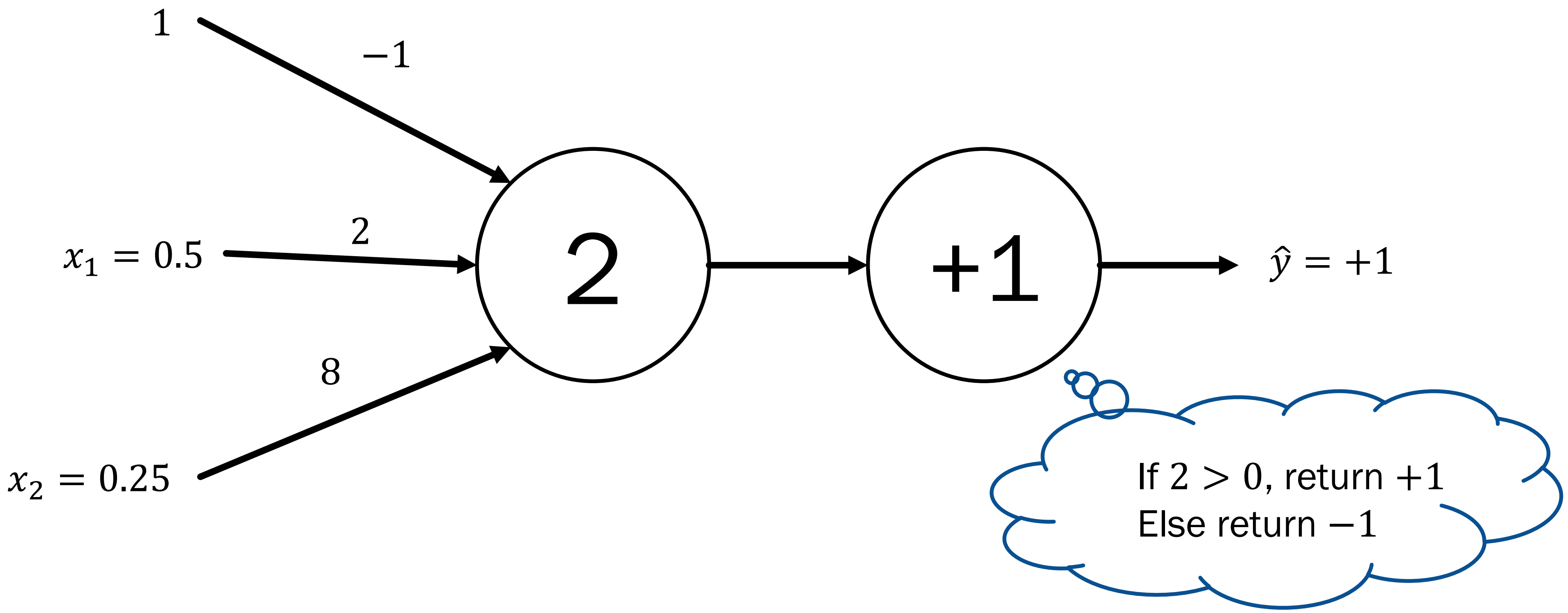

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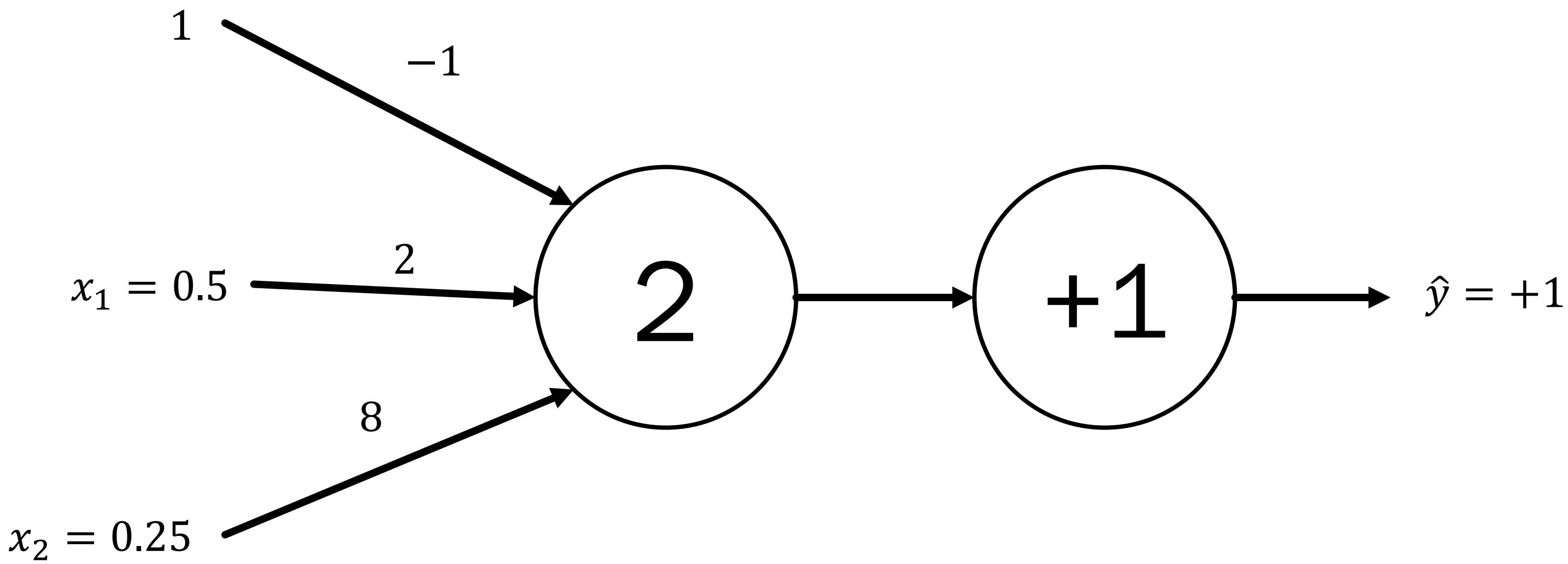

OUR PERCEPTRON'S PREDICTION CAN BE WRITTEN IN A SINGLE LINE

$$\boxed{\hat{y} = \mathrm{sign}\left(\mathbf{w}^{\mathrm{T}}\mathbf{x}\right)}$$

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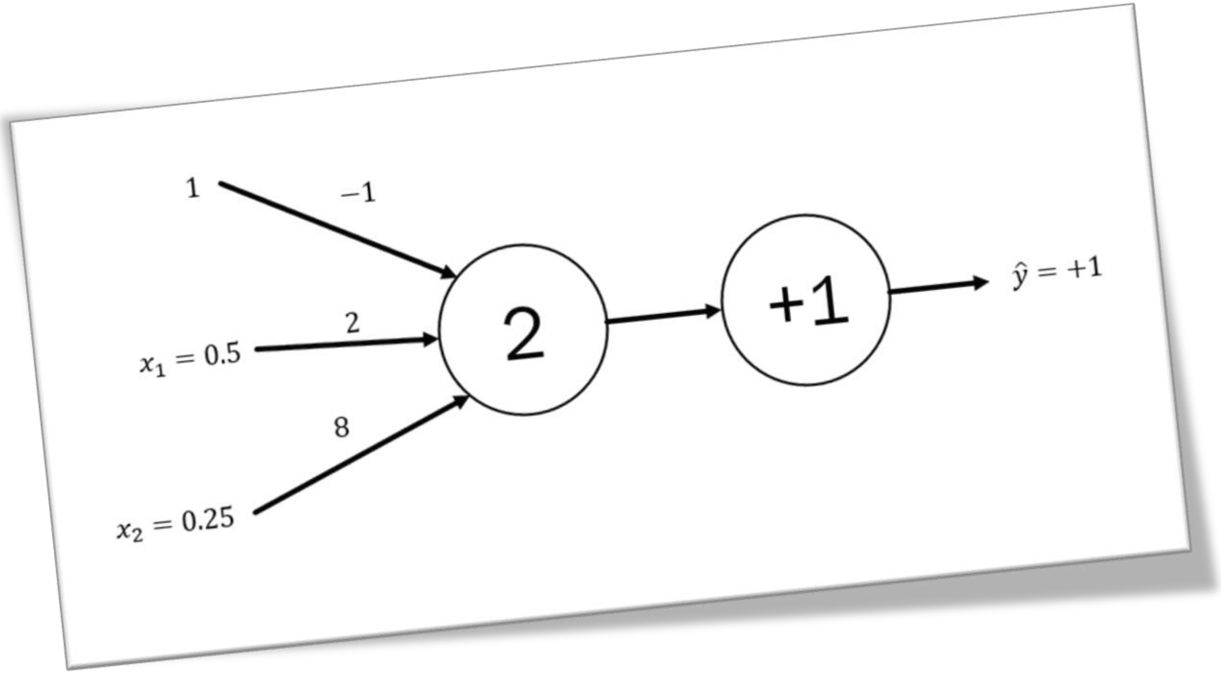


$$\begin{aligned}\hat{y} &= \text{sign}\big((-1)\cdot 1 + 2\cdot 0.5 + 8\cdot 0.25\big)\\ &= \text{sign}(2)\\ &= +1\end{aligned}$$

PERCEPTRON'S CAN HAVE DIFFERENT ACTIVATION FUNCTIONS

$$\boxed{\hat{y} = \mathrm{sign}\left(\mathbf{w}^{\mathrm{T}}\mathbf{x}\right)}$$

Heaviside (step) function

$$\boxed{\hat{y} = \sigma\left(\mathbf{w}^{\mathrm{T}}\mathbf{x}\right)}$$

Sigmoid function

$$\boxed{\hat{y} = \tanh\left(\mathbf{w}^{\mathrm{T}}\mathbf{x}\right)}$$

Hyperbolic tangent function

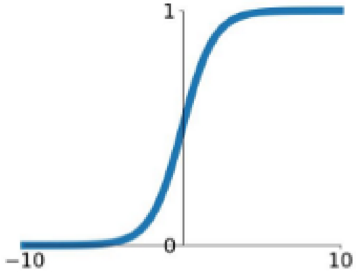
$$\boxed{\hat{y} = \mathrm{ReLU}\left(\mathbf{w}^{\mathrm{T}}\mathbf{x}\right)}$$

Rectified Linear Unit

Activation Functions

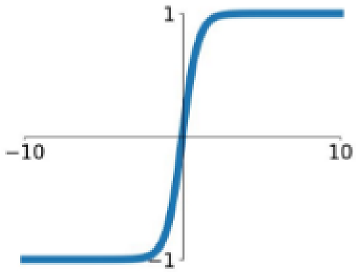
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



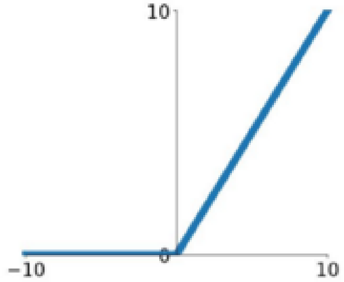
tanh

$\tanh(x)$



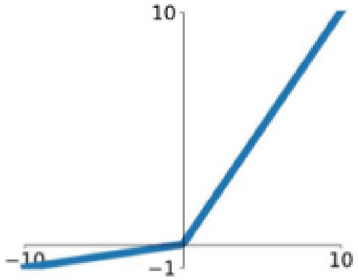
ReLU

$\max(0, x)$



Leaky ReLU

$\max(0.1x, x)$

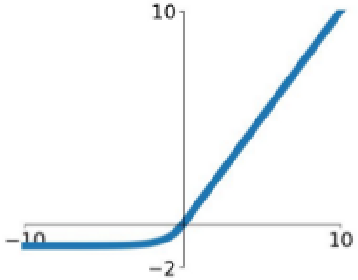


Maxout

$\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

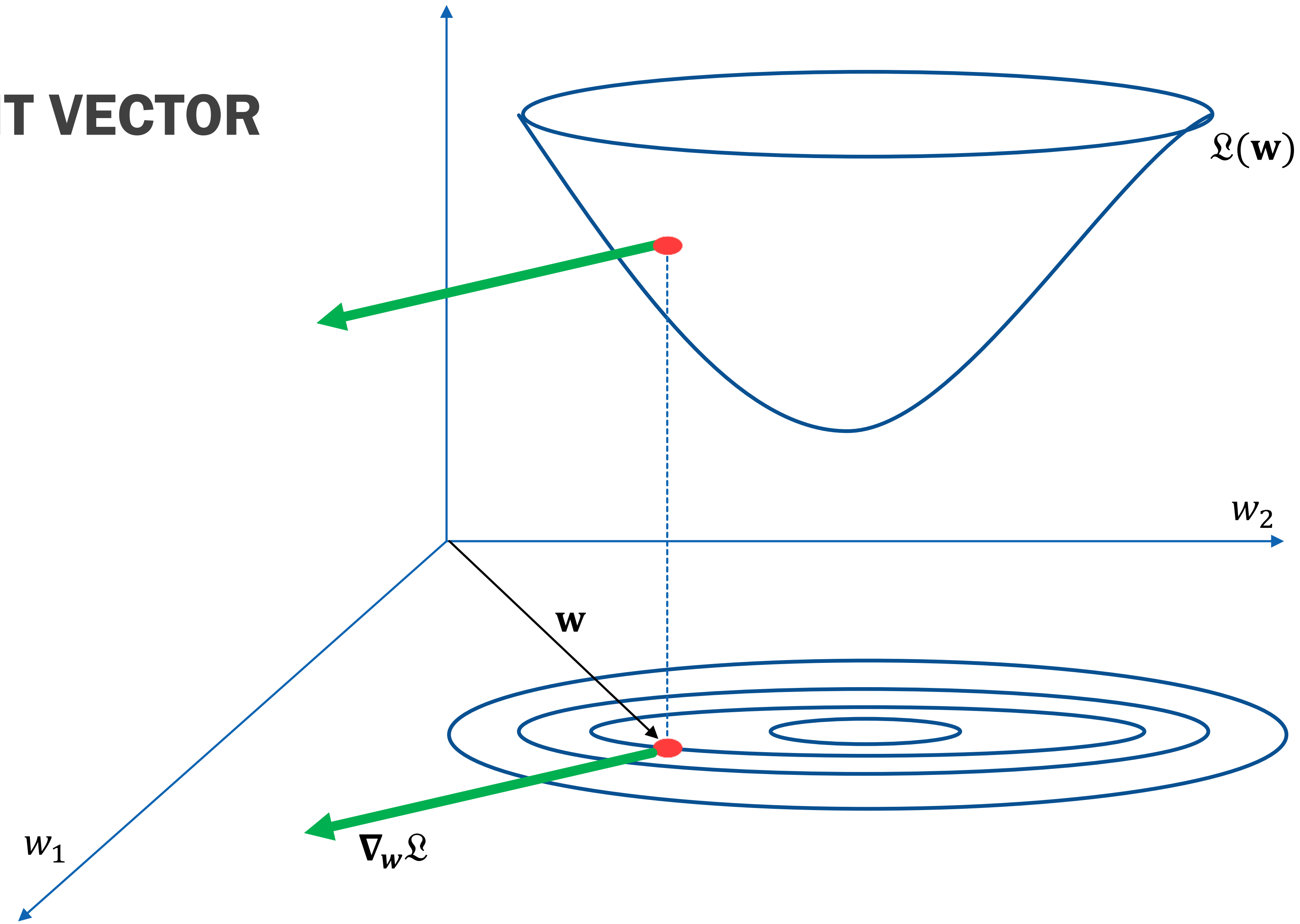
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



HOW CAN THE PERCEPTRON LEARN WHICH WEIGHTS TO USE?

CHOOSE WEIGHTS TO MINIMIZE SOME LOSS FUNCTION

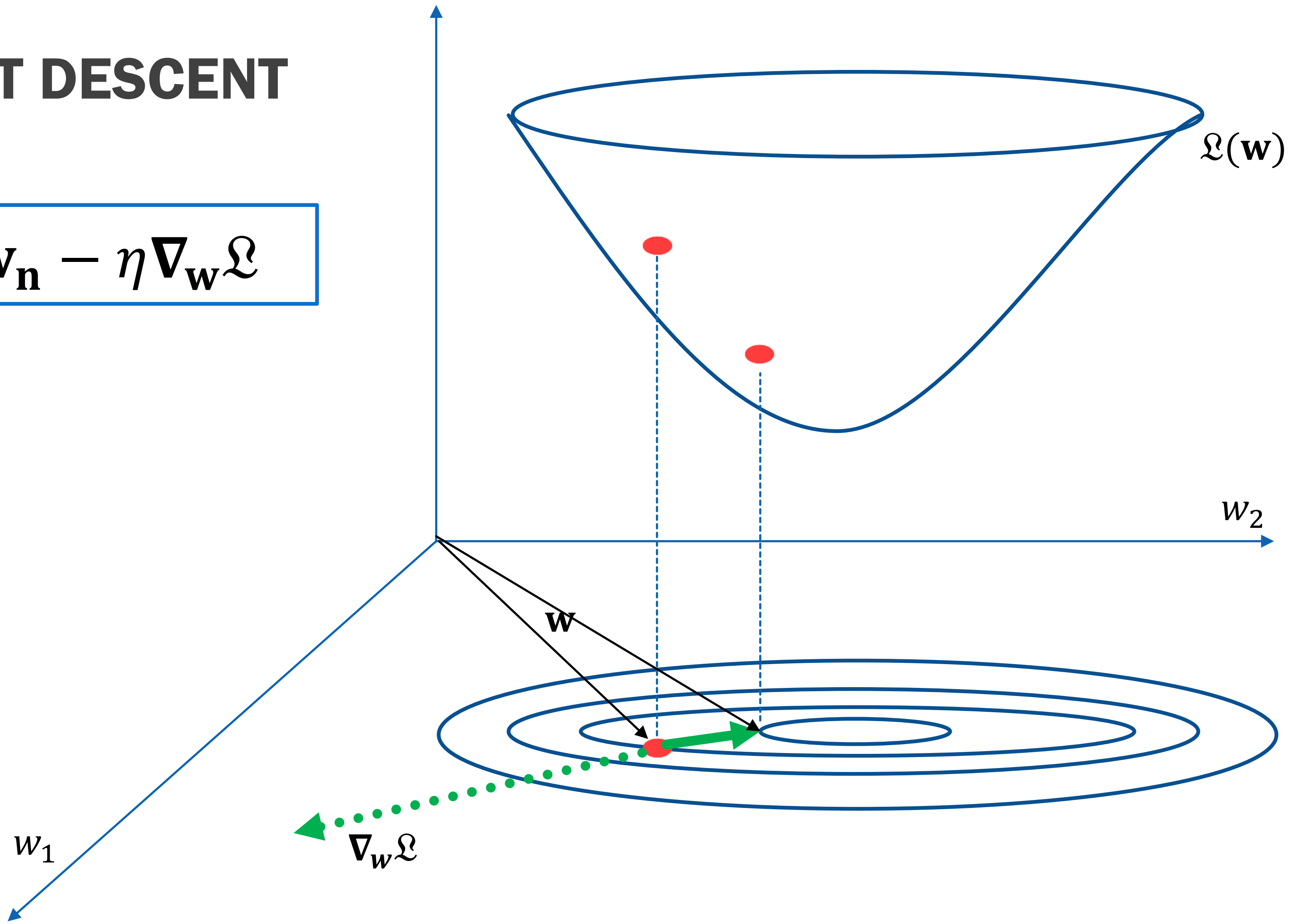
GRADIENT VECTOR



GRADIENT DESCENT

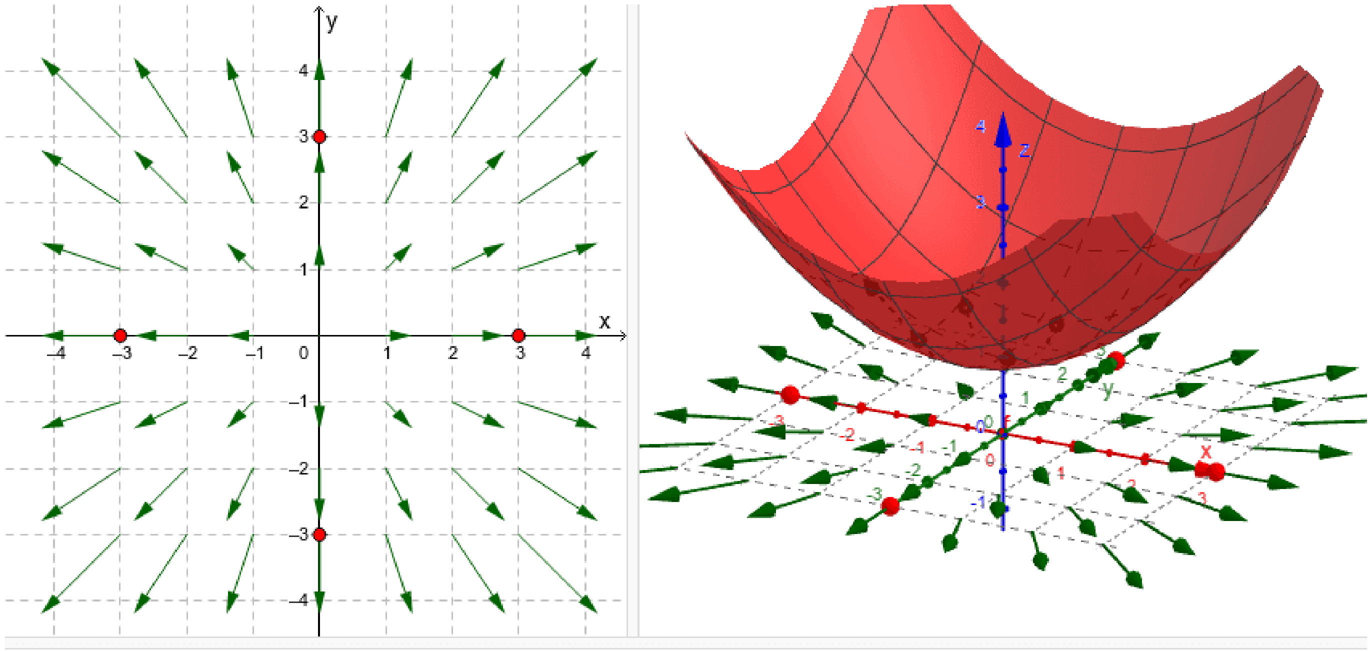
(WITH 1 POINT)

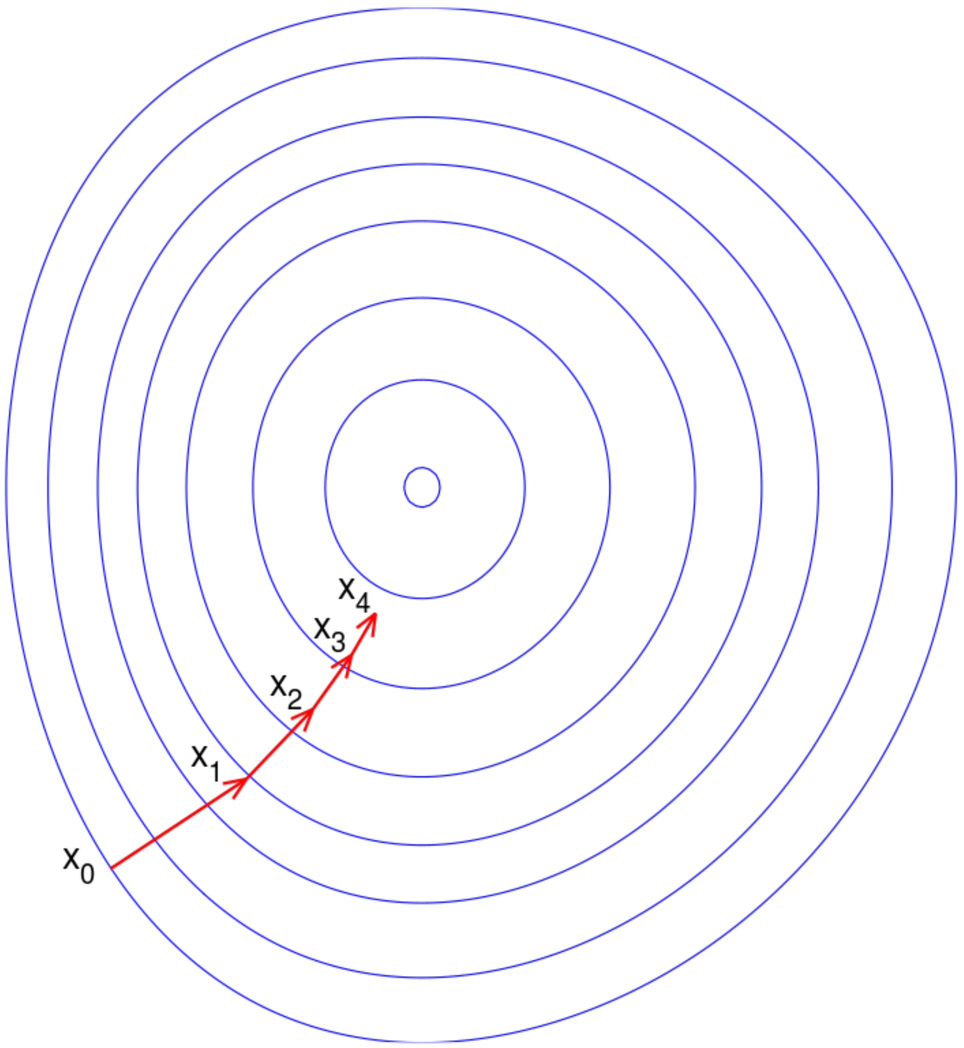
$$\mathbf{w_{n+1}} \leftarrow \mathbf{w_n} - \eta \boldsymbol{\nabla}_{\mathbf{w}} \mathfrak{L}$$



GRADIENT DESCENT

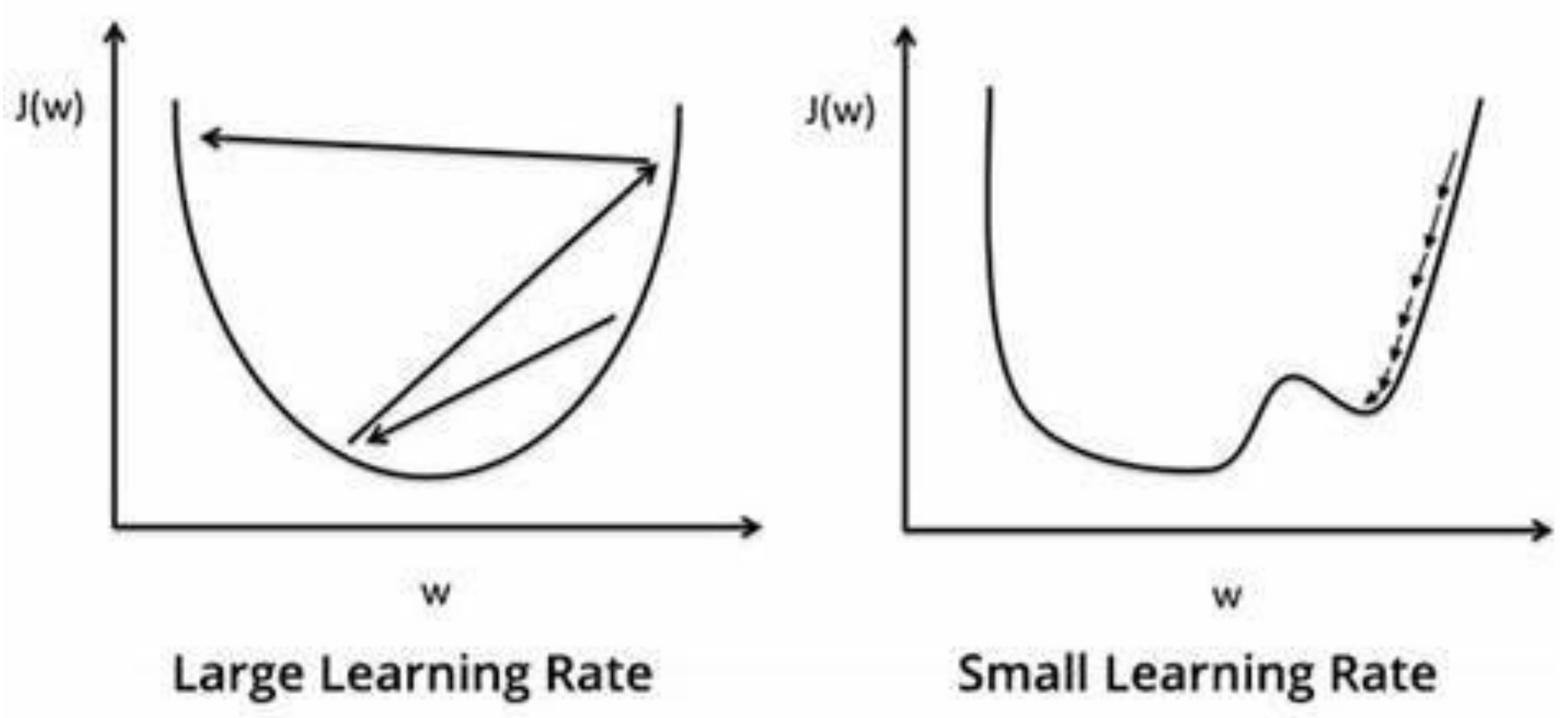
$$\boxed{\mathbf{w_{n+1}} \leftarrow \mathbf{w_n} - \eta \sum_i \nabla_{\mathbf{w}} \mathfrak{L}}$$





x0
x1
x2
x3
x4

EFFECT OF THE LEARNING RATE (η)



EXAMPLE: STEP ACTIVATION FUNCTION WITH HINGE LOSS

$$\mathfrak{L} = \max(0; 1 - y\hat{y})$$
$$\hat{y} = \mathrm{sign}\left(\mathbf{w}^{\mathrm{T}}\mathbf{x}\right)$$
$$\mathbf{w_{n+1}} = \mathbf{w_n} - \eta \boldsymbol{\nabla}_{\mathbf{w}} \boldsymbol{\mathfrak{L}}$$

$$\nabla_{\mathbf{w}} \mathfrak{L} = y\mathbf{x}$$

$$\therefore$$

$$\mathbf{w_{n+1}} \leftarrow \mathbf{w_n} - \eta y \mathbf{x}$$

EXAMPLE: LOGISTIC ACTIVATION FUNCTION WITH HINGE LOSS

$$\mathfrak{L} = \max(0; 1 - y\hat{y})$$
$$\hat{y} = \sigma(\mathbf{w}^{\mathbf{T}}\mathbf{x})$$
$$\mathbf{w}_{\mathbf{n+1}} = \mathbf{w}_{\mathbf{n}} - \eta \boldsymbol{\nabla}_{\mathbf{w}} \mathfrak{L}$$

$$\nabla_{\mathbf{w}} \mathfrak{L} = y\sigma(\mathbf{w}^{\mathbf{T}}\mathbf{x})[\mathbf{1} - \sigma(\mathbf{w}^{\mathbf{T}}\mathbf{x})]\mathbf{x}$$

$\therefore$

$$\mathbf{w}_{\mathbf{n+1}} \leftarrow \mathbf{w}_{\mathbf{n}} - \eta y\sigma(\mathbf{w}^{\mathbf{T}}\mathbf{x})[\mathbf{1} - \sigma(\mathbf{w}^{\mathbf{T}}\mathbf{x})]\mathbf{x}$$

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This tells us about the importance of normalizing the inputs. Why?

$$\nabla_{\mathbf{w}}\mathfrak{L} = y\sigma(\mathbf{w}^{\mathbf{T}}\mathbf{x})[\mathbf{1} - \sigma(\mathbf{w}^{\mathbf{T}}\mathbf{x})]\mathbf{x}$$

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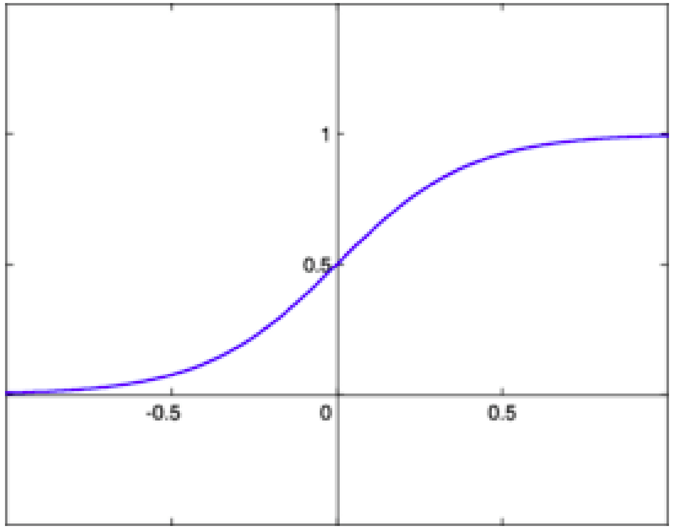
This also offers a nice illustration of the vanishing gradient problem. Why? – and what to do about it?

THE VANISHING GRADIENT PROBLEM

$$\mathbf{w_{n+1}} \leftarrow \mathbf{w_n} - \eta \sum_i \boldsymbol{\nabla}_{\mathbf{w}} \mathfrak{L}$$

When $\nabla_{\mathbf{w}} \mathfrak{L} \rightarrow 0$, learning stops ($\mathbf{w_{n+1}} \approx \mathbf{w_n}$)

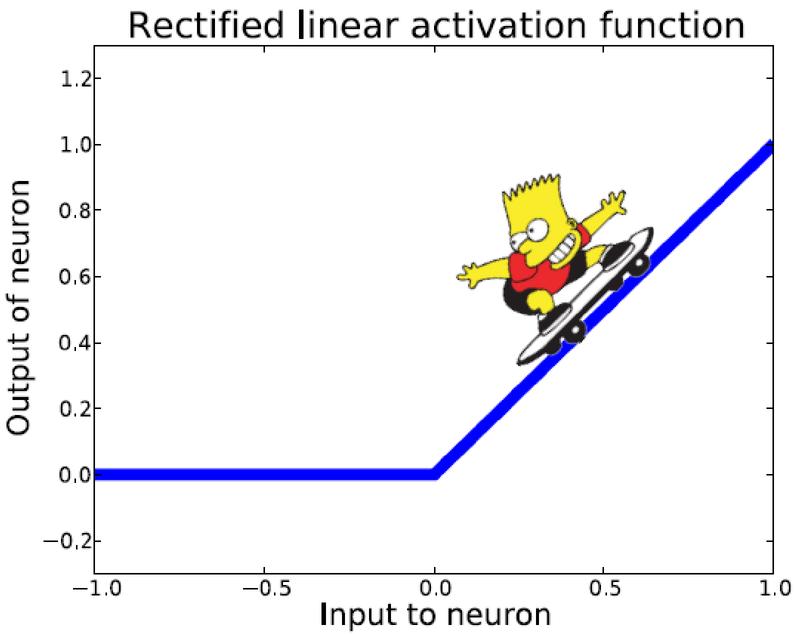
What can we do about it?



SOLUTIONS TO THE VANISHING GRADIENT PROBLEM

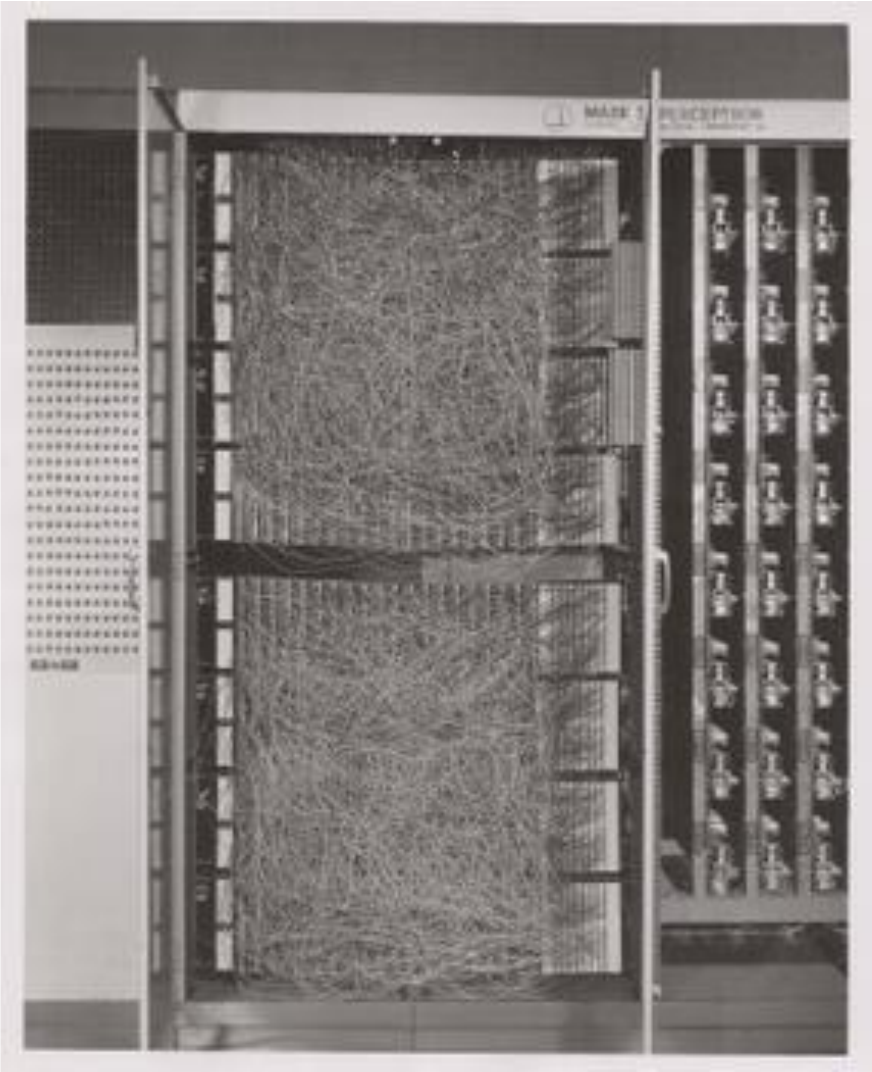
$$x' = \frac{x - \min(x)}{\max(x) - \min(x)}$$

Normalization



Changing the activation function

and other more advanced (or pricy) options…



THE FIRST PERCEPTRON (IBM, 1958)

COMING UP NEXT:

NEURAL NETWORKS