Exercise 19 / 592

1) $y' = F(x; y)$, $y(x_0) = y_0$ step size h at
$x_n = x_{n-1} + h$ $y_n = y_{n-1} + hF(x_{n-1}; y_{n-1})$
where $n = 1, 2, 3 \dots$

- $y' = y$ $y' = F(x, y)$ and $y(0) = 1$ $\Rightarrow x_0 = 0$ $y_0 = 1$

i) $h = 0{,}4$ $y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$

$\underline{y_1 = y_0 + (0.4)F(x_0, y_0)} = 1 + 0.4 \cdot 1 = 1.4$

$\underline{x_1 = x_0 + h} = 0 + 0.4 = 0{,}4$; $y_1 = y(0{,}4) = 1.4$

ii) $h = 0{,}2$

$\underline{x_1 = x_0 + h} = 0 + 0{,}2 = 0{,}2$

$\underline{x_2 = x_1 + h} = 0{,}2 + 0{,}2 = 0{,}4$

$\underline{y_1 = y_0 + hF(x_0; y_0)} = 1 + 0{,}2(y_0) = 1 + 0.2 \cdot 1 = \underline{1.2}$

$\underline{y_2 = y_1 + hF(x_1, y_1)} = 1{,}2 + 0{,}2 \cdot 1{,}2 = \underline{1{,}44}$

iii) $h = 0{,}1$

$\underline{x_1 = x_0 + h} = 0 + 0{,}1 = \underline{0{,}1}$ $\underline{x_2 = x_1 + h} = 0{,}1 + 0{,}1 = \underline{0{,}2}$

$\underline{x_3 = x_2 + h} = 0{,}2 + 0{,}1 = \underline{0{,}3}$ $\underline{x_4 = x_3 + h} = 0{,}3 + 0{,}1 = \underline{0{,}4}$

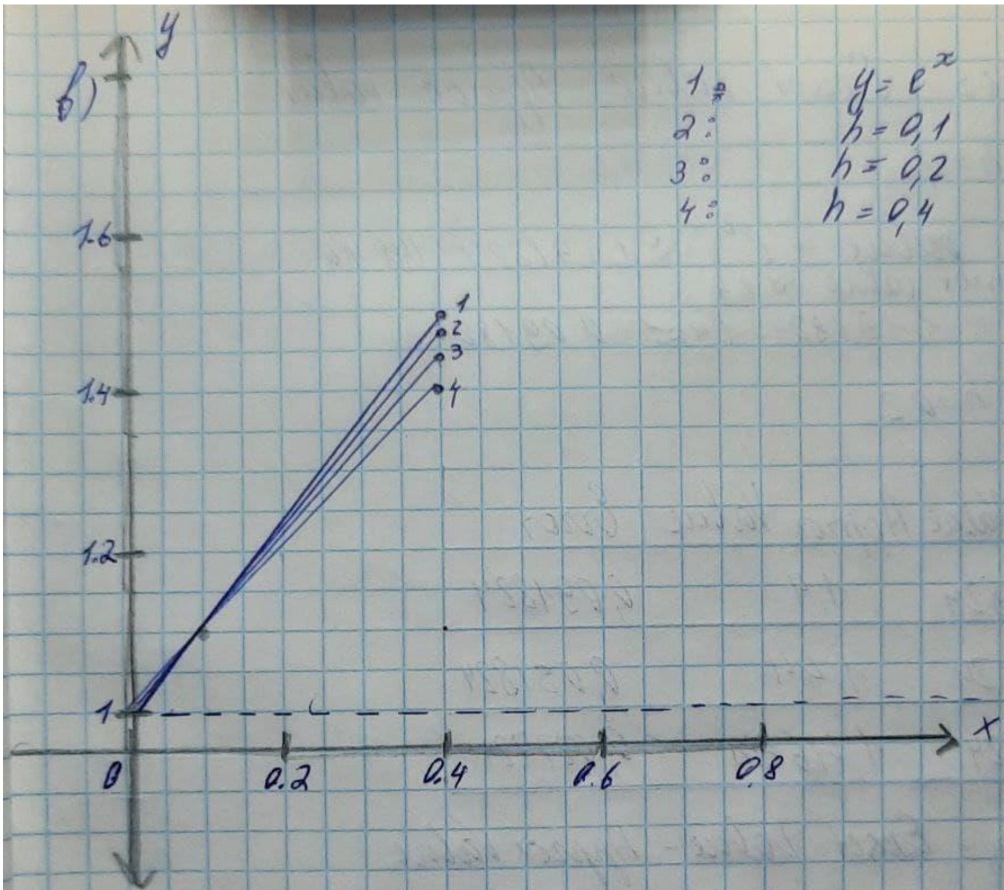
$\underline{y_1} = y_0 + hF(x_0; y_0) = 1 + 0.1(y_0) = \underline{1.1}$

$\underline{y_2} = y_1 + hF(x_1; y_1) = 1.1 + 0.11 = \underline{1.21}$

$\underline{y_3} = y_2 + hF(x_2; y_2) = 1.21 + 0.121 = \underline{1.331}$

$\underline{y_4} = y_3 + hF(x_3; y_3) = 1.331 + 0.1331 = \underline{1.4641}$

b)



① Draw the curve for $y = e^x$

② For $h = 0.4$ $(x_1, y_1) = (0.4; 1.4)$

③ For $h = 0.2$ $(x_1, y_1) = (0.2; 1.2)$ $(x_2, y_2) = (0.4; 1.44)$

④ For $h = 0.1$ $(x_1, y_1) = (0.1; 1.1)$ $(x_3, y_3) = (0.2; 1.21)$

$(x_3, y_3) = (0.3; 1.331)$ $(x_4, y_4) = (0.4; 1.4641)$

As seen in the graph, we see that our estimates in part a are underestimates, because they are bellow the actual curve of $y = e^x$

c) Error = Exact value − Approx. value

- For $h = 0,4$

Exact value $= e^{0.4} \approx 1,491824$

Approx. value ≈ 1.4

Error $= 1,491824 - 1.4 \approx 0,091824$

- For $h = 0,2$

	Exact Value	Approx. Value	Error
$h=0,4$	1,491824	1,4	0,091824
$h=0,2$	1,491824	1,44	0,051824
$h=0,1$	1,491824	1,4641	0,027724

Error = Exact Value − Approx. Value

Exact Value $= e^{0.4} \approx 1,491824$

Conclusion: If the step is halved $\Rightarrow$ the error estimate also appears to be halv

Exercise 25 $\frac{dy}{dx} + 3x^2 y = 6x^2$ $y(0) = 3$

$\frac{dy}{dx}$ at $x=0$ $\Leftrightarrow$ $\frac{dy}{dx} = 6x^2 - 3x^2 \cdot y = 0$

$y(0{,}01) = ?$
Substitute in $y = y_0 + (x - x_0) \cdot \left(\frac{dy}{dx} \text{ at } x=0\right)$

$x_0 = 0$ $y(0) = 3$

h	y(1)
1	3
0.1	2,3928
0.01	2,3701
0.001	2,3681

b) 1 $y = 2 + e^{-x^3}$

$\frac{dy}{dx} = (-3x^2)e^{-x^3}$

$\frac{dy}{dx} + 3x^2 y = 6x^2$

$\Rightarrow -3x^2 e^{-x^3} + 6x^2 + 3x^2 e^{-x^3} = 6x^2 \Rightarrow$

$\boxed{\Rightarrow 6x^2 = 6x^2}$ It means that $y = 2 + e^{-x^3}$ satisfy the given differential equation ✓

2 Check if $y = 2 + e^{-x^3}$ satisfy the given initial ~~value~~ given ~~at~~ condit. $y(0) = 3$

$y = 2 + e^{-x^3}$ At $x = 0$ $y(0) = 2 + e^0 = 2 + 1 = 3$ ✓

c) Error = Exact value − Approx. Value

	Exact Val.	Approx. Val.	Error
h=1	2,3679	3	−0,6321
h=0.1	2,3679	2,3928	−0,0249
h=0.01	2,3679	2,3701	−0,0022
h=0.001	2,3679	2,3681	−0.0002

When H is divided by 10, the error is also divided by approx. 10

```python
x0: int = 0
y0 = 3
# m0=dy/dx
m0 = 3 * x0 ** 2 * (2 - y0)
print(m0)

x001 = 0.01
y001 = y0 + (x001 - x0) * m0
print(y001)
```

These lines of code for each value. Then the next step is to find the new value dy/dx. And then the new Y at=x0.02. And so on until we reach x=1.0

```
scratch_2
0
3.0
```

```
1.
Start

2.
Define
function
f(x, y)

3.
Read values of initial condition(x0 and y0), number of steps(n) and
calculation point(xn)

4.
Calculate step size(h) = (xn - x0) / b

5.Set i = 0

6. Loop  yn = y0 + h * f(x0 + i * h, y0)
y0 = yn
i = i + 1
While
i < n
7.
Display yn as result
8.
Stop
```

```python
#
#  main function
def f(x, y):
    return x + y
# f = lambda x: x+y
# Euler method
def euler(x0, y0, xn, n):
    # Formula for calculating step size
    h = (xn - x0) / n
    print('\n-----------SOLUTION-----------')
    print('------------------------------')
    print('x0\ty0\tslope\tyn')
    print('------------------------------')
    for i in range(n):
        slope = f(x0, y0)
        yn = y0 + h * slope
```

```python
        print('%.4f\t%.4f\t%0.4f\t%.4f' % (x0, y0, slope, yn))
        print('-------------------------------')
        y0 = yn
        x0 = x0 + h

    print('\nAt x=%.4f, y=%.4f' % (xn, yn))
# Inputs
print('Enter initial conditions:')
x0 = float(input('x0 = '))
y0 = float(input('y0 = '))
print('Enter calculation point: ')
xn = float(input('xn = '))
print('Enter number of steps:')
step = int(input('Number of steps = '))

# Euler method call
euler(x0, y0, xn, step)
```