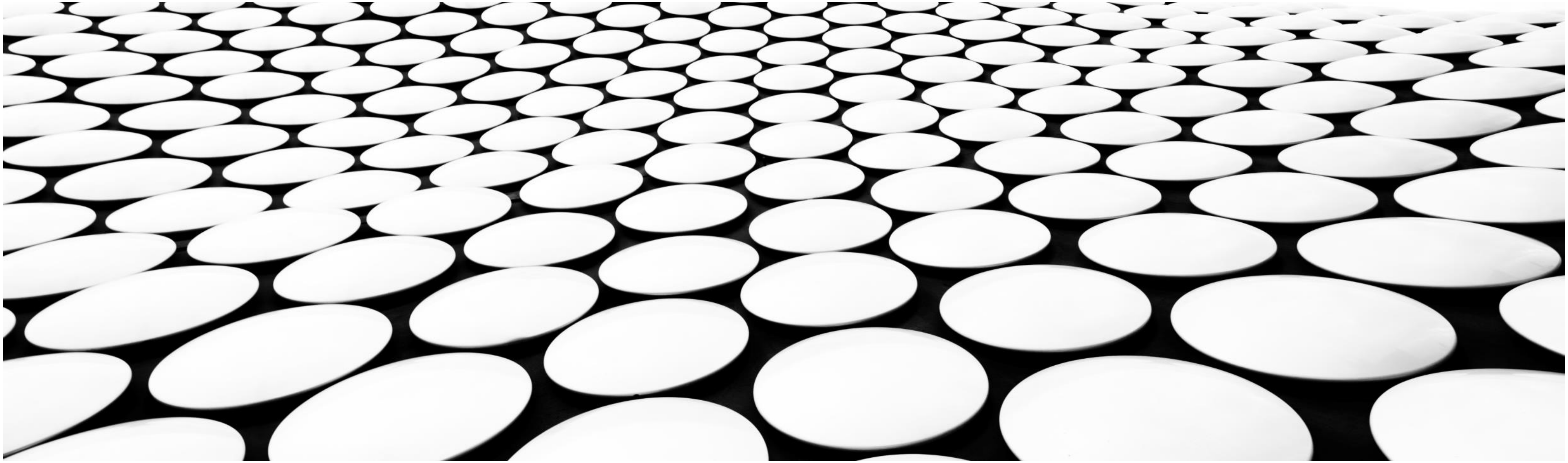
FIXED EFFECTS

FELIPE BUCHBINDER




WELCOME BACK!
TODAY WE'LL TALK ABOUT FIXED EFFECTS.
ENJOY!

IS UNEMPLOYMENT RELATED TO CRIME RATE?

$$\text{Crime Rate}_{it} = \beta_0 + \beta_1 \cdot \text{Unemployment}_{it} + U_i + \epsilon_{it}$$

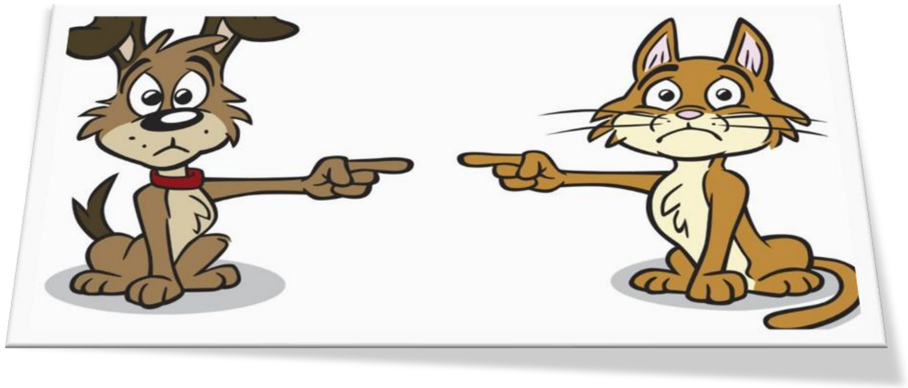
IS UNEMPLOYMENT RELATED TO CRIME RATE?

FIRST DIFFERENCE MODELS COMPARE t WITH $t-1$ TO ELIMINATE U_i

$$\text{Crime Rate}_{it} = \beta_0 + \beta_1 \cdot \text{Unemployment}_{it} + \cancel{U}_i + \epsilon_{it}$$

$$\text{Crime Rate}_{it-1} = \beta_0 + \beta_1 \cdot \text{Unemployment}_{it-1} + \cancel{U}_i + \epsilon_{it-1}$$

$$\Delta\text{Crime Rate} = \beta_1 \cdot \Delta\text{Unemployment}_{it-1} + \Delta\epsilon_{it}$$



WHAT IF THE ERRORS ARE SERIALLY INDEPENDENT, IN WHICH CASE A FIRST DIFFERENCE WON'T WORK?


LET’S TRY A DIFFERENT APPROACH…

LET'S COMPARE EACH CITY'S CRIME RATE WITH THE CITY'S HISTORICAL (AVERAGE) CRIME RATE

In other words, we can use deviations from historical averages

LET'S COMPARE CRIME RATE WITH THE CITY'S HISTORICAL (AVERAGE) CRIME RATE

IN OTHER WORDS, WE CAN USE DEVIATIONS FROM HISTORICAL AVERAGES

$$\text{Crime Rate}_{it} = \beta_0 + \beta_1 \cdot \text{Unemployment}_{it} + \cancel{U}_i + \epsilon_{it}$$
$$\text{Average Crime Rate}_{it} = \beta_0 + \beta_1 \cdot \text{Average Unemployment}_{it} + \cancel{U}_i$$

$$\left(\begin{array}{c} \text{Crime Rate}_{it} \\ - \\ \text{Average Crime Rate}_i \end{array} \right) = \beta_1 \left(\begin{array}{c} \text{Unemployment}_{it} \\ - \\ \text{Average Unemployment}_i \end{array} \right) \cdot + \epsilon_{it}$$

WE CAN ALSO COMPARE CRIME RATE WITH THE CITY'S HISTORICAL (AVERAGE) CRIME RATE

IN OTHER WORDS, WE CAN USE DEVIATIONS FROM HISTORICAL AVERAGES

FIXED EFFECTS

$$\text{Crime Rate}_{it} = \beta_0 + \beta_1 \cdot \text{Unemployment}_{it} + \cancel{U_i} + \epsilon_{it}$$

$$\text{Average Crime Rate}_{it} = \beta_0 + \beta_1 \cdot \text{Average Unemployment}_{it} + \cancel{U_i}$$

$$\left(\begin{matrix} \text{Crime Rate}_{it} \\ - \\ \text{Average Crime Rate}_i \end{matrix} \right) = \beta_1 \left(\begin{matrix} \text{Unemployment}_{it} \\ - \\ \text{Average Unemployment}_i \end{matrix} \right) \cdot + \epsilon_{it}$$

In this context, the U_i are called fixed effects because they does not change through time and are thus eliminated by considering all values relative to their historical averages for each entity

IN THE FIXED EFFECT MODEL, WE ELIMINATE FIXED UNOBSERVED HETEROGENEITIES BY FOCUSING ON HOW THINGS DEVIATE FROM HISTORICAL AVERAGE VALUES.

FIXED EFFECTS REGRESSES

$$\ddot{Y}_{it} \triangleq Y_{it} - \bar{Y}_i$$

ON

$$\ddot{X}_{it} \triangleq X_{it} - \bar{X}_i$$


ANY QUESTIONS?


DO YOU UNDERSTAND HOW FE IS
DIFFERENT THAN FD?

WITHIN TRANSFORMATION

- Transforming a variable by subtracting its historical mean
- $Y_{it} \xrightarrow{\text{within transformation}} Y_{it} - \bar{Y}_i$
- $X_{it} \xrightarrow{\text{within transformation}} X_{it} - \bar{X}_i$
- $\epsilon_{it} \xrightarrow{\text{within transformation}} \epsilon_{it}$

REDUCED EQUATION

- The equation relating within-transformed variables

$$\ddot{Y}_{it} = \beta \ddot{X}_{it} + \epsilon_{\mathrm{it}}$$

REDUCED EQUATION

- The equation relating within-transformed variables

$$\ddot{Y}_{it} = \beta \ddot{X}_{it} + \epsilon_{\mathrm{it}}$$

- Note that it doesn't have an intercept. Why?

A FIXED EFFECTS MODEL WORKS JUST LIKE OLS REGRESSION

BUT WITH WITHIN-TRANSFORMED VARIABLES, RATHER THAN ACTUAL VARIABLES

OLS Regression

- $\mathbf{Y} = \mathbf{X\beta} + \epsilon$
- $\boldsymbol{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$

Fixed Effects Regression

- $\ddot{\mathbf{Y}} = \ddot{\mathbf{X}}\boldsymbol{\beta} + \epsilon$
- $\boldsymbol{\beta} = \left(\ddot{\mathbf{X}}'\ddot{\mathbf{X}}\right)^{-1}\ddot{\mathbf{X}}'\,\ddot{\mathbf{Y}}$

HOW ARE THE RESIDUALS OF THE REDUCED EQUATION RELATED TO THE RESIDUALS OF THE ORIGINAL EQUATION?

HOW SHOULD THE SERIAL CORRELATION OF THE RESIDUALS IN THE ORIGINAL EQUATION BE SO THAT FIXED EFFECTS WORK WELL?

IS INVESTMENT DETERMINED BY COMPANY VALUE? THE GRUNFELD DATASET

#Pooled regression

pooled <- plm(invest ~ value + capital, index=c(“firm”, “year”), data=Grunfeld, model='pooling’)

#Fixed Effects model

fe <- plm(invest ~ value + capital, index=c(“firm”, “year”), data=Grunfeld, model=“within”)

	invest		
	Pooled	First Differences	Fixed Effects
	(1)	(2)	(3)
value	0.115***	0.090***	0.110***
	(0.006)	(0.008)	(0.011)
capital	0.228***	0.291***	0.310***
	(0.024)	(0.051)	(0.017)
Constant	-38.410***	-1.654	
	(8.413)	(3.200)	
Observations	220	209	220
R2	0.818	0.411	0.767
Adjusted R2	0.816	0.405	0.753
F Statistic	487.284*** (df = 2; 217)	71.756*** (df = 2; 206)	340.079*** (df = 2; 207)

EVEN THOUGH FIXED EFFECTS VANISH, WE CAN STILL ESTIMATE THEM

WE'LL HAVE TO USE THE HISTORICAL AVERAGE EQUATION

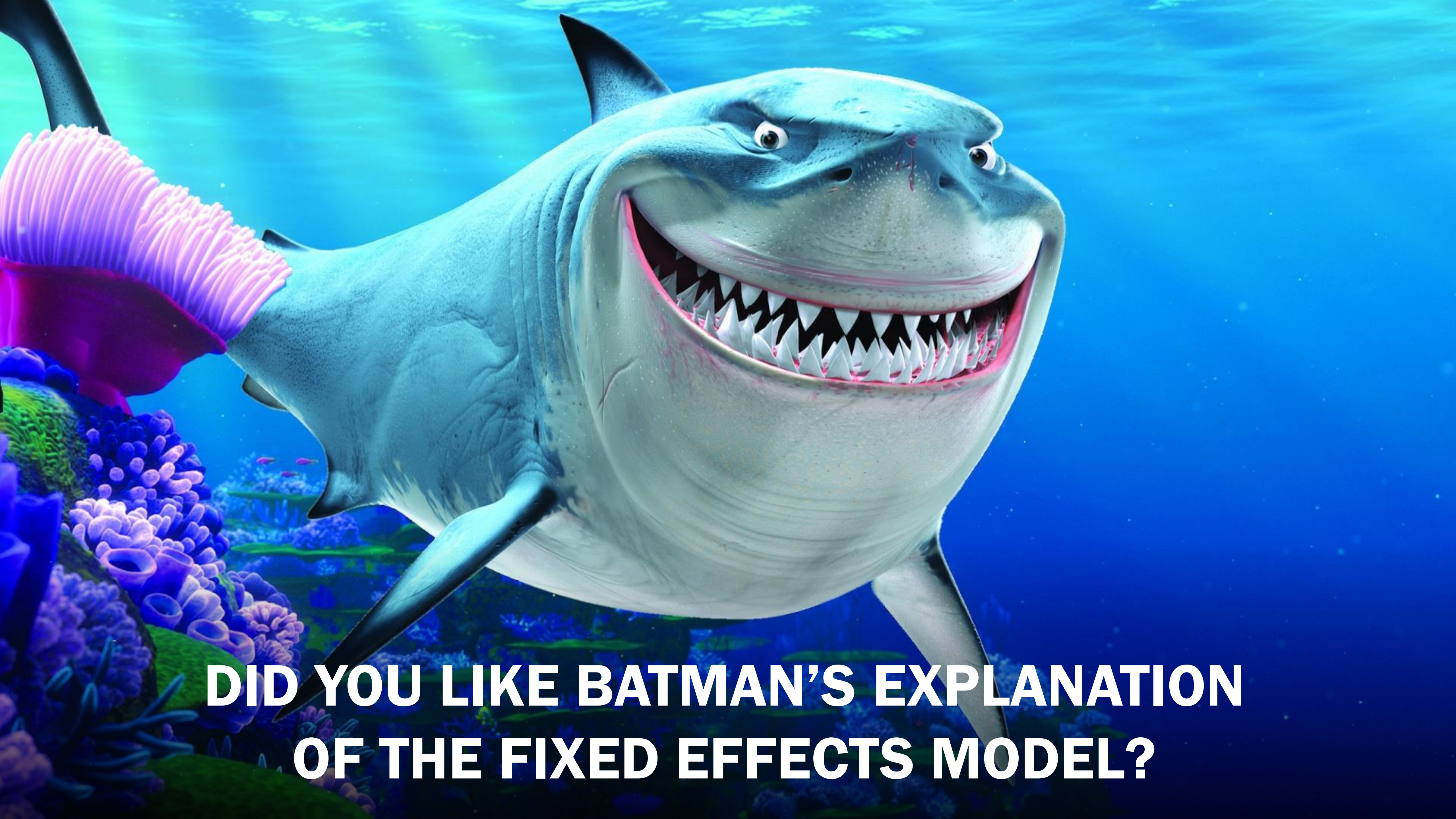
$$\text{Historical Average of } y_{it} = \hat{\beta} \cdot \text{Historical Average of } X_{it} + \widehat{U_i}$$

$$\therefore$$

$$\widehat{U_i} = \text{Historical Average of } y_{it} - \hat{\beta} \cdot \text{Historical Average of } X_{it}$$

KEY TAKEAWAYS

1. Fixed Effects Models are a way to eliminate the unobserved effect U_i in panel data regression
2. In Fixed Effects models, rather than using X_{it} to explain y_{it}, we use deviations from historical averages in X_{it} to explain deviations from historical averages in y_{it}.
3. In other words, we regress $y_{it} - \frac{1}{T}\sum_{t=1}^{T} y_{it}$ on $X_{it} - \frac{1}{T}\sum_{t=1}^{T} X_{it}$
4. Fixed effects models work well when the idiosyncratic error of y_{it} follow the assumptions of normality, homoskedasticity and no serial correlation that are required by classical OLS regression.


DID YOU LIKE BATMAN'S EXPLANATION
OF THE FIXED EFFECTS MODEL?


I ONCE MET BATMAN IN PERSON.
WE MADE A VIDEO:


ZOMBIEBLOODEATH | TUMBLR