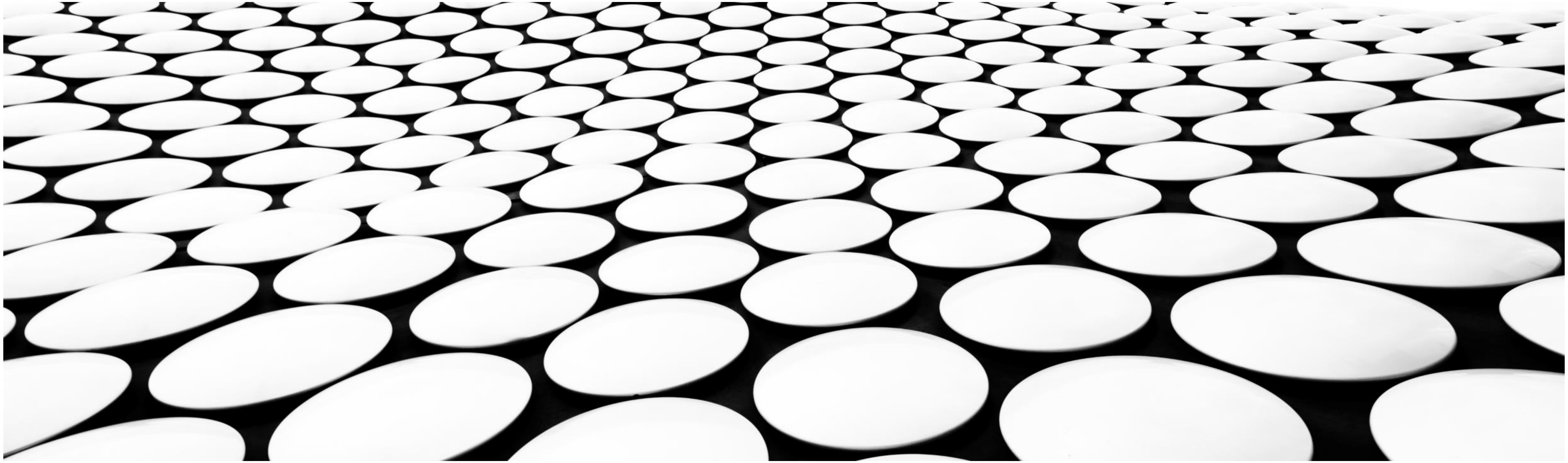
WHEN TO USE FIRST DIFFERENCES OR FIXED EFFECTS?

AND HOW TO DEAL WITH RESIDUALS SERIALLY CORRELATED AS AN AR(1)

FELIPE BUCHBINDER



BEFORE WE BEGIN: WHAT IS AN AR(1) PROCESS?

$$\epsilon_t = \rho \cdot \epsilon_{t-1} + \nu_t$$
$$\nu_t \sim N(0, \sigma^2)$$
$$|\rho| < 1$$

BEFORE WE BEGIN: WHAT IS AN AR(1) PROCESS?

In most practical applications,
$\rho \succ 0$
(Why?)

$$\epsilon_t = \rho \cdot \epsilon_{t-1} + \nu_t$$

$$\nu_t \sim N(0, \sigma^2)$$

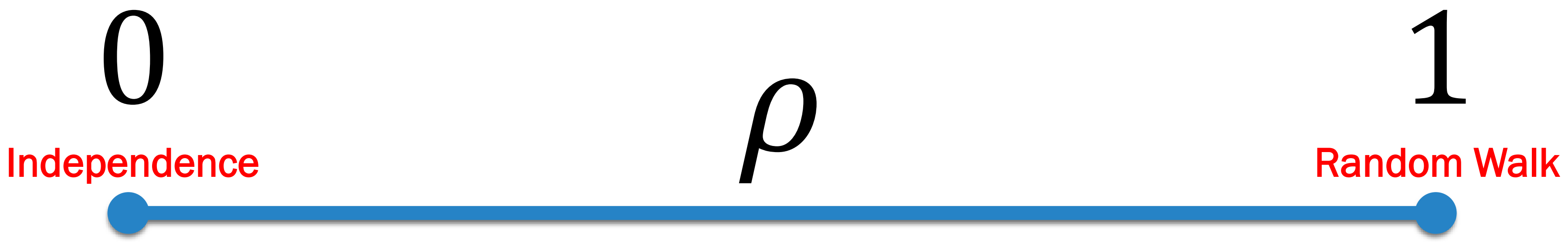
$$|\rho| < 1$$

TESTING FOR SERIAL CORRELATION

- Durbin-Whatson Statistic
- Breusch-Godfrey Lagrange Multiplier test

$$\epsilon_t = \rho \cdot \epsilon_{t-1} + \nu_t$$
$$\nu_t \sim N(0, \sigma^2)$$
$$|\rho| < 1$$

BORDERLINE CASES



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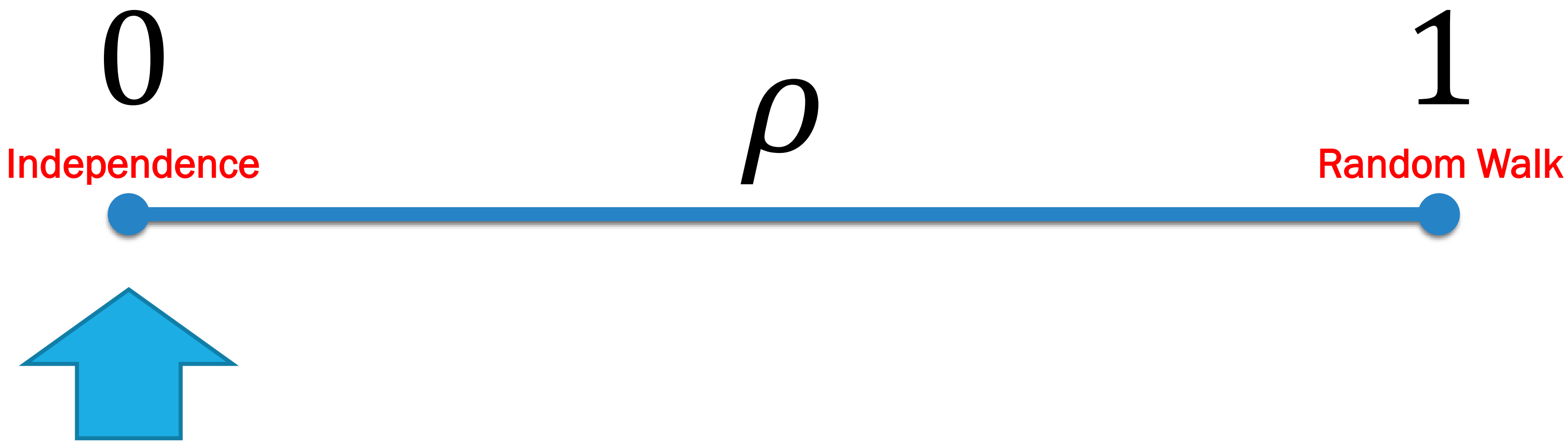
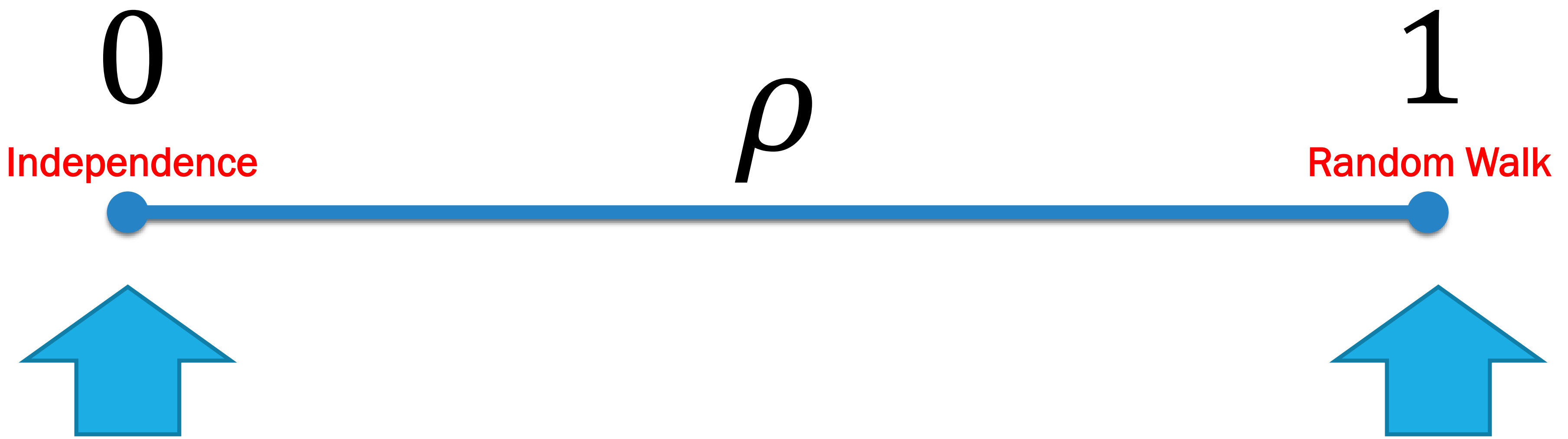
BORDERLINE CASES



OLS works well
when residuals are here

$$\epsilon_t = \rho \cdot \epsilon_{t-1} + \nu_t$$
$$\nu_t \sim N(0, \sigma^2)$$
$$|\rho| < 1$$

FIXED EFFECTS OR FIRST DIFFERENCE?



If FE residuals lie here, used Fixed Effects

If FE residuals lie here, Use First Differences

FIXED EFFECTS OR FIRST DIFFERENCE?

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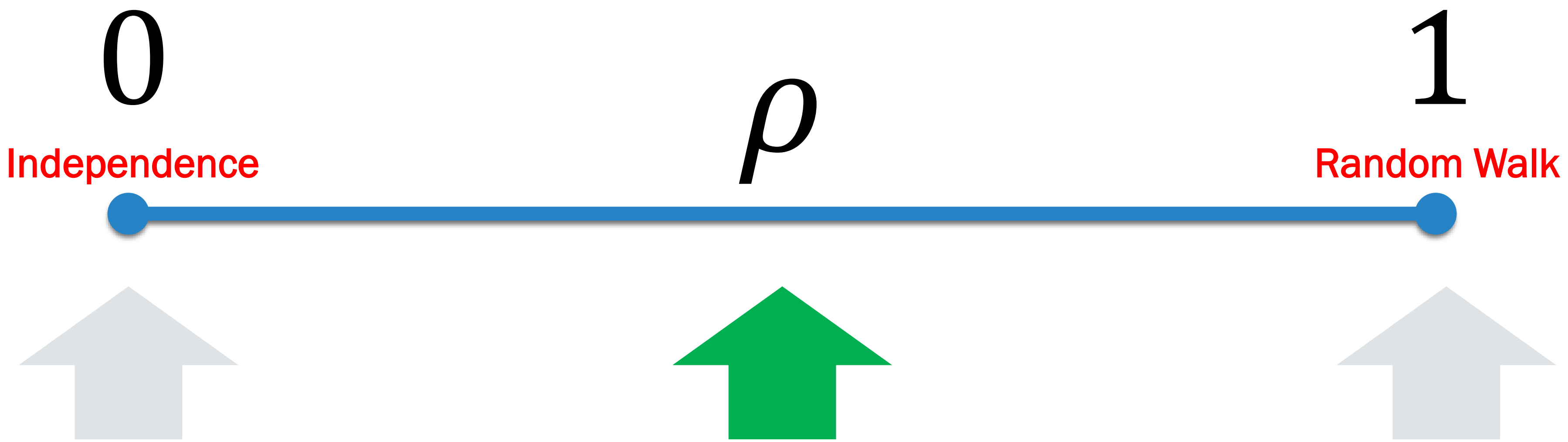

If FE residuals lie here, used Fixed Effects

What if FE residuals lie here?

If FE residuals lie here, Use First Differences

IF WE KNOW THE AR(1) STRUCTURE, WE CAN MAKE SERIALLY INDEPENDENT

THIS PROCESS IS CALLED QUASI-DIFFERENTIATION

$$\mathrm{Y}_{it} = \beta_0 + \beta_1 \cdot \mathrm{X}_{it} + U_i + \epsilon_{it}$$

$$\rho\mathrm{Y}_{it-1} = \rho\beta_0 + \rho\beta_1 \cdot \mathrm{X}_{it-1} + \rho U_i + \rho\epsilon_{it-1}$$

$$\mathrm{Y}_{it} - \rho\mathrm{Y}_{it-1} = \beta_0(1-\rho) + \beta_1 \cdot (X_{it} - \rho X_{it-1}) + (1-\rho)U_i + (\epsilon_{it} - \rho\epsilon_{it-1})$$

WE NOW HAVE A MODEL WHERE RESIDUALS ARE SERIALLY INDEPENDENT…

$$\mathrm{Y}_{it} = \beta_0 + \beta_1 \cdot \mathrm{X}_{it} + U_i + \epsilon_{it}$$
$$\rho \mathrm{Y}_{it-1} = \rho\beta_0 + \rho\beta_1 \cdot \mathrm{X}_{it-1} + \rho U_i + \rho\epsilon_{it-1}$$

$$\mathrm{Y}_{it} - \rho\mathrm{Y}_{it-1} = \beta_0(1-\rho) + \beta_1 \cdot (X_{it} - \rho X_{it-1}) + (1-\rho)U_i + (\epsilon_{it} - \rho\epsilon_{it-1})$$

$$\mathrm{Y}_{it} - \rho\mathrm{Y}_{it-1} = \beta_0(1-\rho) + \beta_1 \cdot (X_{it} - \rho X_{it-1}) + (1-\rho)U_i + \nu_{it}$$

BUT HOW DO WE KNOW THE VALUE OF ρ?

2 VERY SIMILAR APPROACHES AND ONE SLIGHTLY DIFFERENT…

- Cochrane-Orcutt estimation
- Prais-Winsten estimation
- Hildreth-Lu estimation

PROBLEM: WE HAVEN'T ELIMINATED THE UNOBSERVED HETEROGENEITY

(THOUGH WE MIGHT HAVE MADE IT SMALLER, WHICH IS ALREADY A GOOD THING!)

$$\begin{aligned} \mathrm{Y}_{it} &= \beta_0 + \beta_1 \cdot \mathrm{X}_{it} + U_i + \epsilon_{it} \\ \rho \mathrm{Y}_{it-1} &= \rho\beta_0 + \rho\beta_1 \cdot \mathrm{X}_{it-1} + \rho U_i + \rho\epsilon_{it-1} \end{aligned}$$

$$\mathrm{Y}_{it} - \rho \mathrm{Y}_{it-1} = \beta_0(1-\rho) + \beta_1 \cdot (X_{it} - \rho X_{it-1}) + (1-\rho)U_i + (\epsilon_{it} - \rho\epsilon_{it-1})$$

$$\mathrm{Y}_{it} - \rho \mathrm{Y}_{it-1} = \beta_0(1-\rho) + \beta_1 \cdot (X_{it} - \rho X_{it-1}) + (1-\rho)U_i + \nu_{it}$$

SPECIAL CASE: WHEN RESIDUALS FOLLOW A RANDOM WALK, WE GET THE FIRST DIFFERENCES MODEL

IT WORKS: WE HAVE NO HETEROGENEITIES AND SERIALLY UNCORRELATED RESIDUALS

$$\mathrm{Y}_{it} = \beta_0 + \beta_1 \cdot \mathrm{X}_{it} + U_i + \epsilon_{it}$$

$$\rho \mathrm{Y}_{it-1} = \rho\beta_0 + \rho\beta_1 \cdot \mathrm{X}_{it-1} + \rho U_i + \rho\epsilon_{it-1}$$

$$\mathrm{Y}_{it} - \rho \mathrm{Y}_{it-1} = \beta_0(1-\rho) + \beta_1 \cdot (X_{it} - \rho X_{it-1}) + (1-\rho)U_i + (\epsilon_{it} - \rho\epsilon_{it-1})$$

$$\mathrm{Y}_{it} - 1\mathrm{Y}_{it-1} = \beta_0(1-1) + \beta_1 \cdot (X_{it} - 1X_{it-1}) + \cancel{(1-1)U_i} + \nu_{it}$$

KEY TAKEAWAYS

1. Fixed Effects Models work well when residuals are serially independent ($\rho = 0$)
2. First Difference Models work well when residuals follow a Random Walk ($\rho = 1$)
3. Both independence and Random Walk are borderline cases of AR(1) processes
4. When residuals follow a general AR(1) process, this can be corrected by quasi-differentiation.
5. To know if residuals follow an AR(1) process, we can use the Breusch-Godfrey test or the Durbin-Whatson statistic.
6. Quasi-differentiation requires estimating ρ. This can be done using Cochrane-Orcutt, Prais-Winsten or Hildreth-Lu estimation algorithms
7. Quasi-differentiation makes residuals serially uncorrelated but does not completely eliminate the unobserved heterogeneity (though it might make it weaker)
8. In most real-world cases of residuals following an AR(1) process, ρ is positive.