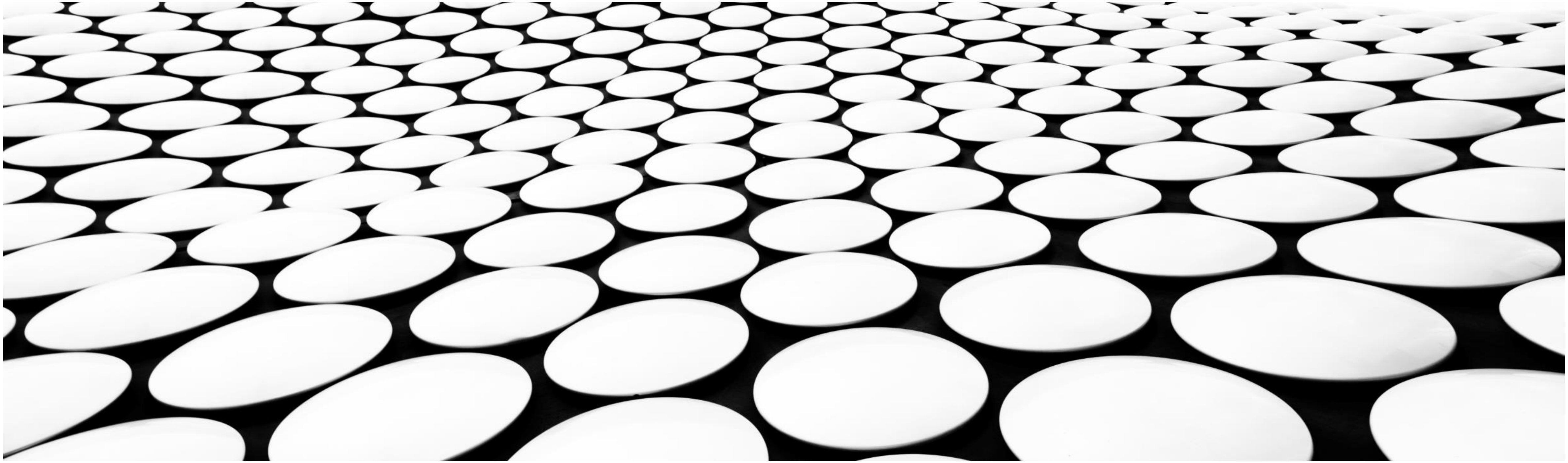
INTRODUCTION TO TIME SERIES

FELIPE BUCHBINDER

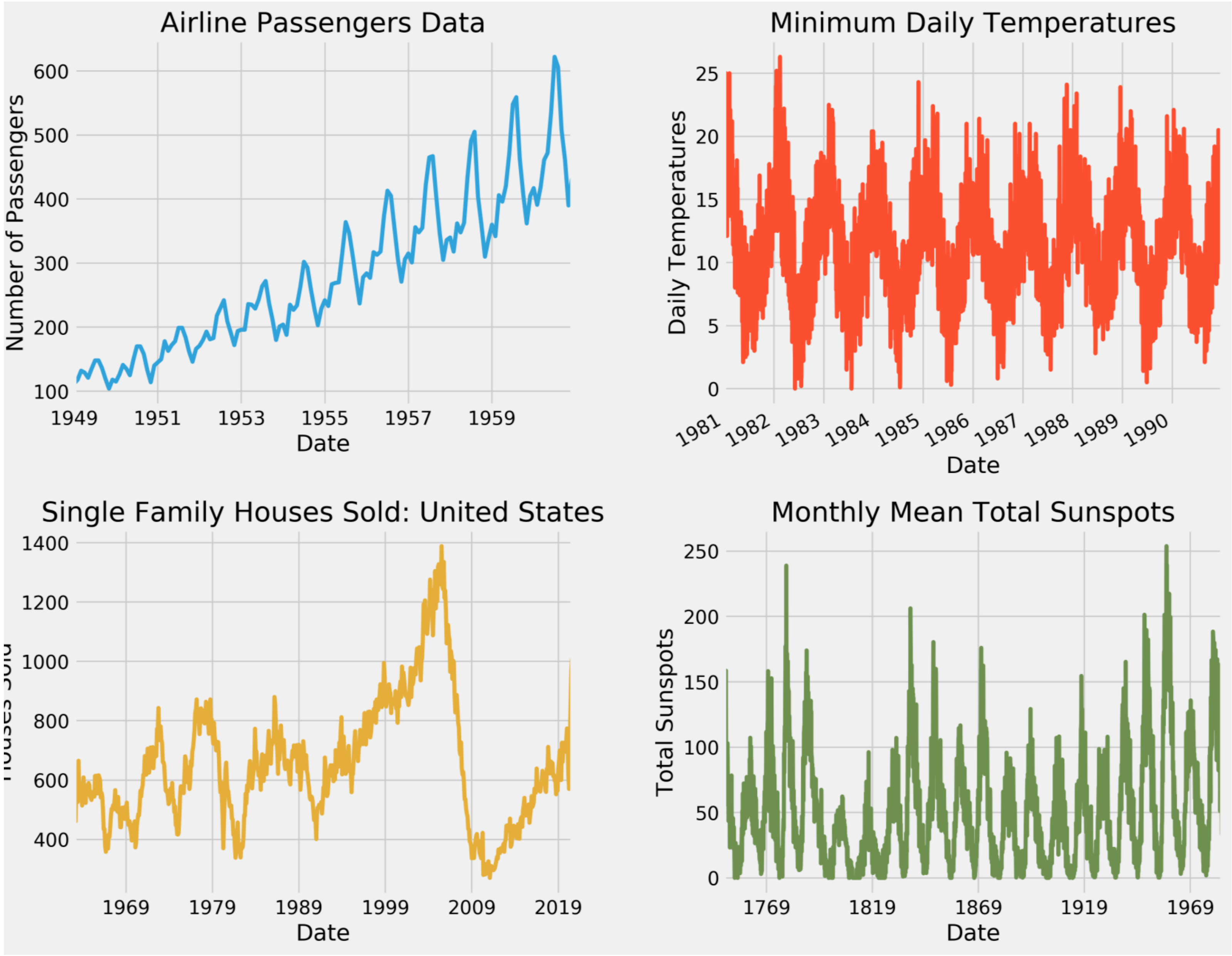


= A DETOUR =

TIME SERIES IS NOT PART OF PANEL DATA.

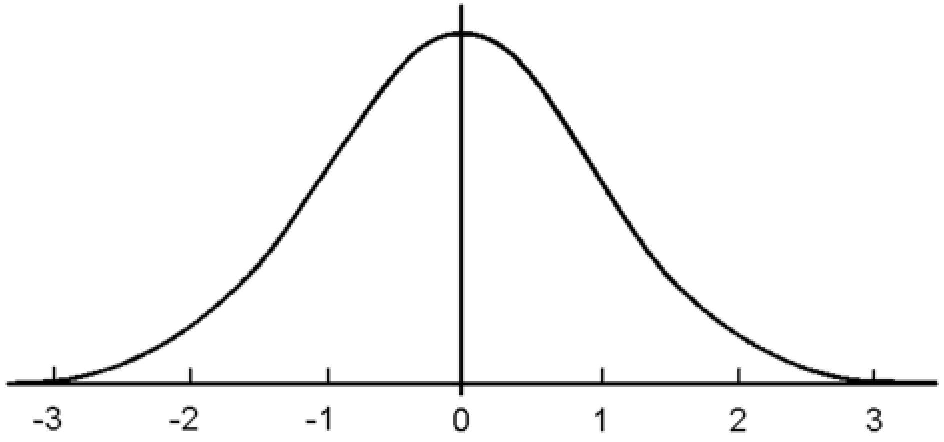
HOWEVER, SINCE WE'LL USE THIS IN OUR NEXT LECTURE, AND SINCE YOU WON'T COVER IT IN ANY OTHER COURSE IN YOUR CURRICULUM (AS FAR AS I KNOW), I'LL TALK ABOUT IT NOW, JUST TO MAKE SURE YOU WON'T GRADUATE WITHOUT EVER HAVING COVERED THIS TOPIC.

THEREFORE, LET'S MAKE A DETOUR TO TALK ABOUT TIME SERIES. WE'LL GET BACK TO PANEL DATA IN OUR NEXT LECTURE.



A **TIME SERIES** IS DATA ON HOW SOMETHING **CHANGES OVER TIME**

WE CAN THINK OF A TIME SERIES AS THE SEQUENCE OF VALUES THAT WE GET FROM DRAWING FROM A PROBABILITY DISTRIBUTION ONCE AT EACH TIME PERIOD



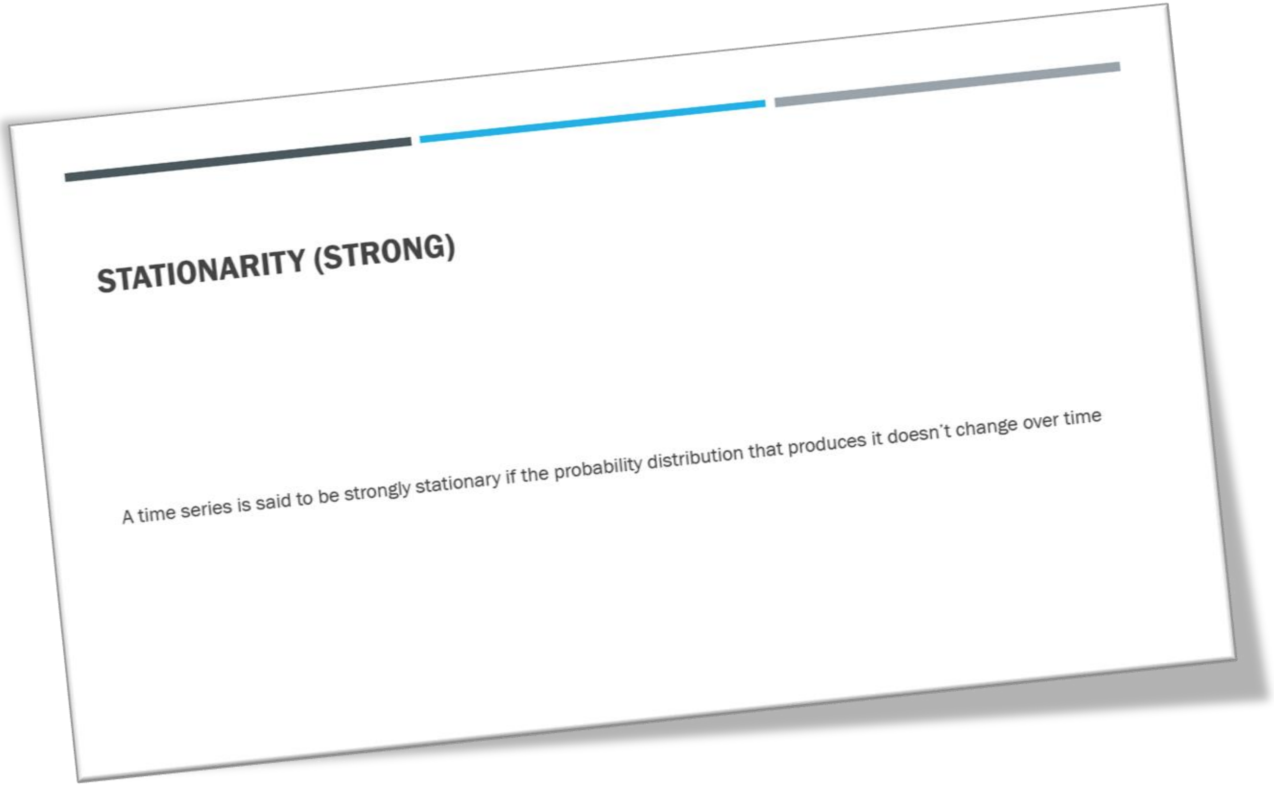
$$\underbrace{\qquad\qquad\qquad}_{}$$

$$y_1, y_2, y_3, y_4, \dots, y_{t-1}, y_t, y_{t+1}, \dots$$

STATIONARITY (STRONG)

A time series is said to be strongly stationary if the probability distribution that produces it doesn’t change over time

STRONG STATIONARITY IS VERY HARD DO PROVE (WHY?)



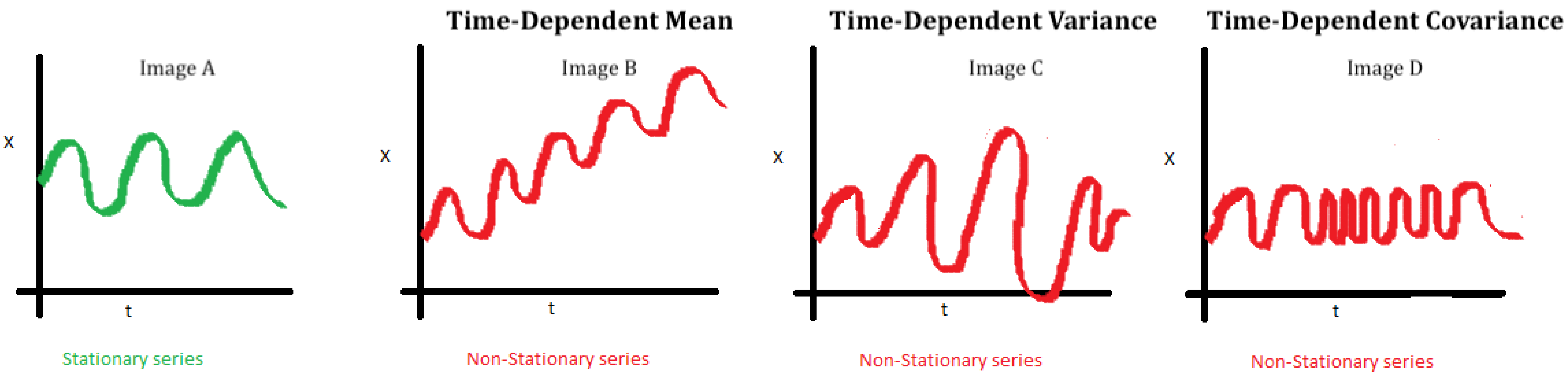
STATIONARITY (WEAK)

A time series is said to be weakly stationary if the mean and variance of the probability distribution that produces it do not change over time and if the covariance between to values depend only on how much time elapsed between these two values, not on when they ocurred.

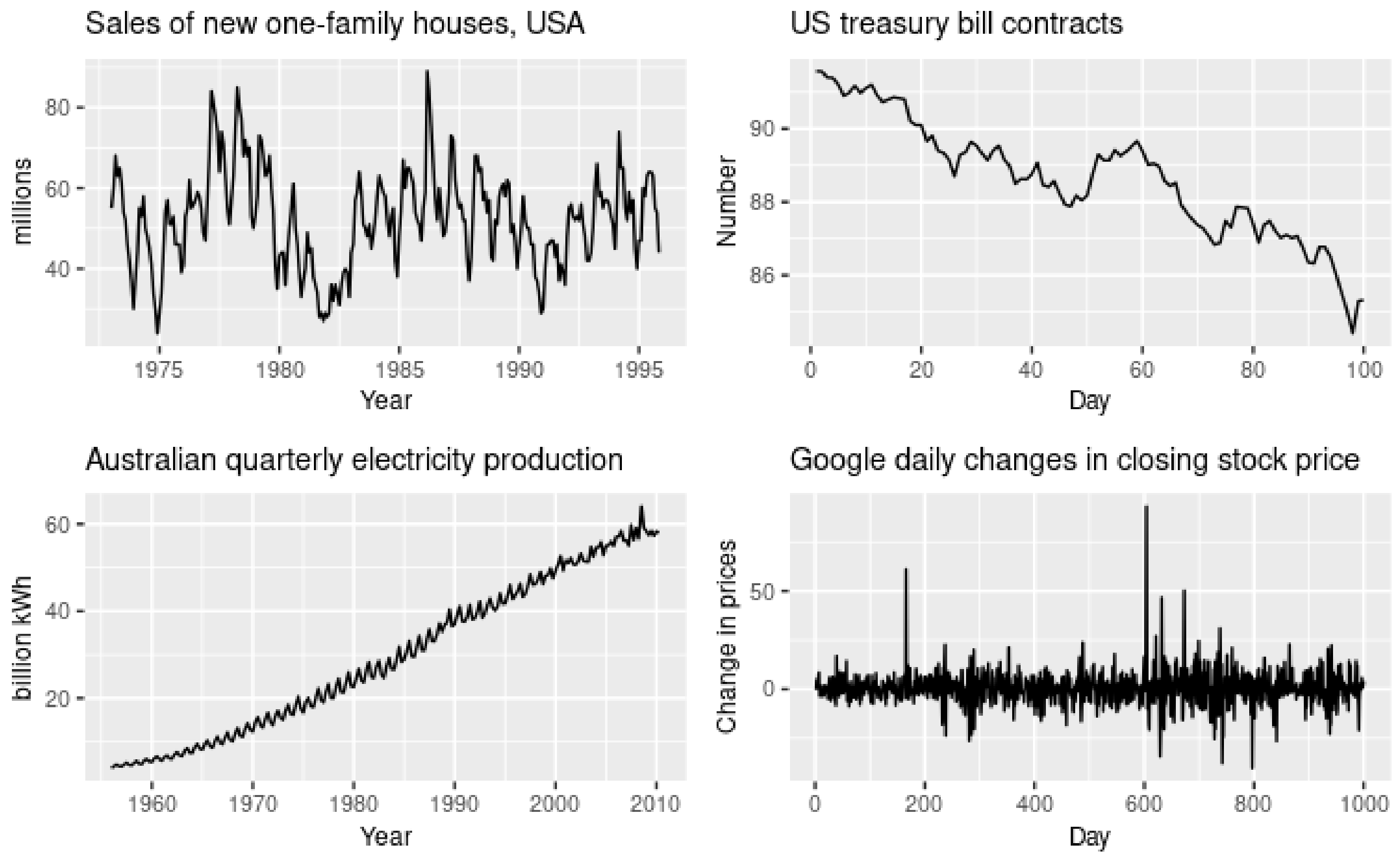
Mathematically…

$$\mathbb{E}(Y_t) = \mu \ \forall t$$
$$\mathbb{V}(Y_t) = \sigma^2 \ \forall t$$
$$\text{Cov}(Y_t; Y_{t+\tau}) = \gamma(\tau) \ \forall t$$

The Principles of Stationarity



REAL-WORLD EXAMPLES



COMPONENTS OF A TIME SERIES

Time Series = Trend + Seasonality + Cycle + Noise

COMPONENTS OF A TIME SERIES

Time Series = Trend + Seasonality + Cycle + Noise

Tackle with regression

COMPONENTS OF A TIME SERIES

add dummy variables to the regression

Time Series = Trend + Seasonality + Cycle + Noise

COMPONENTS OF A TIME SERIES

Time Series = Trend + Seasonality + Cycle + Noise

New stuff!

This is what we'll talk about today

CLASSICAL TIME SERIES MODELS

- Autoregressive models – AR(p)
- Moving Average models – MA(q)
- Autoregressive Moving Average models – ARMA(p,q)
- Autoregressive Integrated Moving Average models – ARIMA(p,d,q)
- Conditionally Heteroskedastic Models – ARCH, GARCH and variations
- Vector Autoregressive Models

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AUTOREGRESSIVE MODELS

WHEN YESTERDAY STILL MATTERS TODAY

AR(*P*)

WHEN THE *P* PREVIOUS DAYS STILL MATTER TODAY

$$y_t = \rho_1 y_{t-1} + \cdots + \rho_p y_{t-p} + \epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

AR(*P*) WITH COVARIATES

WHEN THE *P* PREVIOUS DAYS STILL MATTER TODAY

$$y_t = \beta_0 + \rho_1 y_{t-1} + \cdots + \rho_p y_{t-p} + \beta_1 X_{1t} + \cdots + \beta_k X_{kt} + \epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

AR(1)

WHEN YESTERDAY STILL MATTERS TODAY

$$y_t = \rho y_{t-1} + \epsilon_t$$
$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

AR(1)

CONDITIONS FOR STATIONARITY – CONSTANT MEAN

$$y_t = \rho y_{t-1} + \epsilon_t$$

$$Stationarity \Rightarrow \mathbb{E}(y_t) = \mathbb{E}(y_{t-1}) = \mu$$

$$\therefore$$

$$\mu = \rho\mu$$

$$\therefore$$

$$\mu = \mathbb{E}(y_t) = 0$$

AR(1)

CONDITIONS FOR STATIONARITY – CONSTANT VARIANCE

$$y_t = \rho y_{t-1} + \epsilon_t$$

$$Stationarity \Rightarrow \mathbb{V}(y_t) = \mathbb{V}(y_{t-1}) = \sigma_y^2$$

$$\therefore$$

$$\sigma_y^2 = \rho^2 \sigma_y^2 + \sigma_\epsilon^2$$

$$\therefore$$

$$\sigma_y^2 = \mathbb{V}(y_t) = \frac{\sigma_\epsilon^2}{1 - \rho^2} \Rightarrow \rho \in (-1; 1)$$

AR(1)

CONDITIONS FOR STATIONARITY – CONSTANT COVARIANCE

$$y_t = \rho y_{t-1} + \epsilon_t$$

$$Stationarity \Rightarrow \mathrm{Cov}(y_t, y_{t-\tau}) = \gamma(\tau)$$

$$\therefore$$

$$\gamma(\tau) = \mathrm{Cov}(y_t, y_{t-\tau}) = \mathrm{Cov}(\rho y_{t-1} + \epsilon_t, y_{t-\tau}) = \rho\gamma(\tau - 1)$$

$$\therefore$$

$$\gamma(\tau) \text{ forms a geometric sequence with ratio } \rho \in (-1,1)$$

MOVING AVERAGE MODELS

WHEN SHOCKS DISSIPATE SLOWLY OVER TIME

MA(Q)

WHEN SHOCKS IN THE Q PREVIOUS DAYS STILL MATTER TODAY

$$y_t = \phi_1 \epsilon_{t-1} + \cdots + \phi_q \epsilon_{t-q} + \epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

MA(Q) WITH COVARIATES

WHEN SHOCKS IN THE Q PREVIOUS DAYS STILL MATTER TODAY

$$y_t = \beta_0 + \phi_1\epsilon_{t-1} + \cdots + \phi_q\epsilon_{t-q} + \beta_1 X_{1t} + \cdots + \beta_k X_{kt} + \epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

MA(1)

WHEN SHOCKS YESTERDAY STILL MATTER TODAY

$$y_t = \phi\epsilon_{t-1} + \epsilon_t$$
$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

MA(1)

CONDITIONS FOR STATIONARITY – CONSTANT MEAN

$$y_t = \phi\epsilon_{t-1} + \epsilon_t$$

$$Stationarity \Rightarrow \mathbb{E}(y_t) = \mu$$

$$\therefore$$

$$\mu = \phi \cdot 0 + 0$$

$$\therefore$$

$$\mu = \mathbb{E}(y_t) = 0$$

MA(1)

CONDITIONS FOR STATIONARITY – CONSTANT VARIANCE

$$y_t = \phi \epsilon_{t-1} + \epsilon_t$$

$$Stationarity \Rightarrow \mathbb{V}(y_t) = \sigma_y^2$$

$$\therefore$$

$$\sigma_y^2 = \phi^2 \sigma_\epsilon^2 + \sigma_\epsilon^2$$

$$\therefore$$

$$\sigma_y^2 = (1 + \phi^2)\sigma_\epsilon^2$$

(nothing here)

MA(1)

CONDITIONS FOR STATIONARITY – CONSTANT COVARIANCE

$$y_t = \phi\epsilon_{t-1} + \epsilon_t$$

$$Stationarity \Rightarrow \mathrm{Cov}(y_t, y_{t-\tau}) = \gamma(\tau)$$

$$\therefore$$

$$\gamma(\tau) = \mathrm{Cov}(y_t, y_{t-\tau}) = \mathrm{Cov}(\phi\epsilon_{t-1} + \epsilon_t, \phi\epsilon_{t-\tau-1} + \epsilon_{t-\tau}) = \phi\sigma_\epsilon^2 1_{\tau=1}$$

$$\therefore$$

$\gamma(\tau)$ has a spike for $\tau = 1$ and drops sharply to 0 after that

AUTOREGRESSIVE MOVING AVERAGE MODELS

AND ITS INTEGRATED VERSION

THE BEST OF BOTH WORLDS

ARMA(*P*,*Q*)

AR(P) + MA(Q)

$$y_t = \rho_1 y_{t-1} + \cdots + \rho_{t-p} y_{t-p} + \phi_1 \epsilon_{t-1} + \cdots + \phi_q \epsilon_{t-q} + \epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

ARIMA(*P*,*D*,*Q*)

A time series that needs to be differentiated D times to become an ARMA(P,Q)

(why is this series called "integrated"?)

ARIMA(*P*,*D*,*Q*)

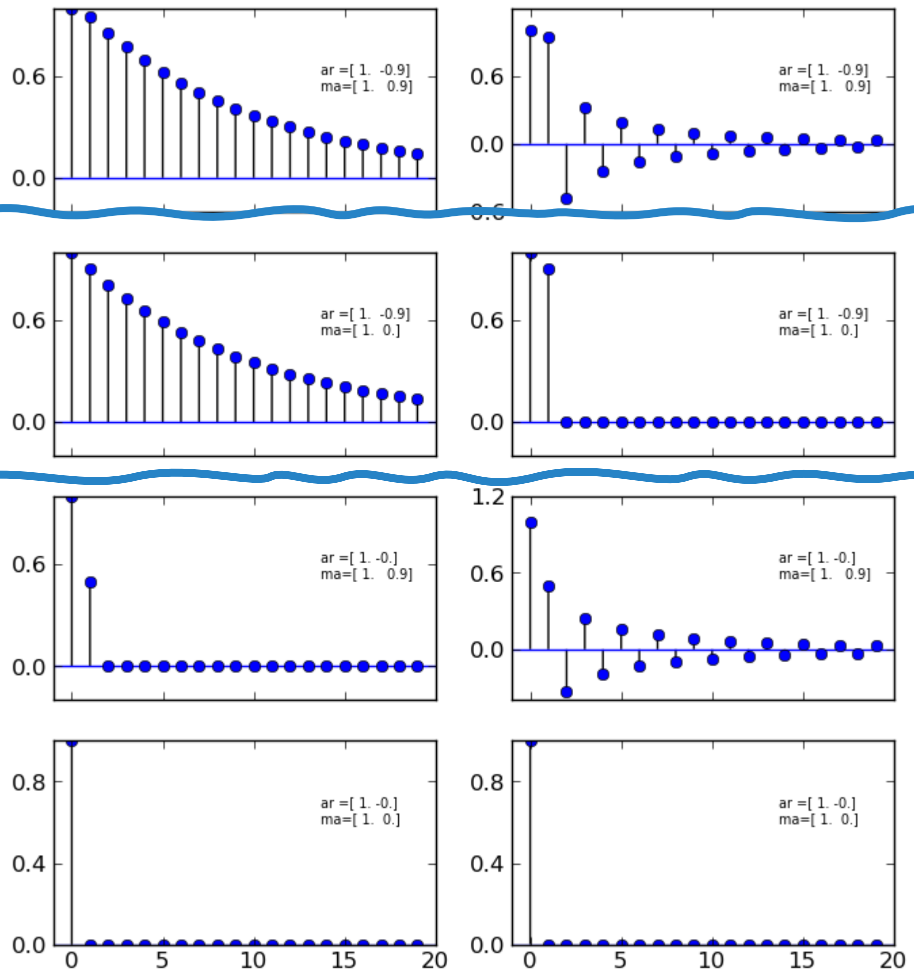
- Why would we possibly need to differentiate a series?
- Pros and cons of differentiation
- Random Walk example
- Unit Roots and how to test for it
  - Dickey-Fuller
  - Augmented Dickey-Fuller
  - Ng-Perron
  - …

BOX-JENKINS METHODOLOGY

A VISUAL GUIDE TO IDENTIFYING IF A SERIES IF AR(P), MA(Q) OR ARMA(P,Q)

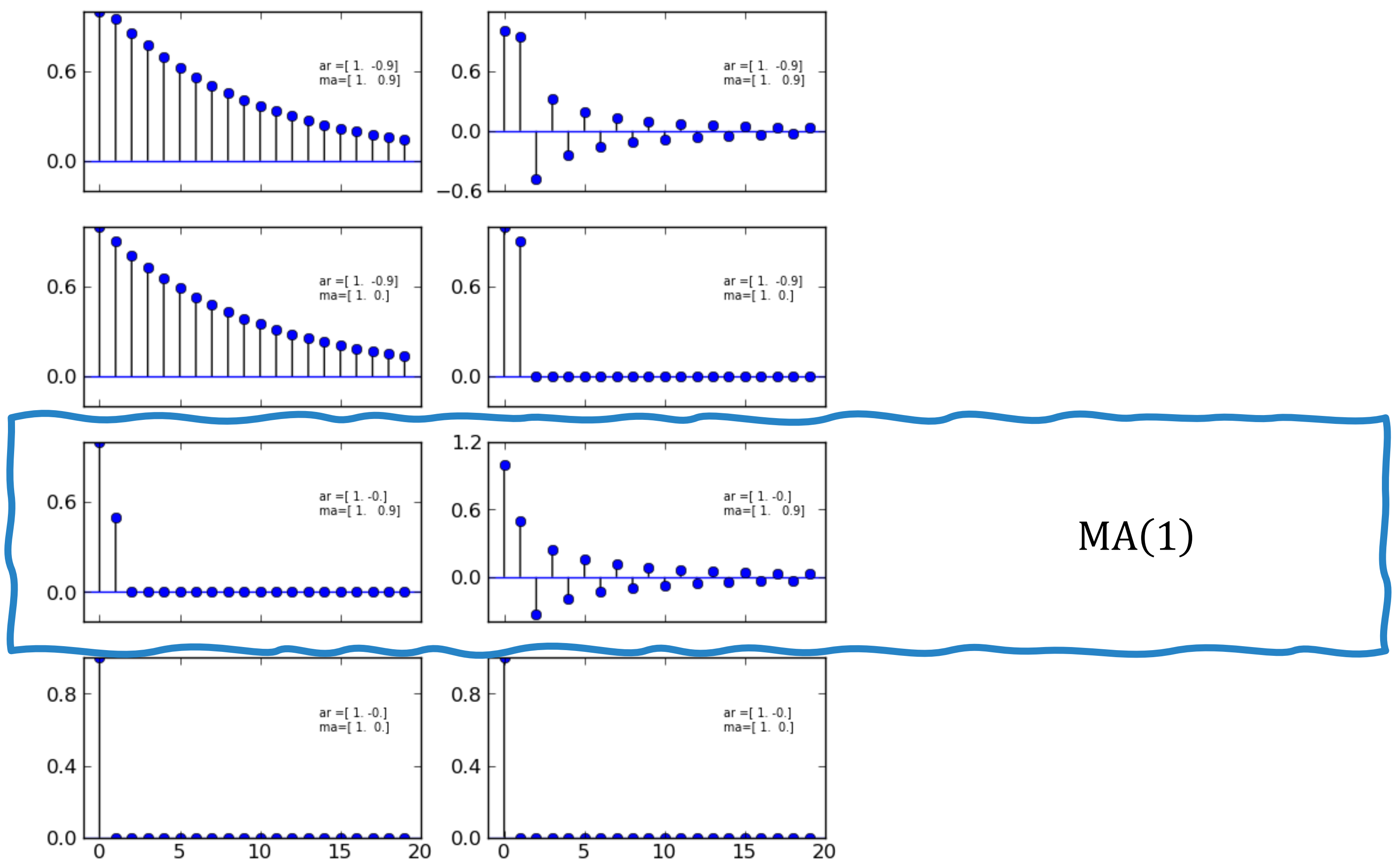
ACFs	PACFs	Model
Decay to zero with exponential pattern	Cuts off after lag p	AR(p)
Cuts off after lag q	Decay to zero with exponential pattern	MA(q)
Decay to zero with exponential pattern	Decay to zero with exponential pattern	ARMA(p, q)

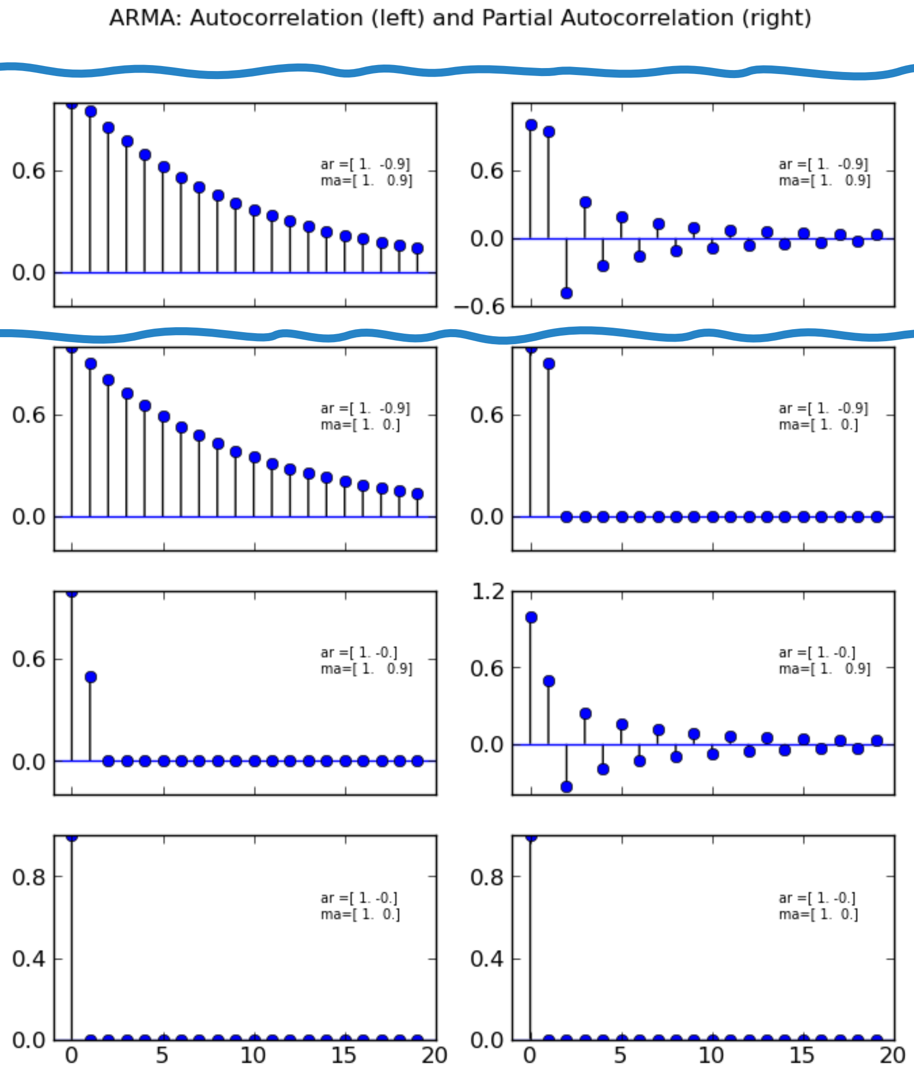
ARMA: Autocorrelation (left) and Partial Autocorrelation (right)



$AR(1)$

ARMA: Autocorrelation (left) and Partial Autocorrelation (right)




ARMA: Autocorrelation (left) and Partial Autocorrelation (right)
ar =[1. -0.9]
ma=[1. 0.9]
ar =[1. -0.9]
ma=[1. 0.]
ar =[1. -0.]
ma=[1. 0.9]
ar =[1. -0.]
ma=[1. 0.]

$ARMA(1,1)$

SCATTERED COMMENTS

- Holt-Winters
- Heteroskedasticity
  ARCH, GARCH and the family
- Vector Autoregression (VAR)
- Cointegration
- Granger causality