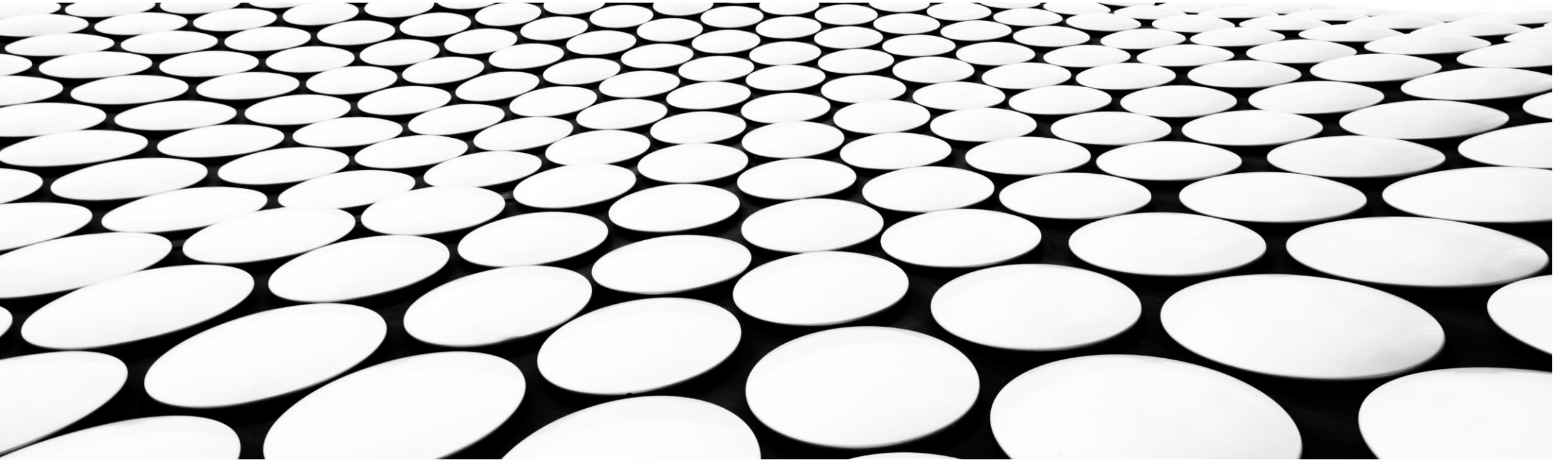

SPATIAL REGRESSION MODELS

FELIPE BUCHBINDER



A vibrant illustration of Aladdin and Jasmine from Disney's Aladdin. They are flying over the city of Agrabah on a magic carpet. Aladdin is wearing his signature white and gold outfit with a blue turban, and Jasmine is in a green and gold dress. They are both smiling and looking towards the viewer. The city below is illuminated with warm lights, and the sky is a deep blue with a large, bright moon and several pink birds flying. The text "A WHOLE NEW WORLD..." is written in white, bold, sans-serif font across the middle of the image, with a thin white horizontal line underneath it.

A WHOLE NEW WORLD...

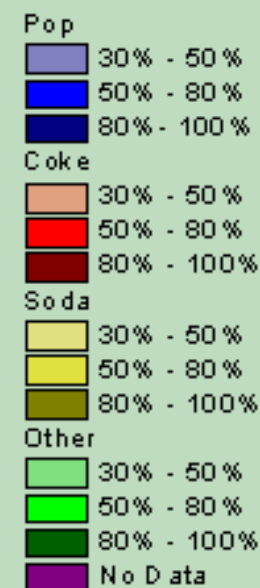
**BUT LET'S SEE THE
BASICS**



GENERIC NAMES FOR SOFT DRINKS BY COUNTY

**TAKE A LOOK AT THIS MAP
ON HOW AMERICANS NAME
SOFT DRINKS**

Most Popular
Term Used



Map by Matthew T. Campbell
Spatial Graphics and Analysis Lab
Department of Cartography and Geography
East Central University (Oklahoma)

Map based upon
120,464 Respondents

Respondents through

**AND THIS MAP ON
SUICIDE RATES OF
WHITE MEN**

Suicide, White Men

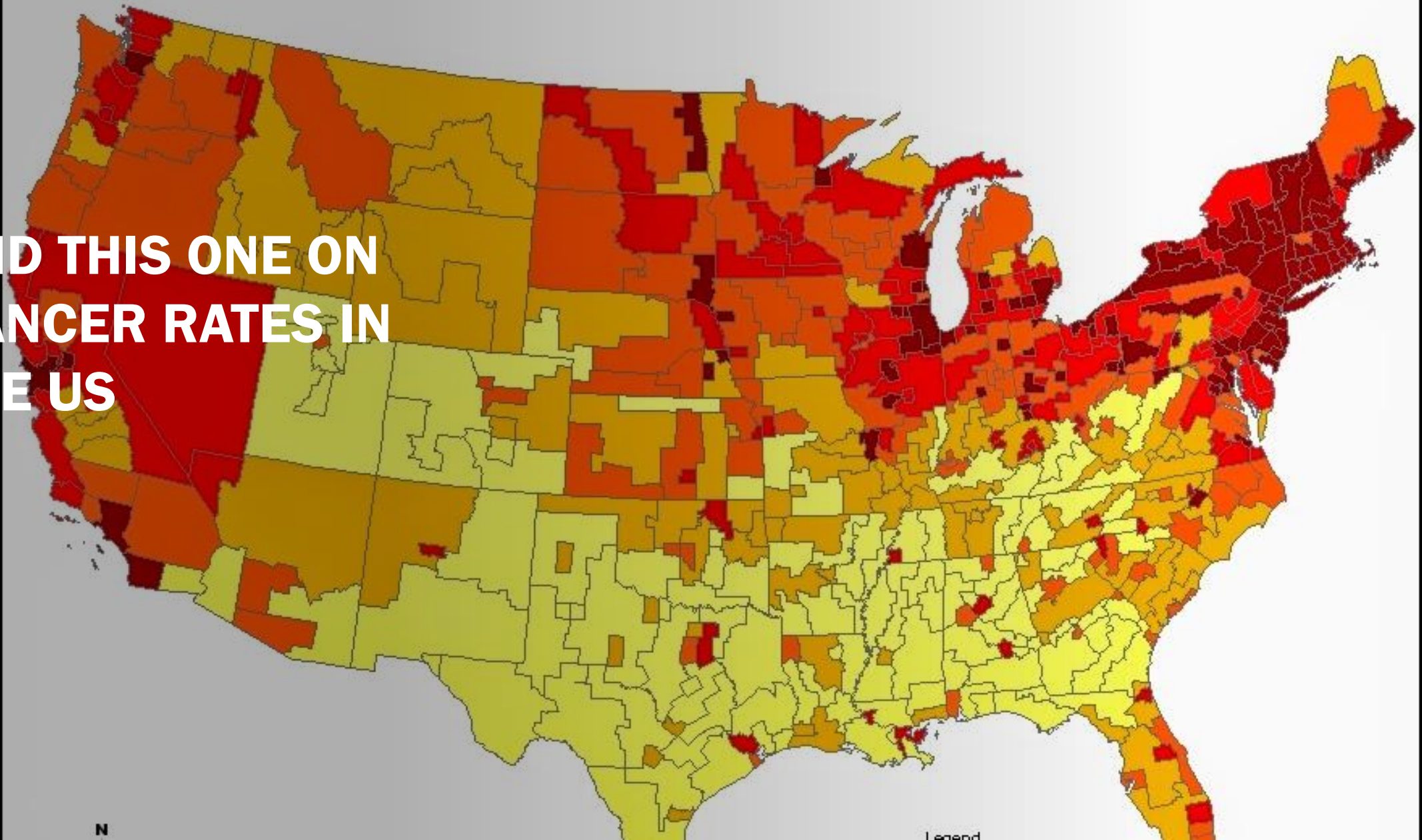
by Health Service Area, 1988–1992

of men



Cancer Rates in the United States

**AND THIS ONE ON
CANCER RATES IN
THE US**



TOBLER'S FIRST LAW OF GEOGRAPHY

“Everything is related to everything else, but near things are more related than distant things”

Mathematically captured by matrix **W**, whose elements W_{ij} decrease as the distance between locations i and j increase.



WEIGHT MATRIX W

Weight Method	Formulation	Definition
Boundary Approach		
Contiguity	$w_{ij} = \begin{cases} 1 : l_{ij} > 0 \\ 0 : l_{ij} = 0 \end{cases}$	l_{ij} : length of shared boundary
Shared boundary	$w_{ij} = \frac{l_{ij}}{\sum_{i \neq j} l_{ij}}$	l_{ij} : length of shared boundary
Distance Approach		
Radial Distance	$w_{ij} = \begin{cases} 1 : 0 \leq d_{ij} \leq d \\ 0 : d_{ij} > d \end{cases}$	d_{ij} : distances between spatial unit d : distance threshold
Power Distance	$w_{ij} = d_{ij}^{\alpha}$	α : any positive exponent
Exponential Distance	$w_{ij} = \exp(-\alpha d_{ij})$	α : any positive value
Double-Power Distance	$w_{ij} = \begin{cases} \left[1 - \left(\frac{d_{ij}}{d}\right)^k\right] : 0 \leq d_{ij} \leq d \\ 0 : d_{ij} > d \end{cases}$	d_{ij} : distances between spatial unit d : distance threshold

Ermagun & Levinson, An Introduction to the Network Weight Matrix, Presentation at 96th Annual Transportation Research Board Meeting, January 2017



SPATIAL REGRESSION MODELS

- Spatial Lag Regression
- Spatial Error Regression
- Spatial Durbin Regression



SPATIAL REGRESSION MODELS

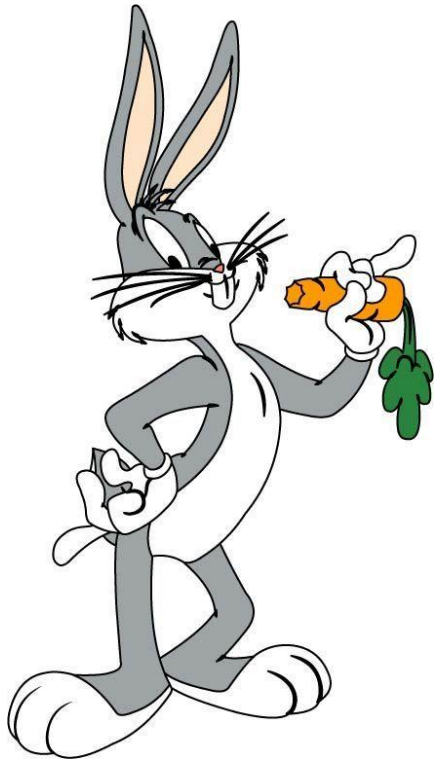
- Spatial Lag Regression
- Spatial Error Regression
- Spatial Durbin Regression

SPATIAL LAG REGRESSION

$$Y_i = \mathbf{X}_i\boldsymbol{\beta} + \rho \sum_{j \neq i} W_{ij} Y_j + \epsilon_i$$

W_{ij} is higher for locations j that are closer to i (Tobler's 1st law of geography)

HOW COULD WE ADD TO THIS MODEL...



... a fixed effect?



... a random effect?



... a dynamic component?

SPATIAL LAG REGRESSION WITH SPATIAL FIXED-EFFECTS

$$Y_i = \mathbf{X}_i \boldsymbol{\beta} + \rho \sum_{j \neq i} W_{ij} Y_j + U_i + \epsilon_i$$

W_{ij} is higher for locations j that are closer to i (Tobler's 1st law of geography)

SPATIAL LAG REGRESSION WITH SPATIAL **RANDOM**-EFFECTS

$$Y_i = \mathbf{X}_i\boldsymbol{\beta} + \rho \sum_{j \neq i} W_{ij} Y_j + U_i + \epsilon_i$$

W_{ij} is higher for locations j that are closer to i (Tobler's 1st law of geography)

$$\text{Cov}(U_i, \mathbf{X}_i) = 0$$

DYNAMIC SPATIAL LAG REGRESSION

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + \rho \sum_{j \neq i} W_{ij} Y_{jt} + U_i + \epsilon_{it}$$

W_{ij} is higher for locations j that are closer to i (Tobler's 1st law of geography)

Fixed Effects: $\text{Cov}(U_i, \mathbf{X}_i) \neq 0$
Random Effects: $\text{Cov}(U_i, \mathbf{X}_i) = 0$

A black and white portrait of Albert Einstein, looking directly at the camera with his hand resting on his chin. He is wearing a dark sweater over a collared shirt. The background is a blurred bookshelf filled with books.

WHICH **TIME** SERIES
MODEL IS
ANALOGOUS TO A
SPATIAL LAG MODEL?

A black and white photograph of Albert Einstein sitting in a chair in a library. He is wearing a light-colored sweater over a collared shirt. He has his characteristic wild hair and a mustache. He is looking slightly to the right of the camera with a thoughtful expression. His hands are resting on his lap, holding a pair of glasses. The background is filled with bookshelves packed with books. The lighting is soft, coming from the left, casting a gentle shadow on his face.

AN AUTOREGRESSIVE MODEL!



SPATIAL REGRESSION MODELS

- Spatial Lag Regression
- Spatial Error Regression
- Spatial Durbin Regression

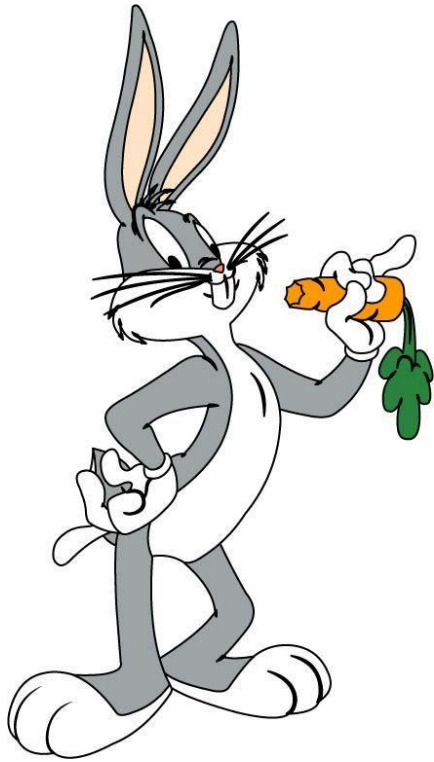
SPATIAL ERROR REGRESSION

$$Y_i = \mathbf{X}_i \boldsymbol{\beta} + \epsilon_i$$

$$\epsilon_i = \phi \sum_{j \neq i} W_{ij} \epsilon_j + \nu_i$$

W_{ij} is higher for locations j that are closer to i (Tobler's 1st law of geography)

HOW COULD WE ADD TO THIS MODEL...



... a fixed effect?



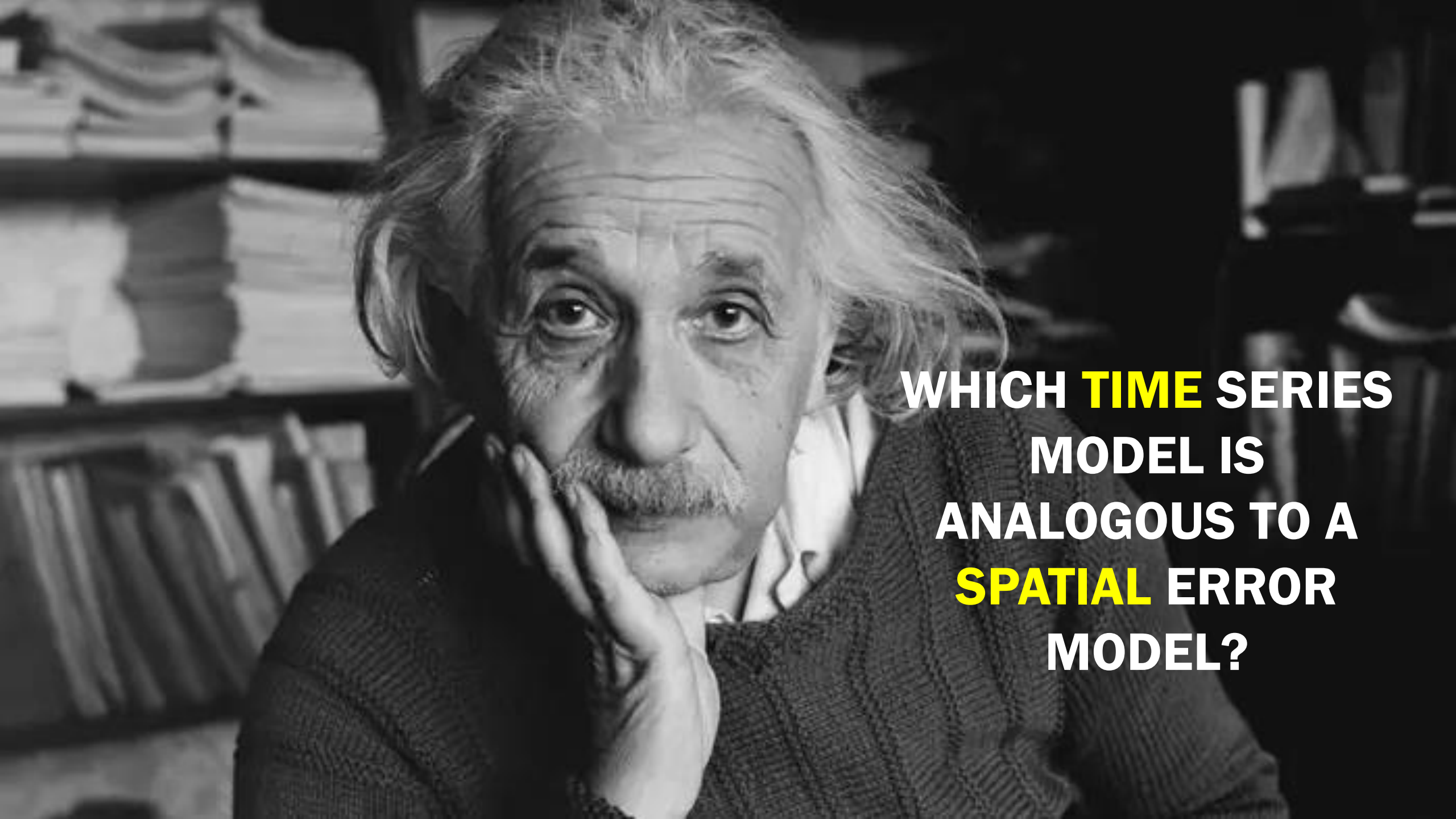
... a random effect?



... a dynamic component?



I LOVE CLASS DISCUSSIONS!

A black and white portrait of Albert Einstein, showing him from the chest up. He has his characteristic wild, white hair and a mustache. He is wearing a dark, textured sweater over a light-colored collared shirt. His right hand is raised to his chin, with his fingers resting against his cheek in a contemplative pose. The background is slightly out of focus, showing what appears to be a bookshelf filled with books.

WHICH **TIME SERIES
MODEL IS
ANALOGOUS TO A
SPATIAL ERROR
MODEL?**

A black and white photograph of Albert Einstein sitting in a chair in a library. He is wearing a light-colored sweater over a collared shirt. He has his characteristic wild hair and a mustache. He is looking slightly to the right of the camera with a thoughtful expression. His hands are resting on his lap, holding a pair of glasses. The background is filled with bookshelves packed with books. In the foreground, to the left, there is a desk with some papers and a rolled-up document. The text "A MOVING AVERAGE MODEL!" is overlaid in white, bold, sans-serif capital letters across the lower middle of the image.

A MOVING AVERAGE MODEL!



SPATIAL REGRESSION MODELS

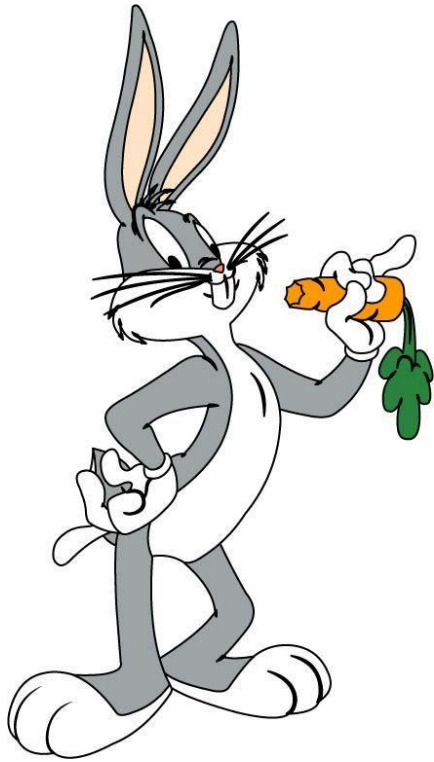
- Spatial Lag Regression
- Spatial Error Regression
- Spatial Durbin Regression

SPATIAL DURBIN MODEL

$$Y_i = \mathbf{X}_i\boldsymbol{\beta} + \theta \sum_{j \neq i} w_{ij} X_j + \rho \sum_{j \neq i} w_{ij} Y_j + \epsilon_i$$

w_{ij} is higher for locations j that are closer to i (Tobler's 1st law of geography)

HOW COULD WE ADD TO THIS MODEL...



... a fixed effect?



... a random effect?



... a dynamic component?



I LOVE CLASS DISCUSSIONS!