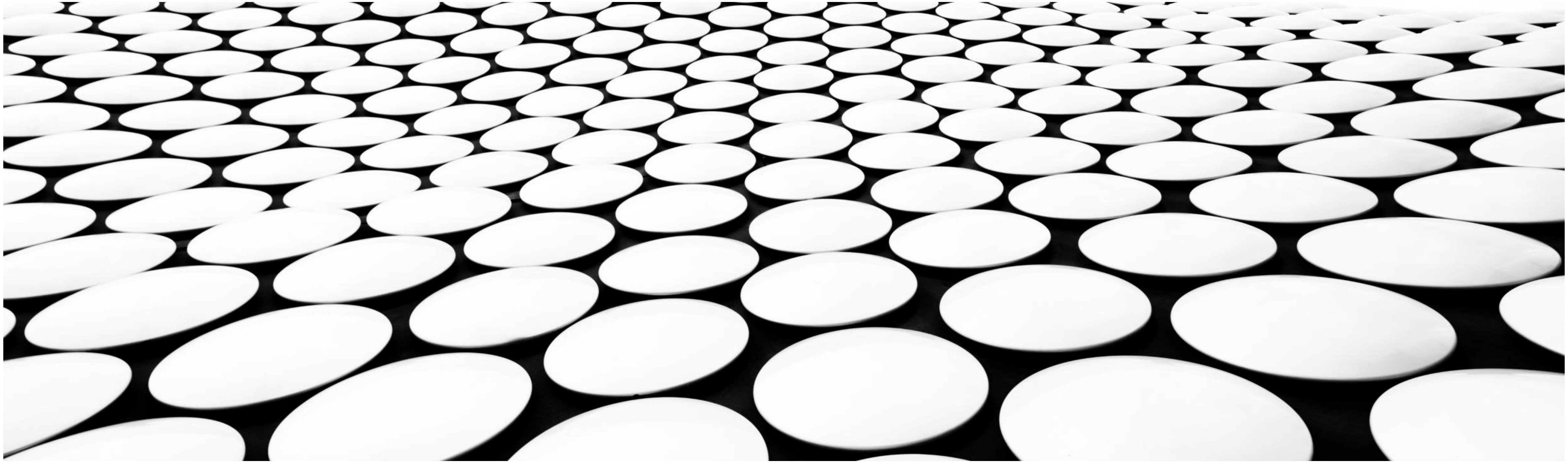
SPATIAL REGRESSION MODELS

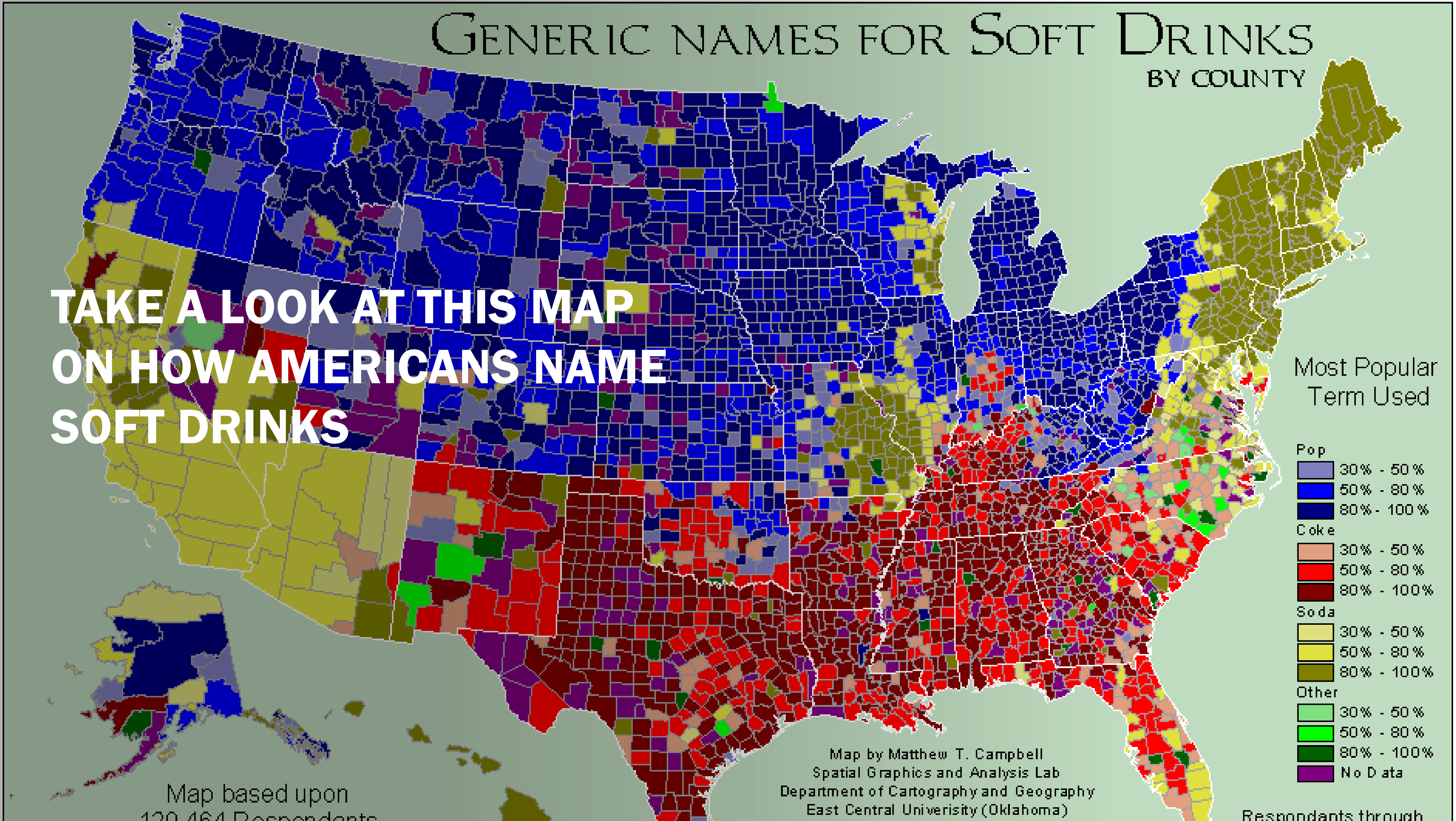
FELIPE BUCHBINDER



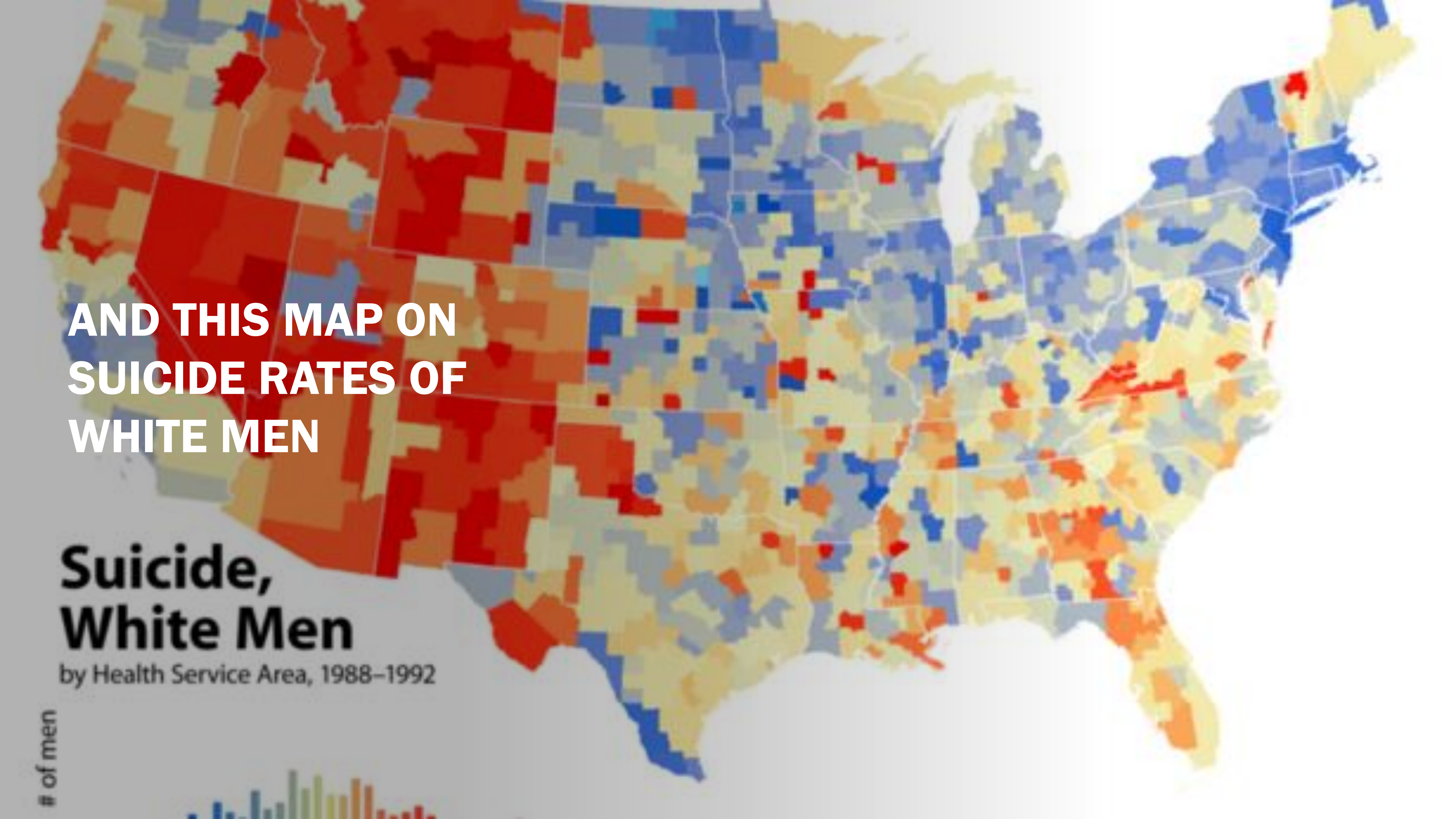
A WHOLE NEW WORLD...

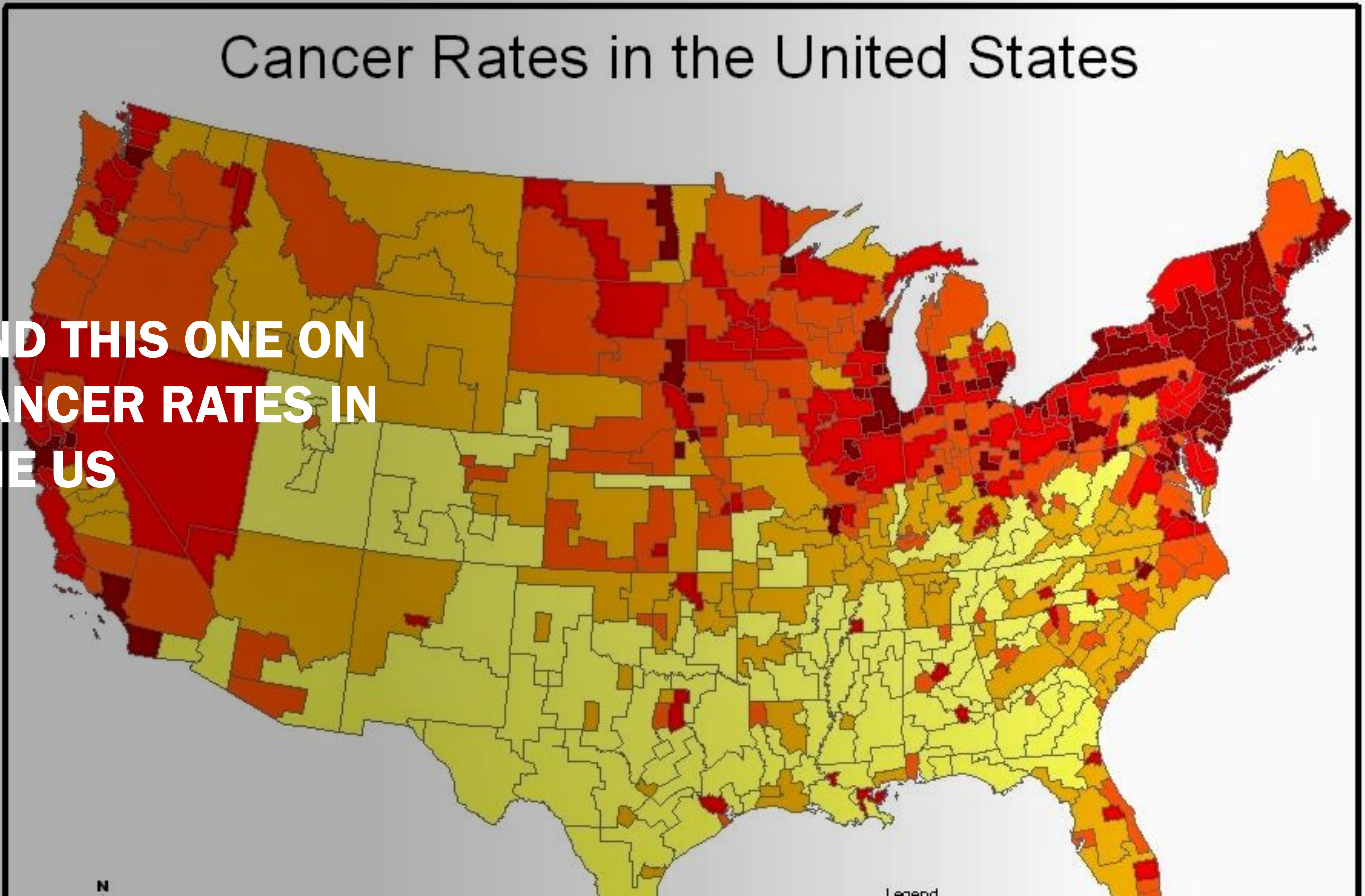
BUT LET’S SEE THE BASICS

TAKE A LOOK AT THIS MAP ON HOW AMERICANS NAME SOFT DRINKS



AND THIS MAP ON SUICIDE RATES OF WHITE MEN





AND THIS ONE ON CANCER RATES IN THE US

TOBLER'S FIRST LAW OF GEOGRAPHY

"Everything is related to everything else, but near things are more related than distant things"

Mathematically captured by matrix $\mathbf{W}$, whose elements W_{ij} decrease as the distance between locations i and j increase.

WEIGHT MATRIX W

Weight Method	Formulation	Definition
Boundary Approach		
Contiguity	$w_{ij} = \begin{cases} 1 : l_{ij} > 0 \\ 0 : l_{ij} = 0 \end{cases}$	l_{ij}: length of shared boundary
Shared boundary	$w_{ij} = \frac{l_{ij}}{\sum_{i \neq j} l_{ij}}$	l_{ij}: length of shared boundary
Distance Approach		
Radial Distance	$w_{ij} = \begin{cases} 1 : 0 \leq d_{ij} \leq d \\ 0 : d_{ij} > d \end{cases}$	d_{ij}: distances between spatial unit d: distance threshold
Power Distance	$w_{ij} - d_{ij}^{\alpha}$	α: any positive exponent
Exponential Distance	$w_{ij} = exp(-\alpha d_{ij})$	α: any positive value
Double-Power Distance	$w_{ij} = \begin{cases} \left[1 - (\frac{d_{ij}}{d})^k\right] : 0 \leq d_{ij} \leq d \\ 0 : d_{ij} > d \end{cases}$	d_{ij}: distances between spatial unit d: distance threshold

Ermagun & Levinson, An Introduction to the Network Weight Matrix, Presentation at 96th Annual Transportation Research Board Meeting, January 2017

SPATIAL REGRESSION MODELS

- Spatial Lag Regression
- Spatial Error Regression
- Spatial Durbin Regression

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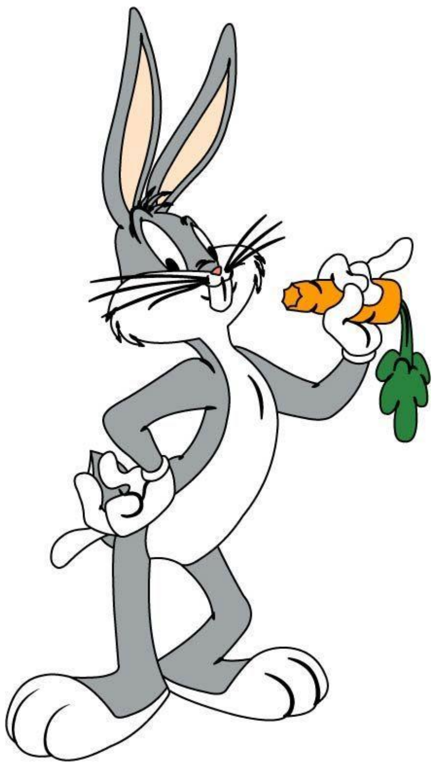
SPATIAL LAG REGRESSION

$$Y_i = \mathbf{X_i \beta} + \rho \sum_{j \neq i} W_{ij} Y_j + \epsilon_i$$

W_{ij} is higher for locations j that are closer to i (Tobler's 1st law of geography)

HOW COULD WE ADD TO THIS MODEL...



... a fixed effect?

... a random effect?

... a dynamic component?

SPATIAL LAG REGRESSION WITH SPATIAL FIXED-EFFECTS

$$Y_i = \mathbf{X_i \beta} + \rho \sum_{j \neq i} W_{ij} Y_j + U_i + \epsilon_i$$

W_{ij} is higher for locations j that are closer to i (Tobler's 1st law of geography)

SPATIAL LAG REGRESSION WITH SPATIAL RANDOM-EFFECTS

$$Y_i = \mathbf{X_i}\boldsymbol{\beta} + \rho \sum_{j \neq i} W_{ij} Y_j + U_i + \epsilon_i$$

W_{ij} is higher for locations j that are closer to i (Tobler's 1st law of geography)

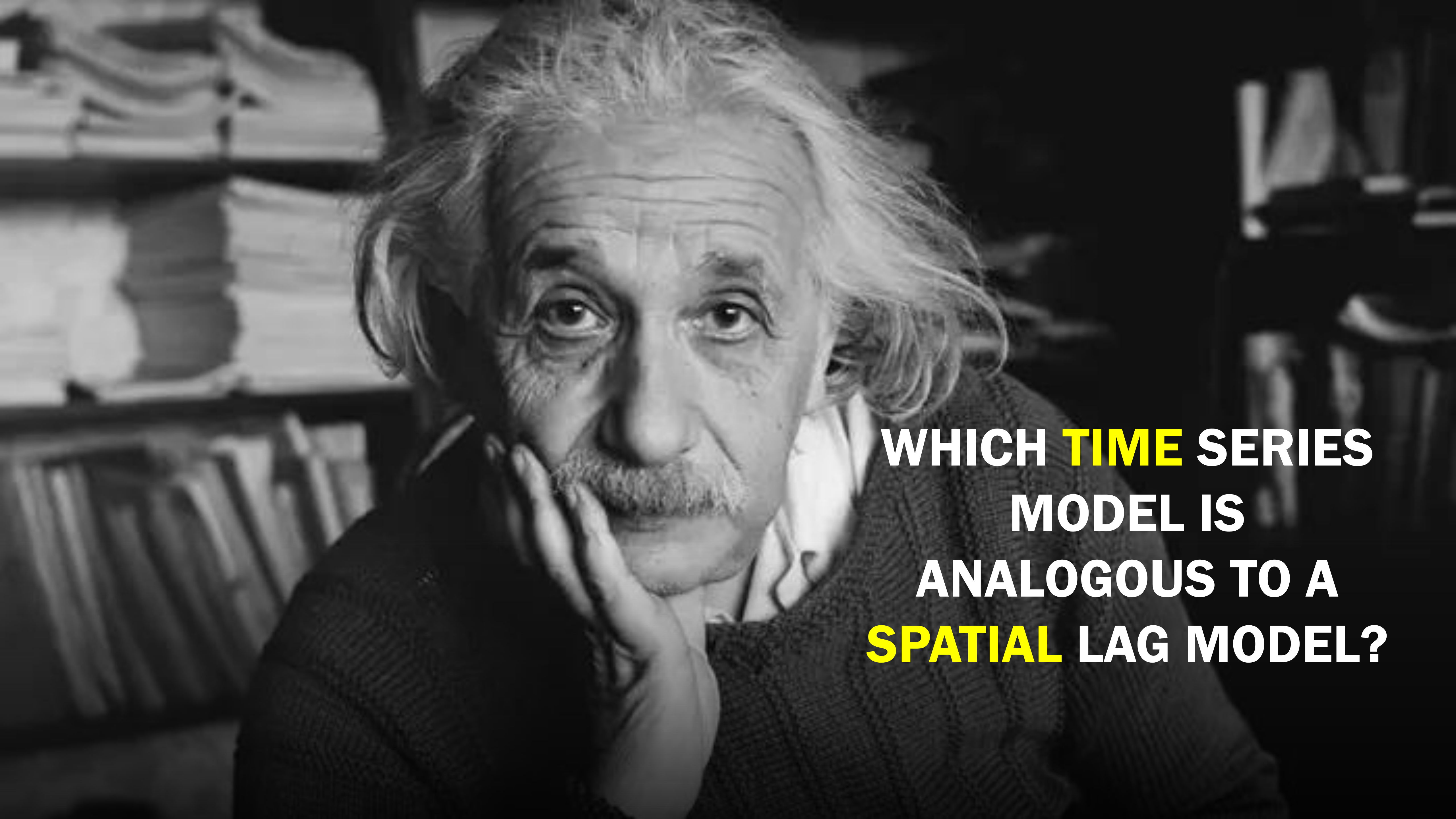
$$\mathrm{Cov}(U_i, \mathbf{X_i}) = 0$$

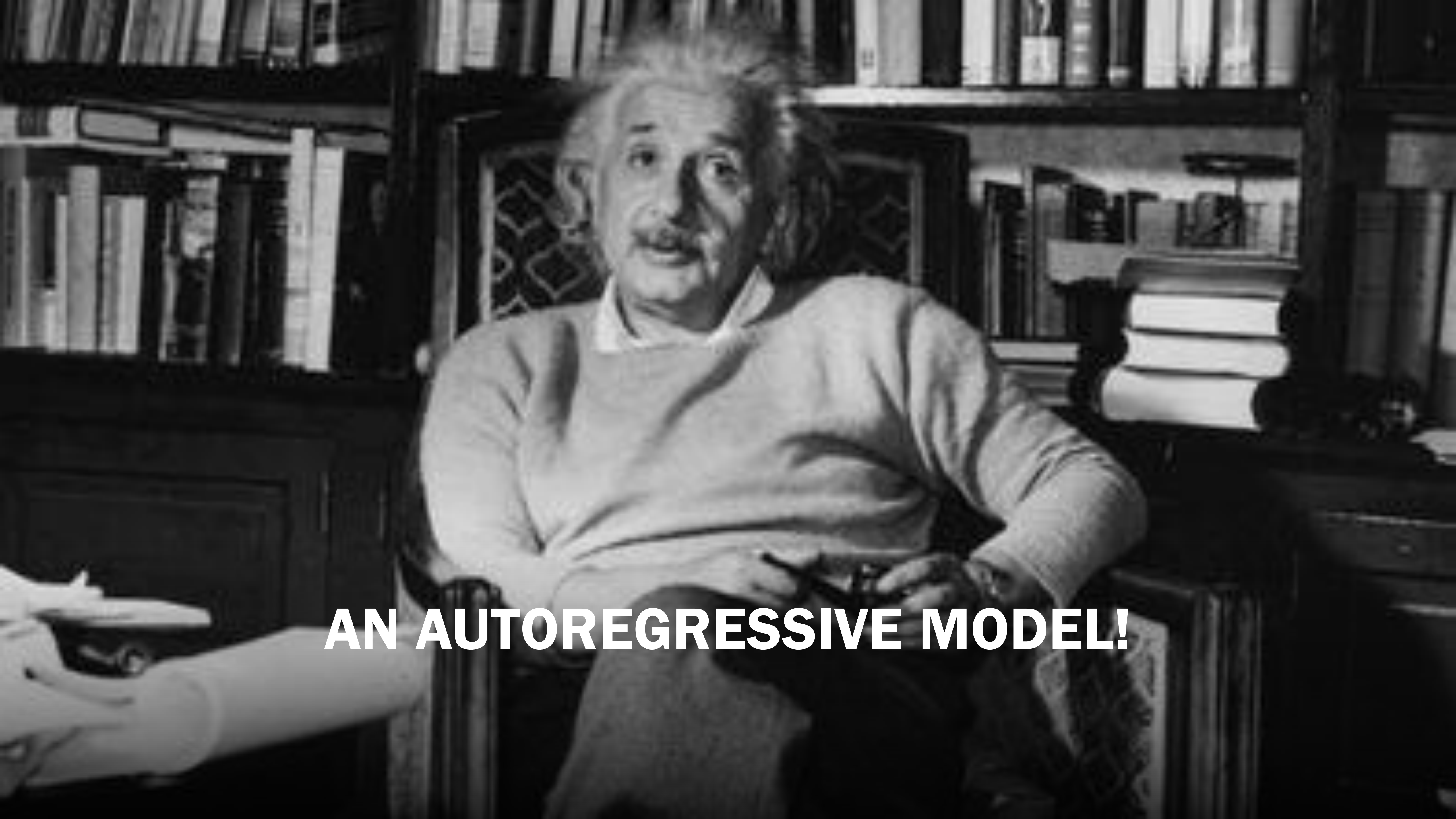
DYNAMIC SPATIAL LAG REGRESSION

$$Y_{it} = \mathbf{X_{it}}\boldsymbol{\beta} + \rho \sum_{j \neq i} W_{ij} Y_{jt} + \boldsymbol{U_i} + \epsilon_{it}$$

W_{ij} is higher for locations j that are closer to i (Tobler's 1st law of geography)

$$\text{Fixed Effects: } \operatorname{Cov}(U_i, \mathbf{X_i}) \neq 0$$
$$\text{Random Effects: } \operatorname{Cov}(U_i, \mathbf{X_i}) = 0$$


WHICH TIME SERIES
MODEL IS
ANALOGOUS TO A
SPATIAL LAG MODEL?


AN AUTOREGRESSIVE MODEL!

SPATIAL REGRESSION MODELS

- Spatial Lag Regression
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- Spatial Durbin Regression

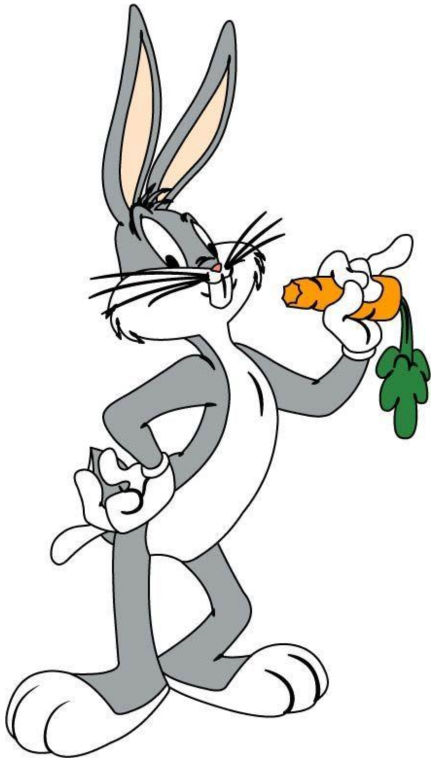
SPATIAL ERROR REGRESSION

$$Y_i = \mathbf{X_i \beta} + \epsilon_i$$

$$\epsilon_i = \phi \sum_{j \neq i} W_{ij} \epsilon_j + \nu_i$$

W_{ij} is higher for locations j that are closer to i (Tobler's 1st law of geography)

HOW COULD WE ADD TO THIS MODEL…



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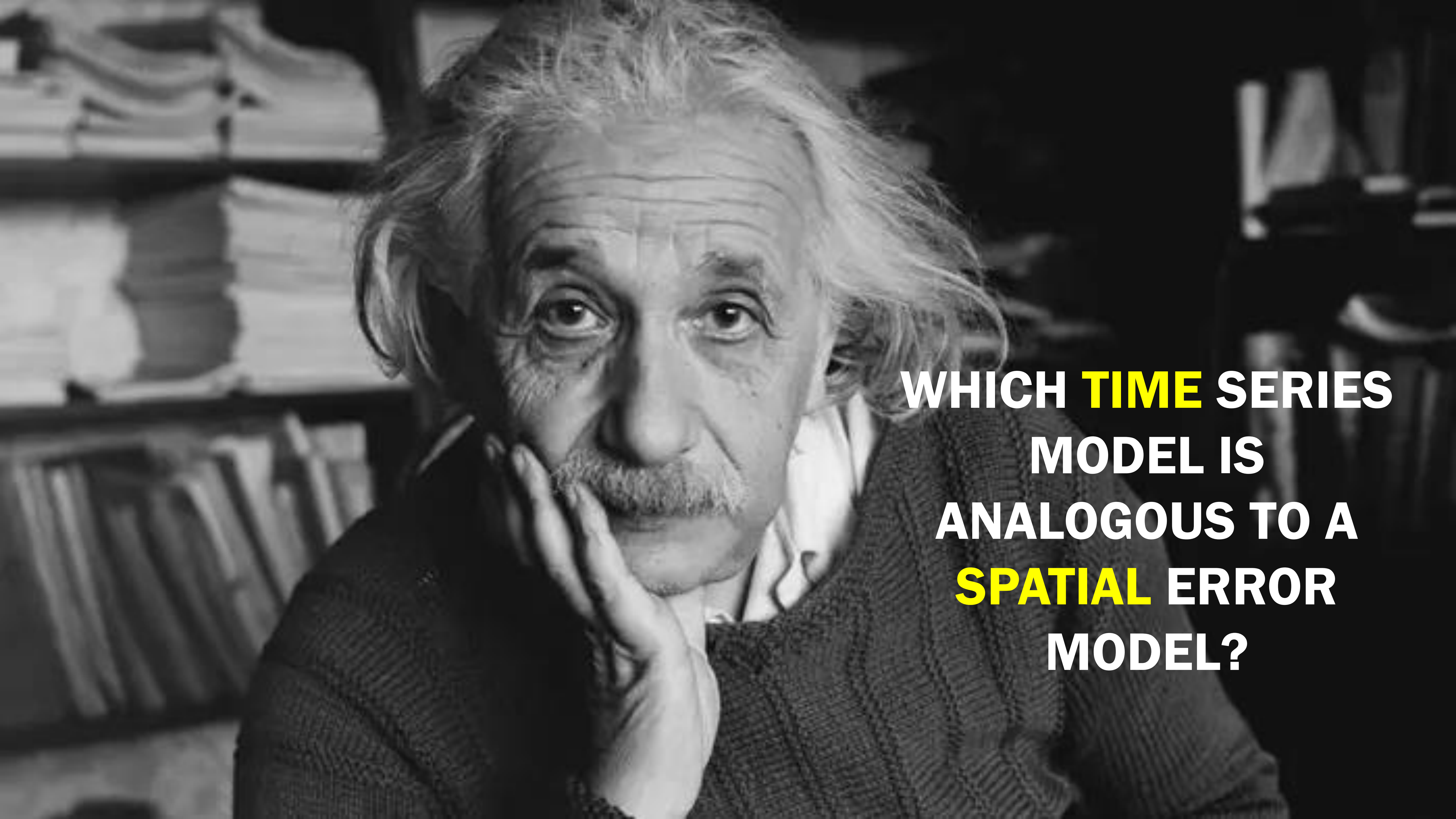
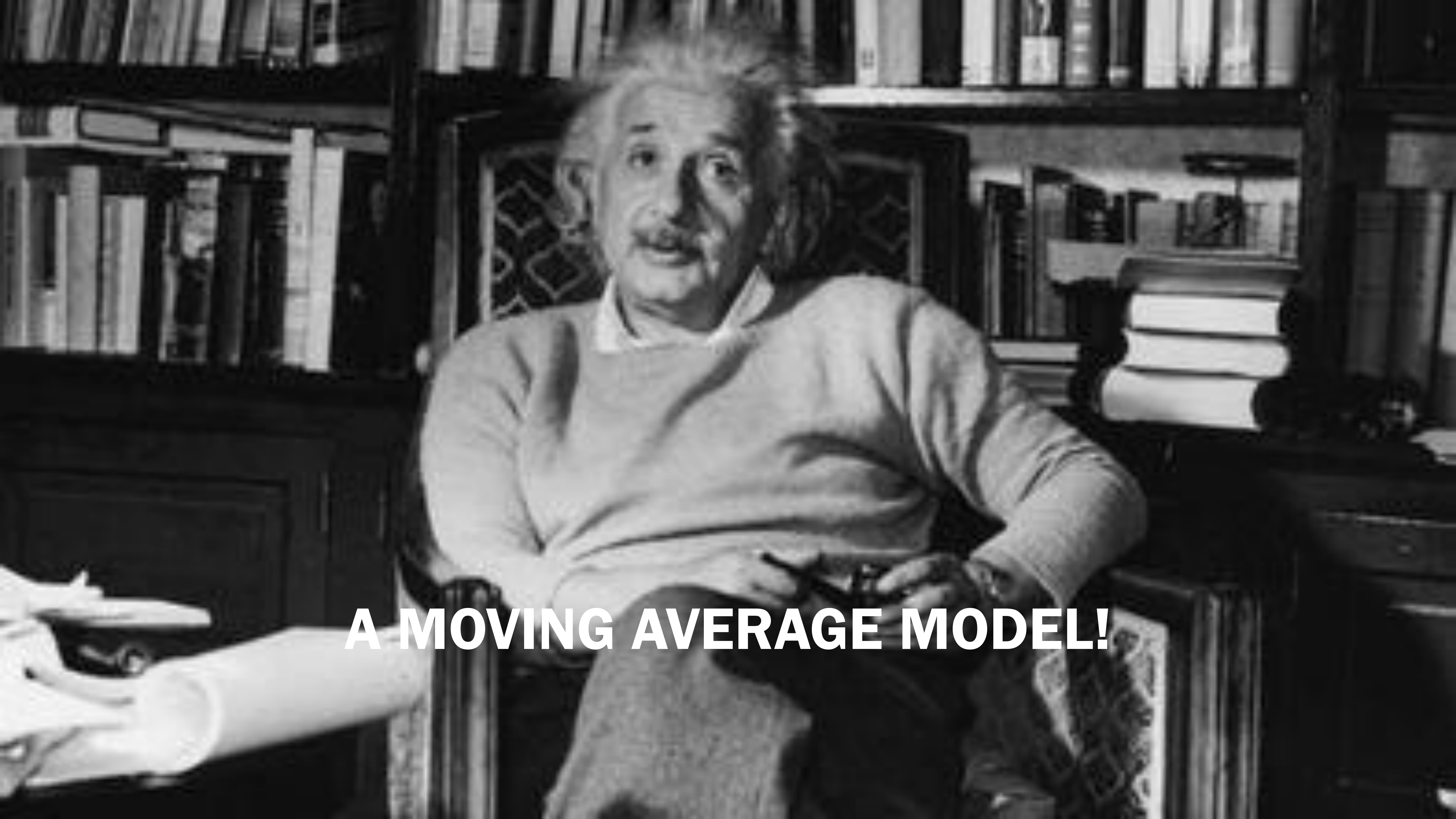
… a dynamic component?


I LOVE CLASS DISCUSSIONS!



WHICH TIME SERIES MODEL IS ANALOGOUS TO A SPATIAL ERROR MODEL?


A MOVING AVERAGE MODEL!

SPATIAL REGRESSION MODELS

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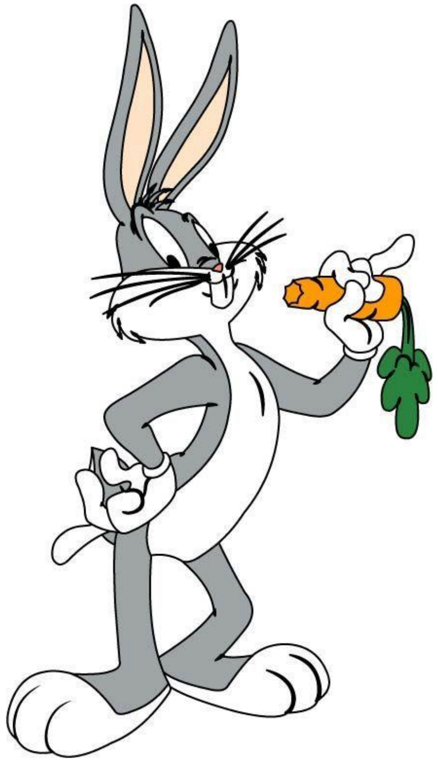
- Spatial Durbin Regression

SPATIAL DURBIN MODEL

$$Y_i = \mathbf{X_i}\boldsymbol{\beta} + \theta \sum_{j \neq i} W_{ij}\mathrm{X_j} + \rho \sum_{j \neq i} W_{ij}Y_j + \epsilon_i$$

W_{ij} is higher for locations j that are closer to i (Tobler's 1st law of geography)

HOW COULD WE ADD TO THIS MODEL…



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