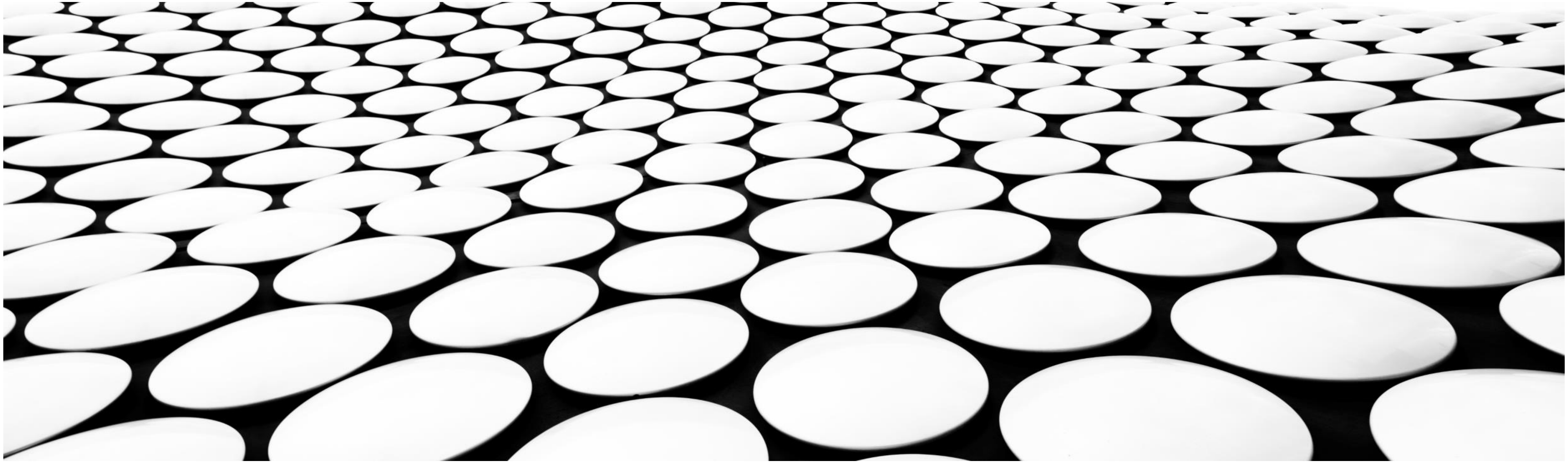
INTRODUCTION TO TIME SERIES

FELIPE BUCHBINDER

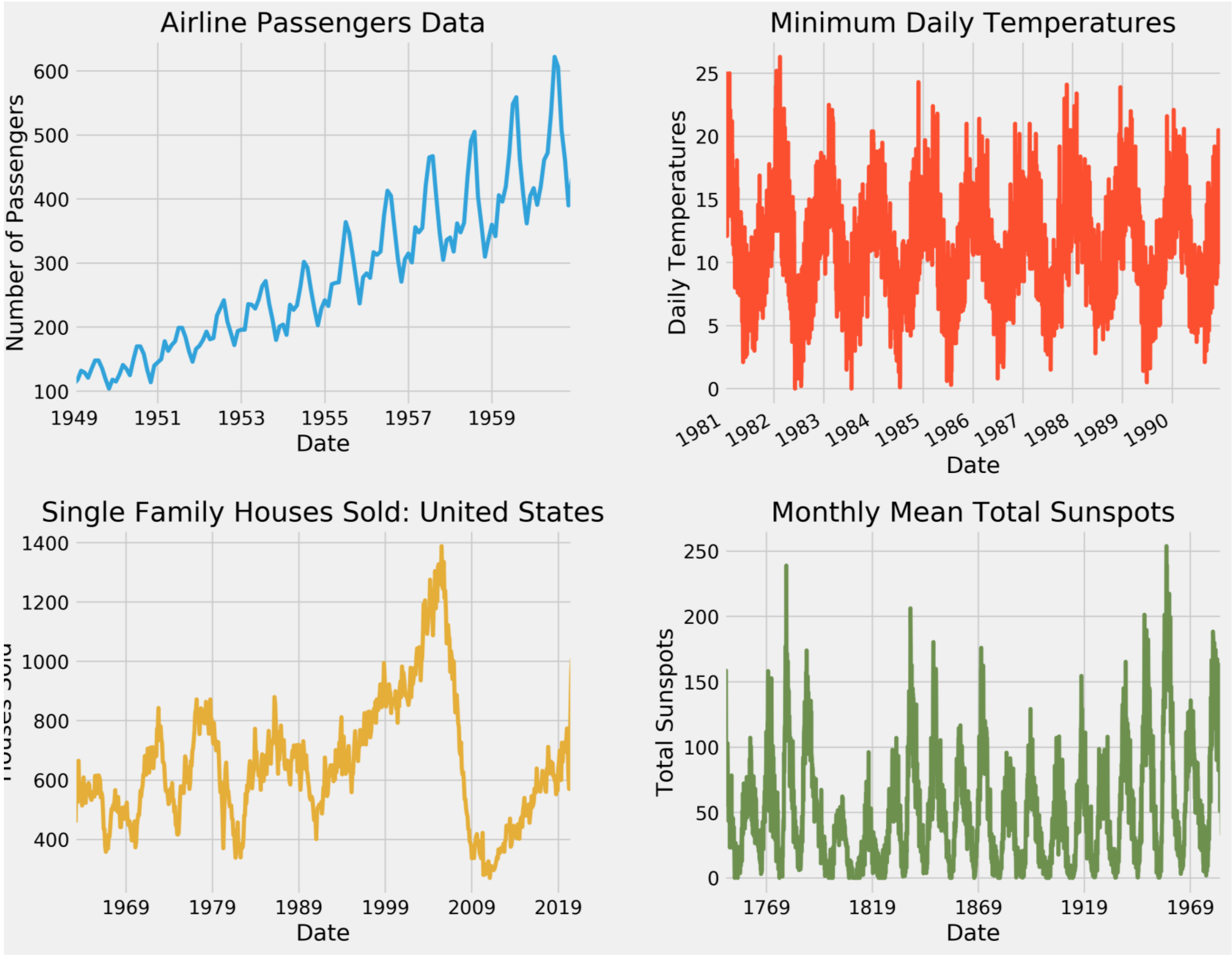


= A DETOUR =

TIME SERIES IS NOT PART OF PANEL DATA.

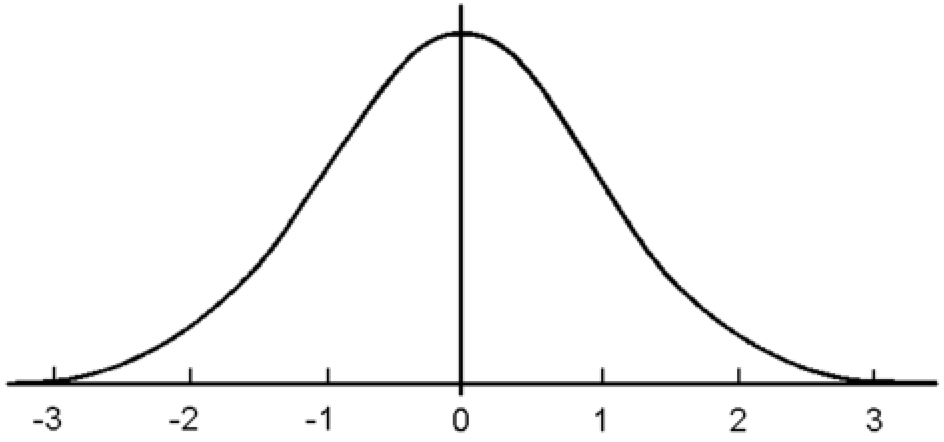
HOWEVER, SINCE WE'LL USE THIS IN OUR NEXT LECTURE, AND SINCE YOU WON'T COVER IT IN ANY OTHER COURSE IN YOUR CURRICULUM (AS FAR AS I KNOW), I'LL TALK ABOUT IT NOW, JUST TO MAKE SURE YOU WON'T GRADUATE WITHOUT EVER HAVING COVERED THIS TOPIC.

THEREFORE, LET'S MAKE A DETOUR TO TALK ABOUT TIME SERIES. WE'LL GET BACK TO PANEL DATA IN OUR NEXT LECTURE.



A TIME SERIES IS DATA ON HOW SOMETHING CHANGES OVER TIME

WE CAN THINK OF A TIME SERIES AS THE SEQUENCE OF VALUES THAT WE GET FROM DRAWING FROM A PROBABILITY DISTRIBUTION ONCE AT EACH TIME PERIOD

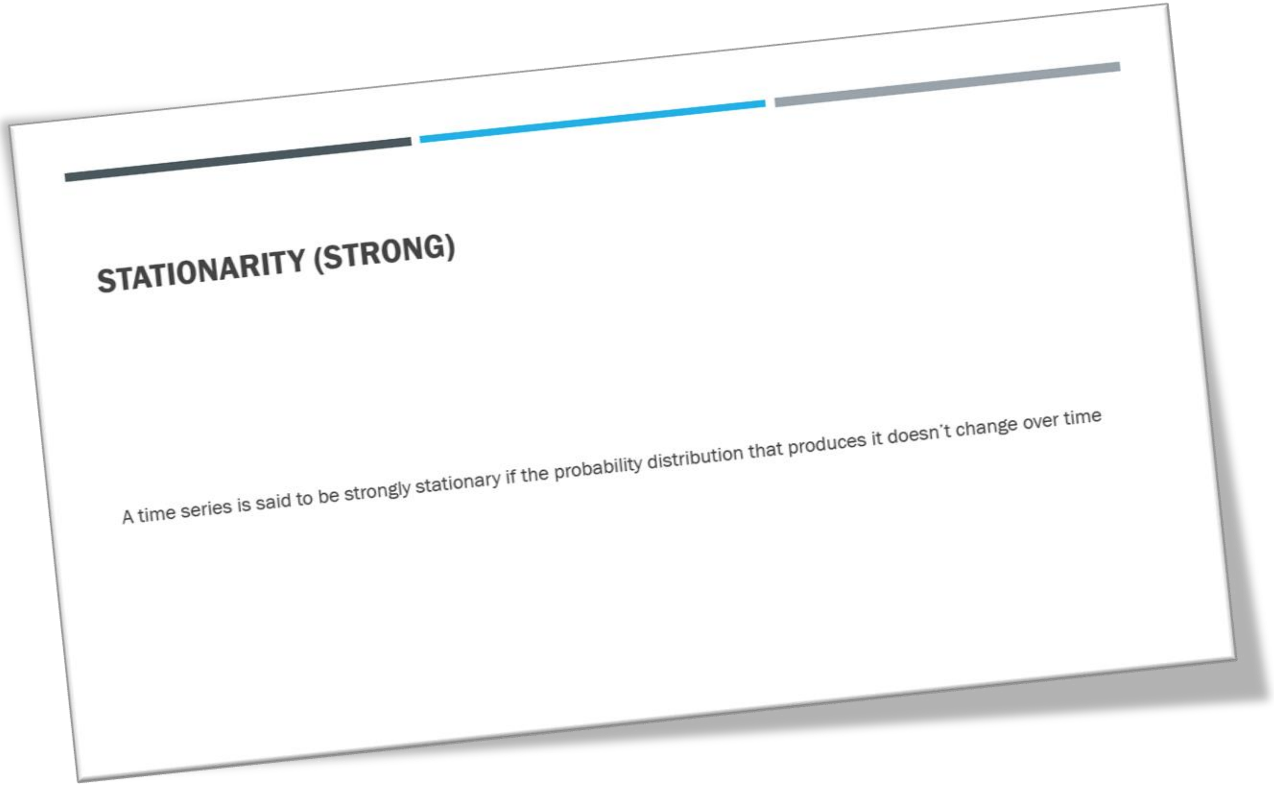


$$y_1, y_2, y_3, y_4, \dots, y_{t-1}, y_t, y_{t+1}, \dots$$

STATIONARITY (STRONG)

A time series is said to be strongly stationary if the probability distribution that produces it doesn’t change over time

STRONG STATIONARITY IS VERY HARD DO PROVE (WHY?)



STATIONARITY (WEAK)

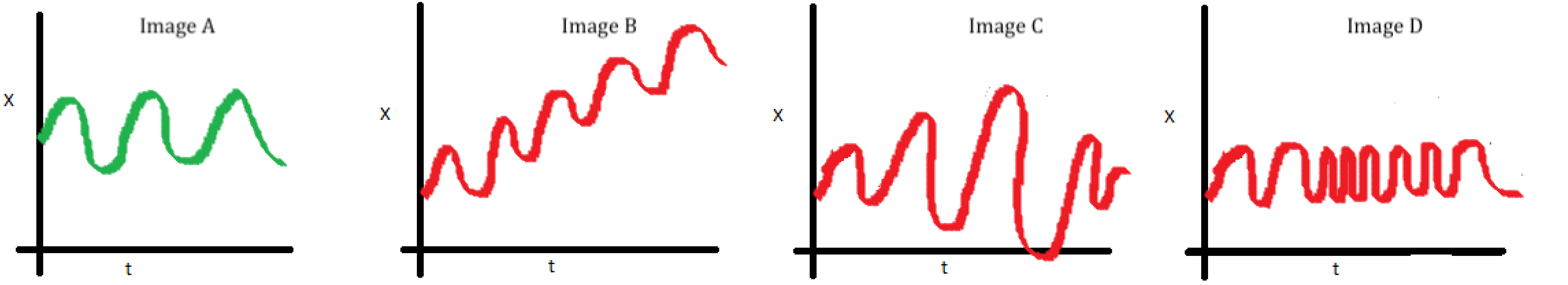
A time series is said to be weakly stationary if the mean and variance of the probability distribution that produces it do not change over time and if the covariance between to values depend only on how much time elapsed between these two values, not on when they ocurred.

Mathematically...

$$\mathbb{E}(Y_t) = \mu \ \forall t$$
$$\mathbb{V}(Y_t) = \sigma^2 \ \forall t$$
$$\mathrm{Cov}(Y_t; Y_{t+\tau}) = \gamma(\tau) \ \forall t$$

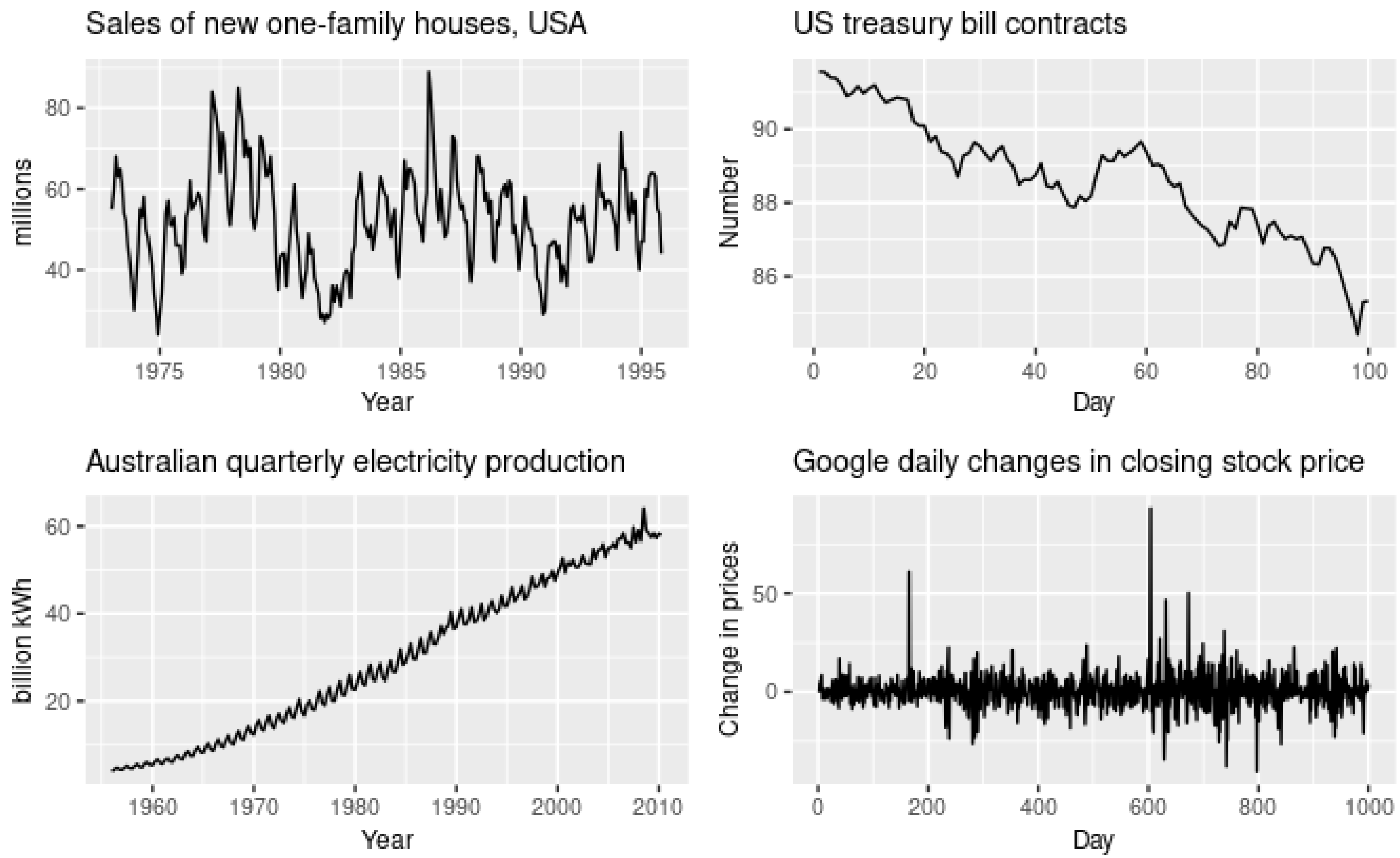
The Principles of Stationarity

Time-Dependent Mean **Time-Dependent Variance** **Time-Dependent Covariance**



Stationary series Non-Stationary series Non-Stationary series Non-Stationary series

REAL-WORLD EXAMPLES



COMPONENTS OF A TIME SERIES

Time Series = Trend + Seasonality + Cycle + Noise

COMPONENTS OF A TIME SERIES

Time Series = Trend + Seasonality + Cycle + Noise

Tackle with regression

COMPONENTS OF A TIME SERIES

add dummy variables to the regression

Time Series = Trend + Seasonality + Cycle + Noise

COMPONENTS OF A TIME SERIES

Time Series = Trend + Seasonality + Cycle + Noise

New stuff!

This is what we'll talk about today

CLASSICAL TIME SERIES MODELS

- Autoregressive models – AR(p)
- Moving Average models – MA(q)
- Autoregressive Moving Average models – ARMA(p,q)
- Autoregressive Integrated Moving Average models – ARIMA(p,d,q)
- Conditionally Heteroskedastic Models – ARCH, GARCH and variations
- Vector Autoregressive Models

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AUTOREGRESSIVE MODELS

WHEN YESTERDAY STILL MATTERS TODAY

AR(P)

WHEN THE P PREVIOUS DAYS STILL MATTER TODAY

$$y_t = \rho_1 y_{t-1} + \cdots + \rho_p y_{t-p} + \epsilon_t$$
$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

AR(*P*) WITH COVARIATES

WHEN THE *P* PREVIOUS DAYS STILL MATTER TODAY

$$y_t = \beta_0 + \rho_1 y_{t-1} + \cdots + \rho_p y_{t-p} + \beta_1 X_{1t} + \cdots + \beta_k X_{kt} + \epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

AR(1)

WHEN YESTERDAY STILL MATTERS TODAY

$$y_t = \rho y_{t-1} + \epsilon_t$$
$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

EFFECT OF ρ

$\rho < 1$

- Effect of today's y will dissipate over time

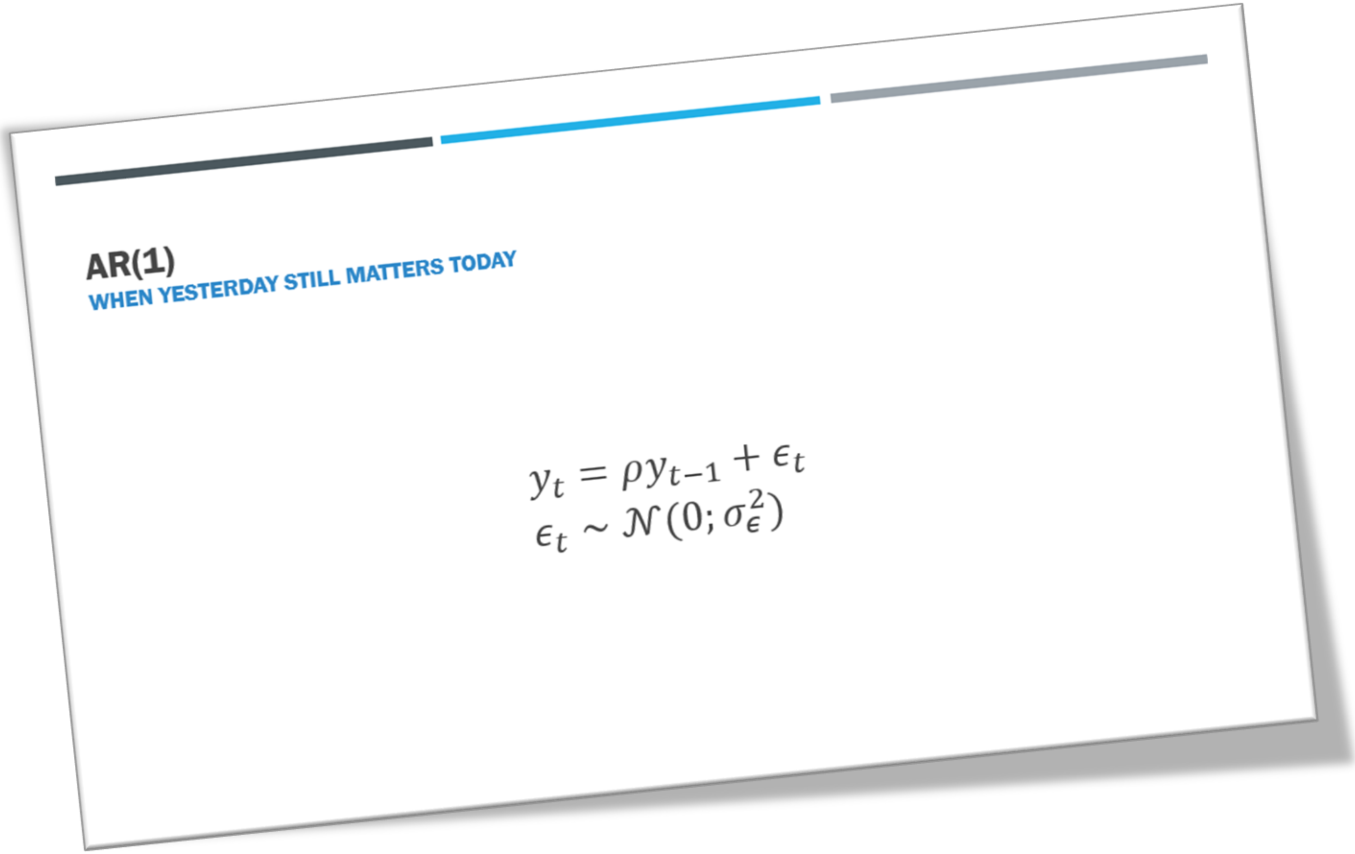
$\rho = 1$

- Effect of today's y will not dissipate over time
- Random Walk

$\rho > 1$

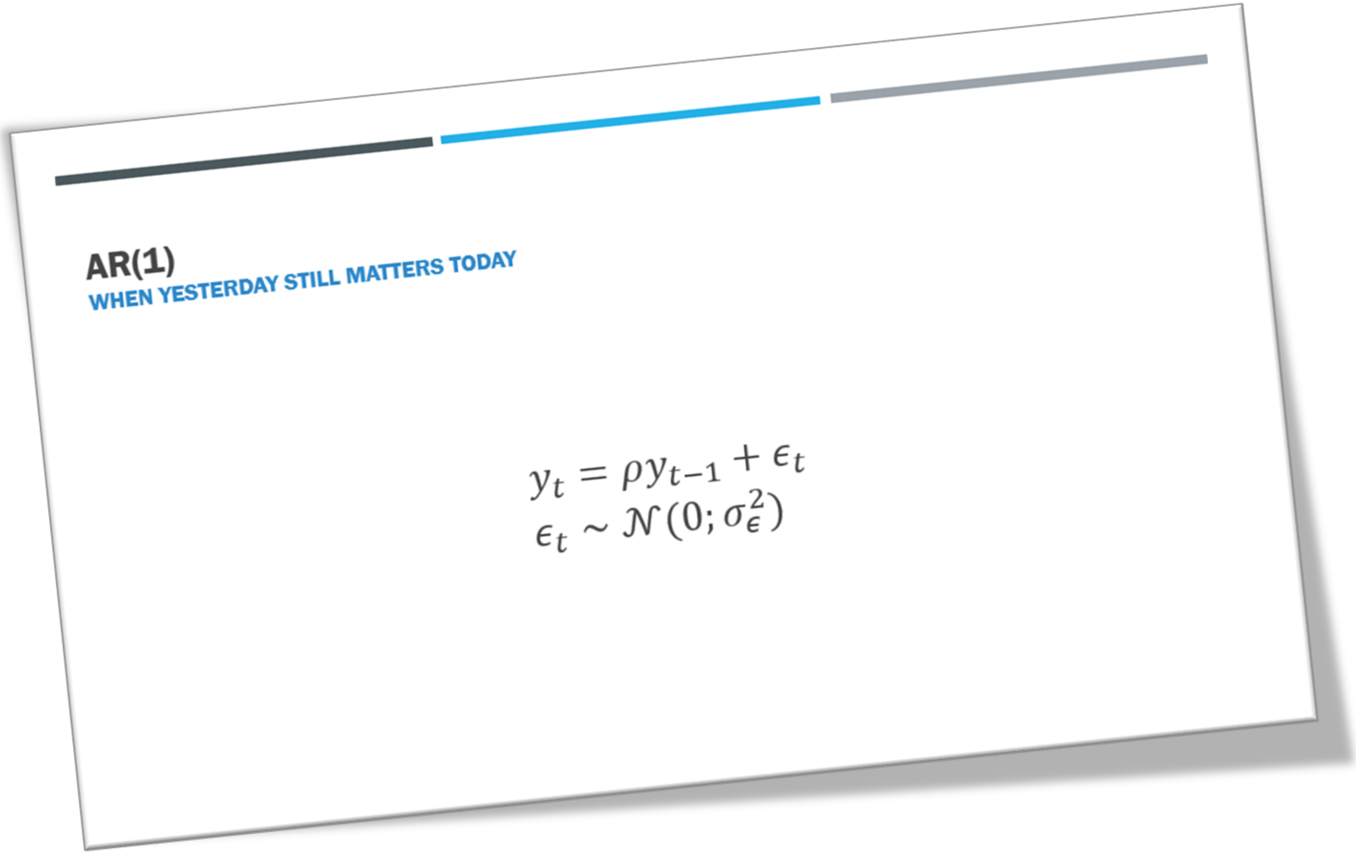
- Effect of today's y will increase over time

CAN A SERIES BE STATIONARY IF…



- … $\rho > 1$?
- … $\rho = 1$?

CAN A SERIES BE STATIONARY IF...

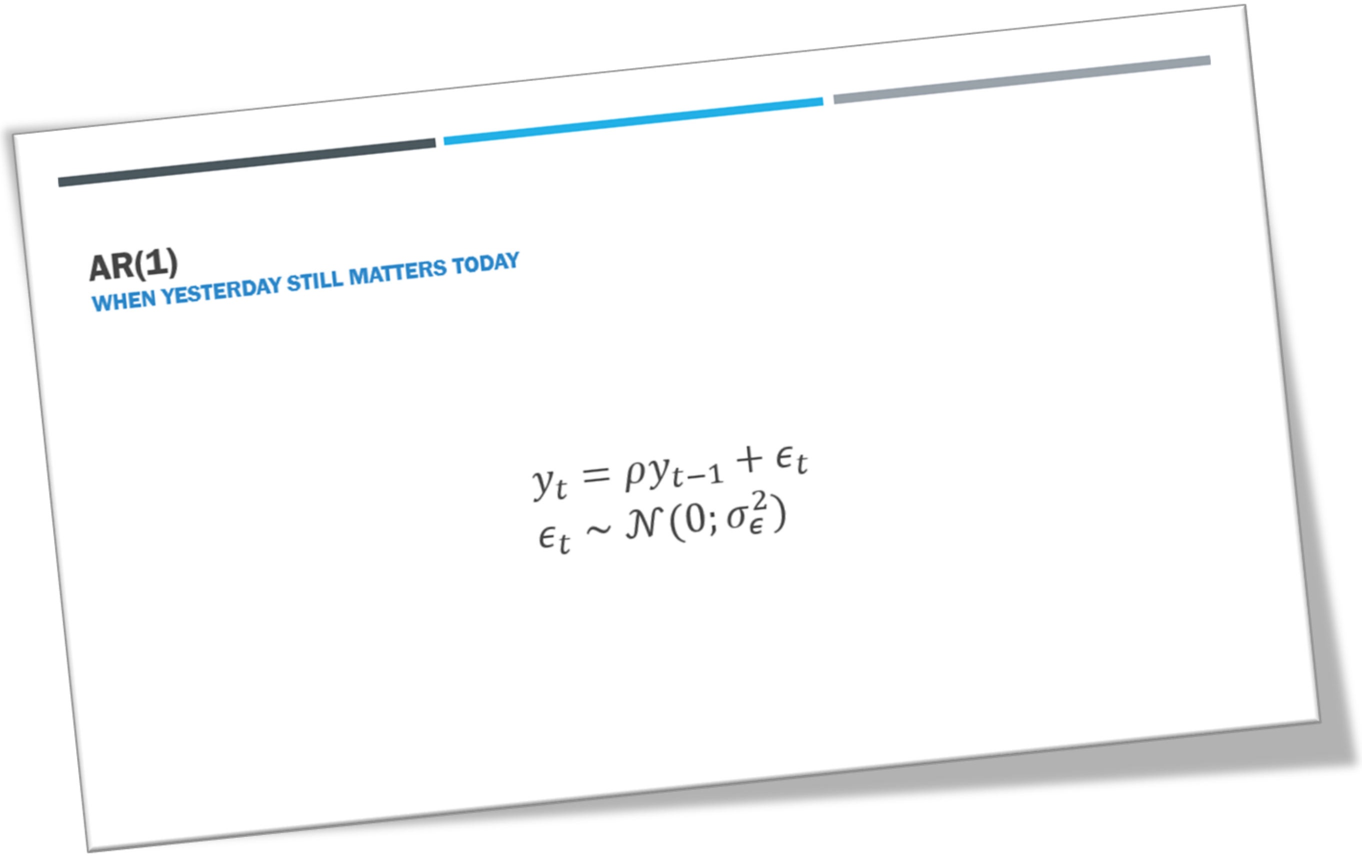


- ... $\rho > 1$?
- ... $\rho = 1$?

No!

For a series to be stationary, we must have $\rho < 1$

CAN A SERIES BE STATIONARY IF…



- … $\rho > 1$?
- … $\rho = 1$?

An AR model cannot have unit roots

PROOF

$$y_t = \rho y_{t-1} + \epsilon_t \Rightarrow \mathbb{V}(y_t) = \mathbb{V}(\rho y_{t-1} + \epsilon_t)$$

$$\therefore$$

$$\mathbb{V}(y_t) = \rho^2 \mathbb{V}(y_{t-1}) + \sigma_\epsilon^2$$

Stationarity implies $\mathbb{V}(y_t) = \mathbb{V}(y_{t-1})$

$$\therefore$$

$$\sigma_y^2 = \rho^2 \sigma_y^2 + \sigma_\epsilon^2$$

$$\sigma_y^2 = \frac{\sigma_\epsilon^2}{1 - \rho^2}$$

Which implies $\rho \neq 1$

AUTOCOVARIANCE FUNCTION OF AN AR(1) TIME SERIES

$$y_t = \rho y_{t-1} + \epsilon_t$$

$$Stationarity \Rightarrow \mathrm{Cov}(y_t, y_{t-\tau}) = \gamma(\tau)$$

$$\therefore$$

$$\gamma(\tau) = \mathrm{Cov}(y_t, y_{t-\tau}) = \mathrm{Cov}(\rho y_{t-1} + \epsilon_t, y_{t-\tau}) = \rho\gamma(\tau - 1)$$

$$\therefore$$

$\boldsymbol{\gamma(\tau)}$ **forms a geometric sequence with ratio** $\boldsymbol{\rho \in (-1, 1)}$

MOVING AVERAGE MODELS

WHEN SHOCKS DISSIPATE SLOWLY OVER TIME

MA(Q)

WHEN SHOCKS IN THE Q PREVIOUS DAYS STILL MATTER TODAY

$$y_t = \phi_1 \epsilon_{t-1} + \cdots + \phi_q \epsilon_{t-q} + \epsilon_t$$
$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

MA(Q) WITH COVARIATES

WHEN SHOCKS IN THE Q PREVIOUS DAYS STILL MATTER TODAY

$$y_t = \beta_0 + \phi_1 \epsilon_{t-1} + \cdots + \phi_q \epsilon_{t-q} + \beta_1 X_{1t} + \cdots + \beta_k X_{kt} + \epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

MA(1)

WHEN SHOCKS YESTERDAY STILL MATTER TODAY

$$y_t = \phi\epsilon_{t-1} + \epsilon_t$$
$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

EFFECT OF ϕ

$\phi < 1$	$\phi = 1$	$\phi > 1$
• Effect of today's shocks will dissipate over time	• Effect of today's shocks will not dissipate over time	• Effect of today's shocks will increase over time

AUTOCOVARIANCE FUNCTION OF AN MA(1) TIME SERIES

$$y_t = \phi\epsilon_{t-1} + \epsilon_t$$

$$Stationarity \Rightarrow \mathrm{Cov}(y_t, y_{t-\tau}) = \gamma(\tau)$$

$$\therefore$$

$$\gamma(\tau) = \mathrm{Cov}(y_t, y_{t-\tau})$$

$$= \mathrm{Cov}(\phi\epsilon_{t-1} + \epsilon_t, \phi\epsilon_{t-\tau-1} + \epsilon_{t-\tau})$$

$$= \phi^2\mathrm{Cov}(\epsilon_{t-1}, \phi\epsilon_{t-\tau-1}) + \phi\mathrm{Cov}(\epsilon_{t-1}, \epsilon_{t-\tau}) + \phi\mathrm{Cov}(\epsilon_t, \epsilon_{t-\tau-1}) + \mathrm{Cov}(\epsilon_t, \epsilon_{t-\tau})$$

All terms are zero except when $\tau = 1$, in which case $\phi\mathrm{Cov}(\epsilon_{t-1}, \epsilon_{t-\tau})$ is non-zero and equals $\phi\sigma_\epsilon^2$. Thus,

$\gamma(\tau)$ has a spike for $\tau = 1$ and drops sharply to 0 after that

AUTOREGRESSIVE MOVING AVERAGE MODELS

THE BEST OF BOTH WORLDS

ARMA(*P*,*Q*)

AR(P) + MA(Q)

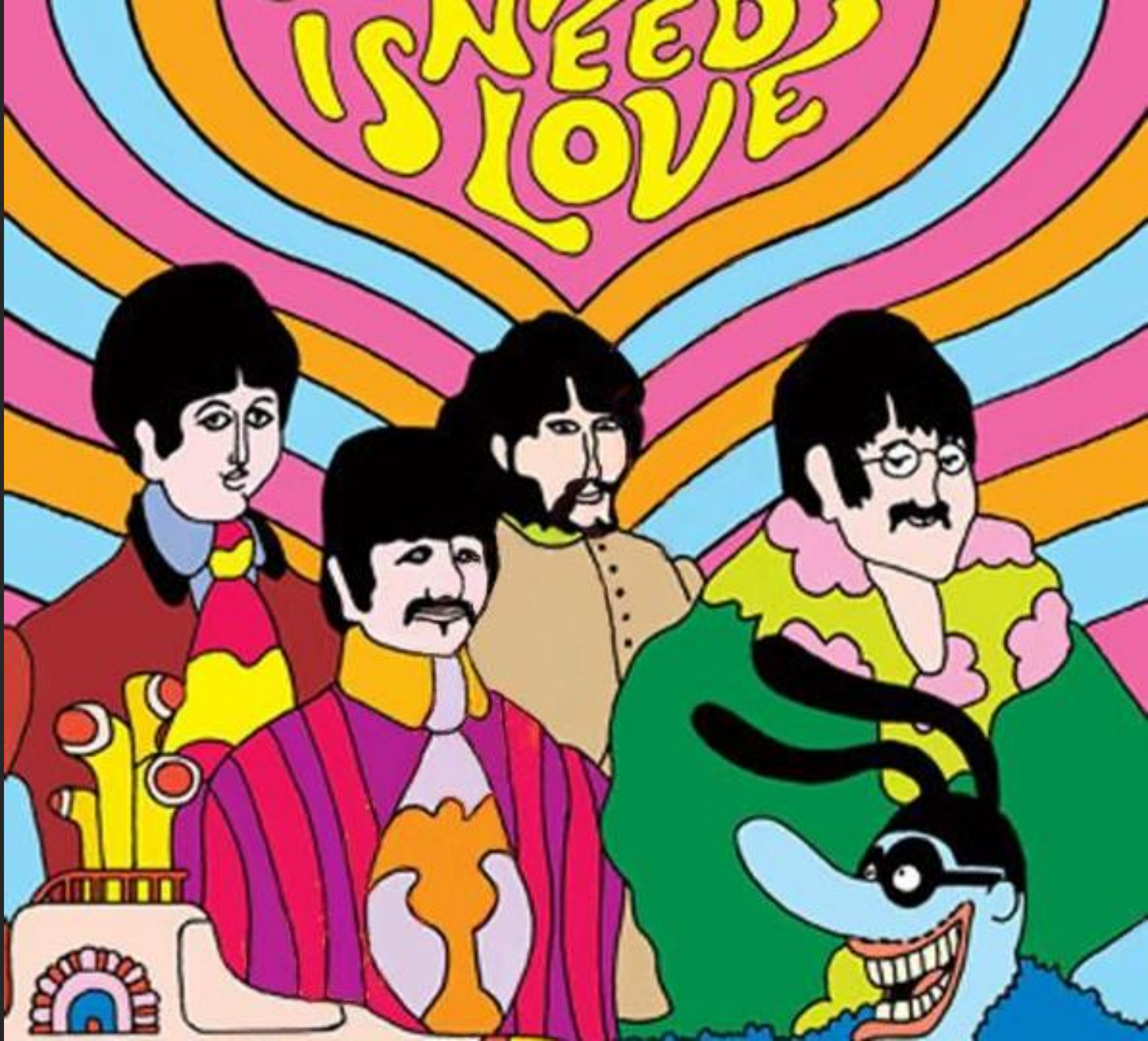
$$y_t = \rho_1 y_{t-1} + \cdots + \rho_{t-p} y_{t-p} + \phi_1 \epsilon_{t-1} + \cdots + \phi_q \epsilon_{t-q} + \epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$$

AN ARMA (P,Q) IS ALL YOU NEED TO EXPRESS ANY STATIONARY TIME SERIES IN DISCRETE TIME

WOLD'S DECOMPOSITION THEOREM

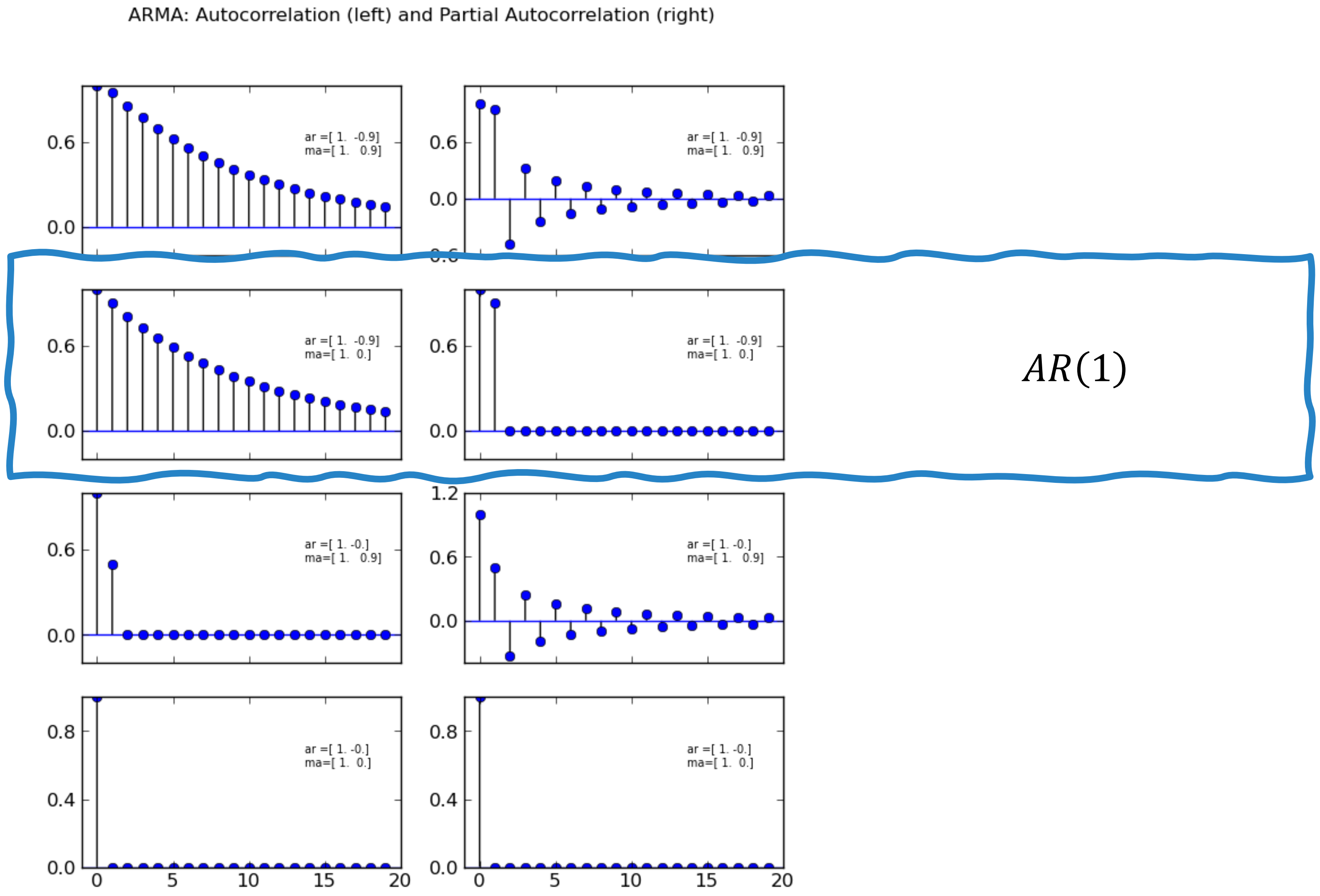
(IN TIME SERIES ANALYSIS)



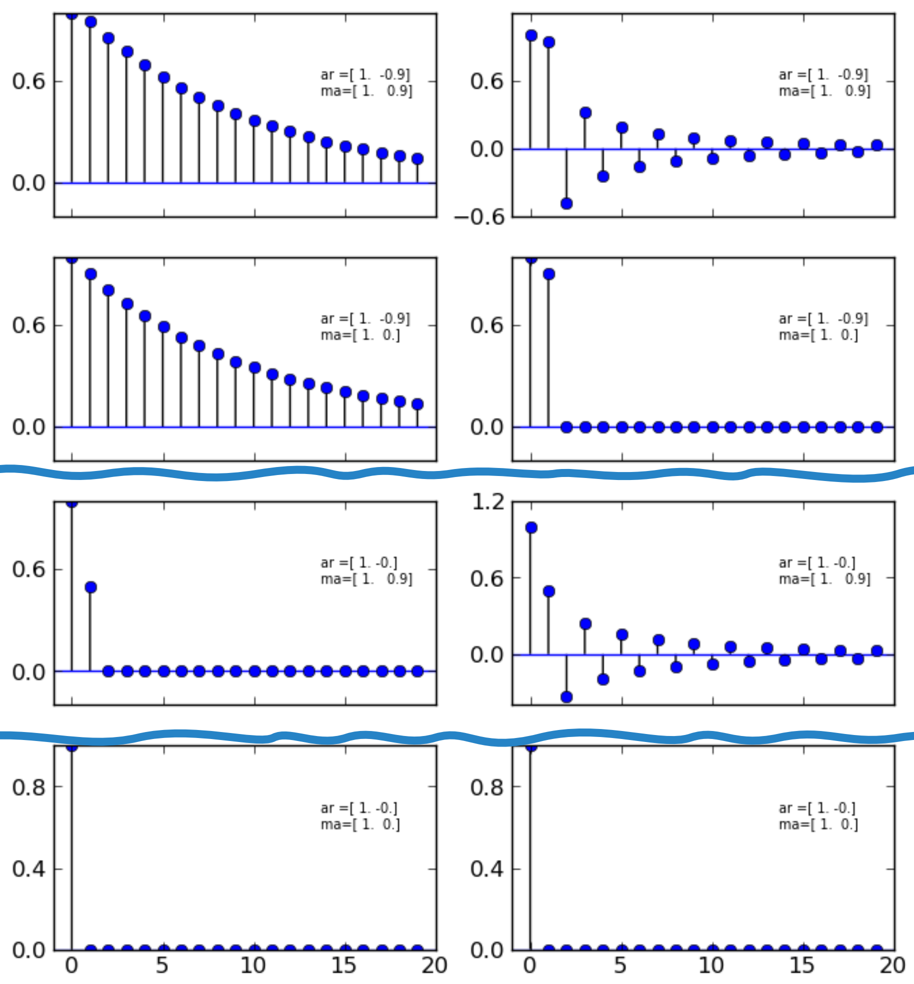
BOX-JENKINS METHODOLOGY

A VISUAL GUIDE TO IDENTIFYING IF A SERIES IF AR(P), MA(Q) OR ARMA(P,Q)

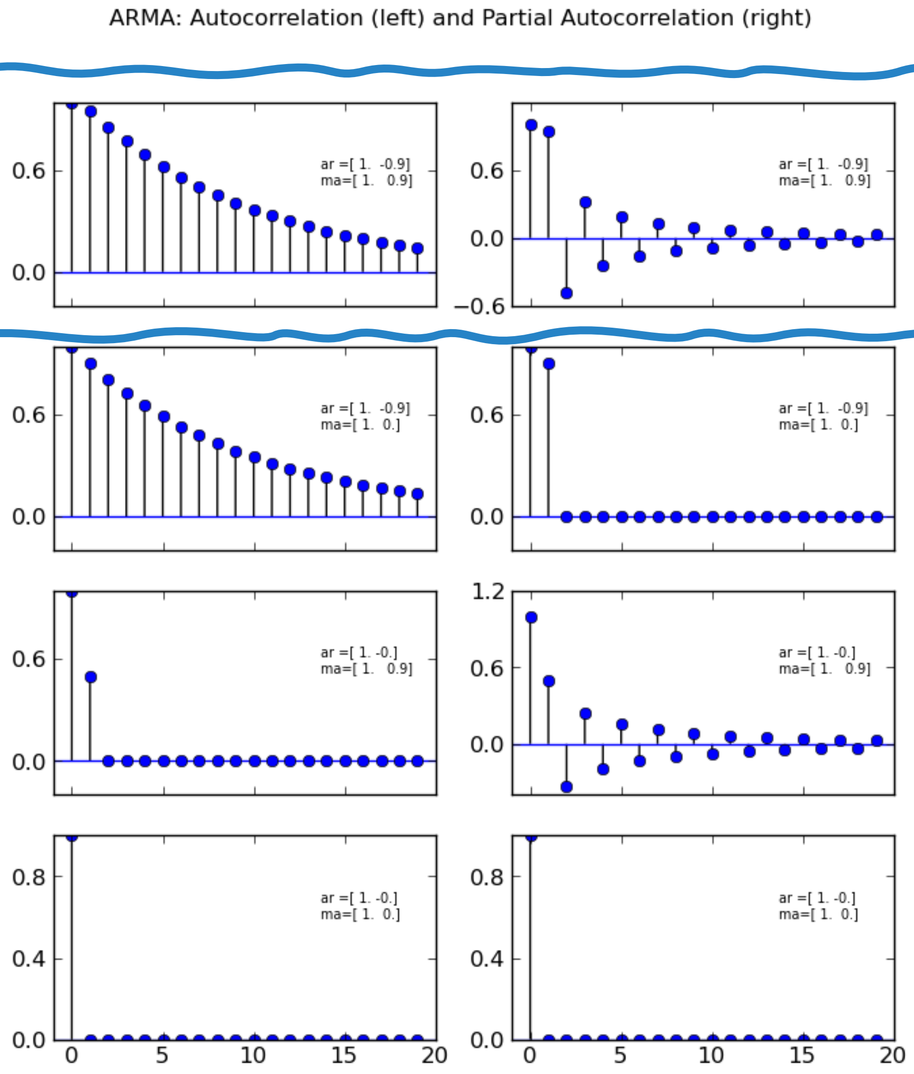
ACFs	PACFs	Model
Decay to zero with exponential pattern	Cuts off after lag p	$\mathrm{AR}(p)$
Cuts off after lag q	Decay to zero with exponential pattern	$\mathrm{MA}(q)$
Decay to zero with exponential pattern	Decay to zero with exponential pattern	$\mathrm{ARMA}(p, q)$


ARMA: Autocorrelation (left) and Partial Autocorrelation (right)
ar =[1. -0.9]
ma=[1. 0.9]
ar =[1. -0.9]
ma=[1. 0.]
ar =[1. -0.]
ma=[1. 0.9]
ar =[1. -0.]
ma=[1. 0.]
AR(1)

ARMA: Autocorrelation (left) and Partial Autocorrelation (right)



MA(1)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$ARMA(1,1)$

AUTOREGRESSIVE INTEGRATED MOVING AVERAGE MODELS

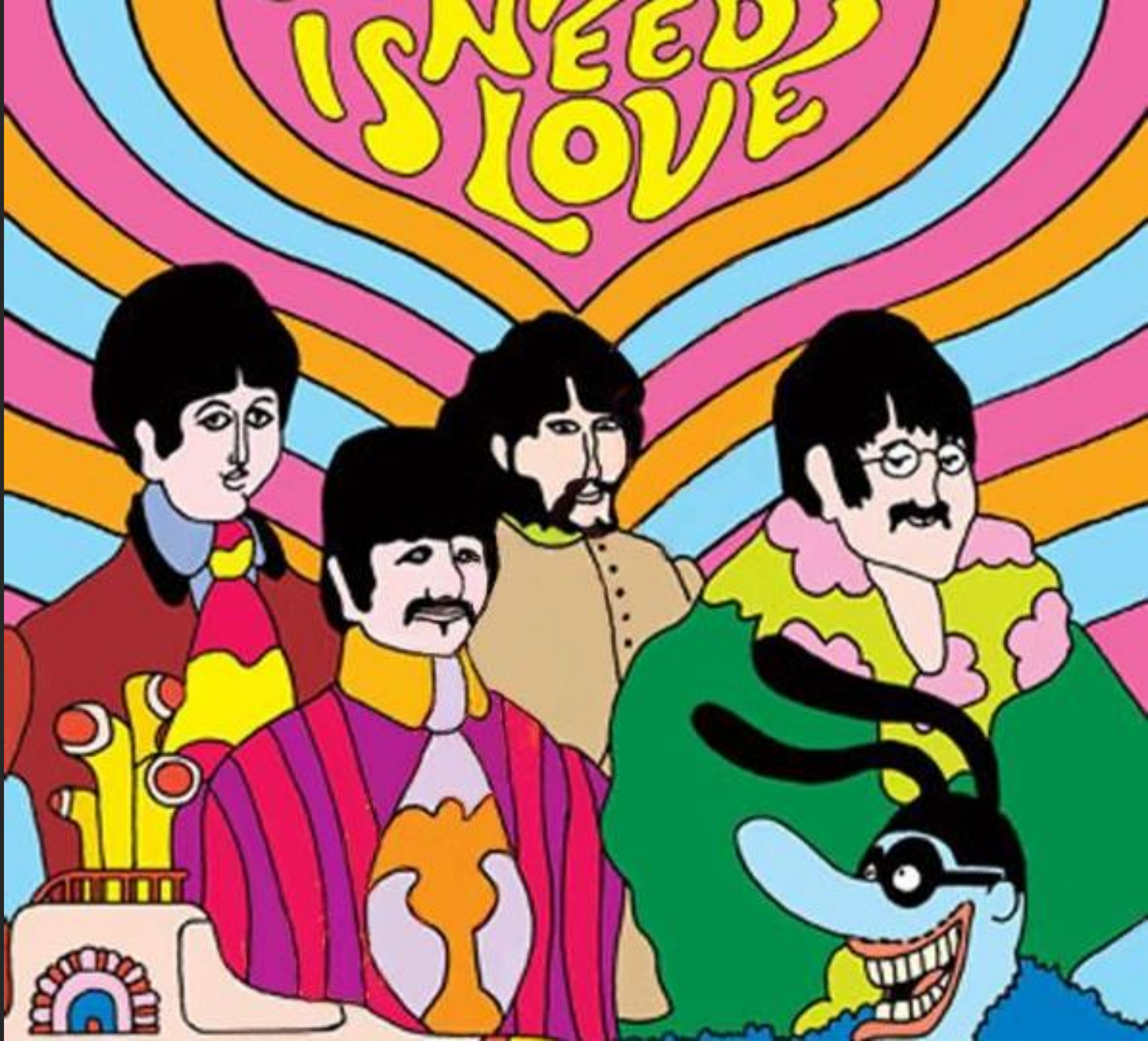
I STILL KNOW WHAT YOU DID LAST SUMMER…

AND I ALWAYS WILL.

AN ARMA (P,Q) IS ALL YOU NEED TO EXPRESS ANY STATIONARY TIME SERIES IN DISCRETE TIME

WOLD'S DECOMPOSITION THEOREM

(IN TIME SERIES ANALYSIS)



WHAT ABOUT TIME SERIES THAT AREN'T STATIONARY?

IDEA:
TRY TO MAKE SERIES STATIONARY BY **DIFFERENTIATION**

DIFFERENTIATION
THE SUNNY SIDE

- Differentiation eliminates linear trend in time-series
- May need to differentiate multiple times
- Order of Integration

ARIMA(*P*,*D*,*Q*)

A time series that needs to be differentiated D times to become an ARMA(P,Q)

(why is this series called “integrated”?)

DIFFERENTIATION
THE MELANCHOLIC SIDE

- Differentiation may add noise and tamper with residual's autocorrelation
- Random Walk example
- Unit Roots and how to test for it
  - Dickey-Fuller
  - Augmented Dickey-Fuller
  - Ng-Perron
  - …

UNIT ROOT VS. TREND-STATIONARITY

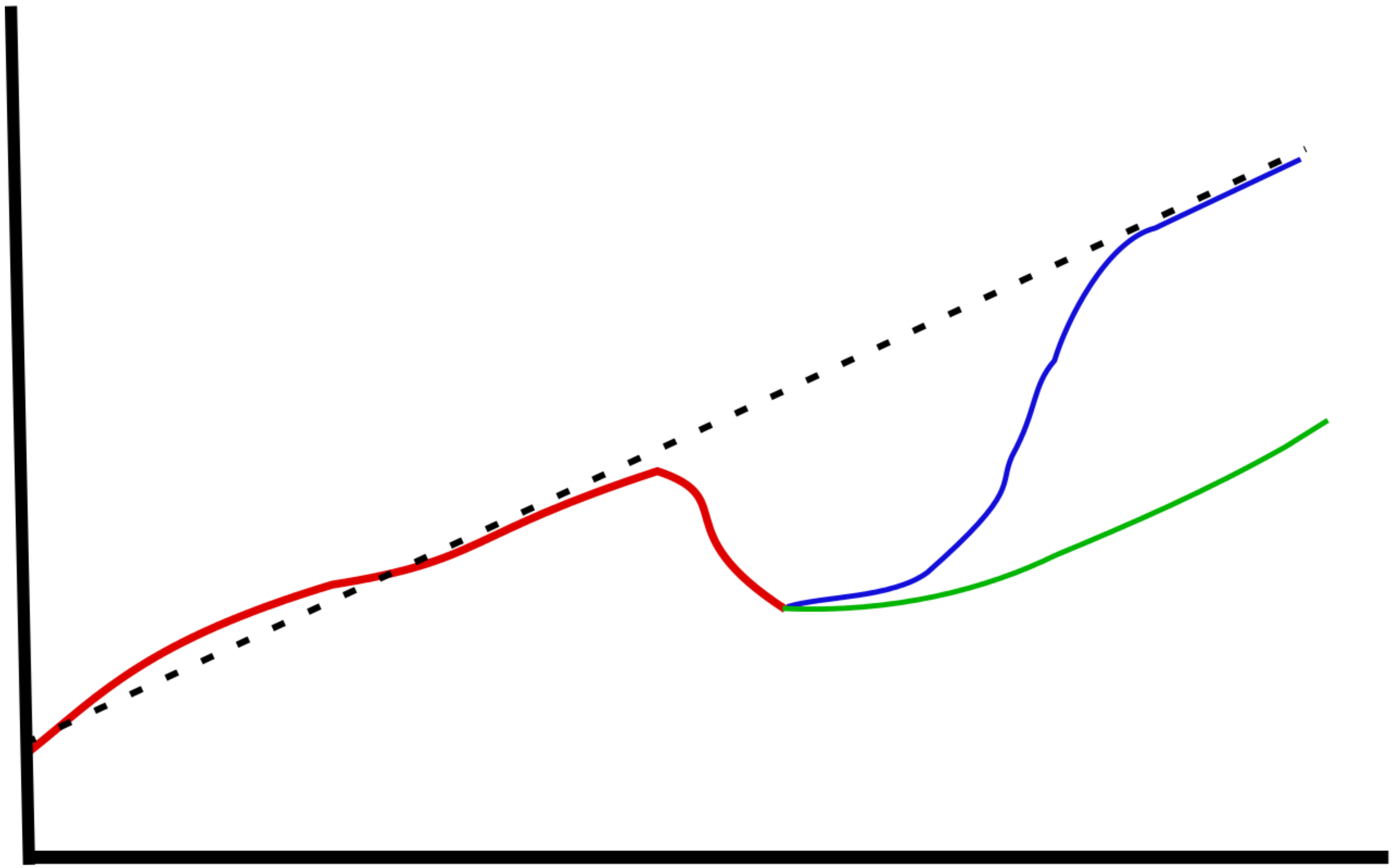
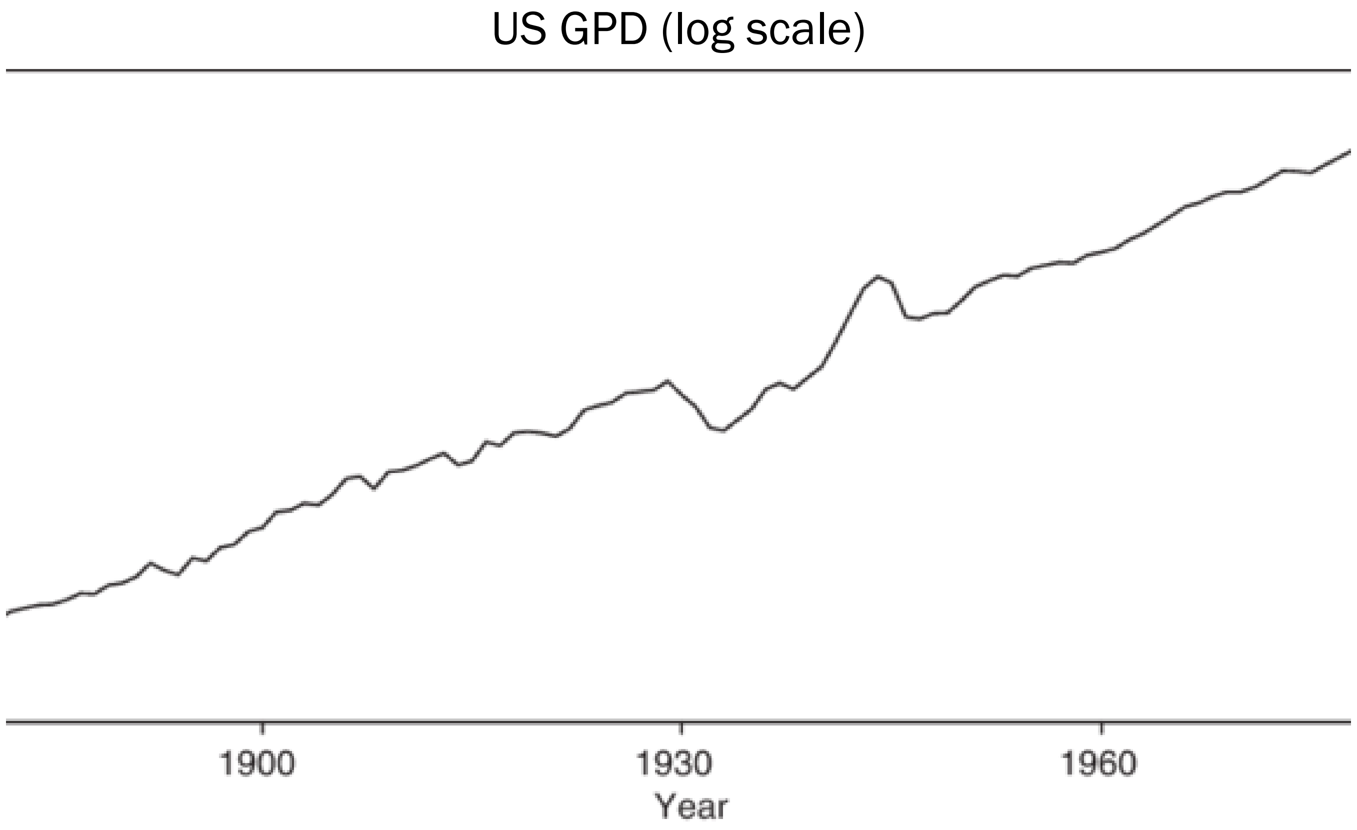


Image source: Pedia, Wiki (2022)

ARE UNIT ROOTS ECONOMICALLY REAL?



SCATTERED COMMENTS

Heteroskedasticity

ARCH, GARCH and the family

Vector Autoregression (VAR)

Cointegration

Granger causality